# Constructing and counting IIb string vacua

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**Abstract**

Based on hep-th/0303194 and

- hep-th/0307049 with Sujay Ashok
- math.CV/0402326 with Bernard Shiffman and Steve Zelditch (Johns Hopkins)
- hep-th/0404116 and to appear with Frederik Denef
- hep-th/0404257 with F. Denef and Bogdan Florea.

# 1. The landscape

- Many string/M theory vacua: choice of theory, of CY, of bundle/branes, of flux, and of phase, *i.e.* particular minimum of effective potential. Not enough dualities to drastically reduce this number.

- Many of these vacua, in particular candidates for our universe, can be described using 4D effective field theory.

- The distribution of these vacua in the space of EFT's can be precisely defined and studied. Thus, the statistics of this distribution are well defined: numbers of vacua, distributions of couplings etc.

- The number of models is so large that the classic problems of "beyond the SM physics" (explaining the spectrum, hierarchy, cosmological constant) may admit statistical solutions (a large enough sample space will contain models which work). Indeed, we know no better solution to the cosmological constant problem.

- We have no powerful *a priori* selection principles, and we do not know if any exist.

# 2. Some questions

- How many (weakly coupled IIb) vacua are there, and how are they distributed?
  - How many metastable (tachyon-free) minima of $V_{eff}$ are there?
  - Do the existing EFT analyses (keeping only fields which were massless before susy breaking) miss tachyons?
  - Are there more subtle instabilities/inconsistencies?
  - How are the broad features distributed?
    - cosmological constant
    - supersymmetry breaking scale
    - hierarchy
    - number and type of hidden sectors, and their couplings to visible sector
    - spectrum and couplings of visible sector
  - Are there correlations between these features ?
  - How many "Standard Models" are there?

- Can the existing string constructions be continued into strong coupling and small volume regimes? What fraction of vacua live there? Are the EFT statistics different there or not?
- Are the known ensembles of vacua "representative" in the senses that
  - They cover an order one fraction of all vacua.
  - Their statistics well approximates the statistics of all vacua.
- Could a single string construction (say IIb or F theory on all CY's) have these properties (if continued to strong coupling and small volume).
- Are vacua "widely spread out" or "tightly packed" in configuration space? For example, if we exit inflation in a small region $R$, what fraction of all vacua might subsequent dynamics reach? What is the "effective dimension" of the configuration space?
- How useful are anthropic considerations?

# 3. Stabilizing moduli

The basic outline of string/M theory compactification with $\mathcal{N} = 1$ supersymmetry was developed in the mid-1980's, in the context of the heterotic string on Calabi-Yau manifolds:

- Compactify 10D supergravity on a Calabi-Yau $K$, with choices of background fields (in heterotic string, a gauge bundle).
- Do Kaluza-Klein reduction to derive an $\mathcal{N} = 1$ supergravity low energy EFT, valid as $\alpha', g_s \to 0$.
- Take into account nonperturbative corrections in $\alpha'$ and $g_s$ as explicit corrections to the EFT. Corrections to the superpotential are exponentially suppressed (and usually come from instantons) and in favorable cases can be found "exactly." Huge progress was made in this direction in the mid-90's. Other corrections, *e.g.* to the Kähler potential, are not under control.
- Expand around a minimum of the resulting effective potential. Ideally, all fields will become massive, and the minimum will exhibit supersymmetry breaking at a hierarchically small scale, produced in some way by the exponential corrections to the superpotential.

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conceptual – we now have good reasons to believe we can understand string theory at intermediate and strong coupling,
and phenomenological, mostly new solutions to the hierarchy problem:

- large extra dimensions
- warped extra dimensions

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Still, steady progress is being made. A particularly useful direction has been compactification with flux (Strominger, Polchinski, Becker, Becker, . . . ). Flux is $p$-form magnetic field strength in the internal dimensions, whose contribution to the vacuum energy depends on the metric moduli (and possibly other fields, such as the dilaton).

