# The Princeton MagnetoRotational Instability Experiment

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Physics of Astrophysical Outflows and Accretion Disks, KITP, 27 May 2005

# Overview of the experiment

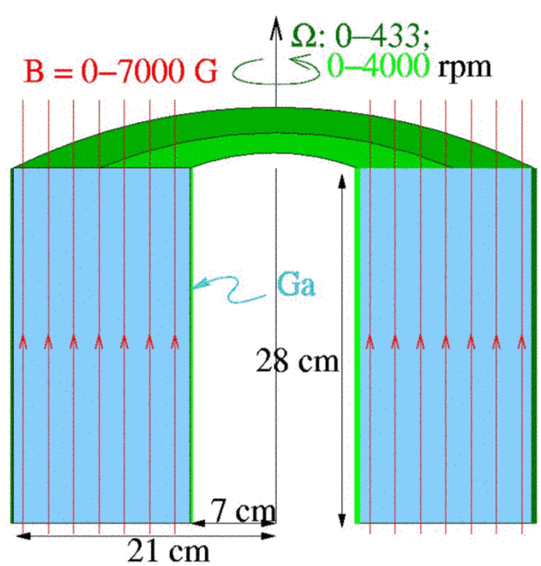
- magnetic Couette flow
  - eutectic gallium
  - axial field
- independently driven cylinders and endcaps

$$\mathrm{Re} \equiv \frac{r_1(r_2-r_1)\Omega_1}{\nu} \approx 10^7$$

$$\mathrm{Re}_M \equiv \frac{r_1(r_2-r_1)\Omega_1}{\eta} \approx 20$$

$$S \equiv \frac{(r_2-r_1)V_A}{\eta} \approx 4$$

$$\mathrm{Pr}_M \equiv \frac{\nu}{\eta} \approx 10^{-6}$$




**Aim to start from a centrifugally stable ($\kappa^2>0$) magnetic steady state ($B\cdot\nabla\Omega=0$)**

# Motivation

- MRI largely theoretical to date
- Explore resistive, low-$Pr_M$, high-$Re$ regime
  - protostellar disks, quiescent dwarf novae...
- Benchmark astrophysical codes
  - ZEUS, ...
- Seek nonlinear hydrodynamic instabilities
  - High-$Re$, centrifugally stable flows are little studied
- Develop laboratory astrophysics

# Outline

- Motivation
- Couette flow
- Linear stability
- Water experiments & Ekman circulation
- ZEUS simulations
- Current status

# Couette flow

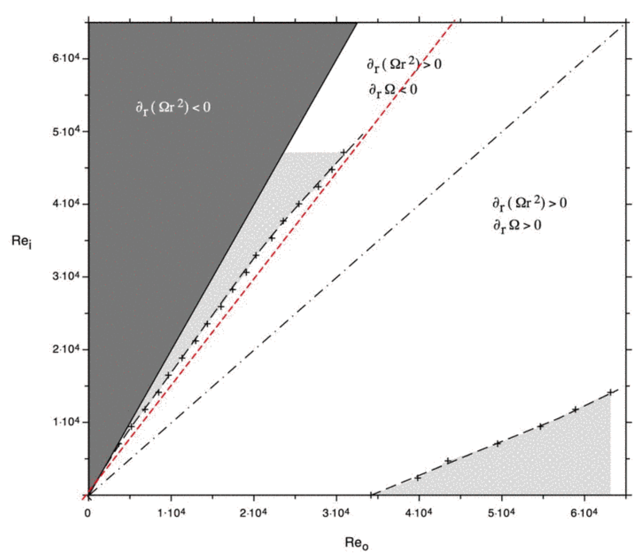
- Pure rotation and $h \to \infty$: $\nu \nabla^2 \left[ r\Omega(r)\hat{\phi} \right] = 0 \;\Leftrightarrow\; \Omega = a + \frac{b}{r^2}$
- Centrifugal (Rayleigh) inviscid axisymmetric stability:

$$\kappa^2 \equiv r^{-3} \frac{d}{dr}\left(r^2\Omega\right)^2 \geq 0 \;\Rightarrow\; ab \geq 0$$

$$i.e.: \quad \Omega_1\Omega_2 \geq 0 \quad \& \quad r_2^2\Omega_2 \geq r_1^2\Omega_1$$

- Taylor number $Ta \equiv -\kappa^2 \Delta r^4 / \nu^2$ controls viscous stability
- **MRI instability: $\Omega_2 < \Omega_1$ and Re, $Re_M$ & S sufficiently large**

