MHD Models of Pulsar Winds

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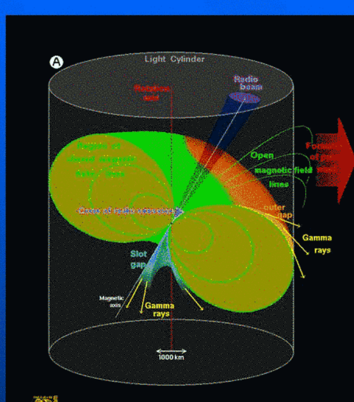
The pulsar magnetosphere



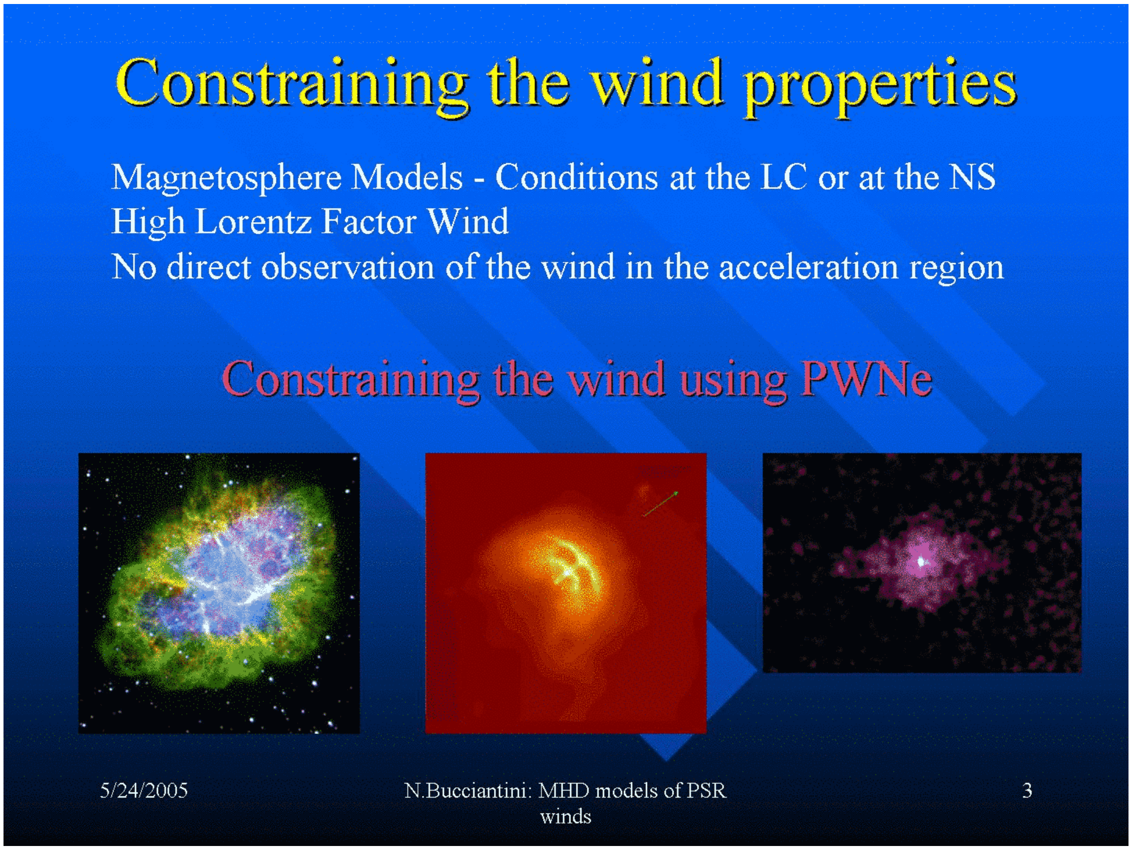
$R_{NS} = 10\ km$

$P = 0.001 - 7\ s$

$B_{surf} = 10^{8-12}\ G$

Acceleration of particle pair plasma- ions from the surface: Initial Lorentz factor ~ 100 Cold wind (Sync. losses)

Pair creations - change density exceeding the GJ value MHD regime (ideal?)