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A general discussion can be phrased in terms of the Gukov-Vafa-Witten superpotential

$$W = \int_K \Omega(z) \wedge G$$

where $\Omega(z)$ is the "special" or calibrating form of the geometry (*e.g.* the holomorphic $n$-form on a Calabi-Yau), and $G$ is the flux. For generic $G$, this is a complicated function of the moduli $z$, and the supersymmetry conditions

$$0 = D_i W = \left( \frac{\partial}{\partial z^i} + \frac{\partial K}{\partial z^i} \right) W$$

have isolated solutions, usually supersymmetric AdS vacua with

$$\Lambda = -3e^K |W|^2 < 0.$$

In most string theories, flux potentials do not depend on all moduli. However there is a class which do, developed by Acharya. It is compactification of M theory on a manifold of $G_2$ holonomy. Such a manifold has a Ricci-flat metric determined by the periods $t^i$ of the "special" three-form

$$\omega^{(3)} = \epsilon^\dagger \Gamma^{(3)} \epsilon$$

$$t^i = \int_{\Sigma_i} C^{(3)} + i\omega^{(3)}.$$

Turning on four-form flux leads to an effective superpotential

$$\begin{aligned} W &= \int G^{(4)} \wedge \left(i\omega^{(3)} + \tfrac{1}{2} C^{(3)}\right) \\ &= \sum_i t^i N_i \end{aligned}$$

This depends on all metric moduli and in combination with the Kähler potential

$$K = -\log \mathrm{Vol}(t)$$

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We can stabilize all moduli if we grant one more ingredient. $G_2$ manifolds can have singularities in codimension $4$, which effectively carry $6+1$ dimensional gauge theory (they reduce to D$6$ branes in IIa theory). By turning on a Wilson line in such a gauge theory, one induces a Chern-Simons term, which Acharya argued would produce a constant shift of $W$. This term (or any similar term) would stabilize all metric moduli.

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In fact, the only framework in which one can explicitly compute the flux potential at present, even in the large volume, weak coupling limit, is Calabi-Yau compactification. Consider IIb orientifolds, in this case

$$\begin{aligned} W &= \int \Omega(z) \wedge (F + \tau H) \\ &= \sum_i \Pi_i(z)\,(N^i_{RR} + \tau N^i_{NS}) \end{aligned}$$

depending on all complex structure moduli $z$, and the dilaton $\tau$. The periods $\Pi_i(z)$ can be evaluated using techniques developed in the study of mirror symmetry (Picard-Fuchs equations).

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On the other hand, Calabi-Yau manifolds necessarily have Kähler moduli as well, which do not appear in the IIb flux superpotential. One can try to dualize to IIa, or compactify on non-Kähler complex manifolds, but at present these possibilities are not well understood.

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Thus, the most promising approach at present for stabilizing all moduli in a computable model, is to find IIb flux compactifications, in which nonperturbative effects stabilize Kähler moduli, as suggested by Kachru, Kallosh, Linde and Trivedi.

# 4. Kähler stabilization

It is not hard to find IIb orientifold models with nonperturbative contributions to the superpotential which depend on Kähler moduli, because the gauge couplings on brane world-volumes in IIb theory depend on Kähler moduli. For example, in the related type I theory, the 4D gauge coupling is

$$\frac{1}{g_{YM}^2} = \frac{V_6}{l_s^6 g_s}$$

and $V_6$, the volume of the CY, is a Kähler modulus.

To get supersymmetric IIb flux vacua, we need to use models with O3/O7 planes (orientifold limits of F theory) rather than type I, as discussed by Giddings, Kachru and Polchinski. This is because of the Chern-Simons term

$$\int C^{(4)} \wedge F^{(3)} \wedge H^{(3)}$$

which makes the flux a source for the D3 tadpole, which must be cancelled. Fortunately D3/D7 couplings also depend on Kähler moduli.

Nonperturbative effects in a $U(N_c)$ D7 world-volume theory wrapped on a cycle $\Sigma$ would be expected to produce a superpotential

$$W \sim b e^{-V(\Sigma)/N_c}.$$

A naive parameter counting would suggest that we need $h^{1,1}$ different such gauge theories, to produce a superpotential depending on all Kähler moduli.

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However, not all supersymmetric gauge theories lead to superpotentials. There is a constraint on the massless matter content: for representations $R_i$, we require ($C_2$ is the index)

$$\sum_i C_2(R_i) < C_2(\mathrm{adj}).$$

For example, for $SU(N_c)$ SYM with fundamental matter, we need $N_f < N_c$.

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And, in most brane configurations which cancel tadpoles, the branes have a lot of matter. For example, one can look through the exhaustive study of brane and orientifold configurations in the Gepner models $3^5$ (the quintic) and $1^3 2^2$ (a hypersurface in $\mathbb{WP}^{1,1,1,2,2}$) of Brunner, Hori, Hosomichi and Walcher, hep-th/0401137. No suitable configurations appear.