# Nonlinear hydrodynamic instability




Stability in plane of $\Omega_1$ versus $\Omega_2$
red line=keplerian
*(Richard 2001)*

- Exp'ts of Wendt (1933), Taylor (1936), Richard (2001) suggest anomalous torque even when $\kappa^2 > 0$
- Richard & Zahn (1999):

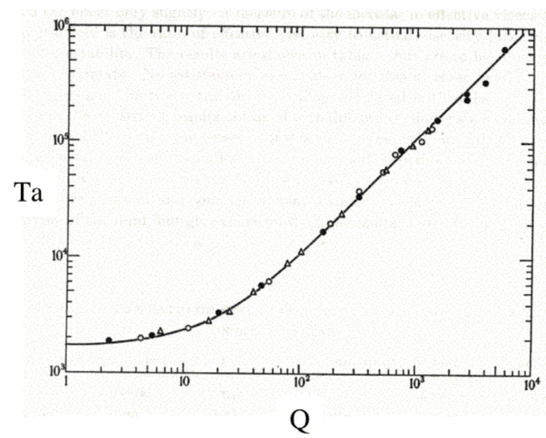
$$\nu_T \approx -\beta r^3 \frac{\partial \Omega}{\partial r}; \; Re_{crit} \approx \frac{1}{\beta}; \; \beta \approx 3 \times 10^{-5}$$

  - should be measurable in Pct. exp't $Re \sim 10^7$, $\Gamma_T \sim 6$ N-m
  - nonlocal if applicable to disks
- However, the torque might be a wall or Ekman effect
  - Direct shearing-box simulations laminar for $\zeta > 0.05$ (Hawley, Balbus, & Winters 1999)
  - Empirical closure scheme suggests laminar flow for $\zeta > 0.33$ (Ogilvie 2005). Keplerian: $\zeta = 0.5$

# Magnetized T-C Flow

- **Velikhov (1959) & Chandrasekhar (1960) discovered ideal MRI**
- **Chandrasekhar (1961) made first thorough analysis of magnetized Couette flow, followed by many other authors.**
  - **Concentrated on magnetic stabilization of Rayleigh-unstable flows: critical Ta increases with B or Chandrasekhar number $Q=(V_A d)^2/\nu\eta$**
  - **Term $r\Omega'\delta B_r$ crucial to MRI was omitted (but OK in this case)**
- **Confirming experiments followed: Donnelly & Ozima (1960, 1962) and more carefully by Donnelly & Caldwell (1964).**
- **Recent non-ideal MRI analyses for infinite/periodic cylinders: Goodman & Ji (2002), Rüdiger & Shalybkov (2002)**

$$\begin{aligned}
\delta\dot{B}_r - B\partial_z\delta v_r &= \eta(\partial_r\partial_r^\dagger + \partial_z^2)\delta B_r \\
\delta\dot{B}_\theta - B\partial_z\delta v_\theta - \delta B_r r\partial_r\Omega &= \eta(\partial_r\partial_r^\dagger + \partial_z^2)\delta B_\theta \\
\partial_r^\dagger\delta B_r + \partial_z\delta B_z &= 0 \\
\delta\dot{v}_r - 2\Omega\delta v_\theta + \partial_r\delta\tilde{p} - V_A\partial_z\delta B_r &= \nu(\partial_r\partial_r^\dagger + \partial_z^2)\delta v_r \\
\delta\dot{v}_\theta + \delta v_r\partial_r^\dagger(r\Omega) - V_A\partial_z\delta B_\theta &= \nu(\partial_r\partial_r^\dagger + \partial_z^2)\delta v_\theta \\
\partial_r^\dagger\delta v_r + \partial_z\delta v_z &= 0
\end{aligned}$$



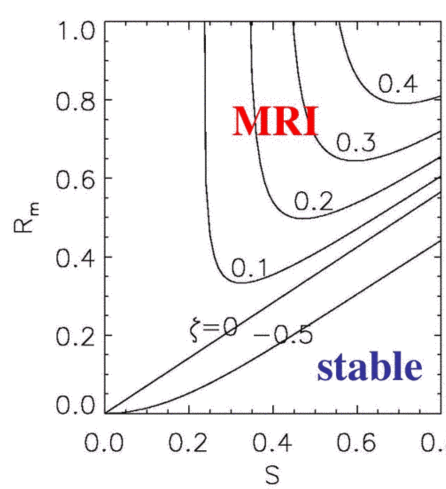

# MRI Stability Limits ($\nu << \eta$)

**WKB works well:** $$k^2V_A^2 + \frac{\kappa^2(\eta k^2)^2}{(k_z V_A)^2} + \frac{\partial\Omega^2}{\partial\ln r} \geq 0: \quad k_z \approx \frac{\pi}{h},\ k_r \approx \frac{\pi}{d}$$

