Numerical Simulations of Black Hole Magnetospheres

Seeking answers to basic questions on magnetic extraction of rotational energy of black holes

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3+1 Splitting of Space-time

$ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ x^ν - arbitrary coordinates of space-time

$$ds^2 = (\beta^2 - \alpha^2) dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

t - global time, x^i - space coordinates, γ_{ik} – metric tensor of space

$n^\mu = \frac{1}{\alpha}(1, -\beta^i)$ - 4-velocity of an observer at rest in space (FIDO)

$\alpha = d\tau/dt$ - "lapse function"

$\beta^i = -dx^i/dt$ - 3-velocity of the spatial grid relative to FIDO ("shift vector")

Kerr black holes allow coordinates {t,φ,r,θ} such that

$$\partial_\phi g_{\nu\mu} = \partial_t g_{\mu\nu} = 0$$

e.g. the Boyer-Lindquist coordinates and the Kerr-Schild coordinates

3+1 Splitting of Electrodynamics

Landau & Lifshitz (1971), Macdonald &Thorne (1982), Komissarov(2004)

$$\nabla_\beta {}^*F^{\alpha\beta} = 0 \qquad \nabla_\beta F^{\alpha\beta} = I^\alpha$$

${}^*F^{\alpha\beta}$ - Faraday tensor

$F^{\alpha\beta}$ - Maxwell tensor

I^α - Electric current

$$\partial_t \boldsymbol{B} + \nabla\times\boldsymbol{E} = 0 \qquad -\partial_t \boldsymbol{D} + \nabla\times\boldsymbol{H} = \boldsymbol{J}$$

$$\nabla\cdot\boldsymbol{B} = 0 \qquad \nabla\cdot\boldsymbol{D} = \rho$$

3+1 equations of black hole electrodynamics

Constitutive equations:

$$\boldsymbol{E} = \alpha\boldsymbol{D} + \boldsymbol{\beta}\times\boldsymbol{B}$$

$$\boldsymbol{H} = \alpha\boldsymbol{B} - \boldsymbol{\beta}\times\boldsymbol{D}$$

Empty space behaves as magnetically active medium

Electric charge conservation

$$\partial_t\rho + \nabla\cdot\boldsymbol{J} = 0$$

Electrostatic and vector potentials

$$\boldsymbol{E} = -\nabla\Phi - \partial_t\boldsymbol{A}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}$$

Energy conservation

$$\partial_t e + \nabla\cdot\boldsymbol{S} = -(\boldsymbol{E}\cdot\boldsymbol{J})$$

$$e = -\alpha T^t{}_t = \frac{1}{2}(\boldsymbol{E}\cdot\boldsymbol{D} + \boldsymbol{B}\cdot\boldsymbol{H})$$

e – volume density of "energy at infinity"

$$\boldsymbol{S} = \boldsymbol{E}\times\boldsymbol{H}$$

Angular momentum conservation

$$\partial_t l + \nabla\cdot\boldsymbol{L} = -(\rho\boldsymbol{E} + \boldsymbol{J}\times\boldsymbol{B})\cdot\boldsymbol{m}$$

$$l = \alpha T^t{}_\phi = (\boldsymbol{D}\times\boldsymbol{B})\cdot\boldsymbol{m}$$

1 - volume density of "angular momentum at infinity"

$$\boldsymbol{L} = -(\boldsymbol{E}\cdot\boldsymbol{m})\boldsymbol{D} - (\boldsymbol{H}\cdot\boldsymbol{m})\boldsymbol{B} + \frac{1}{2}(\boldsymbol{E}\cdot\boldsymbol{D} + \boldsymbol{B}\cdot\boldsymbol{H})\boldsymbol{m}$$

$$\boldsymbol{m} = \partial_\phi$$

Quantities in FIDO's frame (with hats):

Magnetic field: $\check{\boldsymbol{B}} = \boldsymbol{B}$

Electric field: $\check{\boldsymbol{E}} = \boldsymbol{D}$

Electric charge density: $\check{\rho} = \rho$

Electric current density: $\check{\boldsymbol{j}} = (\boldsymbol{J} + \rho\boldsymbol{\beta})/\alpha$

Far away from the black hole $\alpha = 1$, $\beta = 0$ and

$$\check{\boldsymbol{B}} = \boldsymbol{B} = \boldsymbol{H}$$
$$\check{\boldsymbol{E}} = \boldsymbol{E} = \boldsymbol{D}$$
$$\check{\boldsymbol{j}} = \boldsymbol{J}$$

Steady-state axisymmetric magnetospheres. I. Vacuum solutions

Steady state

No currents

$$\nabla \times \boldsymbol{H} = 0$$
$$\nabla \times \boldsymbol{E} = 0$$

axisymmetry:

$$H_\phi = E_\phi = 0$$

$\boldsymbol{S}_p = (\boldsymbol{E} \times \boldsymbol{H})_p = 0$ - no energy extraction

$\boldsymbol{L}_p = -(E_\phi)\boldsymbol{D}_p - (H_\phi)\boldsymbol{B}_p = 0$ - no angular momentum extraction

Steady-state axisymmetric magnetospheres. II. Force-free solutions

$\boldsymbol{\nabla}\times\boldsymbol{H}=\boldsymbol{J} \longrightarrow H_\phi = I/2\pi \qquad I=\int \boldsymbol{J}\cdot d\boldsymbol{S}$ - current within a flux tube