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The problem is that most cycles one can wrap branes on, have moduli. As the simplest example, D3 branes always have moduli. Since each modulus leads to an massless adjoint field, we need rigid cycles (no moduli). Among curves in a CY, only $\mathbb{P}^1$ with normal bundle $\mathcal{O}(-1) \oplus \mathcal{O}(-1)$ is rigid. Surfaces can be rigid, but most bundles on surfaces have moduli.

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- It could be that in some examples, there are enough rigid cycles. Since this is not generic, we need systematic methods to find them.
- It could be that classical brane world-volume superpotentials are present, which lead to nonperturbative quantum effects. For example, an adjoint multiplet $\phi$ with

$$W = \mathrm{Tr}\ \phi^k$$

will lead to a nonperturbative superpotential.
However, such cases are not generic; varying the moduli will typically give mass to the matter. Thus, a systematic approach as above should see most such cases.

- It could be that the background fluxes will give mass to the matter. For example, the D7 world-volume Born-Infeld action

$$\int d^8x_{\mathbb{R}^{3,1}\times\Sigma}\ \sqrt{\det(g+F-B)},$$

in the presence of a non-trivial $H = dB$, would be expected to depend on the moduli of the surface $\Sigma$, and thus lift these moduli.

  This has not yet been worked out in detail. Although it could work, one needs to check that the fluxes which lift the charged matter, do not also force gauge symmetry breaking. Generic F theory fluxes, which correspond to magnetic fields on the D7 branes, will force gauge symmetry breaking.

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In any case, we need a systematic approach, to get any feeling for whether models which stabilize Kähler moduli are common or rare, and whether this condition is correlated with other features of interest.

A systematic approach to finding these nonperturbative corrections was developed around 1997 using the relation to F theory, by Witten, Donagi and Grassi.

A good physical starting point is to compactify M theory on a Calabi-Yau fourfold $X$. This will lead to a 3D theory with four supercharges, related to F theory and IIb if $X$ is $T^2$-fibered, by taking the limit $\mathrm{vol}(T^2) \to 0$. The complex modulus of the $T^2$ becomes the dilaton-axion varying on the base $B$.

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In M theory, an M5 brane wrapped on a divisor $D$ (essentially, a hypersurface), will produce a nonperturbative superpotential, if $D$ has arithmetic genus one:

$$1 = \chi(\mathcal{O}_D) = h^{0,0} - h^{0,1} + h^{0,2} - h^{0,3}.$$

Each of these complex cohomology groups leads to two fermion zero modes; an instanton contributes to $W$ if there are exactly two.

In the F theory limit, only divisors which wrap the $T^2$ contribute, and these correspond to D3-instantons wrapping surfaces in $B$. They come in two types:

- "gauge type," which sit in the singularities of the $T^2$ fibration, and are thus associated to D7 branes. These are the D-brane counterparts of small instantons in the D7 world volume theories.
- "instanton type," which are pullbacks of smooth divisors on the base, and need not correspond to any nonabelian gauge theory.

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- "instanton type," which are pullbacks of smooth divisors on the base, and need not correspond to any nonabelian gauge theory.

In both cases, the a. g. one condition can also be understood from the F theory analysis, in the examples which have been studied in detail. The rough correspondence is

- $h^{0,0} = 1$ counts gauge symmetries on the brane
- $h^{0,1}$ counts adjoint matter from brane gauge fields
- $h^{0,3}$ counts adjoint matter from varying brane embedding
- $h^{0,2}$ counts possible superpotential constraints on matter

and thus we require enough superpotential constraints to lift all the matter. This analysis suggests that $\chi(D) > 1$ might work as well, but no examples are known. (In 3D, one can argue that this cannot work.)

In any case, to get examples which clearly work, we look for F theory compactifications on an elliptically fibered fourfold $X$ with "enough" divisors of arithmetic genus one, so that a superpotential

$$W = W_{flux} + \sum_i b_i e^{-\vec{t}\cdot D_i}$$

will lead to non-trivial solutions to $DW = 0$ by balancing the exponentials against the dependence coming from the Kähler potential.

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In fact, the necessary condition for this to work is somewhat weaker: a linear combination

$$\mathcal{R} = r_{\vec{n}} D_{\vec{n}}$$

of the contributing divisors must exist so that $\mathcal{R}$ lies within the Kähler cone (*i.e.* values of moduli such that the volumes of all curves are positive). However we did not need this generalization to find our examples.