Constraining the wind properties
Magnetosphere Models - Conditions at the LC or at the NS
High Lorentz Factor Wind
No direct observation of the wind in the acceleration region
Constraining the wind using PWNe
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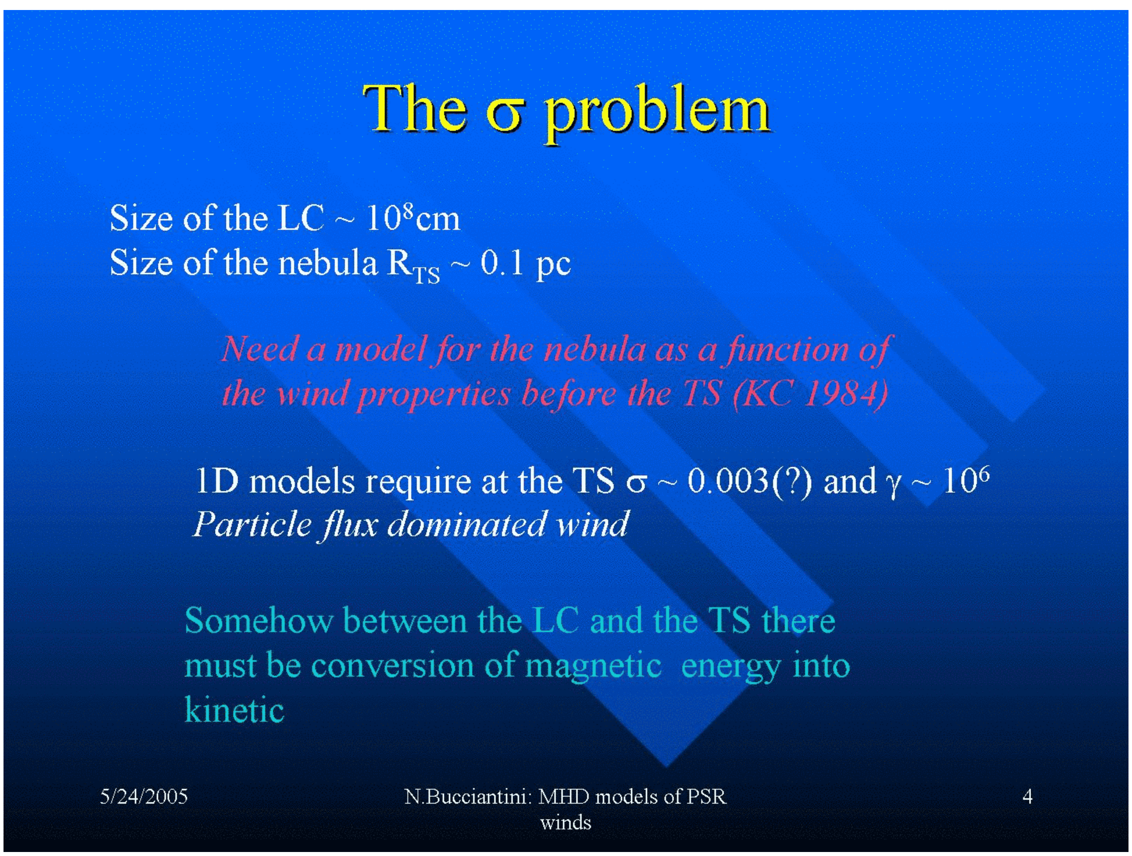
The σ problem
Size of the LC ~ 10^8cm
Size of the nebula R_TS ~ 0.1 pc
Need a model for the nebula as a function of the wind properties before the TS (KC 1984)
1D models require at the TS σ ~ 0.003(?) and γ ~ 10^6
Particle flux dominated wind
Somehow between the LC and the TS there must be conversion of magnetic energy into kinetic
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1D monopole

Michel 1969; Kennel et al. 1983; Daigne & Drenkhahn 2002

Cold wind -> Fast point is located at infinity Low efficiency
Pressure -> closer FP (higher efficiency but still low)

$$\sigma \approx \Phi^2 \Omega / \dot{M}$$

Terminal $\gamma = \sigma^{1/3}$ Above equipartition at the FP

Strict monopole does not work

Diverging flux tube

Radial flux tube - no efficient acceleration
Relaxing the radial assumption.

Simplified 1D non radial model require strong divergence
- Fountain like or sheet like transition
- Continuously opening flux tube $A \propto r^{2+\delta}$

Divergence has to take place after the LC

The problem of magnetic energy conversion is related to the general issue of collimation of a relativistic wind
2D model

At best we require a 2D axisymmeric RMHD "Steady state"

Solving the Grad-Shafranov + Transverse Equilibrium

No exact solution even for the simple monopole-like case

Study the wind in some specialized conditions

- Asymptotic behavior and collimation
- Trans fast regimes
- Self similar models
- Perturbation on the FF solution

$$\gamma_\infty \sim \sigma^{1/3} \qquad \sigma_\infty \sim \sigma^{2/3}$$

Conversion of magnetic energy is logarithmic after the fast p.

No collimation for a relativistic wind

Relativistic MHD equations

- Covariant form (fluid eqs. + Maxwell eqs. + freeze-in):

