Finite conductivity in protostellar disks

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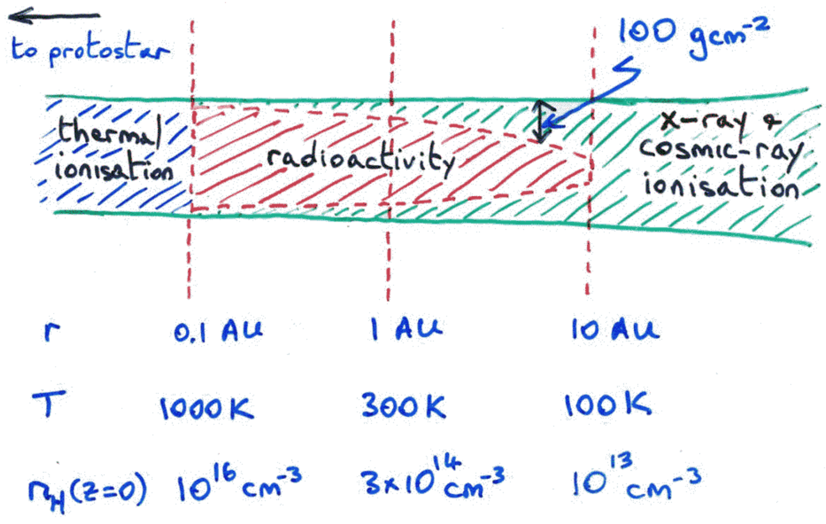
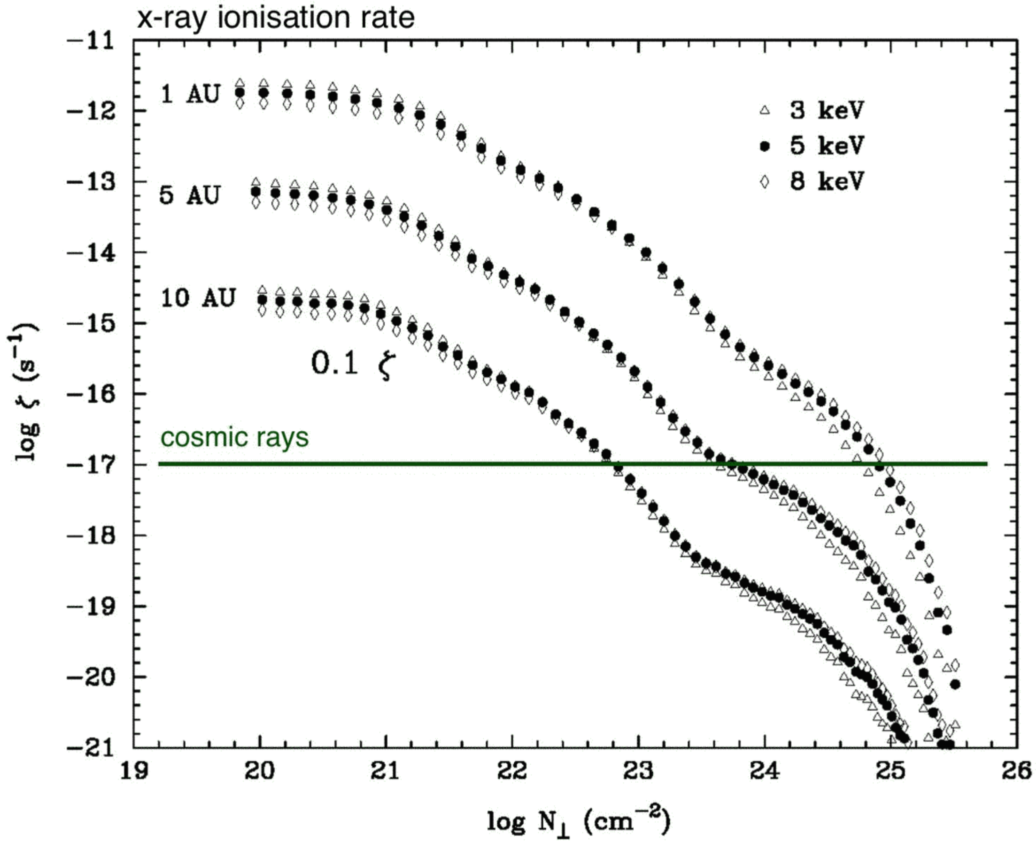
Protostellar disks are poorly conducting

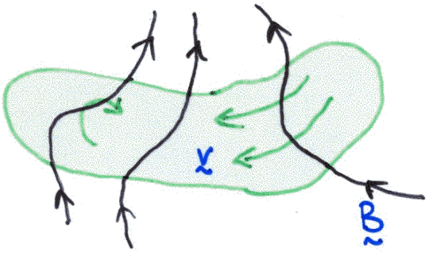
- high density implies low conductivity
  - recombinations relatively rapid
  - drag on charged particles
- deeper layers shielded from ionising radiation for $r < 5$ AU
  - x-ray attenuation column ~10 g/cm^2
  - cosmic ray attenuation column ~100 g/cm^2





Igea & Glassgold 1999

MHD with finite conductivity

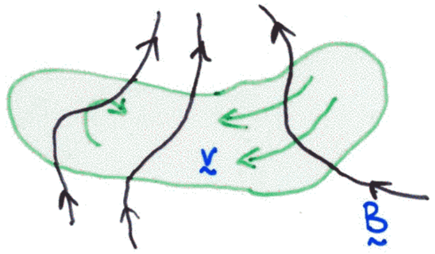


Given ρ, $\underset{\sim}{v}$, $\underset{\sim}{B}$ how does the fluid evolve?