In the math literature, there is a very general relation between divisors of a.g. one, and contractions of manifolds. This allows proving the following relation in many cases (toric or Fano base):

$$D \cdot \Sigma < 0$$

for $\Sigma$ an effective curve. This implies that the instanton action for to the divisor $D$ must have at least one negative coefficient (in some basis),

$$W \sim b_D \exp 2\pi i(c_1 t_1 + \ldots - c_n t_n).$$

This implies that no model with one Kähler modulus can stabilize Kähler moduli (see also Robbins and Sethi, 0405011).

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This implies that no model with one Kähler modulus can stabilize Kähler moduli (see also Robbins and Sethi, 0405011).

However, this is not a problem if the CY threefold has more than one Kähler modulus, $h^{1,1} > 1$.

Using the very complete study of divisors of a. g. one of A. Grassi, math.AG/9704008, we have found $6$ models with toric Fano threefold base which can stabilize all Kähler moduli, and could be analyzed in detail using existing techniques.

The simplest, $\mathcal{F}_{18}$, has $89$ complex structure moduli. According to the AD counting formula, it should have roughly $\epsilon \times 10^{307}$ flux vacua with all moduli stabilized, where

$$\epsilon = g_s^2 \times \frac{|W|}{m_s^4}\Big|_{max}.$$

Models which stabilize all Kähler moduli are not generic, because a.g. one divisors are not. However, they are not uncommon either; there are 29 out of 100 with toric base, and probably many more with $\mathbb{P}^1$ fibered base. This last class of model should be simpler, in part because these have heterotic duals, but analyzing them requires better working out the D7 world-volume theories.

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We expect one can add antibranes or D breaking as in the KKLT discussion to get de Sitter vacua, but have not analyzed this. Also, there are many more moduli to stabilize (brane and bundle moduli). We have general arguments that these live in compact moduli spaces, so a generic potential would stabilize them.

# 5. Flux vacua

The simplest example of IIb flux vacua is to consider a rigid CY, *i.e.* with $b^{2,1} = 0$ (for example, the orbifold $T^6/\mathbb{Z}_3$). Then the only modulus is the dilaton $\tau$, with Kähler potential $K = -\log \mathrm{Im}\ \tau$.

The flux superpotential reduces to

$$W = A\tau + B; \qquad A = a_1 + \Pi a_2; B = b_1 + \Pi b_2$$

with $\Pi = \int_{\Sigma_2} \Omega^{(3)} / \int_{\Sigma_1} \Omega^{(3)}$, a constant determined by CY geometry.

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Now it is easy to solve the equation $DW = 0$:

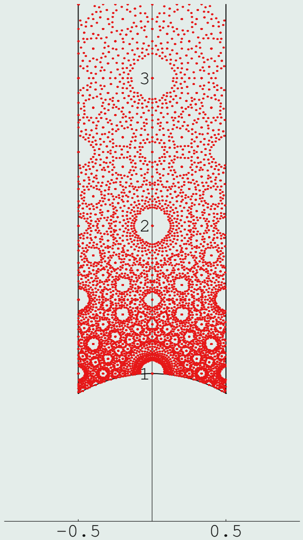
$$\begin{aligned} DW &= \tfrac{\partial W}{\partial \tau} - \tfrac{1}{\tau - \bar{\tau}} W \\ &= \tfrac{-A\bar{\tau} - B}{\tau - \bar{\tau}} \end{aligned}$$

so $DW = 0$ at

$$\bar{\tau} = -\frac{B}{A}$$

where $\bar{\tau}$ is the complex conjugate.

Here is the resulting set of flux vacua for $L = 150$ and $\Pi = i$:




This graph was obtained by enumerating one solution of $a_1 b_2 - a_2 b_1 = L$ in each $SL(2,\mathbb{Z})$ orbit, taking the solution $\tau = -(b_1 - ib_2)/(a_1 - ia_2)$ and mapping it back to the fundamental region.

The total number of vacua is $N = 2\sigma(L)$, where $\sigma(L)$ is the sum of the divisors of $L$. Its large $L$ asymptotics are $N \sim \pi^2 L/6$.

A similar enumeration for a Calabi-Yau with $n$ complex structure moduli, would produce a similar plot in $n+1$ complex dimensions, the distribution of flux vacua. It could (in principle) be mapped into the distribution of possible values of coupling constants in a physical theory.

