Edge & Surface States of Topological Superfluids

J. A. Sauls

Northwestern University

• Hao Wu (Northwestern) • Supported by NSF Grant DMR-1106315

• K. Kono (RIKEN) • J. Parpia (Cornell) • J. Saunders (RHUL) • W. Halperin (NU)

- Topology of Superfluid ^{3}He-A
- Chiral Edge States in 2D ^{3}He-A
- Detecting Chiral Edge Fermions
- ^{3}He-B - 3D Topological SF
- Strong Confinement: $D \lesssim 10\xi_0$
- Edge State Interference
- Chiral "Crystalline" Phase
- Signatures of Majorana Modes

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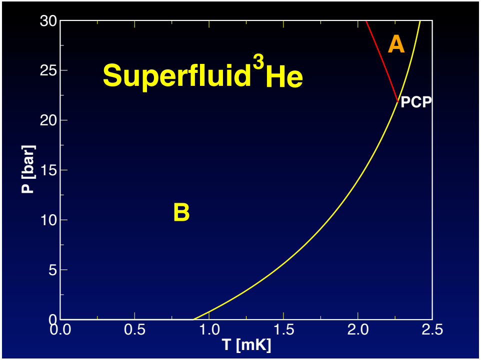
- M. Stone, R. Roy, Phys. Rev. B 69, 184511 (2004)
- J. A. Sauls, Phys. Rev. B 84, 214509 (2011)
- S. B. Chung and S.-C. Zhang, PRL 103, 235301 (2009)
- T. Mizushima, Phys. Rev. B 86 094518, (2012)
- Hao Wu, J. A. Sauls, Phys. Rev. B 88, 18 184506 (2013)

Symmetry Group of Normal ^{3}*He*: $\mathtt{G} = \mathtt{SO(3)_S} \times \mathtt{SO(3)_L} \times \mathtt{U(1)_N} \times \mathtt{P} \times \mathtt{T}$

Phase Diagram of Bulk ^{3}He



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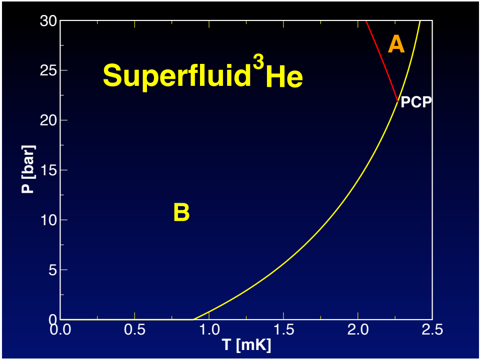

Spin-Triplet, P-wave Order Parameter :

$$\Delta_{\alpha\beta}(\mathbf{p}) = \vec{\mathbf{d}}(\mathbf{p}) \cdot (i\vec{\boldsymbol{\sigma}}\sigma_y)_{\alpha\beta} \rightsquigarrow \mathbf{d}_\mu(\mathbf{p}) = \mathcal{A}_{\mu i}\, \mathbf{p}_i$$

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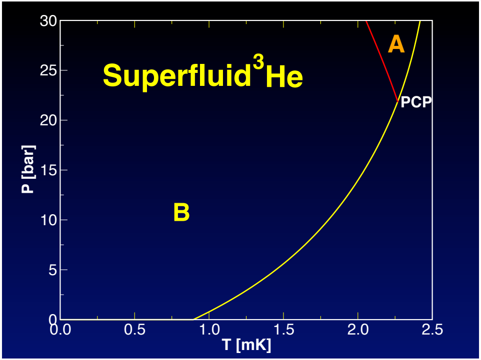
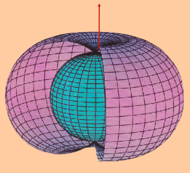