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho v) = 0 \qquad \nabla^2 \Phi = 4\pi G \rho$$

$$\rho\frac{\partial v}{\partial t} + \rho(v\cdot\nabla)v + \nabla P + \rho\nabla\Phi = \frac{J\times B}{c} \qquad J = \frac{c}{4\pi}\nabla\times B$$

(dissipation, internal excitation, PdV, cooling) $\qquad \dfrac{\partial B}{\partial t} = -c\nabla\times E$



Given $\rho, \mathbf{v}, \mathbf{B}$ how does the fluid evolve?

$\nabla\times\mathbf{B} \Rightarrow \mathbf{J}$ $\qquad$ $\mathbf{J} = \frac{c}{4\pi}\nabla\times\mathbf{B}$

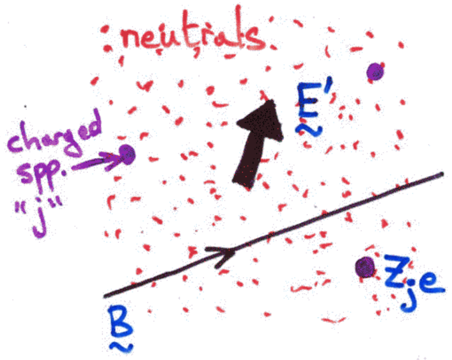
$\Rightarrow \mathbf{E}'$ (fluid) $\qquad$ $\boxed{\mathbf{J} = \boldsymbol{\sigma}\,\mathbf{E}'}$

$\Rightarrow \mathbf{E}$ (observer) $\qquad$ $\mathbf{E} = \mathbf{E}' - \frac{\mathbf{v}}{c}\times\mathbf{B}$

$\Rightarrow \frac{\partial \mathbf{B}}{\partial t}$ $\qquad$ $\frac{\partial \mathbf{B}}{\partial t} = -c\,\nabla\times\mathbf{E}$

$$= \nabla\times(\mathbf{v}\times\mathbf{B}) - \frac{c^2}{4\pi}\nabla\times(\boldsymbol{\sigma}^{-1}\nabla\times\mathbf{B})$$

Conductivity



Apply an electric field $\mathbf{E}'$ to a weakly-ionised, magnetised gas.

What is the resulting $\mathbf{J}$?

$$\underbrace{Z_j e\,\mathbf{E}'}_{\text{electric}} + \underbrace{Z_j e\,\frac{\mathbf{v}_j}{c}\times\mathbf{B}}_{\text{mag.}} - \underbrace{\gamma_j m_j \rho\,\mathbf{v}_j}_{\text{drag}} = 0$$

Solve for $\mathbf{v}_j$, form $\mathbf{J} = \sum_j n_j e Z_j \mathbf{v}_j$

Magnetic diffusion

$$\frac{\partial B}{\partial t} = -c\,\nabla\times \underset{\uparrow\ \text{lab frame}}{E} = -c\,\nabla\times\Big(\underset{\uparrow\ \text{fluid frame}}{E'} + \frac{v}{c}\times B\Big)$$

$$= \nabla\times\left[\underbrace{v\times B}_{\text{inductive}} - \underbrace{\frac{J}{\sigma_{\parallel}}}_{\text{ohmic}} - \underbrace{\frac{\sigma_H}{\sigma_\perp^2}\frac{J\times B}{B}}_{\text{Hall}} - \underbrace{\left(\frac{\sigma_P}{\sigma_\perp^2} - \frac{1}{\sigma_{\parallel}}\right)\frac{(J\times B)\times B}{B^2}}_{\text{ambipolar}}\right]$$

$$= \nabla\times(v\times B) - \nabla\times\Big[\underset{\text{Ohmic}}{\eta_{\parallel}\nabla\times B} + \underset{\text{Hall}}{\eta_H(\nabla\times B)\times\hat{B}} + \underset{\text{Ambipolar}}{\eta_A(\nabla\times B)_\perp}\Big]$$

Motivation

- Is the magnetic field coupled to the matter?
  - □ $\eta < h\,c_s$?
- Which diffusion components dominate?
  - – ohmic, hall or ambipolar?
- What are the consequences of different diffusion regimes?
  - – vector evolution of B shows fundamental differences
  - – hall diffusion reverses sign under global field reversal (yikes)
- Diffusion and magnetocentrifugal jet launching
  - – loading of mass onto field lines
  - – constrains bending of field lines within disk
  - – radial drift of field
- Diffusivity depends on location
  - – vertical stratification of ionisation rate and density
  - – inconvenient radial variations of microphysics