The intricate distribution we just described has some simple properties. For example, one can get exact results for the large $L$ asymptotics, by computing a continuous distribution $\rho(z,\tau;L)$, whose integral over a region $R$ in moduli space reproduces the asymptotic number of vacua which stabilize moduli in the region $R$, for large $L$,

$$\int_R dz d\tau \; \rho(z,\tau;\; L) \sim_{L\to\infty} N(R).$$

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Explicit formulas for these densities can be found, in terms of the geometry of the moduli space $\mathcal{C}$. The simplest such result computes the index density of vacua:

$$\rho_I(z,\tau) = \frac{(2\pi L)^{b_3}}{b_3!\pi^{n+1}} \det(-R - \omega \cdot 1)$$

where $\omega$ is the Kähler form and $R$ is the matrix of curvature two-forms. This is a lower bound for the number of vacua.

The index density can be defined as

$$\rho_I(z,\tau) = \sum_{\substack{dB,dC \\ L=\int dB\wedge dC}} (-1)^F \delta_{z,\tau}(DW(z,\tau))$$

where

$$(-1)^F = \text{sgn} \;\; \det \begin{pmatrix} \bar\partial_{\bar i} D_j W(z) & \partial_i D_j W(z) \\ \bar\partial_{\bar i} \bar D_{\bar j} W^*(z) & \partial_i \bar D_{\bar j} W^*(z) \end{pmatrix}.$$

This is $(-1)^{n+1}$ for Minkowski vacua $W=0$, and is the Morse sign for the function $e^K|W|^2$ for $W\neq 0$.

The index is invariant under small variations of the data $K$ and $W$ and is thus the obvious candidate for a topological field theory computation. Indeed, if $\mathcal{C}$ had been a compact manifold, it would be invariant under arbitrary variations,

$$[c_{n+1}(\Omega\mathcal{C}\otimes\mathcal{L})] = \frac{1}{\pi^{n+1}}\int_{\mathcal{C}} \det(-R-\omega\cdot 1).$$

The index density can be integrated over a fundamental region of the moduli space to estimate the total number of flux vacua. For example, for $T^6$ (with symmetrized period matrix), $K = b_3 = 20$, and

$$I = \int \rho^I = \frac{7 \cdot (2\pi L)^{20}}{4 \cdot 181440 \cdot 12 \cdot 20!} \sim 4 \cdot 10^{21} \qquad \text{for } L = 32.$$

Since $r \sim 1$ in the bulk of moduli space, the condition $L > K/r^2$ for the validity of this estimate should be satisfied, but it is worth looking for subtleties here!

More examples: the "mirror quintic" (Greene and Plesser; Candelas et al) with a one parameter moduli space $\mathcal{M}_c(\tilde{Q})$. The integral

$$\frac{1}{\pi^2}\int_{\mathcal{C}} \det(-R-\omega) = \frac{1}{12}\chi(\mathcal{M}_c(\tilde{Q})) = \frac{1}{60}.$$

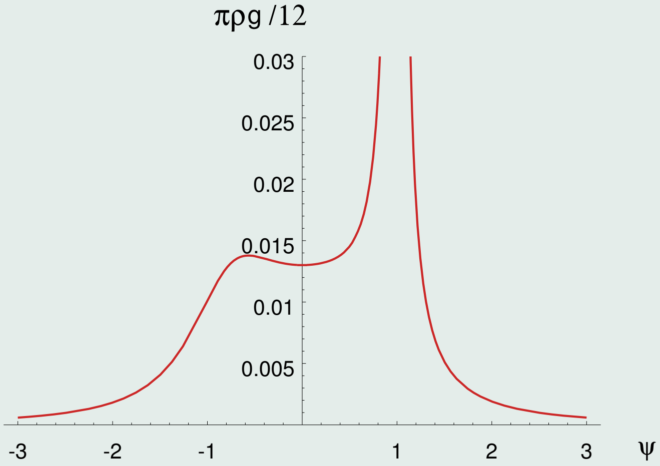
And, the quintic, with its $101$ parameter moduli space (Z. Lu),

$$\int_{\mathcal{M}_c(Q)} \frac{\omega^{101}}{101!} = 5^{-24}\frac{1}{|\Gamma|}$$

where $\Gamma$ is a residual discrete symmetry group.

# 6. Distributions of flux vacua

Let us look at the details of the distribution of flux vacua on the mirror quintic ($K = 4$ and $n = 1$), as a function of complex structure modulus:




Note the divergence at $\psi = 1$. This is the conifold point, with a dual gauge theory interpretation. It arises because the curvature $R \sim \partial\bar{\partial} \log \log |\psi - 1|^2$ diverges there. The divergence is integrable, but a finite fraction of all the flux vacua sit near it.