Chiral ABM State $\vec{l} = \hat{\mathbf{m}} \times \hat{\mathbf{n}}$



$$\mathcal{A}_{\mu i} = \Delta\, \hat{\mathbf{d}}_\mu\, (\hat{\mathbf{m}} + i\hat{\mathbf{n}})_i$$

$$L_z = 1,\ S_z = 0$$

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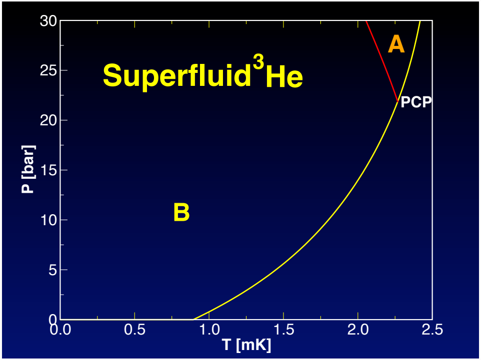

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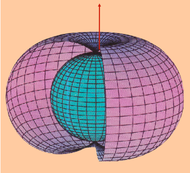

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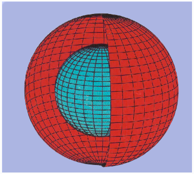
"Isotropic" BW State



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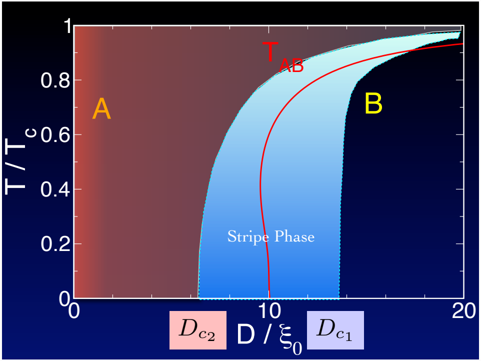
A. Vorontsov & JAS, PRL, 2007



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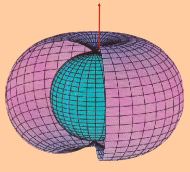

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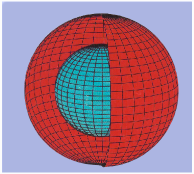

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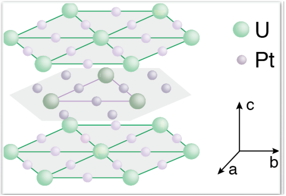
UPt_3 T_c=0.56 K



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- ``f-wave'', L_z = 2

$$\Delta(\mathbf{p}) \sim p_z\,(p_x + ip_y)^2$$

- chiral

multiple SC phases

JAS, Adv. Phys. 43, 113(1994)

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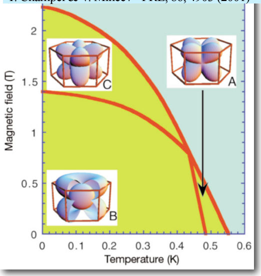

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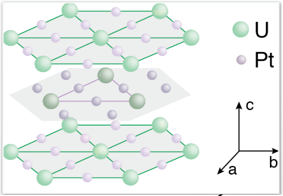

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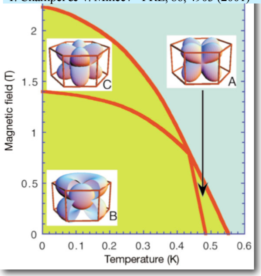

Josephson Interferometry

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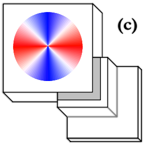

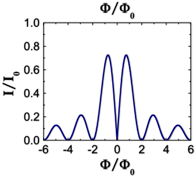

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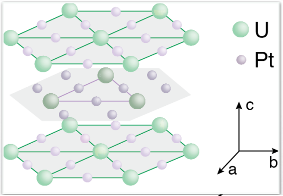

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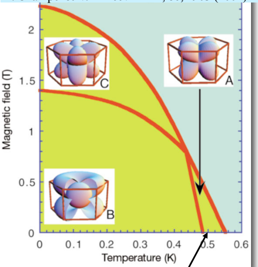
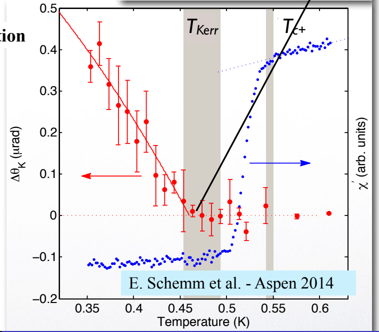