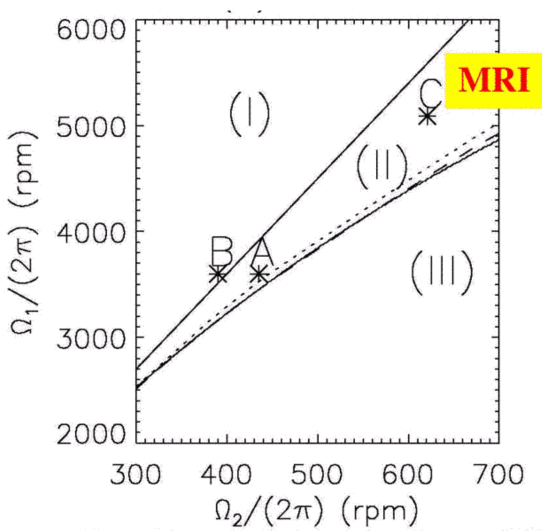
$$R_m^2 \leq \frac{S^4\left[1+(h/r)^2\right]}{2\left[(2-\varsigma)S^2-\varsigma\right]}$$

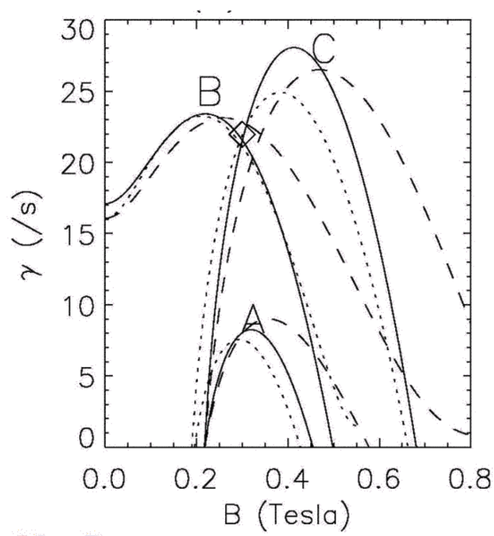
$$\zeta \equiv \frac{\partial\ln(r^2\Omega)}{\partial\ln r}$$

**minimum critical $R_m$ at some S when $\zeta > 0$**



# Stability Diagram




(I): Unstable but can be stabilized by B

(II): Stable but can be destabilized by B: MRI

(III): Always stable

$r_1$=5 cm

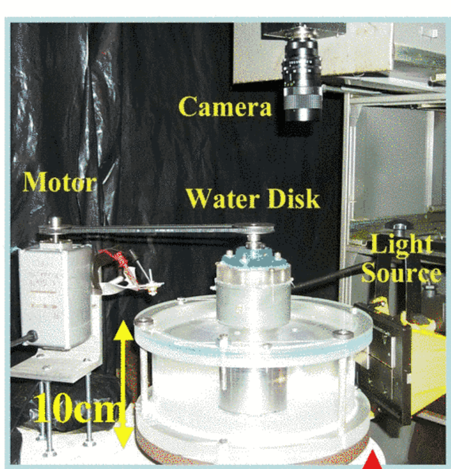
$r_2$=15 cm

h=10 cm

***WKB agrees well with global analysis***

# Prototype water experiment (2002)

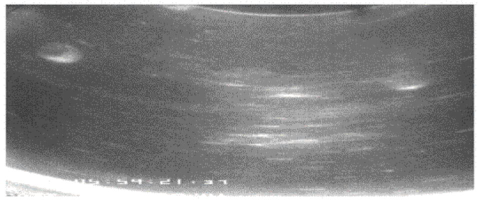
- **Independent rotation of inner and outer cylinders**
- **Endcaps corotate with outer cylinder**
- **Very short geometry: h/r = 1**
- **Seed particles to monitor stability and to measure flow**