In a bit more detail, let $v$ be a modulus near the conifold point $v = 0$. The metric near $v = 0$ is

$$g_{v\bar{v}} \approx c \ln \frac{\mu^2}{|v|^2}, \tag{1}$$

and the third derivative of the prepotential or "Yukawa coupling" is

$$\mathcal{F} = \partial^3 \mathcal{F}(v) = g_{v\bar{v}}^{-3/2} e^K \left( i \textstyle\int_A \hat{\Omega}\ \partial_v^3 \int_B \hat{\Omega} + \text{anal.} \right) \approx i \left( c \ln \frac{\mu^2}{|v|^2} \right)^{-3/2} \frac{c}{v}, \tag{2}$$

so $\mathcal{F} \to \infty$ when $v \to 0$. The same is true for $\rho \approx |\mathcal{F}|^2/\pi^2$. However, the density integrated over the fundamental $\tau$-domain and $|z| < R$ remains finite. For small $R$:

$$\int d^2\tau\, g_{\tau\bar{\tau}} \int d^2v\, g_{v\bar{v}}\, \rho \approx \frac{1}{12 \ln \frac{\mu^2}{R^2}}. \tag{3}$$

The number of susy vacua with $L \leq L_*$ and $|v| \leq R$ is

$$\mathcal{N}_{vac} = \frac{\pi^4 L_*^4}{18 \ln \frac{\mu^2}{R^2}}. \tag{4}$$

For example, with $L_* = 100$ and $\mu = 1$, there are about one million susy vacua with $|v| < 10^{-100}$ (about $0.1\%$).

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Vacua close to conifold degenerations are interesting for model building, as they provide a natural mechanism for generating large scale hierarchies (Randall and Sundrum, etc.). They may also enable controlled constructions of de Sitter vacua by adding anti-D3 branes, as proposed by KKLT However, for the latter it is also necessary that the mass matrix at the critical point is positive.

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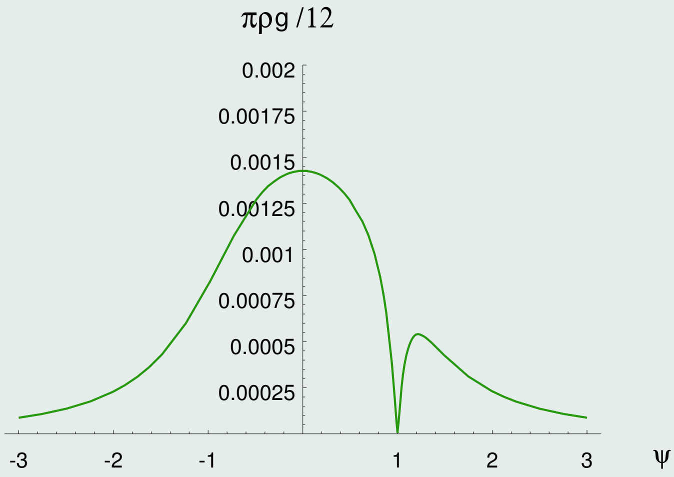
The distribution of tachyon-free D breaking vacua is



In fact, most D breaking vacua near the conifold point have tachyons (for one modulus CY's), so we get suppression, not enhancement.

This is not hard to understand: assuming $DW = 0$, the bosonic mass matrix is (as in supersymmetric AdS),

$$M = H^2 - 3e^{K/2}|W|H, \qquad H = 2\,d^2 e^{K/2}|W|. \tag{5}$$

This means that eigenvalues of $H$ (the fermion mass matrix) between $0$ and $3e^{K/2}|W|$ lead to tachyons. In general, small $|W|$ makes tachyonic moduli unlikely.

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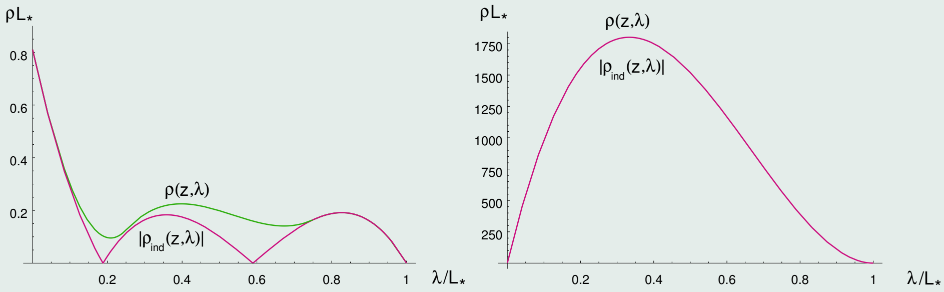
However, in a one modulus model, the matrix $H$ takes the form

$$\Delta H = \begin{pmatrix} 0 & S \\ \bar{S} & 0 \end{pmatrix}, \qquad S = \frac{\bar{W}}{|W|}\begin{pmatrix} 0 & Z \\ Z & \mathcal{F}\bar{Z}. \end{pmatrix} \tag{6}$$

($Z$ is a random parameter). When $\mathcal{F}$ is large, its eigenvalues are approximately $e^K|W| \pm \mathcal{F}^{\pm Z}$, and there is always a small positive eigenvalue.