Kerr rotation



E. Schemm et al. - Aspen 2014

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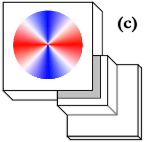
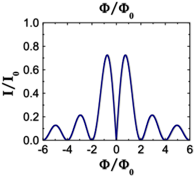
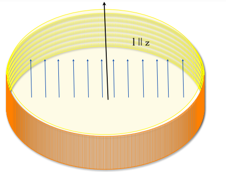




Ground-State for Chiral P-wave BEC Molecules or BCS Pairs Composed of N Fermion atoms:



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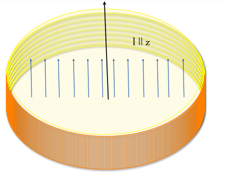

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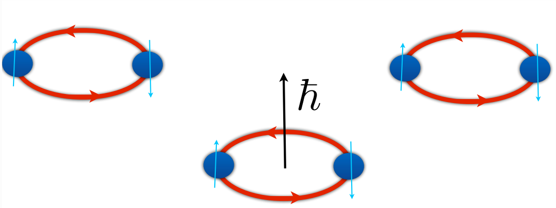

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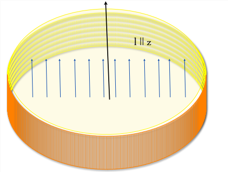

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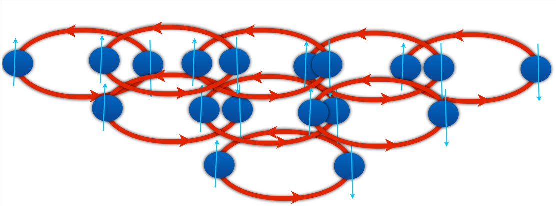

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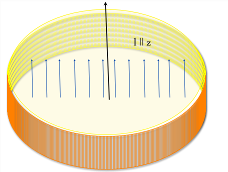

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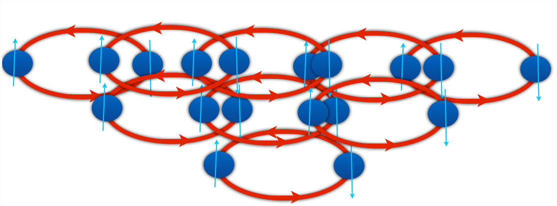

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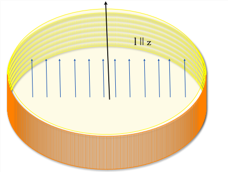

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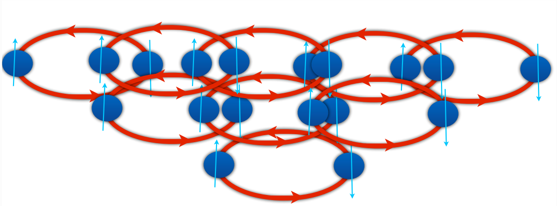

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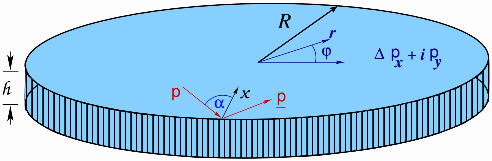
Currents are confined on the Edge

^{3}He-A confined in a cylindrical cavity with $h \ll \xi_0$ and $R \gg \xi_0$.