$\boldsymbol{\nabla}\times\boldsymbol{E}=0 \longrightarrow E_\phi = 0$

$\boldsymbol{E}=-\Omega\boldsymbol{m}\times\boldsymbol{B} \qquad \boldsymbol{m}=\partial_\phi$

$\rho\boldsymbol{E}+\boldsymbol{J}\times\boldsymbol{B}=0 \longrightarrow \boldsymbol{E}\cdot\boldsymbol{B}=0$

Ω - angular velocity of magnetic field lines

$B^i\partial_i\Omega = B^i\partial_i\Phi = B^i\partial_i H_\phi = 0$

Ω, Φ, H_φ are constant along the magnetic filed lines

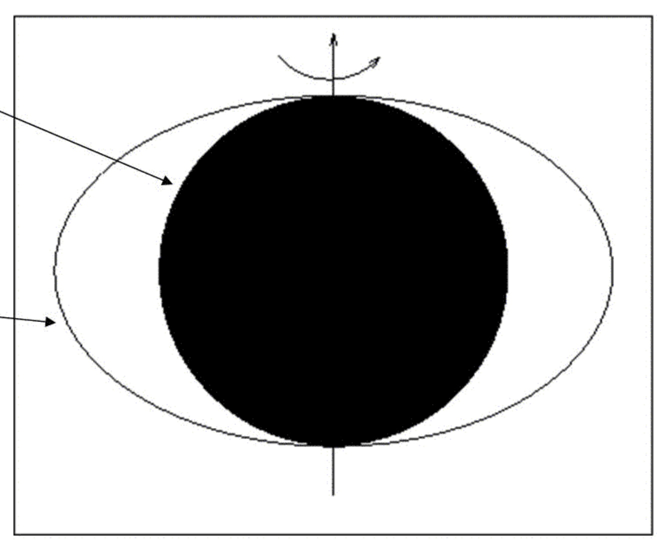
Energy and angular momentum conservation:

$$\partial_t e + \boldsymbol{\nabla}\cdot\boldsymbol{S}=0 \qquad \boldsymbol{S}_p = -(H_\phi\Omega)\boldsymbol{B}_p$$

$$\partial_t l + \boldsymbol{\nabla}\cdot\boldsymbol{L}=0 \qquad \boldsymbol{L}_p = -H_\phi\boldsymbol{B}_p$$

- poloidal fluxes

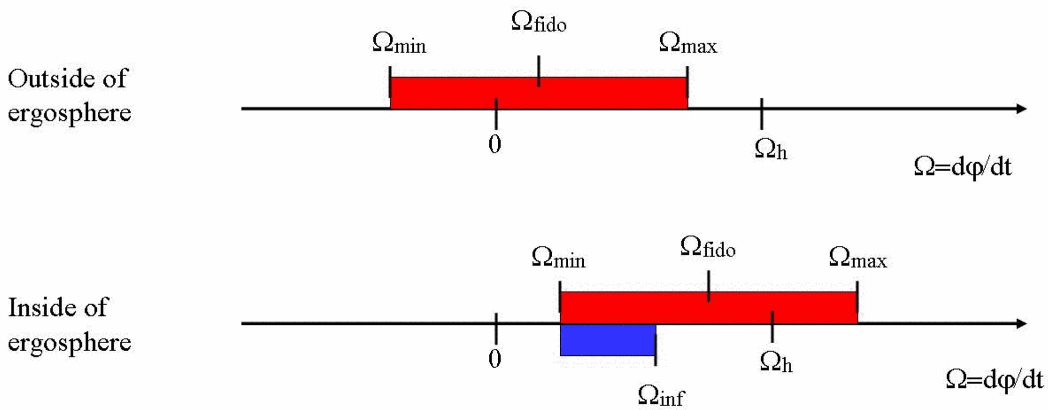
Event Horizon and Ergosphere



horizon
($\alpha = 0$ in BL-coordinates)

ergosphere
($\alpha^2 = \beta^2$ both in the BL- and KS-coordinates)

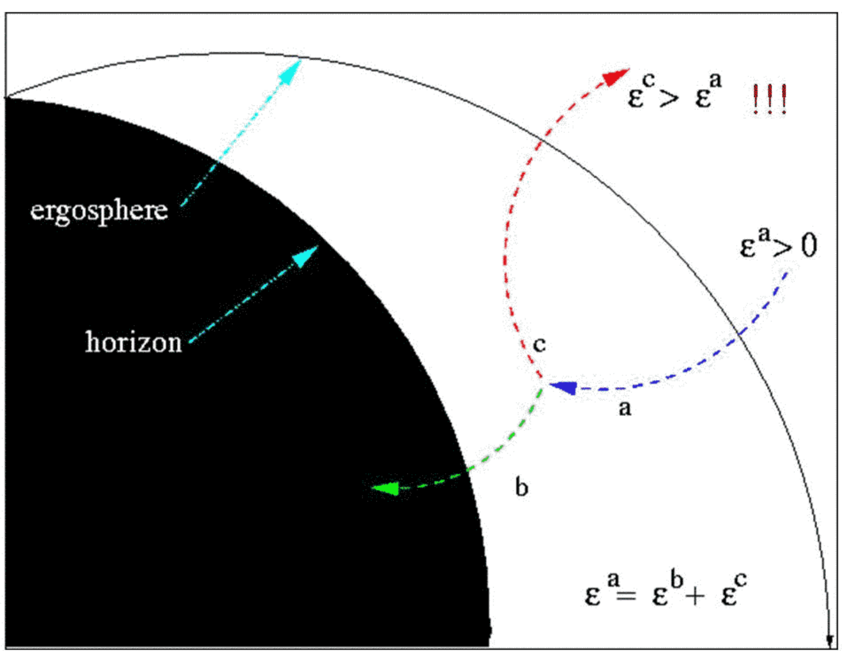
Inertial frames dragging and negative energy



1) all particles are forced to rotate in the same sense as the BH;
2) their "energy at infinity" can be negative.