It remains to be seen whether this is also true in multi-modulus models.

Here is the distribution of cosmological constants, both at generic points (left) and near the conifold point (right). Note that at generic points it is fairly uniform, all the way to the string scale. On the other hand, imposing small c.c. competes with the enhancement of vacua near the conifold point.




The left hand graph compares the total number of vacua (green) with the index (red). The difference measures the number of Kähler stabilized vacua, vacua which exist because of the structure of the Kähler potential, not the superpotential.

# 7. Large complex structure

Another simple universal property: In the large complex structure limit, $R \sim \omega$ and the density of vacua is reasonably well approximated by $\det \omega$. This can be computed from the Kähler potential, which is cubic in the LCS limit,

$$K = -\log c_{\alpha\beta\gamma} y^\alpha y^\beta y^\gamma$$

where $y^\alpha = \text{Im } t^\alpha$ is a Kähler modulus, giving

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Taking the determinant, one finds $\det \omega \sim y^{-2n}$, and this leads to a universal falloff in the large complex structure limit,

$$\int_{V>V_0} \rho \sim V_0^{-n/3}$$

where $V$ is the "mirror volume" (the mirror of the LCS is the large volume limit). For large $n$, this is a drastic falloff, and typically there are no vacua in this regime.

This suppression can be understood directly in terms of the mirror IIa derivation of the metric on Calabi-Yau moduli space. In this case it is the metric on Kähler moduli space, which at large volume comes from the metric on the space of metrics,

$$\langle \delta g_{ij}, \delta g_{kl} \rangle = \frac{1}{V} \int_{CY} \sqrt{g} g^{ik} g^{jl} \delta g_{ij} \delta g_{kl}$$

where $g_{ij}$ is the metric on the CY, and the $1/V$ factor (which compensates the $\sqrt{g}$) comes from the standard derivation of the kinetic term in KK reduction on CY. Because of the inverse factors of the metric, this falls off with volume as $V^{-1/3}$. This factor appears for each modulus, leading to a dramatic effect with many moduli.

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A possible physical application of this: we know how to stabilize complex structure moduli using fluxes in IIb. Suppose we can use T-duality to get a corresponding class of models in IIa with stabilized Kähler moduli. Then, the mirror interpretation of this result is the number of vacua which stabilize the volume of the compact dimensions at a given value.

Ignoring the geometric factor, and writing $V \sim R^6$, we find a number of vacua

$$N \sim \frac{(2\pi L)^K R^{-K}}{K!}$$

so large $K$ disfavors large volume in this case, and the maximum volume one expects is of order

$$V \sim \left(\frac{2\pi L}{K}\right)^6 .$$

The parameter $L = \chi/24$ in F theory compactification on four-folds and can reach values of several thousand. This will provide the upper bound on values of $V$. Will it reach the $\sim 10^{30}$ of the "large extra dimensions" scenario?

# 8. Susy breaking vacua

D breaking vacua (with $DW = 0$) are described by the same results, just we require the vacua to be tachyon free and have near zero c.c. As we saw, the distribution of the parameter $-3e^K|W|^2$ in flux vacua is fairly uniform, all the way to the string scale. This means that an arbitrary supersymmetry breaking contribution to the vacuum energy,

$$V = \sum_\alpha D_\alpha^2 - 3e^K|W|^2$$

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Thus, the need to get small c.c. does not favor a particular scale of susy breaking. The simplest assumption for the distribution of D breaking scales is that it is uniform. In such a distribution, the number of vacua with susy breaking scale $D^2 < M_{susy}^4$ goes as $M_{susy}^4/M_P^4$, and high scales are favored.

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If several D terms can be present, since the overall breaking scale is $\sum D^2$, the analogous distribution favors high scale breaking even more (this observation is also made by Susskind, hep-th/0405189). While this suggests that a high scale of supersymmetry breaking is preferred, this could easily be in a hidden sector.