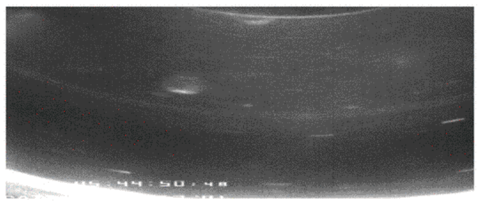

**Potter's wheel**

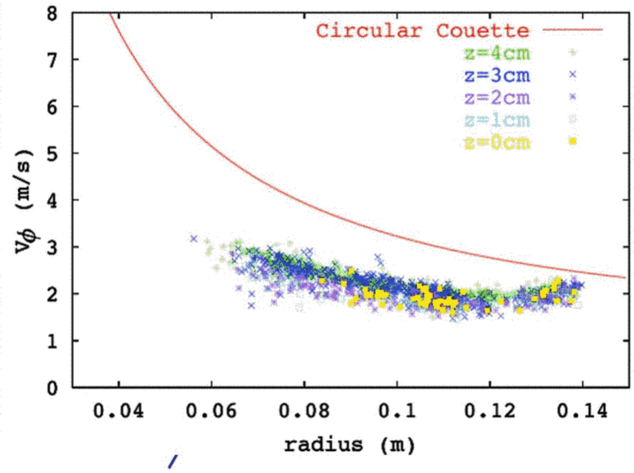
# Stability and Flow Measurements

**Unstable flow**



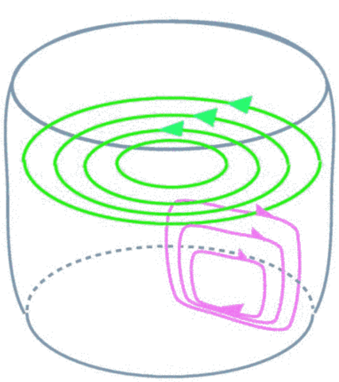
**Stable flow**

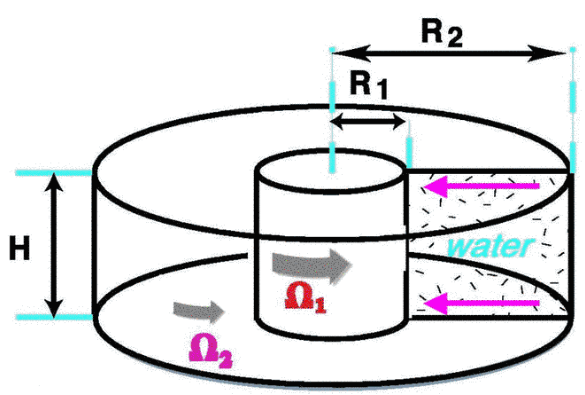





- **Significant deviation from Couette profile due to *Ekman circulation*.**
- **Unstable to Rayleigh mode near the inner wall.**

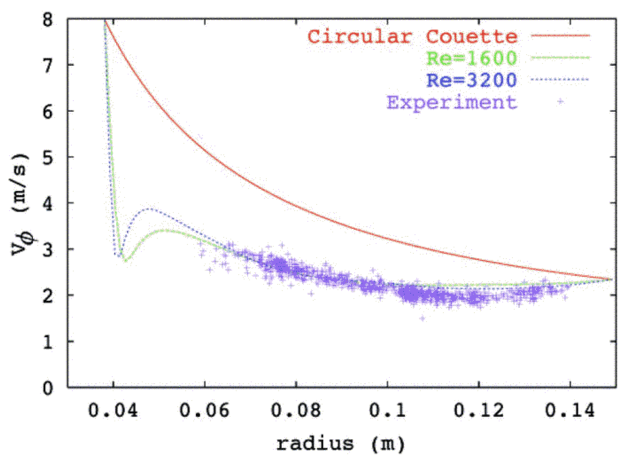
# Ekman Circulation



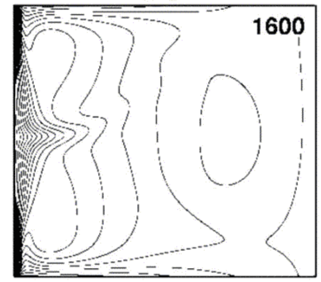



- **Significant vertical transport of angular momentum**
- **Timescale ~ $h/(\nu\Omega)^{1/2} \sim \Omega^{-1}\,Re^{1/2}$**