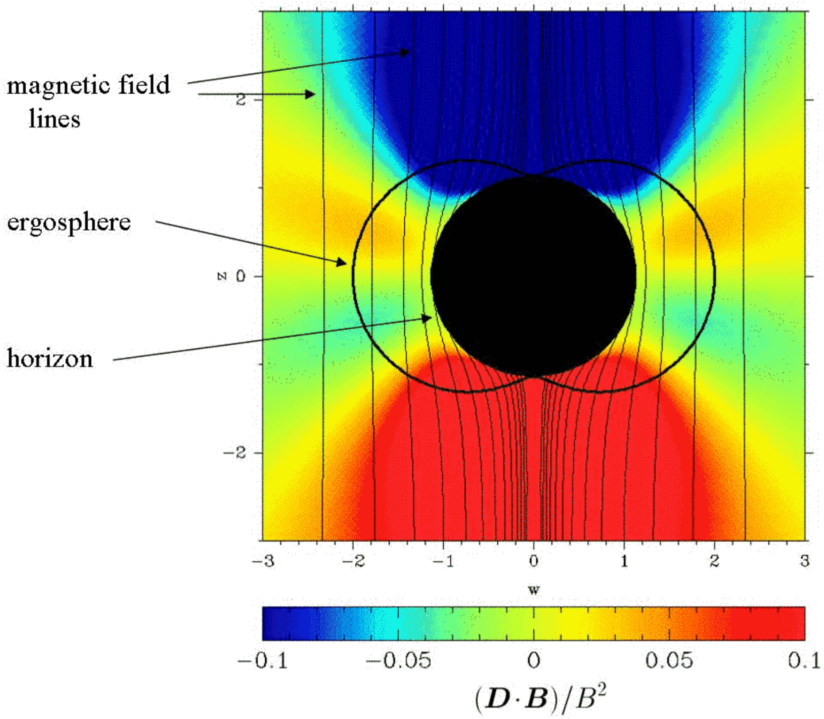
$$\varepsilon_{oo} = -mu_t < 0 \quad \text{if} \quad \Omega < \Omega_{inf}$$

Mechanical Penrose mechanism



Penrose (1969)

Blandford-Znajek mechanism



Wald (1974)

Vacuum solution for a Kerr black hole in a uniform magnetic field.

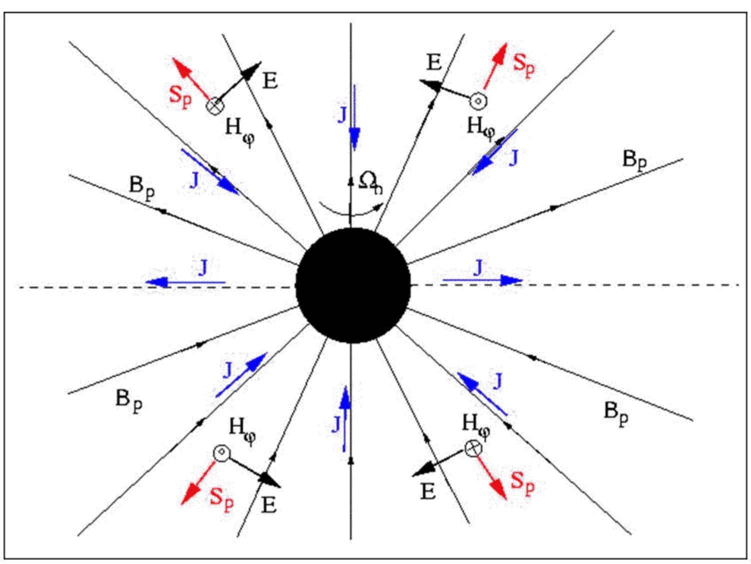
Gravitationally induced electric field with

$$\boldsymbol{D}\cdot\boldsymbol{B} \neq 0$$

$$(\check{\boldsymbol{E}} = \boldsymbol{D})$$

Similarity with the vacuum solution for rotating neutron stars.

Blandford-Znajek mechanism



Force-free solution.

Split-monopole radial poloidal magnetic field with

$$B^r = B_0 \sin\theta / \sqrt{\gamma}$$

$$\Omega = 0.5\Omega_h$$

$$H_\phi = -\frac{\Omega_h B_0}{2}\sin^2\theta$$

(in the northern hemisphere)

Hence, outflows of energy

$$\boldsymbol{S}_p = -(H_\phi \Omega)\boldsymbol{B}_p$$

and angular momentum

$$\boldsymbol{L}_p = -H_\phi \boldsymbol{B}_p$$

Blandford & Znajek (1977)
Macdonald & Thorne (1982), Phinney (1983)

Criticism of the Blandford-Znajek mechanism

Punsly & Coroniti (1990a,b)

1) Vacuum electric field cannot drive stationary electric currents as it gets screened by plasma; capacitor