With Denef, we have analyzed F breaking flux vacua in orientifolds in some detail. These satisfy

$$0 = \partial_I V = Z_I \bar{Y}^0 + \mathcal{F}_{IJK} \bar{Y}^J \bar{Z}^K - 2 Y_I \bar{X} \tag{7}$$

where $X = |W|$, $Y_I = D_I W$, $Z_I = D_0 D_I W$ and $\mathcal{F}_{IJK}$ are the special geometry cubic couplings.

The simplest way to satisfy this is to have $W = D^2 W = 0$, giving an "anti-supersymmetric branch" of vacua, whose flux is anti-self-dual. These are probably not physical as they have large positive c.c.

The more interesting F breaking vacua are the "mixed branches," in which the parameters $Y = DW$ satisfy relations allowing $X = |W| \neq 0$ (the matrix implicit in (7) is reduced in rank).

Such vacua do exist, but (rather surprisingly, and in the limits we considered) all with $\Lambda = V < 0$; we did not find de Sitter vacua. This may simply be because stabilizing de Sitter vacua requires two points of inflection in the potential, and in the limits we considered (large complex structure and conifold) the potential is too simple to obtain this.

# 9. Conclusions

The value of the statistical approach to string compactification depends on whether vacua which realize each of the many features needed to reproduce real world physics can be constructed within the general frameworks we now have, and whether we can capture interesting features of the real distribution of vacua in distributions which are simple enough to work with.

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Regarding the first point, we have found explicit examples of IIb orientifold compactifications in which all Kähler moduli are stabilized, and vacuum counting estimates which suggest that all moduli can be stabilized. So far, it appears that such vacua are not generic, but they are not uncommon either: about a third of our sample of F theory models with Fano threefold base should work.

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In DDF, we considered $\mathcal{F}_{18}$, with 89 complex structure moduli, in some detail. If we had reasons to think this model were important, a detailed analysis could be pushed through, but it is complicated. We believe simpler models can be found given better analysis of the brane world-volume theories. One good candidate is the orientifold of a hypersurface in $\mathbb{WP}^{1,1,1,6,9}$ (a CY elliptically fibered over $\mathbb{P}^2$).

Regarding the second point, we have explicit results for distributions of flux vacua of many types: supersymmetric, non-supersymmetric, tachyon-free. They display a lot of structure, with suggestive phenomenological implications:

- Enhanced numbers of vacua near conifold points (possibly inconsistent with metastability and small c.c.).
- Correlations with the cosmological constant
- Falloff in numbers at large volume and large complex structure
- Uniform distributions involving many susy breaking parameters favor high scales of supersymmetry breaking. The condition of cancelling the c.c. does not compensate for this.

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To make any serious statement about a statistically preferred scale of supersymmetry breaking, besides sharpening up the last point, one would need to discuss the communication of supersymmetry breaking to the observable sector. One might argue that the simplest models, in which this is fairly direct (so one gets high superpartner masses) are most numerous, but this is not at all clear.

Simple distributions for the gauge group and matter spectrum probably can be found as well. For example, one might hypothesize that the total rank of the gauge group is distributed as $r^{-\alpha}$, or the number of generations of matter is distributed as $N^{-\beta}$. Such hypotheses could be explicitly checked in brane constructions, or other classes of compactifications.

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Comparing these distributions between different dual classes of constructions (say type II and heterotic) will provide a highly non-trivial check of duality for $N = 1$ and non-supersymmetric theories.

If two classes of construction produce the same statistics, that is evidence that both are representative of the statistics of the full ensemble of string/M theory vacua. We could then start to make interesting statements about the distribution of all the vacua.

The most optimistic interpretation of such results will come out if it turns out that, on reaching the end of the problem and constructing fully consistent vacua, the total numbers of vacua are not too large, say of the order $10^{100}$ that we wanted to solve the c.c. problem. Based on present numbers, for this to come out, we must hope that many of the seemingly consistent tachyon-free vacua we just discussed in the end turn out to be unstable or inconsistent, or that some sort of cosmological selection applies.

In this case, such distribution results could imply that certain regions of moduli space in fact have no vacua which satisfy all the other constraints of the problem.

For example, if we found that approximately $10^{40}$ vacua with a high scale of supersymmetry breaking) should match the SM, and $10^{-40}$ with a low scale should match, we would have evidence for the claim that low scale supersymmetry breaking is not possible in string theory. Thus we might imagine someday making testable predictions which could falsify string theory (this particular one would be very ironic, of course).