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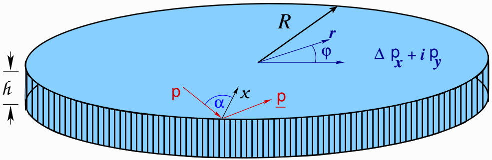

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Bogoliubov Equations for Fermionic Excitations:

$$\left(-\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2 - \mu\right) u + \sigma_x \frac{\hbar}{i}\left(\Delta_1 \frac{\partial}{\partial x} + i\Delta_2 \frac{\partial}{\partial y}\right) v = \varepsilon\, u$$
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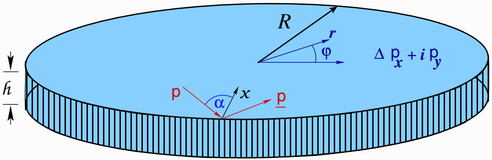

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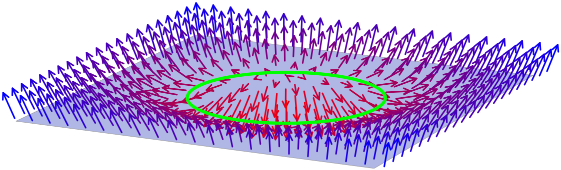
Nambu-Momentum Representation with particle-hole (iso-spin) matrices $\widehat{\vec{\tau}} = (\widehat{\tau}_1, \widehat{\tau}_2, \widehat{\tau}_3)$

$$\widehat{H} = \left(|\mathbf{p}|^2/2m - \mu\right)\widehat{\tau}_3 + \sigma_x \left[\Delta_1\, p_x\, \widehat{\tau}_1 \mp \Delta_2\, p_y\, \widehat{\tau}_2\right]/p_f = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$$

Nambu-Bogoliubov Hamiltonian for 2D ^{3}He-A : $\widehat{H} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\boldsymbol{\tau}}}$

$\rightsquigarrow$ $\vec{\mathbf{m}} = (\, cp_x \, , \, \mp cp_y \, , \xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0 \;, \mu \neq 0$



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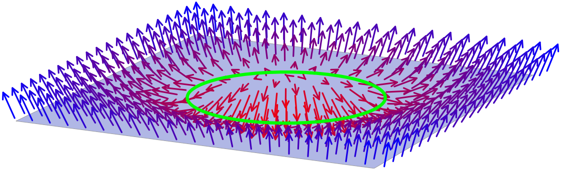
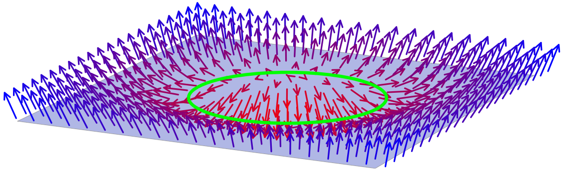


Topological Invariant for 2D ^{3}He-A $\leftrightarrow$ QED in d = 2+1 [G.E. Volovik, JETP 1988]:

$$N_{2D} = \pi \int \frac{d^2p}{(2\pi)^2}\, \hat{\mathbf{m}}(\mathbf{p}) \cdot \left(\frac{\partial \hat{\mathbf{m}}}{\partial p_x} \times \frac{\partial \hat{\mathbf{m}}}{\partial p_y}\right) = \begin{cases} \pm 1 \;; & \mu > 0 \text{ and } \Delta \neq 0 \\ 0 \;; & \mu < 0 \text{ or } \Delta = 0 \end{cases}$$

Nambu-Bogoliubov Hamiltonian for 2D ^{3}He-A : $\widehat{H} = \vec{\mathbf{m}}(\mathbf{p}) \cdot \widehat{\vec{\tau}}$

$\rightsquigarrow$ $\vec{\mathbf{m}} = (\,cp_x\,,\,\mp cp_y\,,\xi(\mathbf{p}))$ with $|\vec{\mathbf{m}}(\mathbf{p})|^2 = \left(|\mathbf{p}|^2/2m - \mu\right)^2 + c^2|\mathbf{p}|^2 > 0\,,\mu \neq 0$



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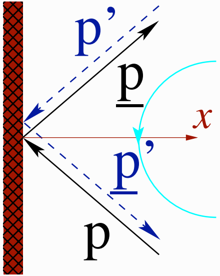
“Vacuum” $(\Delta = 0)$ with $N_{2D} = 0$ | ^{3}He-A$(\Delta \neq 0)$ with $N_{2D} = 1$