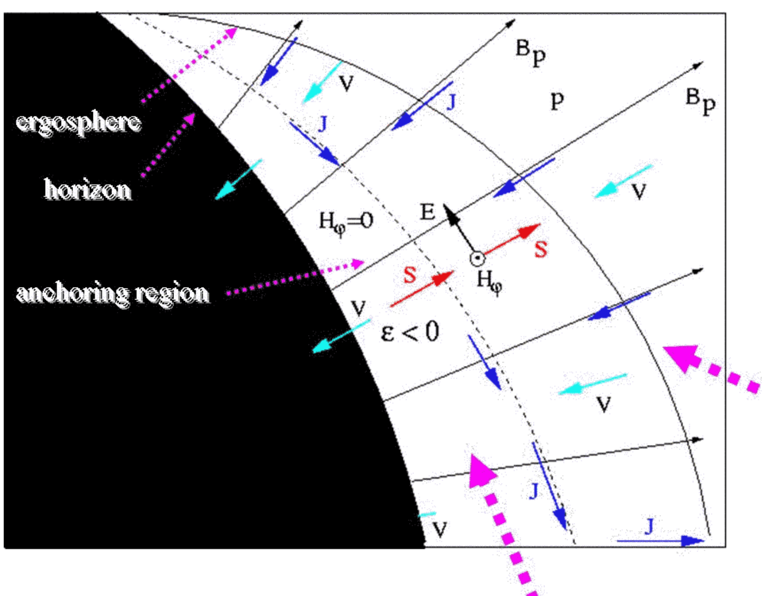
2) BZ-model lacks proper unipolar induction; Faraday Disc

3) Setting boundary condition (Znajek 1977) on the event horizon is incorrect as it is out of causal contact with the magnetosphere;

Conclusions:

I. BZ-solution is non-causal and non-physical. Unstable?
II. Force-free approach is fundamentally fraud.
III. Particle inertia has to be included.

MHD model of Punsly and Coroniti



Punsly & Coroniti (1990a,b)

"MHD Penrose process"

Within the ergosphere plasma must rotate. If magnetic field is frozen into this plasma it will rotate as well.

Magnetically dominated region. Outgoing Poynting flux.

Particle dominated region. Influx of particles of negative energy.

return

Numerical simulations of force-free magnetospheres
Setup

(Komissarov, 2001,2004);

Main goal: test stability of BZ solution;

Equations: a) Time-dependent force-free electrodynamics (Komissarov, 2001); FFDE
and
b) Electrodynamics with generalised Ohm's law (Komissarov 2004); OL

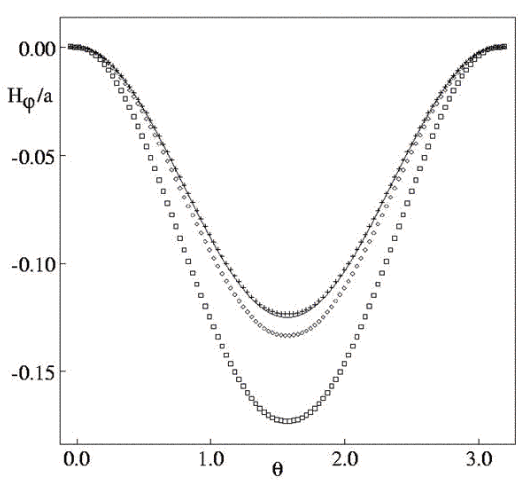
Details: 2D; axisymmetric; Kerr-Schild coordinates; inflow inner boundary inside the event horizon; Kerr black hole with a=0.1, 0.5, 0.9;

Initial solution: the same monopole poloidal field as in BZ-solution;
but E=0 and hence Ω=0; (different setups give the same outcome)

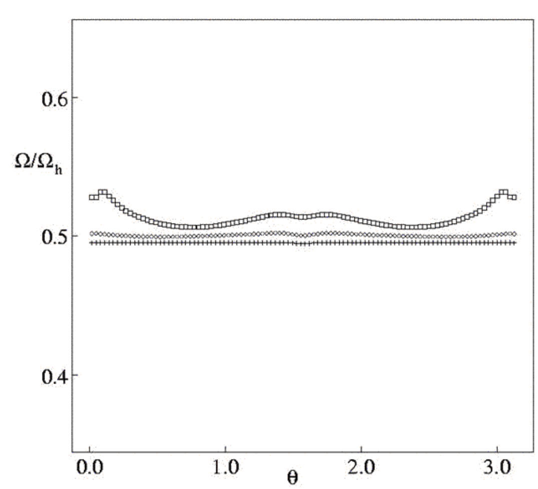
Time evolution: towards the steady-state solution of Blandford and Znajek (1977). 2D plot

Numerical simulations of force-free magnetospheres
Results

(Komissarov, 2001);



- BZ-solution;
+ - numerical, a=0.1;
◇ - numerical, a=0.5;
□ - numerical, a=0.9.



BZ-solution is an attractor !!!

Electromotive Force of the BZ-mechanism

There are no steady-state axisymmetric solutions for the electromagnetic field supported by remote electric currents that simultaneously satisfy both

$$\boldsymbol{D}\cdot\boldsymbol{B} = 0 \qquad \text{and} \qquad B^2 - D^2 > 0$$

along the magnetic field lines penetrating the black hole ergosphere.

Gravitationally induced electric field, $\check{\boldsymbol{E}} = \boldsymbol{D}$, cannot be screened within the ergosphere by any static distribution of electric charge !

This field can drive stationary poloidal currents – battery effect.

Not the event horizon but the ergosphere plays the key role in BZ-mechanism.