$$\nabla_\mu (\rho u^\mu) = 0$$

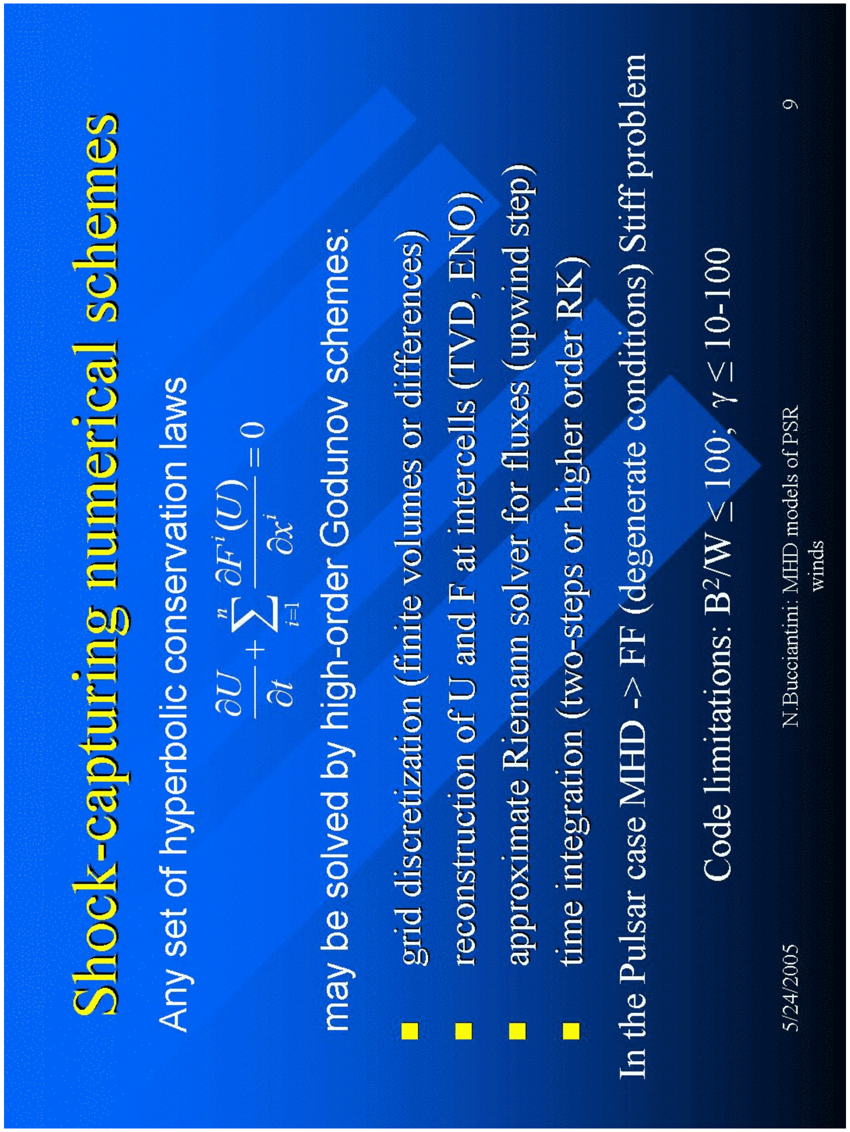
$$\nabla_\mu [(w + b^2) u^\mu u^\nu - b^\mu b^\nu + (p + b^2/2) g^{\mu\nu}] = 0$$

$$\nabla_\mu (u^\mu b^\nu - u^\nu b^\mu) = 0$$

- where (c=1, 4π→1):

$$x^\mu = (t, x^j), \; u^\mu = (\gamma, \gamma v^j), \; b^\mu = (\gamma v_i B^i, B^j/\gamma + \gamma v_i B^i v^j)$$