Criterion for coupling

$$\frac{\partial B}{\partial t} = \nabla\times(v\times B) - \nabla\times\left[\underbrace{\eta_{\parallel}\nabla\times B}_{\text{Ohmic}} + \underbrace{\eta_H(\nabla\times B)\times\hat{B}}_{\text{Hall}} + \underbrace{\eta_A(\nabla\times B)_{\perp}}_{\text{Ambipolar}}\right]$$

η small $\Rightarrow$ flux freezing

η large $\Rightarrow$ field diffusion

* for a disk in Keplerian rotation,

$$\left.\begin{aligned} \nabla\times(v\times B) &\sim \Omega B \\ \eta\nabla^2 B &\sim \frac{\eta B}{h^2} \end{aligned}\right\} \Rightarrow \text{Field is coupled to shear when } \eta \lesssim \frac{h^2 v_K}{r} = h c_s$$

$h = \frac{c_s}{v_K} r$

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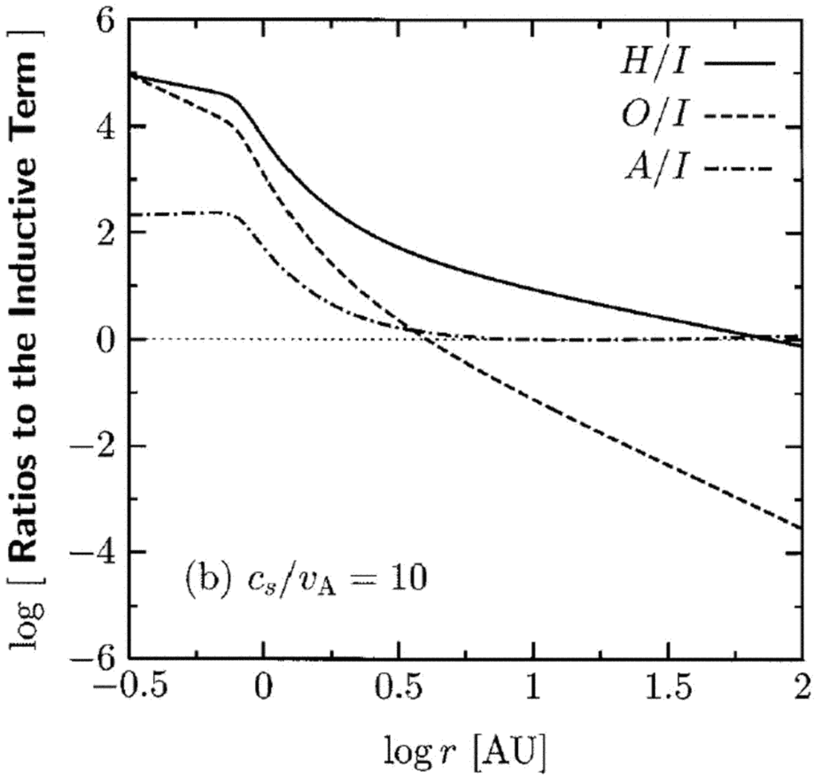

Sano & Stone 2002a

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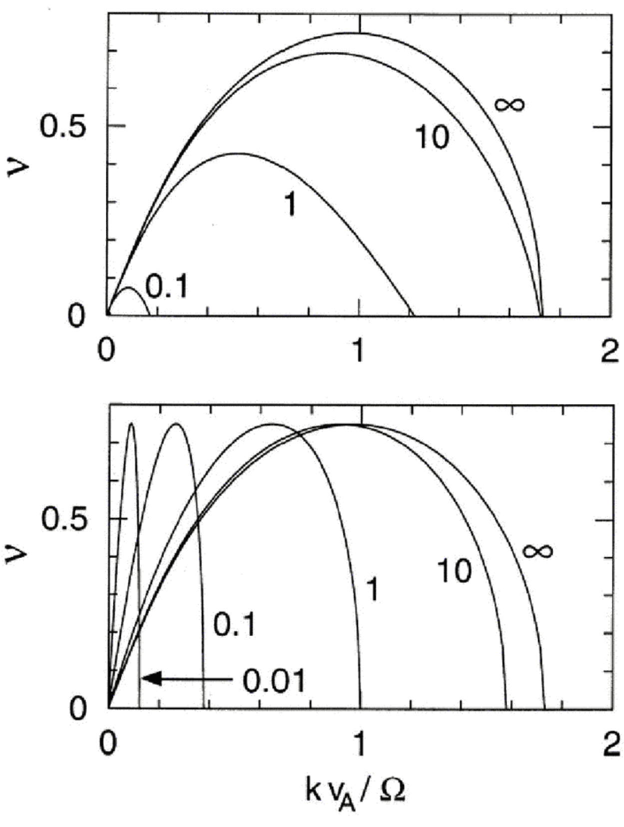
MRI in non-ideal MHD