Proof

Assume that $\boldsymbol{D}\cdot\boldsymbol{B} = 0$ and show that $(B^2 - D^2) < 0$ inside the ergosphere

a) $\boldsymbol{D}\cdot\boldsymbol{B} = 0$ and $\boldsymbol{E} = \alpha\boldsymbol{D} + \boldsymbol{\beta}\times\boldsymbol{B}$ → $\boldsymbol{E}\cdot\boldsymbol{B} = 0$

steady-state axisymmetry → $E_\phi = 0$

$\boldsymbol{E}\cdot\boldsymbol{B} = 0$, $E_\phi = 0$ → $\boldsymbol{E} = -\Omega\boldsymbol{m}\times\boldsymbol{B}$ $\qquad \boldsymbol{m} = \partial_\phi$

$\boldsymbol{E} = -\Omega\boldsymbol{m}\times\boldsymbol{B}$, $\nabla\times\boldsymbol{E} = 0$, $\nabla\cdot\boldsymbol{B} = 0$ → $B^i\partial_i\Omega = 0$ → Ω is constant along magnetic field lines;

b) Far away from the hole $\boldsymbol{E} = 0$ and hence $\Omega = 0$. → $\boldsymbol{E} = 0$ everywhere;

$$\boldsymbol{D} = \frac{1}{\alpha}(\boldsymbol{E} - \boldsymbol{\beta}\times\boldsymbol{B}) = -\frac{1}{\alpha}\boldsymbol{\beta}\times\boldsymbol{B}$$

c) $\boldsymbol{D} = -\frac{1}{\alpha}\boldsymbol{\beta}\times\boldsymbol{B}$

Boyer-Lindquist coordinates

$$\alpha^2(B^2-D^2) = -B^2(\beta^2-\alpha^2) + \left(\frac{\beta^\phi}{\alpha}\right)^2 H_\phi^2$$

If $H_\phi = I/2\pi$ vanish (vacuum or no poloidal current) then

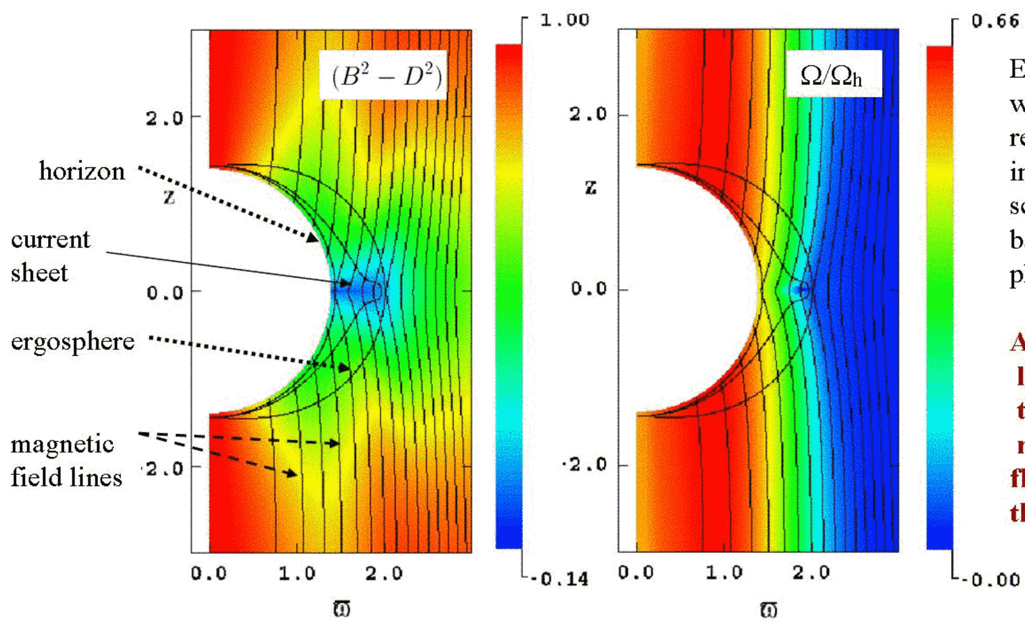
$$B^2 - D^2 = -\frac{\beta^2-\alpha^2}{\alpha^2}B^2$$

Within ergosphere $(\beta^2-\alpha^2) > 0$ and hence $(B^2-D^2) < 0$

QED

Plasma version of the Wald problem.

Komissarov(2004)



Electrodynamics with radiative resistivity – inverse Compton scattering of background photons.

All magnetic field lines that enter the ergosphere rotate! Energy flows away along these lines.

In the current sheet plasma emits negative energy photons that are absorbed by the black hole as suggested in Punlsy & Coroniti (1990).

MHD simulations of magnetically dominated magnetospheres

Koide et al. (2002), Koide (2003,2004),
Komissarov (2004,2005),
McKinney & Gammie (2004),
De Villiers et al.(2005).

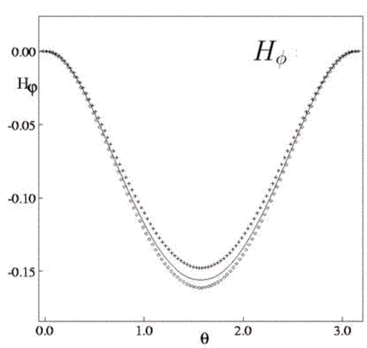
Ideal relativistic MHD
in Kerr metric. MHD

Key questions: Will inertial effects lead to strong deviations from the force-free results?
Will partically-dominated regions develop in black hole ergospheres?
Can the influx of negative mechanical energy become significant?