$$\gamma = (1 - v_i v^i)^{-1/2}, \; w = e + p, \; e = \rho + p/(\Gamma - 1)$$


Shock-capturing numerical schemes
Any set of hyperbolic conservation laws
$\frac{\partial U}{\partial t} + \sum_{i=1}^{n} \frac{\partial F^i(U)}{\partial x^i} = 0$
may be solved by high-order Godunov schemes:
grid discretization (finite volumes or differences)
reconstruction of U and F at intercells (TVD, ENO)
approximate Riemann solver for fluxes (upwind step)
time integration (two-steps or higher order RK)
In the Pulsar case MHD -> FF (degenerate conditions) Stiff problem
Code limitations: $B^2/W \leq 100$; $\gamma \leq 10$-100
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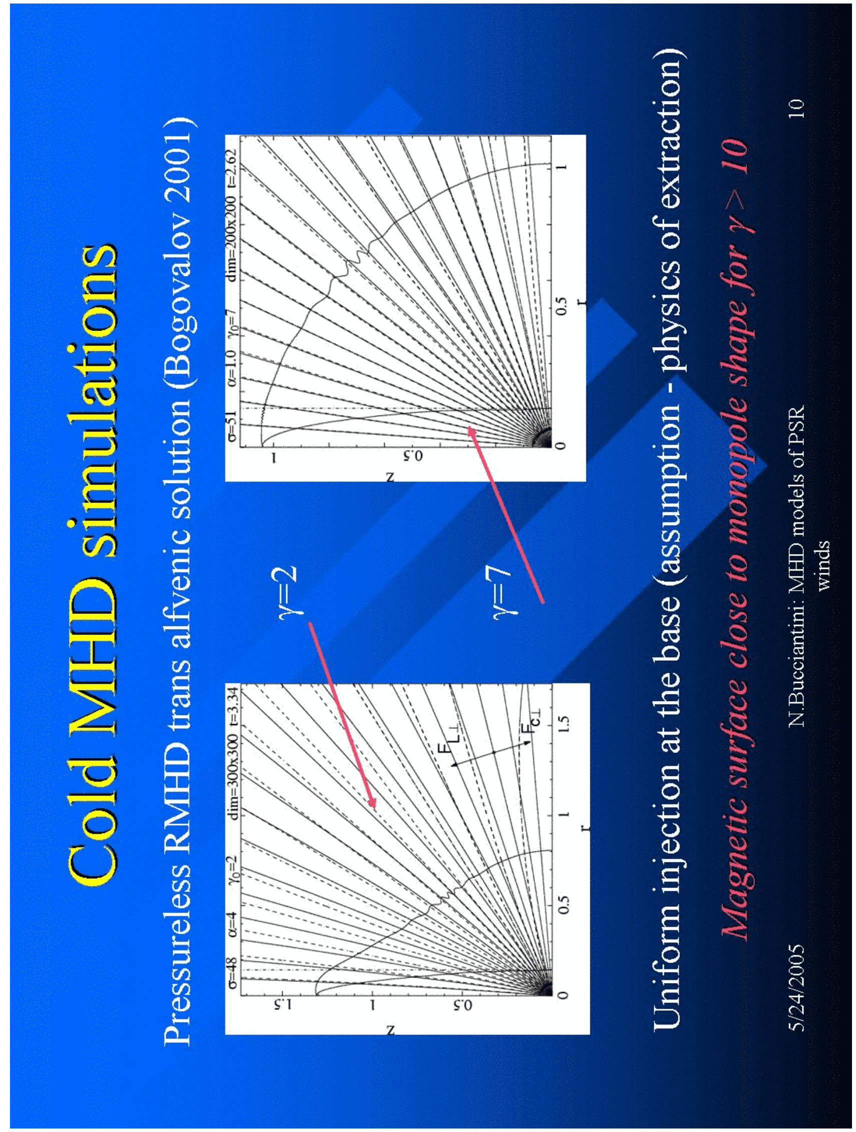
Cold MHD simulations
Pressureless RMHD trans alfvenic solution (Bogovalov 2001)
σ=48 α=4 γ0=2 dim=300x300 t=3.34
σ=51 α=1.0 γ0=7 dim=200x200 t=2.62
z
r
γ=2
γ=7
Uniform injection at the base (assumption - physics of extraction)
Magnetic surface close to monopole shape for γ > 10
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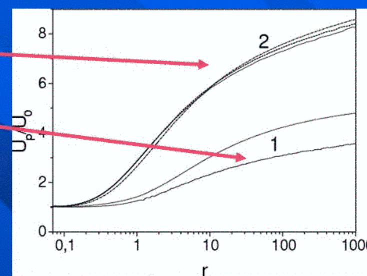
Jet and Acceleration

A cylindrical collimated core is formed along the axis

- For high γ $R_{jet} << R_{fast}$
- Mass flux in the jet ~ 10^{-3} total flux

2- $\gamma = 2$
1- $\gamma = 7$

Energy conversion is inefficient for relativistic outflows



Split monopole model

$$\gamma(\theta) = \gamma_0[\alpha + (1-\alpha)\sin^2(\theta)] \qquad B(r,\theta) = B_0(r_0/r)\sin(\theta)$$

Acceleration of a relativistic wind from a fast rotator

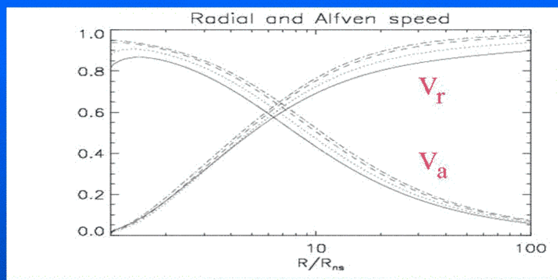
Magnetar model - initial neutrino driven wind (Thompson et al. 2003)

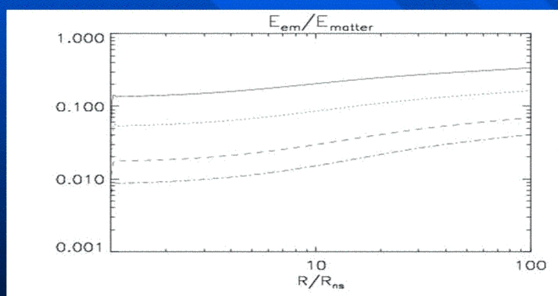
Rotating magnetar (Ω ~0.2)
Non cold atmosphere (C_s ~ 0.1c, H ~ 0.1 R_{NS})
Surface Density ~ 10^{13} g/cm^3
Surface Magnetic field B ~ 10^{14-16} G

- *The solution has to pass all 3 critical points*
- *Particle injection conditions are not fixed*
- *Gravity is included using Schwarzschild*