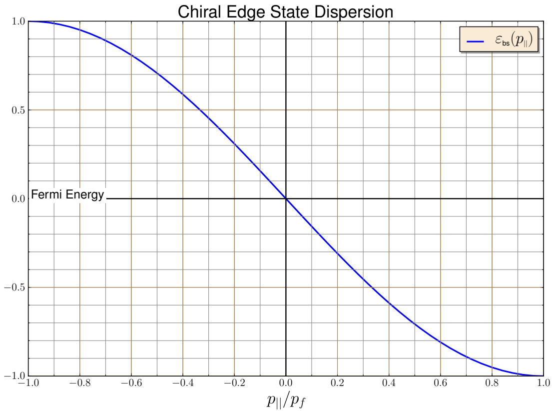
Zero Energy Fermions $\uparrow$ Confined on the Edge

Propagator for Edge Fermions: $g^{\mathrm{R}}_{\mathrm{edge}}(\mathbf{p},\varepsilon;x) = \dfrac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\mathrm{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$

Confinement on $\xi_\Delta = \hbar v_f/2\Delta \approx 10^3\,\mathring{A} \gg \hbar/p_f$

- $\varepsilon_{\mathrm{bs}} = -c\,p_{||}$ with $c = \Delta/p_f \ll v_f$
- Broken P & T $\rightsquigarrow$ Edge Current





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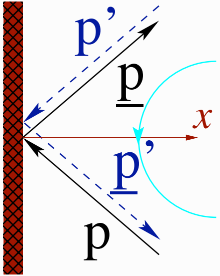
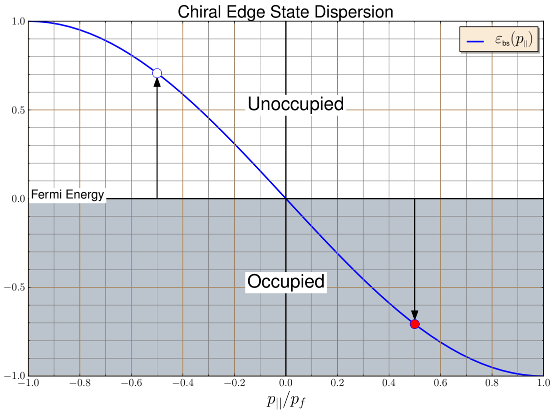
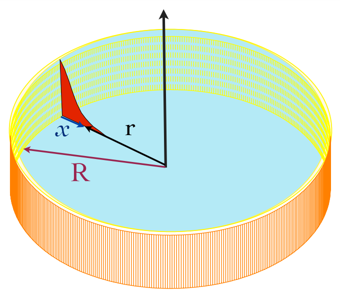




Ground-State Current Density: $\vec{J}(x) = \int_{-1}^{+1} \frac{dp_{||}}{p_f} \int_{-\infty}^{0} \vec{J}(\mathbf{p}, x; \varepsilon)$



Bound-State Contribution ($R \gg \xi_\Delta$):

$$\begin{aligned} J_\varphi(\mathbf{p}, x; \varepsilon) &= 2N_f\, v_f\, \Delta\, |p_x|\, p_\varphi\, e^{-x/\xi_\Delta} \\ &\times \left[\delta(\varepsilon - \varepsilon_{\text{bs}}(\mathbf{p}_{||})) - \delta(\varepsilon - \varepsilon_{\text{bs}}(\mathbf{p}'_{||}))\right] \end{aligned}$$

Bound-State Edge Current: $\int_0^\infty dx J_\varphi(x) = \frac{1}{2}\, n\, \hbar$

Mass Current: $v_f \rightarrow p_f \rightsquigarrow \vec{J} \rightarrow \vec{g}$

- $L_z^{\text{bs}} = \int_V d^2r\, [r\, g_\varphi(\mathbf{r})]$ $= N\,\hbar$ $\times 2$ Too Large vs. MT