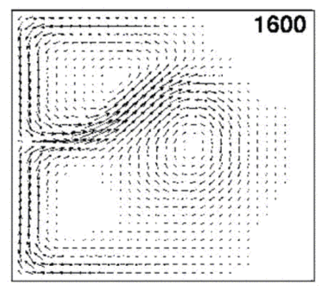
# Measurements Explained by Simulations




Kageyama et al. (2004)




**Toroidal flow**




**Poloidal flow**

**Agreements are remarkable since simulations are in 2D with Re << Re in exp**

# Spin-down Measurements

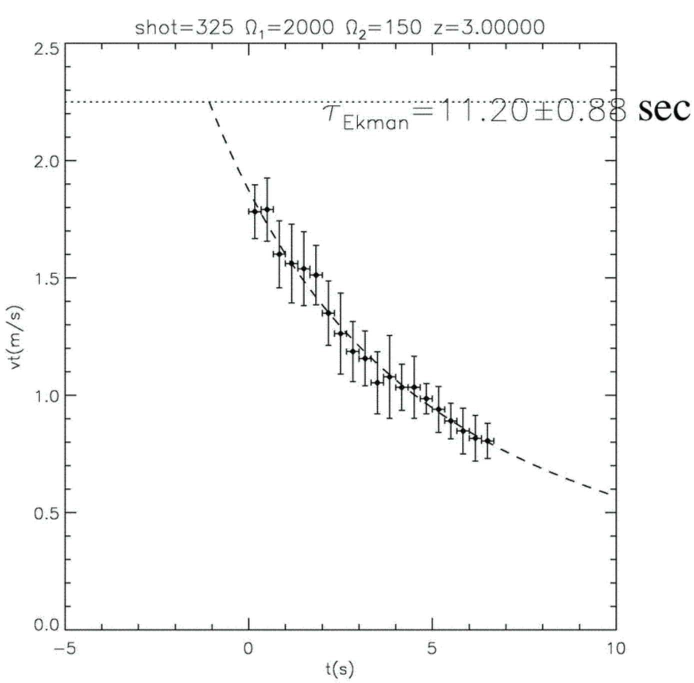
- Ekman Spin-down time:

$$\tau_E = \frac{h}{2\delta_E \overline{\Omega}} = \frac{h}{2\sqrt{\nu\overline{\Omega}}}$$

$$\delta_E = \sqrt{\frac{\nu}{\overline{\Omega}}}$$

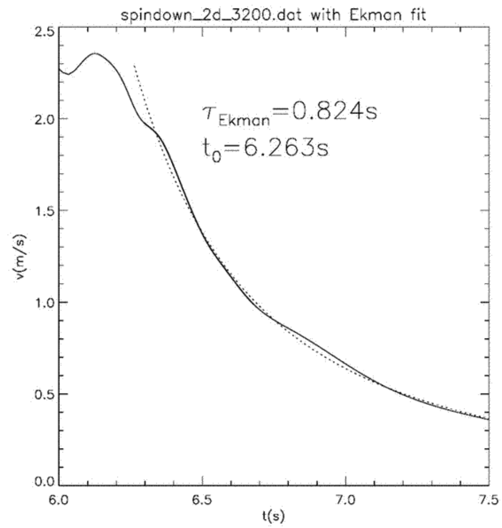
$$\frac{d\overline{\Omega}}{dt} = -\frac{\overline{\Omega}}{\tau_E} \propto -\overline{\Omega}^{3/2}$$

$$\overline{\Omega}(t) = \frac{\overline{\Omega}(t_0)}{\left(1 + \frac{t - t_0}{\tau}\right)^2}$$