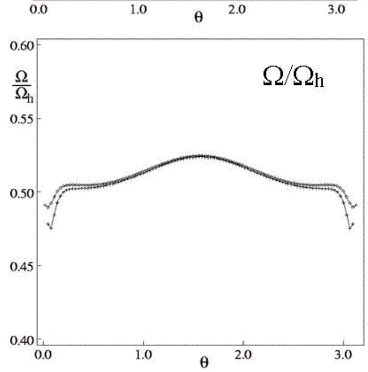
Punsly & Coroniti (1990) PC

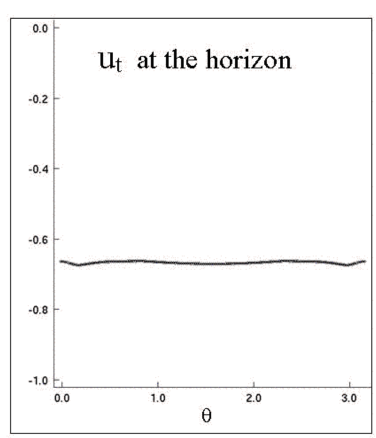
Monopole magnetospheres

Komissarov (2004)



—— - BZ solution
\+ - MHD at r = 5M
◇ - MHD at r = 50M





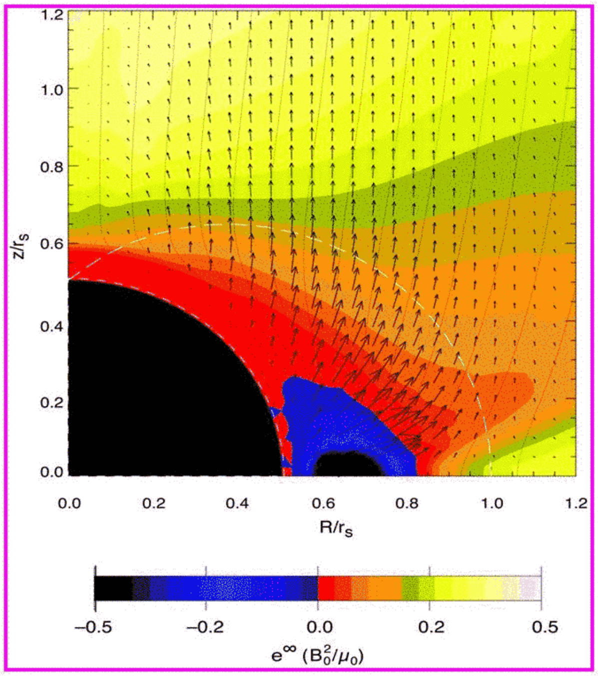
Inertial effect do not become significant within the ergosphere;

Particles do not attain negative mechanical energy;

Force-free approximation of Blandford-Znajek is perfectly fine.

t=170M a=0.9

MHD version of the Wald problem.



Solution at $t \sim 14\ r_g/c$ (~ one period of the black hole)

Koide et al.(2002), Koide(2003)

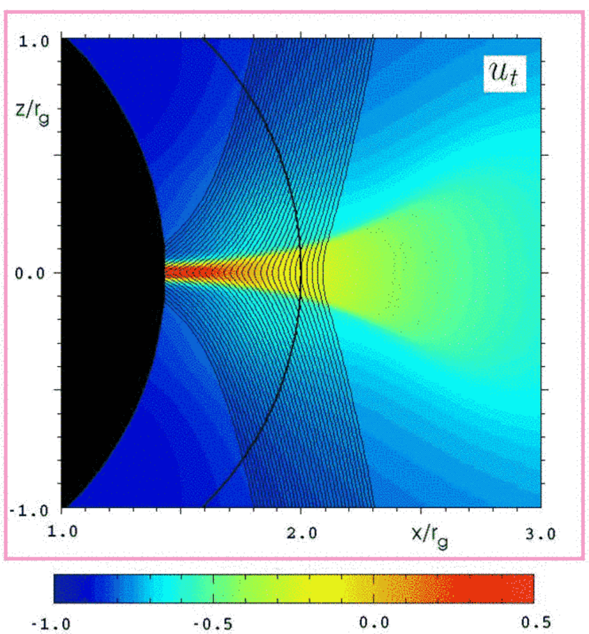
Region of negative mechanical energy develops within the ergosphere;

Near the horizon the electromagnetic energy flux is similar to the hydrodynamic energy flux;

New version of MHD Penrose process!

However, a steady state is not reached! Could this be only a transient phase?

MHD version of the Wald problem.



Magnetic field structure and u_t at $t = 6\ r_g/c$

Komissarov (2005)

The setup is similar to Koide et al.(2002) Setup

During the initial phase the solution is similar to that of Koide et al. (2002)

$$e = -\alpha T^t{}_{t_{(m)}} = \alpha(-w u_t u^t + p)$$

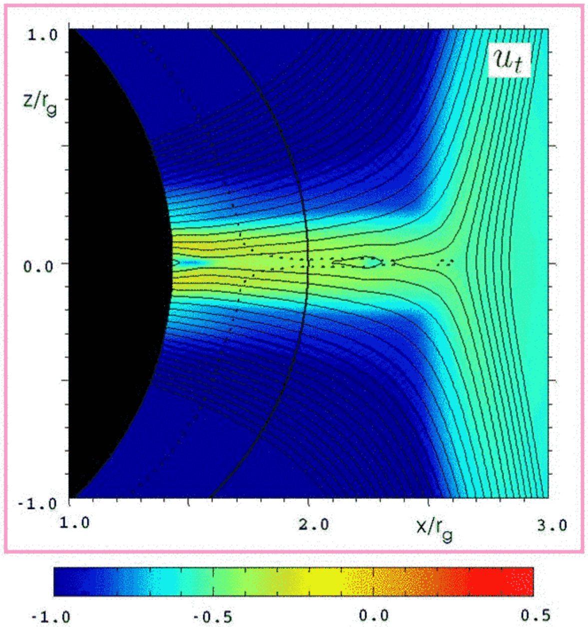
- hydrodynamic energy density

$$S^i = -\alpha T^i{}_{t_{(m)}} = -\alpha w u_t u^i$$

- hydrodynamic energy flux density

MHD version of the Wald problem.