Can a magnetar trigger a GRB (efficient acceleration)
Does mass flux depend on the value of B (i.e. acceleration)
How energy and angular momentum losses scale with B^2/ρ

1D monopole





B=2 x 10^{15} G

$\dot{M}=3.5\cdot 10^{-4} M_{sol}/s$

$\dot{M}=1.3\cdot 10^{-4} M_{sol}/s$

$\dot{M}=3.8\cdot 10^{-5} M_{sol}/s$

$\dot{M}=1.8\cdot 10^{-5} M_{sol}/s$

$\gamma = 2.4$

$\gamma = 3.6$

$\gamma = 5.1$

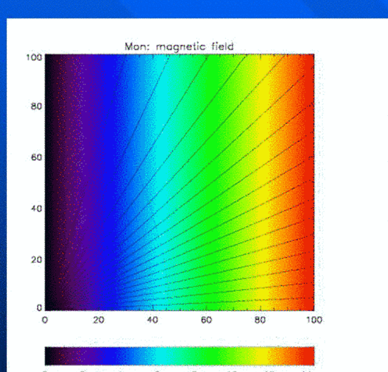
$\gamma = 8.0$

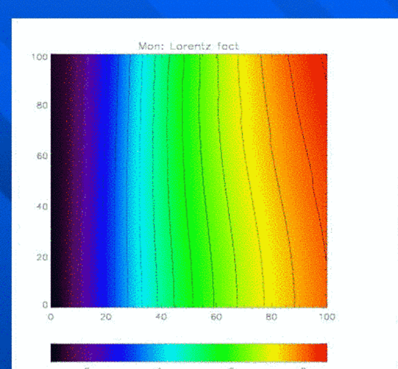
2D Monopole

Uniform radial magnetic field, density and pressure

At the star boundary Vr, Bθ, Bϕ, are extrapolated

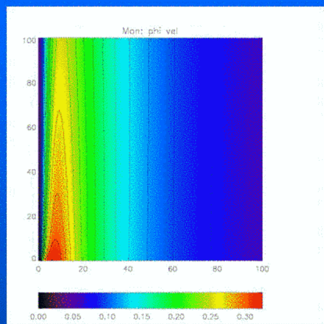
$V_\theta = V_r \cdot B_\theta / B_r \qquad V_\phi = \Omega r/\alpha + V_r \cdot B_\phi / B_r$

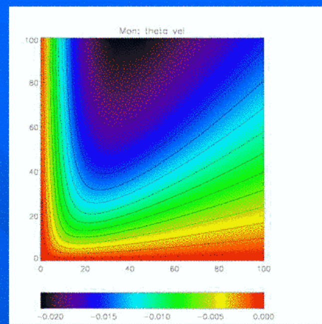


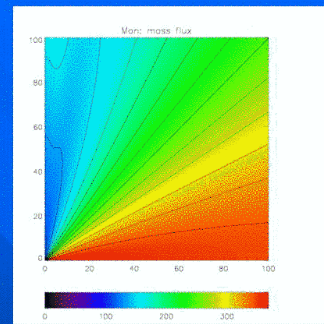


(B=2 10^{15}G, r=10^{10}g/cm^3, c_s=0.2c) - 10^{-5} $M_\odot$/s

$V\phi$ | $V\theta$ | M_{flux}



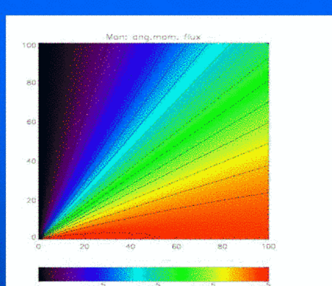


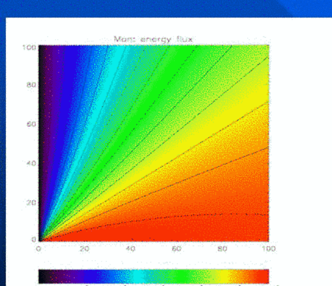


The maximum of V_ϕ if achieved at the LC, beyond $V_\phi \sim 1/R$
From V_θ - residual collimation of the flow toward the axis -
$V_\theta = V_\theta(\theta)$ increasing toward the pole
Matter flux is higher on the equator ~ $C + A \sin^2(\theta)$

Formation of a overdense core on the axis - collimation (bc?)