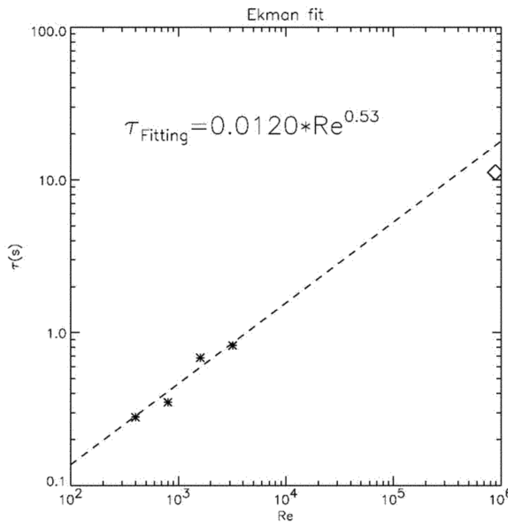


# Measured Spin-down Time Consistent with Simulations

## Simulated spin-down:

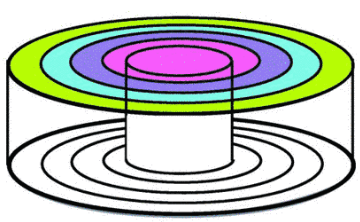


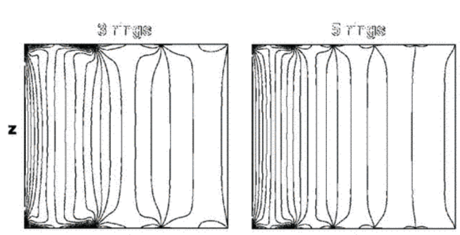

## Scaling with Re:

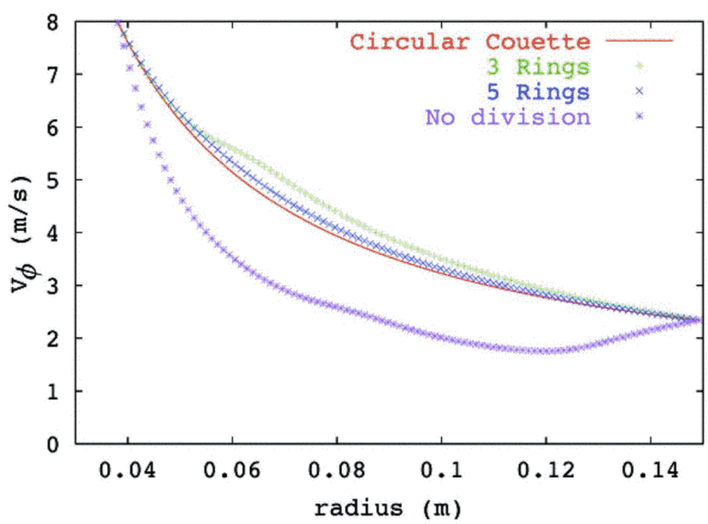



$$\tau_E = \frac{h}{2\sqrt{\nu\overline{\Omega}}} = \frac{h}{2\sqrt{R_1(R_2 - R_1)}\sqrt{\Omega_1\overline{\Omega}}}\sqrt{\mathrm{Re}} = 0.0110\mathrm{Re}^{0.5}\,s$$

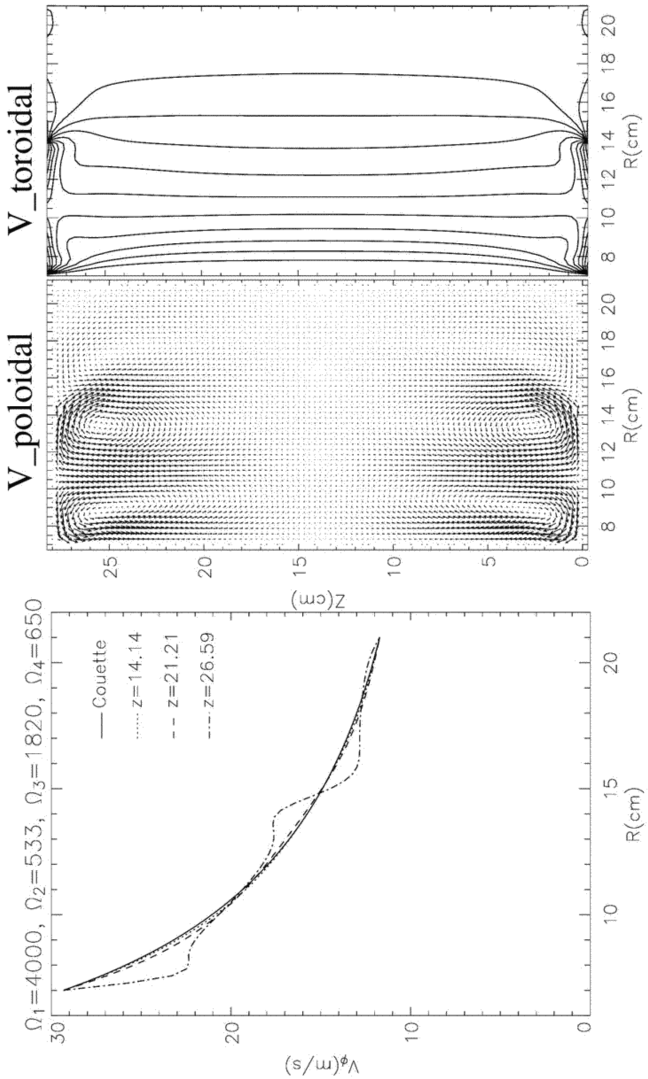
# Solution: Multiple Driven Rings at Each End