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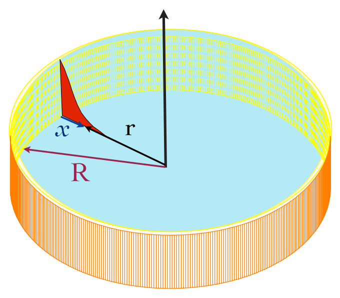

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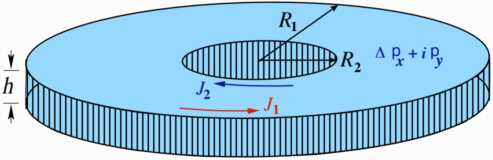
- Continuum ($\varepsilon < -\Delta$): $J_\varphi^{\text{C}} = 2N_f\, v_f\, |p_x| \left(\frac{\Delta^2\, p_\varphi^2}{\varepsilon^2 - \varepsilon_{\text{bs}}^2(\mathbf{p}_{||})}\right)$ $\sin\left(2\sqrt{\varepsilon^2 - \Delta^2}\, x/v_x\right)$

- $L_z^{\text{C}} = \int_V d^2r \left[r\, g_\varphi^{\text{C}}(\mathbf{r})\right]$ $= -\frac{1}{2} N\,\hbar$ $\rightsquigarrow$ $L_z^{\text{total}} = (N/2)\hbar$ - MT Result Recovered!

^{3}He-A confined in a toroidal cavity



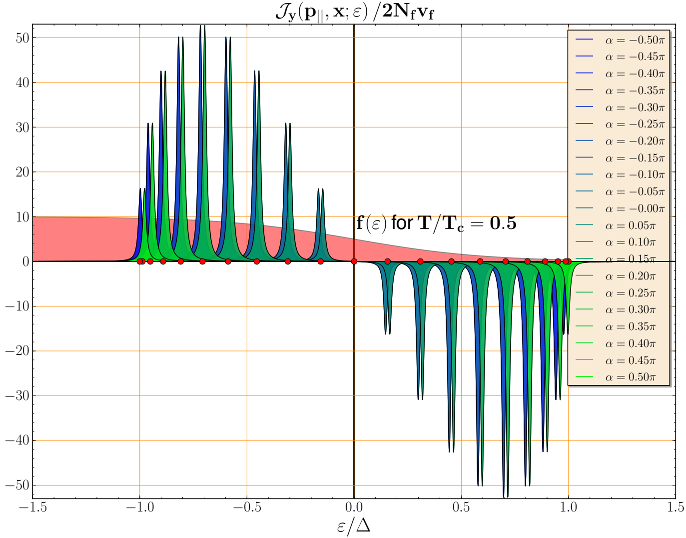
- $R_1, R_2, R_1 - R_2 \gg \xi_0$
- Volume: $V = h\pi(R_1^2 - R_2^2)$

- Sheet Current: $J = \frac{1}{4}\, n\, \hbar$ $(n = N/V =$ ^{3}He density)
- Counter-propagating Edge Currents: $J_1 = -J_2 = \frac{1}{4}\, n\, \hbar$
- Angular Momentum:

$$L_z = 2\pi\, h\, (R_1^2 - R_2^2) \times \frac{1}{4}\, n\, \hbar = (N/2)\, \hbar$$