Komissarov (2005)



Magnetic field structure and u_t at
$t = 60\ r_g/c$

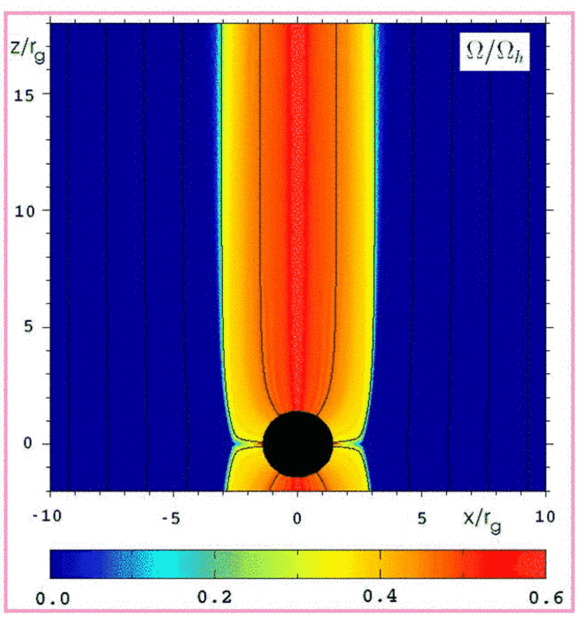
Later the solution settles to a steady state with a split-monopole configuration of magnetic fields;

The region with negative mechanical energy (positive u_t) disappears;

Only the Blandford-Znajek process operates in the steady state.

MHD version of the Wald problem.

Komissarov (2005)



Magnetic field structure and Ω at
$t = 60\ r_g/c$

$\Omega \sim 0.4–0.6\ \Omega_h$ inside the rotating "column" of ergospheric field lines;

$\Omega \sim 0.0$ outside of it.

Conclusions

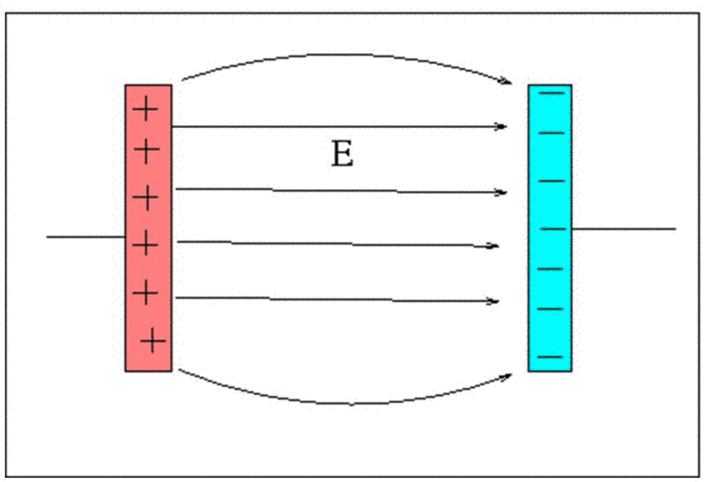
1. The Blandford-Znajek solution is an asymptotically stable steady-state solution that is perfectly causal and physical. This is confirmed both by the force-free and the full MHD simulations;

2. In the Blandford-Znajek mechanism the potential drop of the black hole battery is induced gravitationally when its ergosphere is threaded by the lines of external magnetic field.
   i) This is a purely GR effect that is specific to the properties of space-time within the ergosphere;
   ii) It is not caused by the inertial frame dragging effect - the rotation of magnetospheric plasma within the ergosphere is not important;
   iii) It is very robust;

3. Contrary to the theory of Punsly-Coroniti the numerical simulations have shown no evidence for development of particle-dominated anchoring regions at the bases (within the ergosphere) of magnetically dominated magnetospheres.

4. However, the simulations have also indicated the feasibility of mixed processes where production of massive and massless particles of negative mechanical energy facilitates magnetic extraction of rotational energy of black holes.

Capacitors do not support currents



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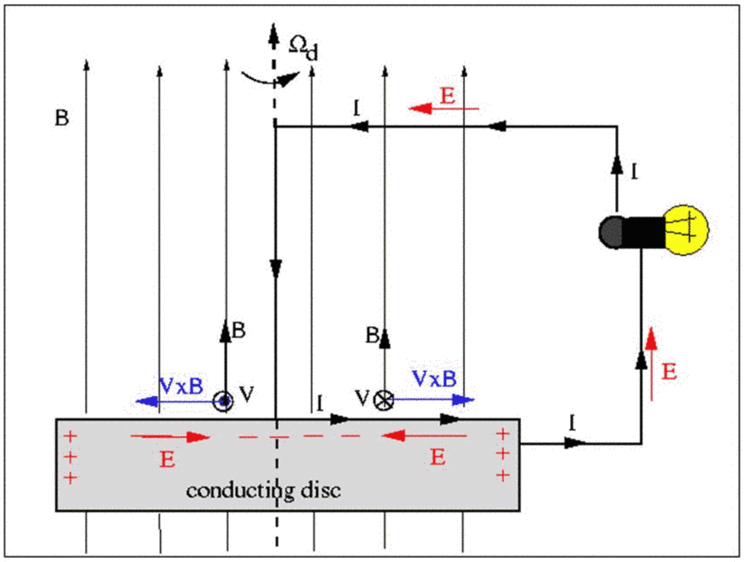
Unipolar induction in Faraday disc