Ambipolar or ohmic diffusion

Hall diffusion

Wardle 1999

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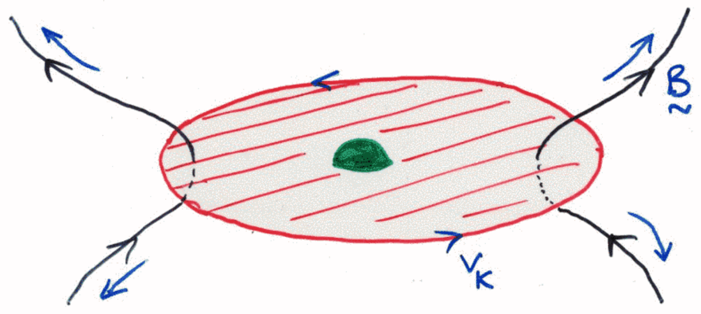
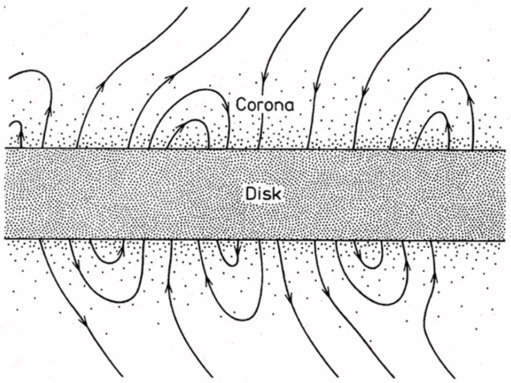
Disk-driven winds