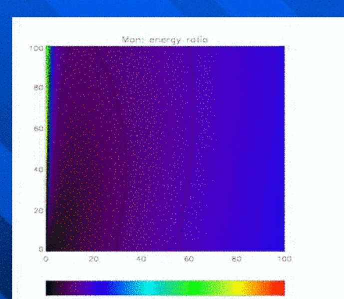
Energy and angular momentum





Angular momentum flux ~ $\sin^2(\theta)$
Energy flux ~ $\sin(\theta)$

Poynting flux dominated outflow

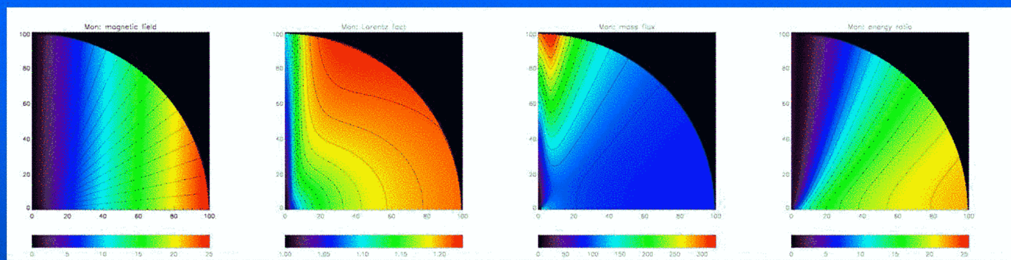


$E_{matter}/E_{em} \sim 0.1$

The ratio of particle to magnetic energy ~ $A + B \sin(\theta)$

Within the computational dom. Conversion more efficient than log (~ $r^{1/3}$)

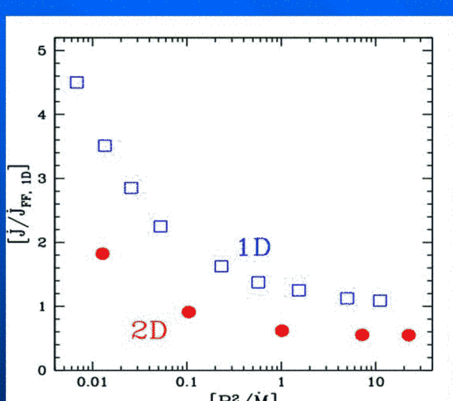
Mass loaded case



$B\phi$ | γ | M_{flux} | Eff.

- *Magnetic structure close to the magnetic dominated case*
- *Lorentz factor is higher at $\theta \sim 70^o$*
- *Mass and energy flux higher on the axis (strong collimation)*

Convergence to the Force-Free solution



$(B \sim 2\cdot 10^{15}\,G, \dot{M} \sim 2\cdot 10^{-4}\,M_{sol}/s)$

In the 1D case:

$$\dot{J} = d + k\cdot(B^2/\dot{M})^{-1/2}$$
$$d \to \dot{J}_{ff}$$
$$\dot{J}/\dot{J}_{ff} \propto (B^2/\dot{M})^{-1/3}$$

In the 2D case the fit is not as good but it is still possible to extrapolate the ff value

$$\dot{J}_{ff,2D} \approx 0.5\cdot \dot{J}_{ff,1D}$$

Dipole

More realistic configuration for the close pulsar magnetosphere
Magnetic surfaces diverge more than radial - Higher acceleration?

Existence of a closed zone in equilibrium
- *ambiguity in a GS approach*
- *injection condition for a cold gas*
- *extrapolation problem at the stellar surface*

Presence of current sheets in the computational domain and at the boundaries
- *strong shear at the open-closed zone boundary*
- *resolution of the Y point*
- *reconnection on the equatorial sheet*

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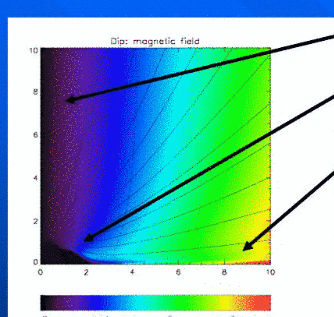
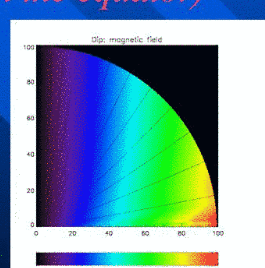
Preliminary results

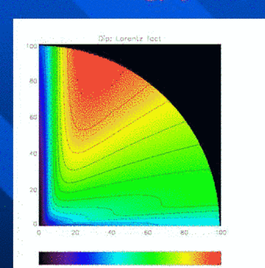
(Corresponding to the high mass loading monopole case $B^2/\rho c^2=0.06$)



Collimation is efficient
Closed zone ($R_c \sim 2\ R_{NS}$)
Equatorial channel (most of the mass, angular mom., and energy flux in the equator)

The far zone resemble a monopole - lower equatorial velocity





γ

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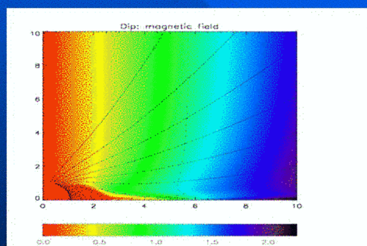
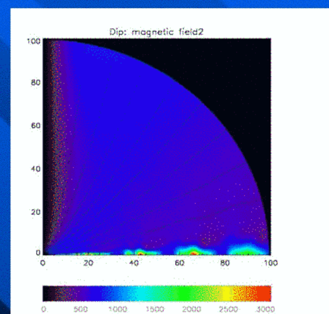
Instability at the current sheet and formation of plasmoids