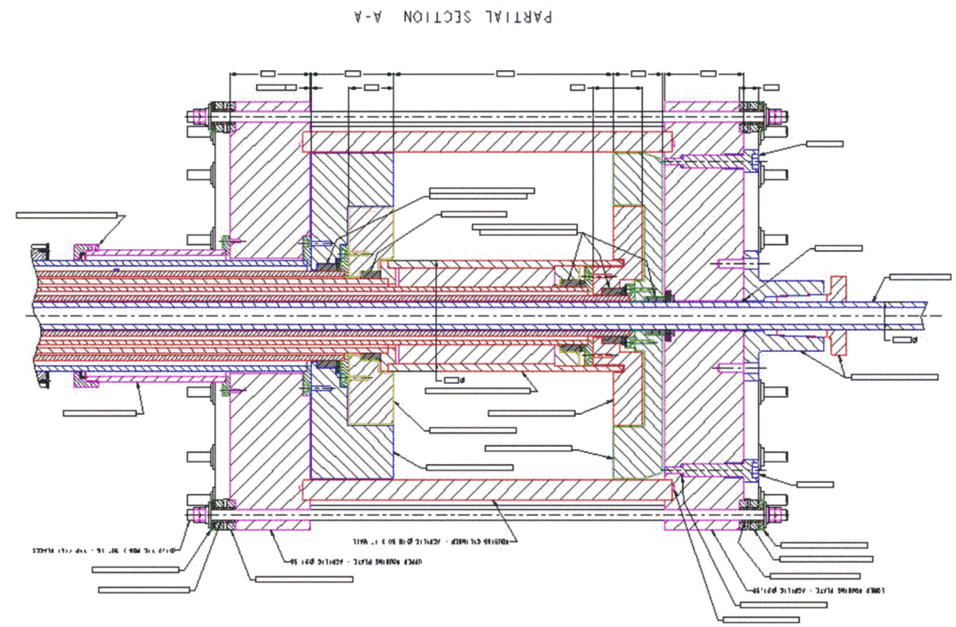


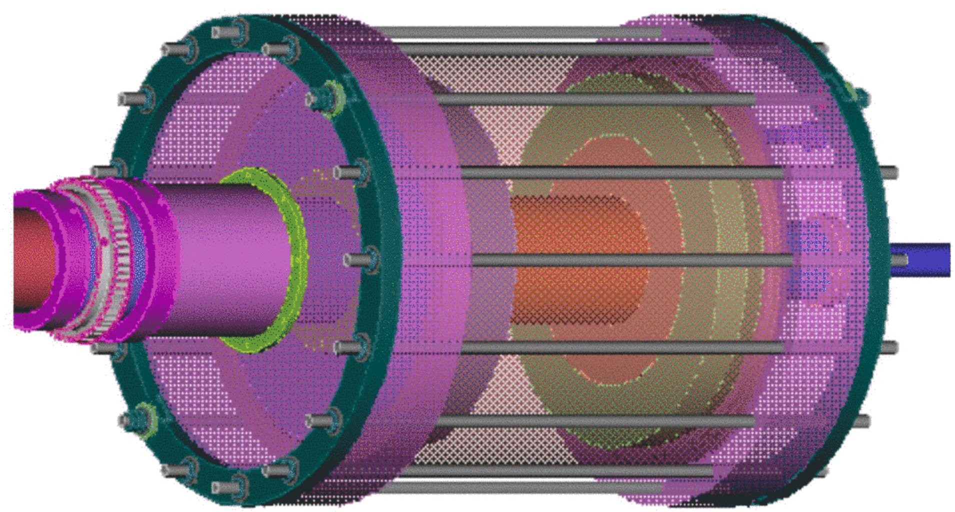



# Final 2-ring Design




2-rings, r1=7cm, r2=20.3cm, h=28cm




# Diagnostics

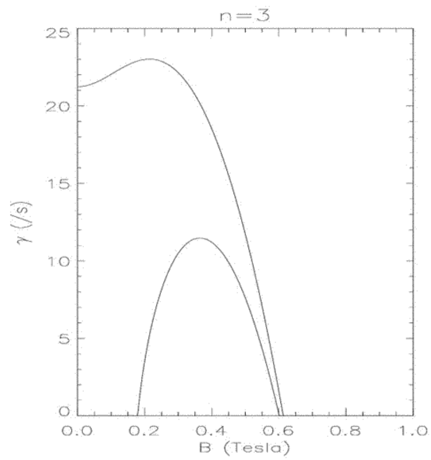
- Torque couplings between inner cylinder and other rotating parts
- Surface and internal magnetic perturbations
- Surface pressure perturbations
- Internal flows by ultrasound