Blandford & Payne 1982

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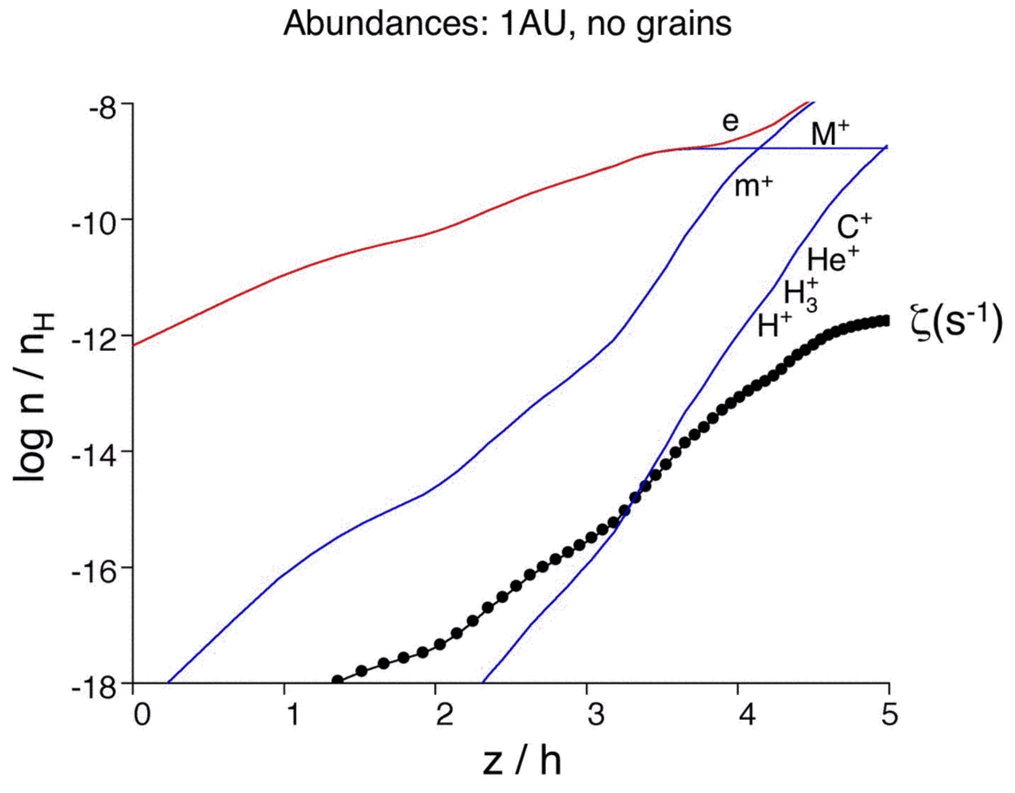
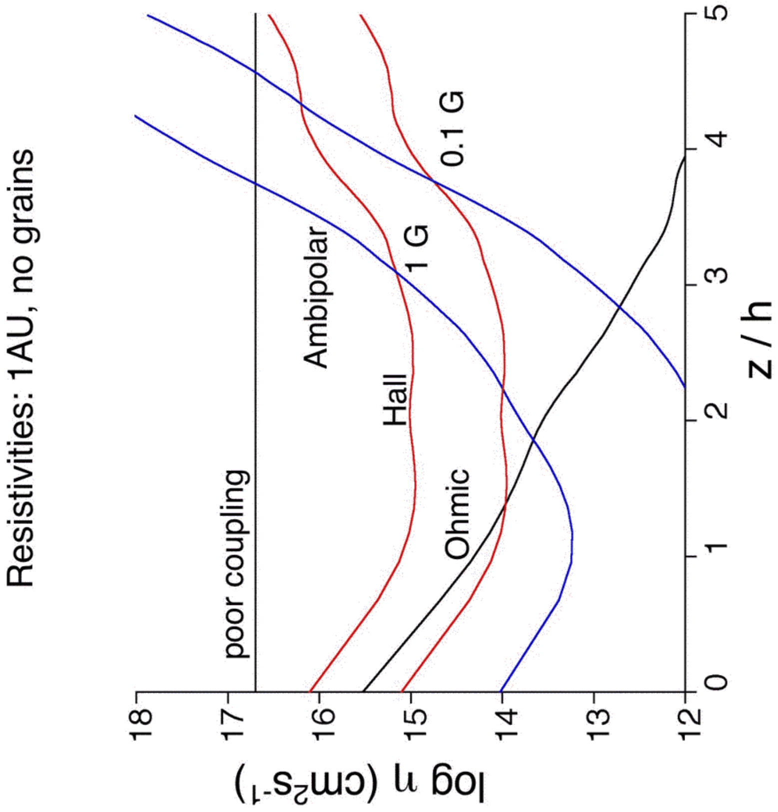
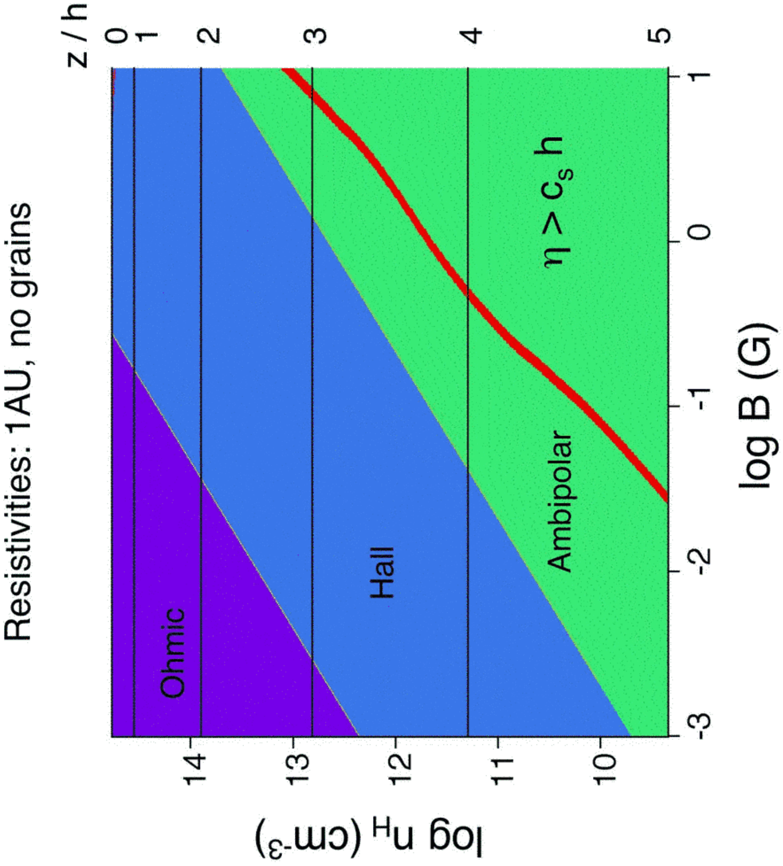
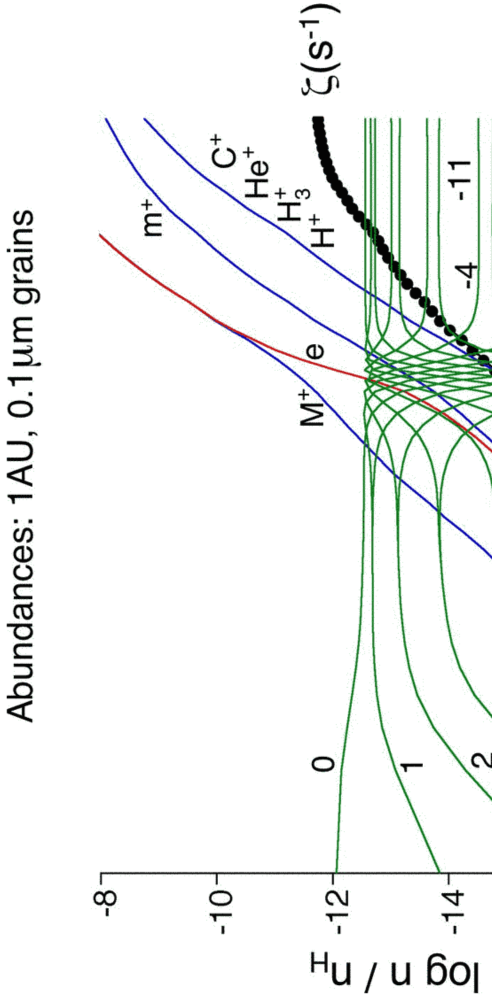
Resistivity calculations