Increasing the magnetic field on the surface of a factor 100 causes instability to develop on the equatorial current sheet:

- Periodic reconnection close to the Alfven-LC surface
- Formation of plasmoids that carry most of the energy and angular momentum
- Possible noisy torque on the star

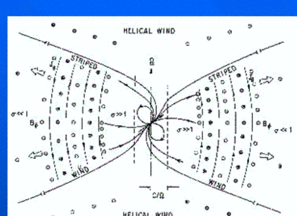




$R_c \sim 3R_{NS}$

Non ideal MHD effects

Striped wind model - Dissipation of the current sheet



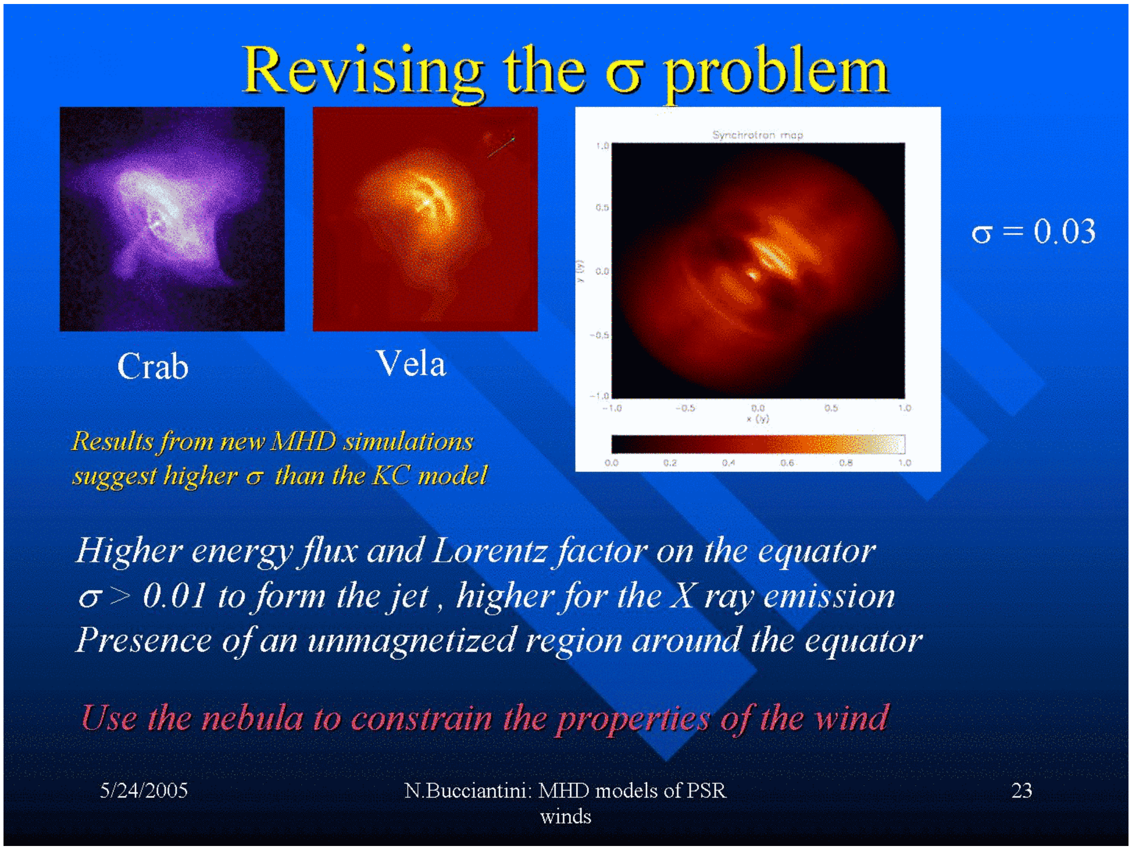
Magnetic filed reverse sign every half period
Presence of a folded current sheet

If dissipation of the magnetic field in the striped wind region is efficient high Lorentz factors might be reached

How does dissipation work?

LK 2001 - no efficient dissipation before the TS
KS 2003 - high pair flux required

Could dissipation take place at the TS?