V – rotational velocity; VxB force drives electric current against the electric force and across the magnetic field lines !

return

MAGNETODYNAMICS* is *MAGNETOHYDRODYNAMICS

without the *HYDRO* part

Condition: $T^{\alpha\beta}_{(m)} \ll T^{\alpha\beta}_{(ef)}$

Evolution equations:

$$\nabla_\mu \boldsymbol{T}^{\nu\mu}_{(f)} = 0$$

$$\nabla_\beta {}^*F^{\alpha\beta} = 0$$

Perfect conductivity:

$$F_{\mu\nu} {}^*F^{\mu\nu} = 0$$

$$F_{\mu\nu} F^{\mu\nu} > 0$$

(Komissarov 2002)

return

Generalised Ohm's Law:

Komissarov (2004)

$$\boldsymbol{j} = \sigma_{||}\boldsymbol{E}_{||} + \sigma_{\perp}\boldsymbol{E}_{\perp} + \boldsymbol{j}_d$$

radiative resistivity;
no particle inertia.

$$\boldsymbol{j}_d = \rho\frac{\delta^2}{1+\delta^2}\frac{\boldsymbol{E}\times\boldsymbol{B}}{B^2}$$ *- drift current*

$$\sigma_{\perp} = \frac{1}{1+\delta^2}\sigma_{||}$$ *- anisotropic conductivity*

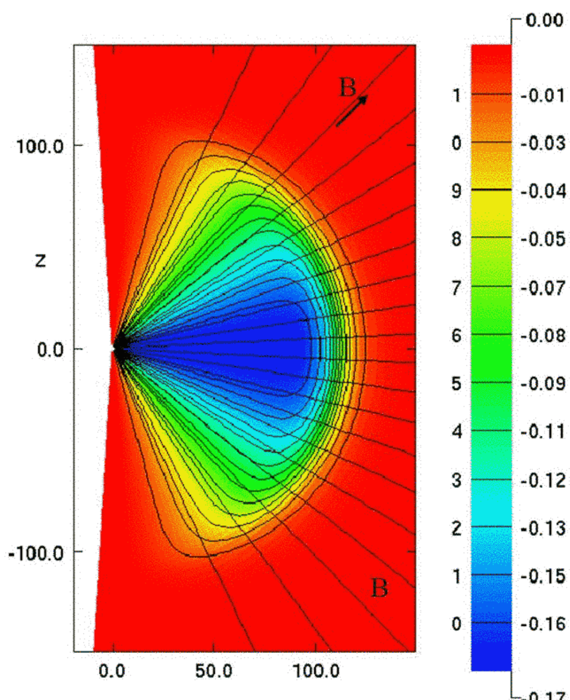
Typical conditions of BH and pulsar magnetospheres: $\sigma_{||} \gg 1,\ \delta \gg 1,\ \sigma_{\perp} \ll 1$ ⟶ $\rho\boldsymbol{E} + \boldsymbol{j}\times\boldsymbol{B} = 0$

In current sheets: $E_{\perp} \geq B, \quad \sigma_{\perp} > 1$ *and even* $\sigma_{\perp} \simeq \sigma_{||}$

return

Time-dependent magnetodynamic simulations

Komissarov (2001)



H_{ϕ} at time t=120

Establishing of the global current system of the BZ-magnetosphere

2D axisymmetric simulations;
Kerr black hole, a=0.9;
Kerr-Schild coordinates;
Monopole magnetic field;

return

Ideal relativistic MHD

$\partial_t(\alpha\sqrt{\gamma}\rho u^t) + \partial_i(\alpha\sqrt{\gamma}\rho u^i) = 0,$ - continuity equation

$\partial_t(\alpha\sqrt{\gamma}T^t{}_\nu) + \partial_i(\alpha\sqrt{\gamma}T^i{}_\nu) = \frac{\alpha\sqrt{\gamma}}{2}\partial_\nu(g_{\alpha\beta})T^{\alpha\beta},$ - energy-momentum equation

$\partial_t(B^i) + e^{ijk}\partial_j(E_k) = 0.$ - induction equation;

$T^{\mu\nu}_{(e)} = F^{\mu\gamma}F^\nu{}_\gamma - \frac{1}{4}(F^{\alpha\beta}F_{\alpha\beta})g^{\mu\nu},$ -stress-energy-momentum of the electromagnetic field;

$T^{\mu\nu}_{(m)} = wu^\mu u^\nu - pg^{\mu\nu},$ - stress-energy-momentum of matter;

return

MHD version of the Wald problem. Setup.

1) Coordinate system: Kerr-Schild; a = 0.9;

2) Computational domain: 2D; $(0 < \theta < \pi)$; $(1.35r_g < r < 52r_g)$;
400x401 cells;
$0 < t < 60\ r_g/c$;

3) Boundary conditions: axisymmetric at $\theta = 0,\pi$;
inflow at $r = 1.35\ r_g$;
free flow at $r = 52\ r_g$;

4) CPU time (1.4GHz processor): 22 hours;

5) Numerical scheme: Godunov-type; linear Riemann solver;
artificial viscosity/diffusivity/resistivity.

return