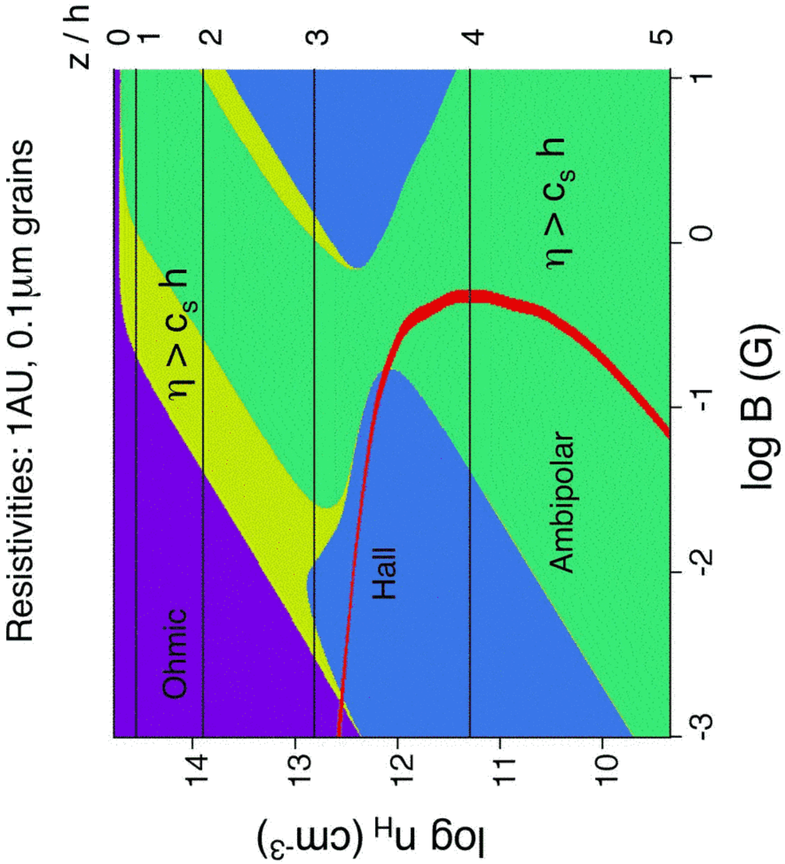
- minimum solar nebula
  - assume isothermal in z-direction
- ionisation by cosmic rays and x-rays from central star
- simple reaction scheme following Nishi, Nakano & Umebayashi (1993)
  - H^+,H_3^+,He^+,C^+,molecular (M^+) and metal ions (M^+), e^-, and charged grains
  - extended to allow high grain charge (T larger than in molecular clouds)
- adopt model for grains
  - none, single size grains, MRN size distribution, MRN + ice mantles, etc
  - results for “no grains” or 0.1 μm grains presented here
- evaluate resistivity components
  - when can the field couple to the shear in the disc?
  - which form of diffusion is dominant?