# Experimental Protocols

- **Time scales:** $\tau_\eta \sim \Omega^{-1} \sim 10\,\mathrm{ms}$ $\quad \tau_A \equiv \frac{h}{2V_A} \sim 24\,\mathrm{ms}$

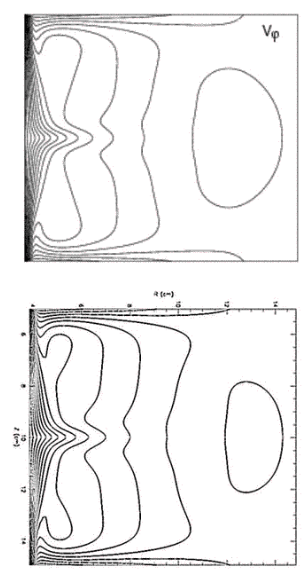
  $\tau_{\mathrm{MRI}} \sim 50\,\mathrm{ms}$ $\quad \tau_{\text{spin-up}} \sim \tau_{\mathrm{E}} \sim 20\mathrm{s}$

  $\tau_\nu \sim 5000\mathrm{s}$
- **MRI:**
  - **Spin up Couette flow without B**
  - **Impose B on penetration time $\tau_\eta$**
  - **MRI grows in $\tau_{\mathrm{MRI}}$**
- **Magnetic Ekman circulation:**
  - **Setup MRI stable flows and impose B**




# Modelling with ZEUS2D

- Wei Liu added viscosity and resistivity to ZEUS2D
  - excellent agreement with incompressible hydro code if run at peak Mach number M=1/4
  - MRI growth rates as expected
  - explicit resistivity is expensive, but that can be fixed
- Currently exploring MRI saturation and magnetic Ekman circulation

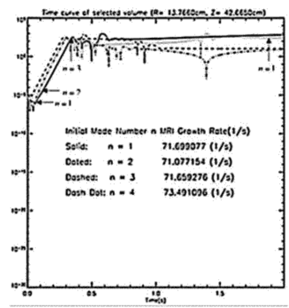



Vø from incompressible code (top) *versus* ZEUS2D at Mach number 1/4. Re=1600, endcaps corotating with outer cylinder.

# Vertically periodic flow

$Re=Re_M=400, B=0.5\ T$

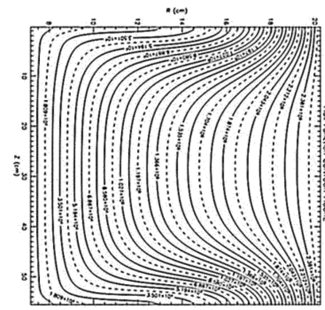
Growth and saturation. $\gamma \approx 70\ sec^{-1}$ n=3 mode



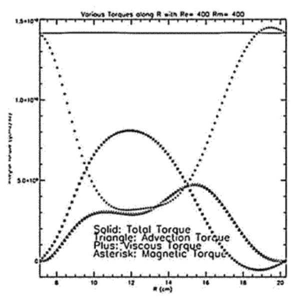

Poloidal field lines

r ->

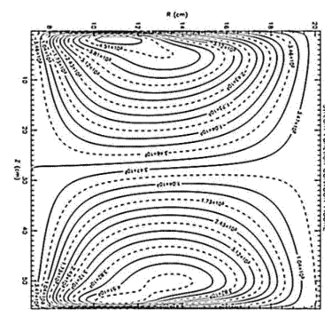
z




Viscous, magnetic, & advective torques in final state.




Poloidal stream lines




# Current status and plans

- Couette-flow apparatus is in hand
  - balance, bearings, seals almost debugged (?)
- This summer: take data in water
  - torque measurements
  - flow visualization
- This Fall: begin experiments in gallium
- Simulations and modeling
  - implement insulating boundaries for finite height
  - speed up resistive time step
  - move to 3D