Revising the σ problem
Crab
Vela
σ = 0.03
Synchrotron map
Results from new MHD simulations
suggest higher σ than the KC model
Higher energy flux and Lorentz factor on the equator
σ > 0.01 to form the jet , higher for the X ray emission
Presence of an unmagnetized region around the equator
Use the nebula to constrain the properties of the wind
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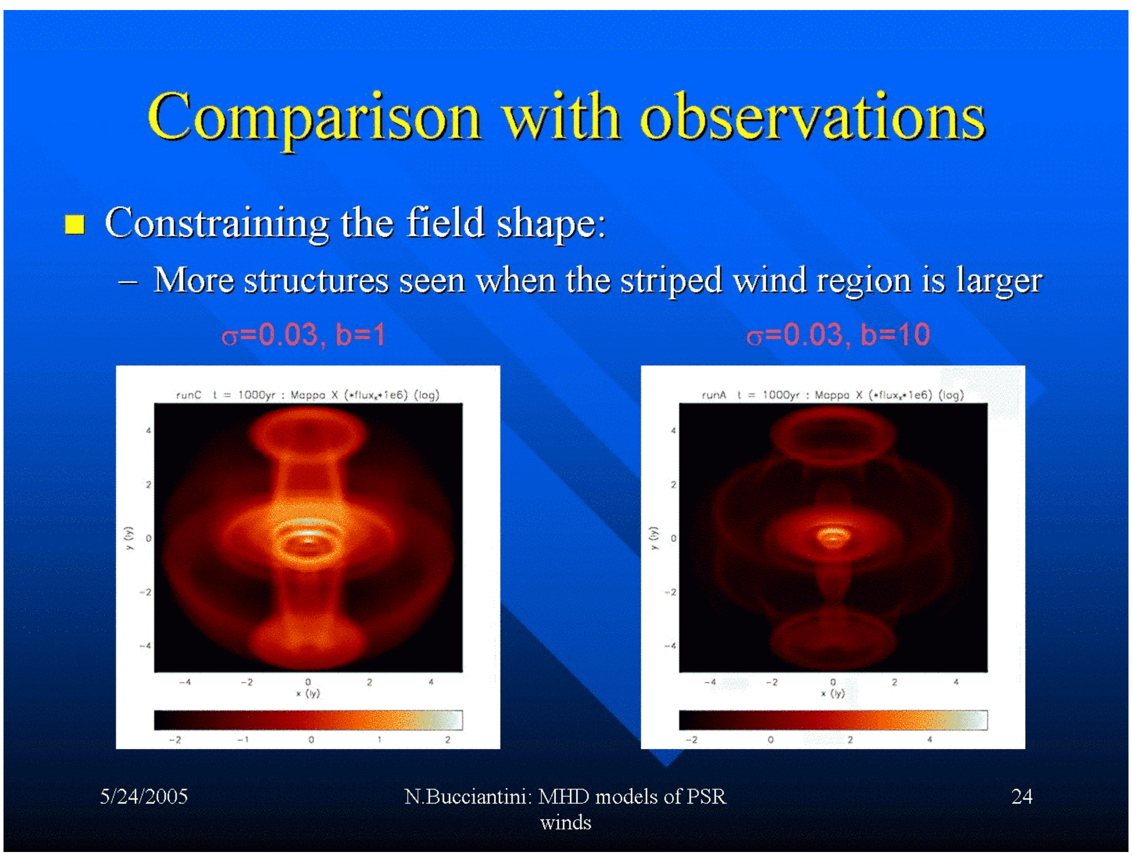
Comparison with observations
Constraining the field shape:
– More structures seen when the striped wind region is larger
σ=0.03, b=1
σ=0.03, b=10
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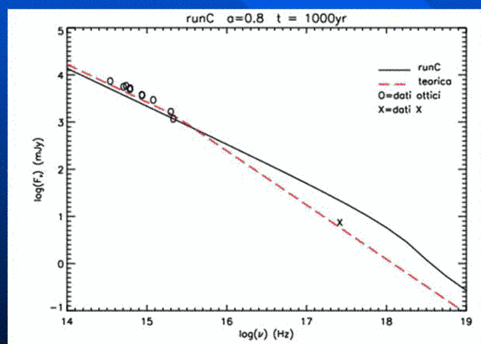
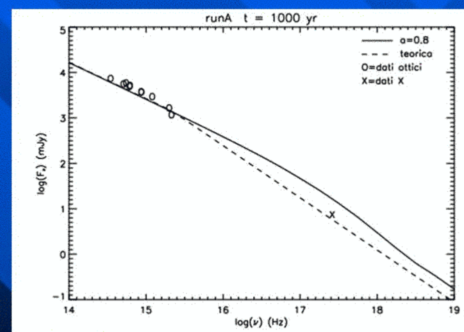
Comparison with observations

- Simulated integrated spectra:
  - Reasonable results for higher magnetization
  - Still problems to get the right spectral break

σ=0.03, b=1



σ=0.03, b=10



Conclusions

- Collimation and acceleration of the wind are related problems
- Strong divergence of the flux tube is required after the AS
- FF simulation - Magnetic field structure - no acceleration
- RMHD simulations to handle multi-D cases - consistent acceleration
- Very robust codes for PSR magnetosphere (FF limit)
- 1D monopole is highly inefficient
- 2D monopole is inefficient for $\gamma > 10$
- Structure of the monopolar wind in agreement with PSR simulation
- Convergence of the integrated quantities for $(B - 2\cdot10^{15}\,G, \dot{M} - 2\cdot10^{-4}\,M_{sol}/s)$
- Dipolar structure more realistic - stronger divergence in the inner zone
- Far zone - monopolar structure -> low acceleration
- Problems with the stability of the current sheet - tearing modes
- No ideal effect - dissipation of the magnetic field in the striped region - numerical model for the synchrotron emission

Thank you