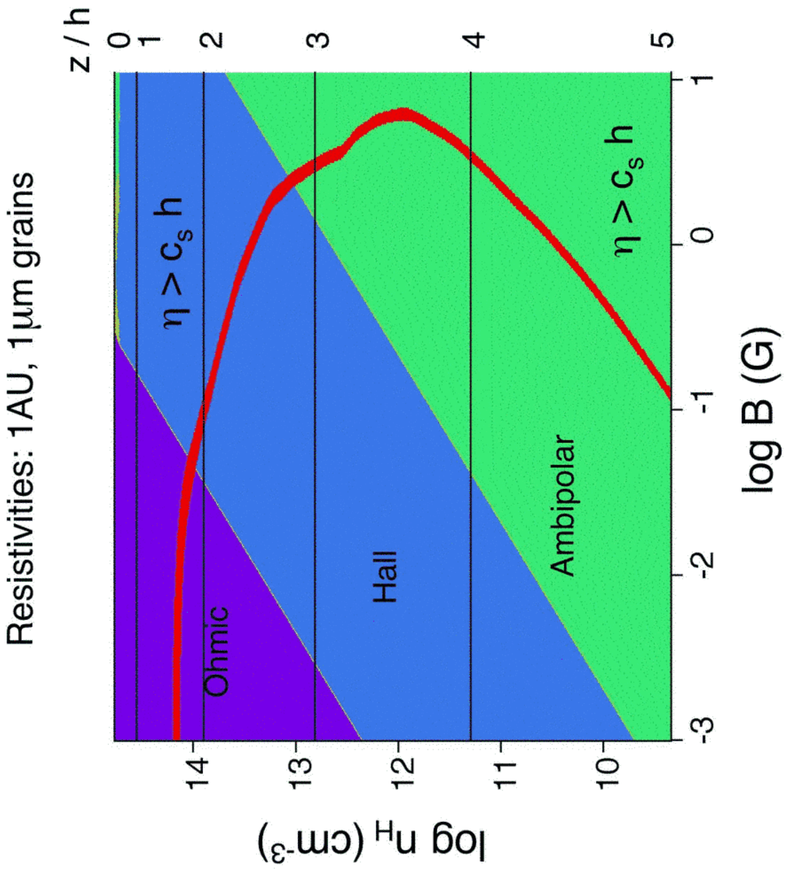


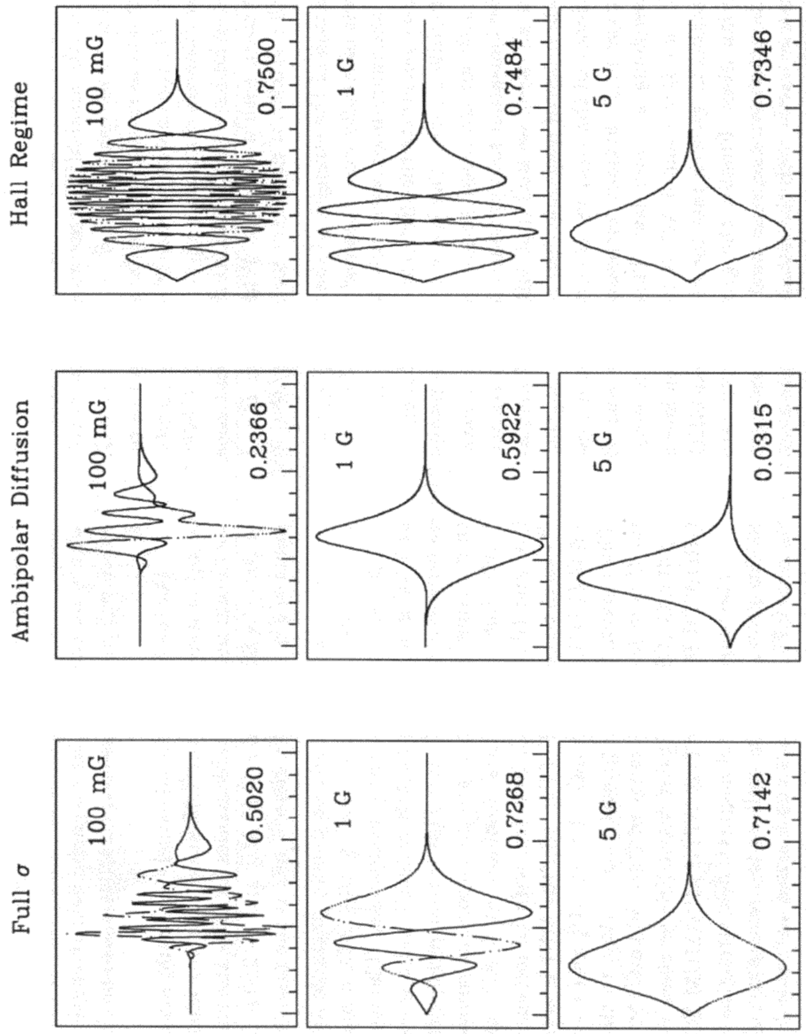

Resistivities: 1AU, no grains
poor coupling
Ambipolar
Hall
Ohmic
1 G
0.1 G
log η (cm²s⁻¹)
18
17
16
15
14
13
12
z / h
0
1
2
3
4
5


Resistivities: 1AU, no grains
z / h
0
1
2
3
4
5
Ohmic
Hall
Ambipolar
η > c_s h
log n_H (cm⁻³)
14
13
12
11
10
log B (G)
-3
-2
-1
0
1

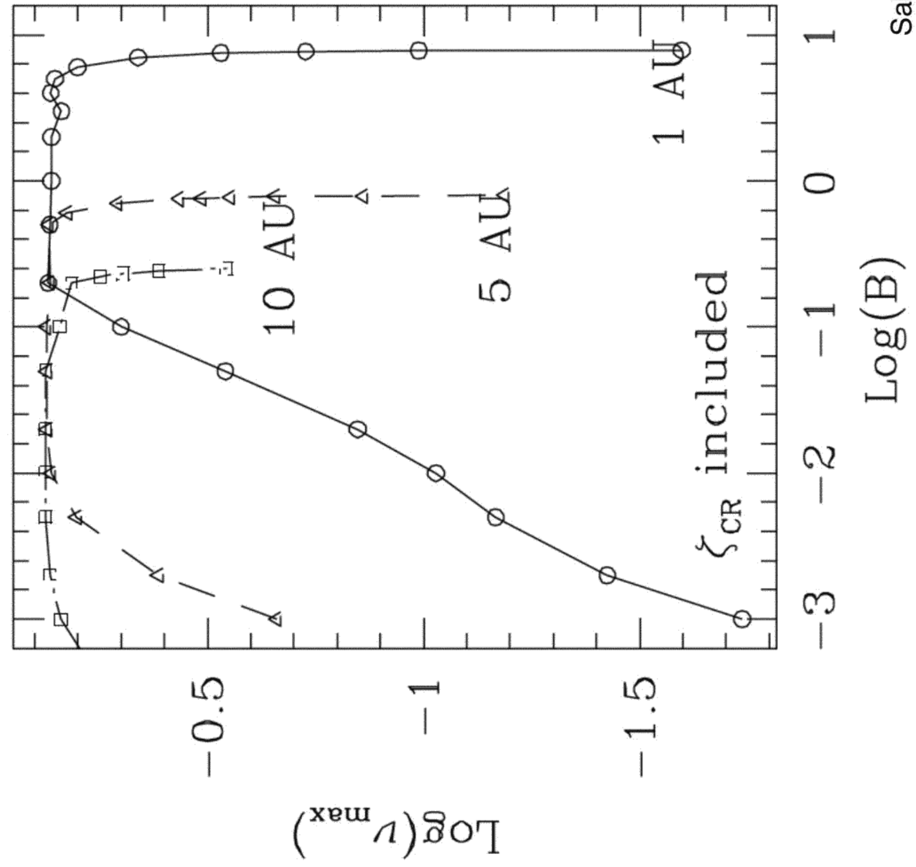

Abundances: 1AU, 0.1μm grains
log n / n_H
z / h
ζ(s⁻¹)
m⁺
C⁺
He⁺
H₃⁺
H⁺
M⁺
e
0
1
2
3
-4
-3
-2
-11
-12
-13
-14
-8
-10
-12
-14
-16
-18
0
1
2
3
4
5


Resistivities: 1AU, 0.1μm grains
log n_H (cm⁻³)
log B (G)
z / h
Ohmic
Hall
Ambipolar
η > c_s h
η > c_s h
14
13
12
11
10
-3
-2
-1
0
1
0
1
2
3
4
5

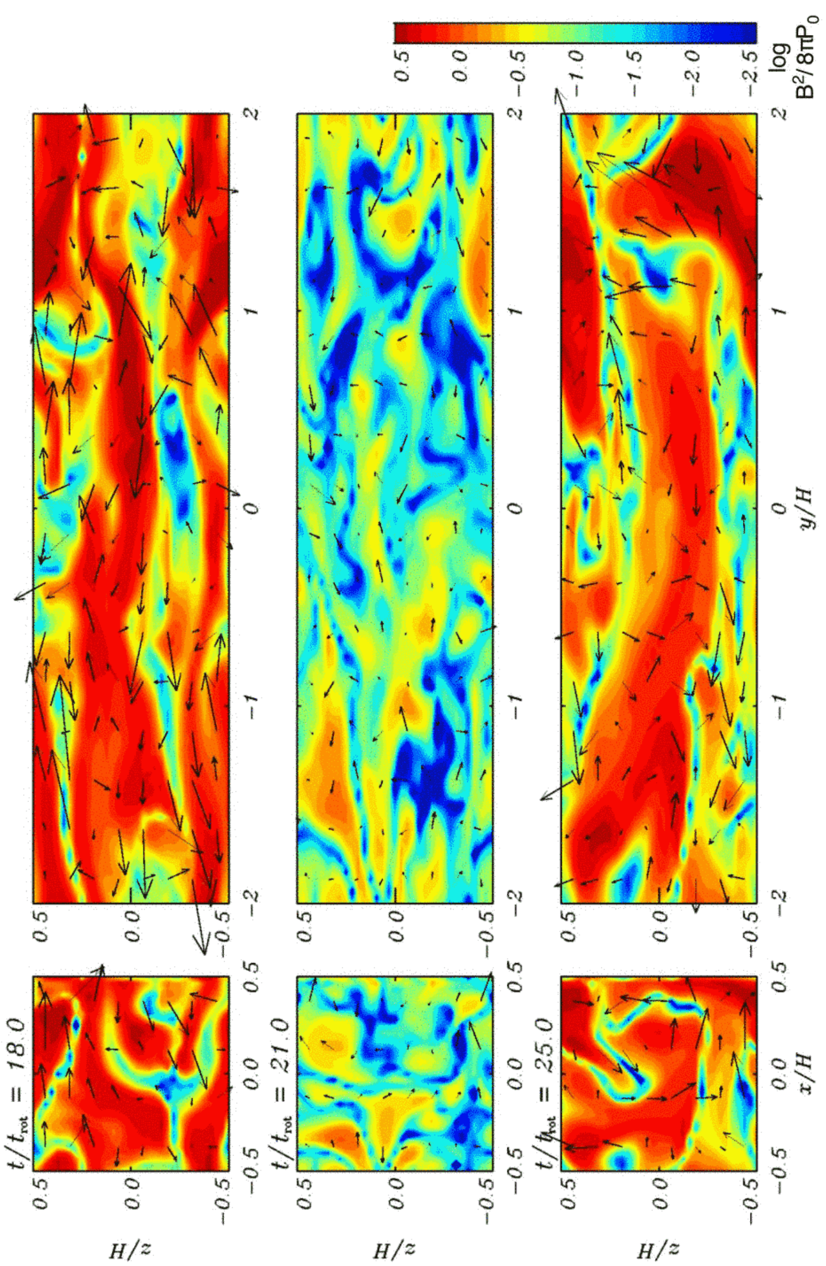

Resistivities: 1AU, 1μm grains



Salmeron & Wardle MNRAS in press



Salmeron & Wardle



Sano & Stone 2002b

Summary

- Resistivity is a microphysically-determined tensor
- Magnetic activity is stratified
  - entire disk cross-section is magnetically active in the absence of grains
  - when grains are present, coupling occurs at ~3 scale heights
    - “dead zone” and active layers
    - accretion energy deposited near surface of the disk
- Hall diffusion of B is unavoidable
  - generally dominates in the active regions
  - modifies field geometry, reduces extent of dead zone
  - affects vector evolution of B; sign-of-B_z effects?
  - ohmic diffusion is dominant only at very low field strengths
  - ambipolar diffusion important for strong fields or large heights
- Ambipolar and hall resistivities depend on field strength
  - potential for interesting behaviour (eg. Sano & Stone 2002b)