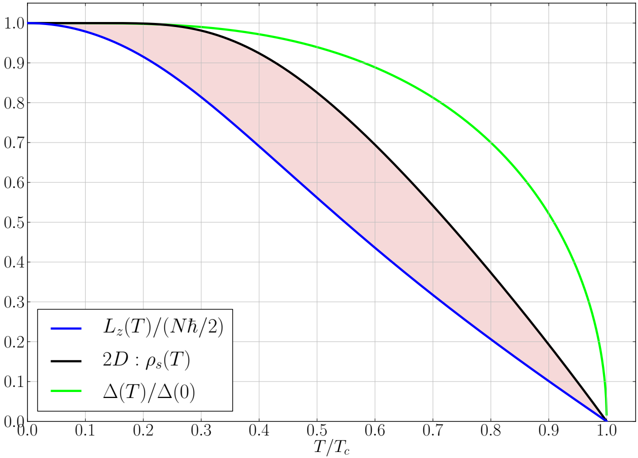
McClure-Takagi Result

Thermally Excited Edge Fermions Carry the Opposite Current



Angular Momentum of ^{3}He-A vs. Temperature

$$L_z = (N/2)\hbar \times \mathcal{Y}_{L_z}(T) \qquad \mathcal{Y}_{L_z}(T) \approx 1 - c\,(T/\Delta)^2\,, \quad T \ll \Delta$$



- JAS, Phys. Rev. B 84, 214509 (2011)
- Y. Tsutsumi et al., PRB 85, 100506(R) (2012)

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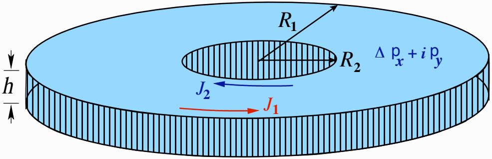

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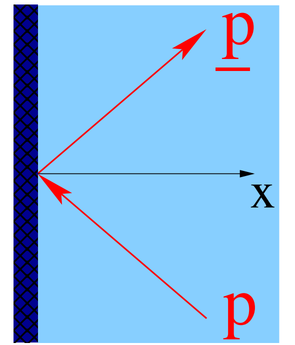
McClure-Takagi Result

Magnitude of Edge Currents are Protected by Symmetry, not Topology

Specular Reflection



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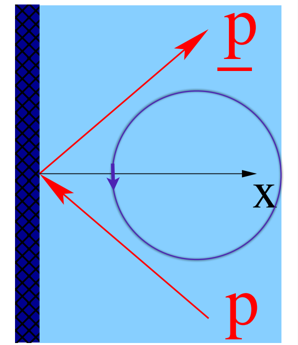

Propagating Chiral Fermions:

$$g^{\mathsf{R}}(\mathbf{p}, \varepsilon; x) = \frac{\pi\Delta|\mathbf{p}_x|}{\varepsilon + i\gamma - \varepsilon_{\mathsf{bs}}(\mathbf{p}_{||})}\, e^{-x/\xi_\Delta}$$

Edge Current: $J = \frac{1}{4}\, n\, \hbar$

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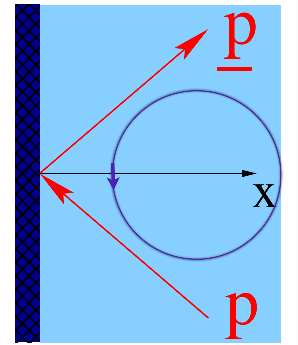

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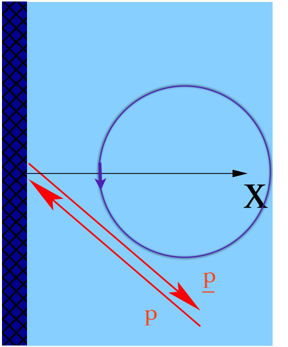

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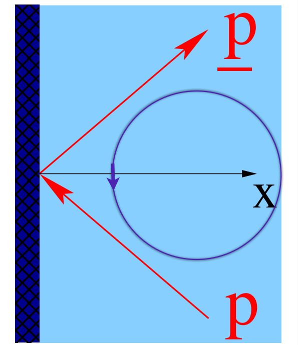

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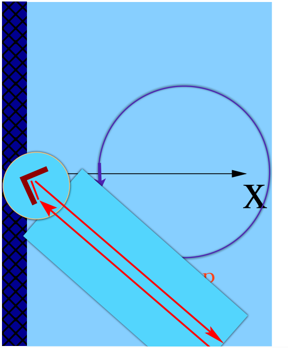

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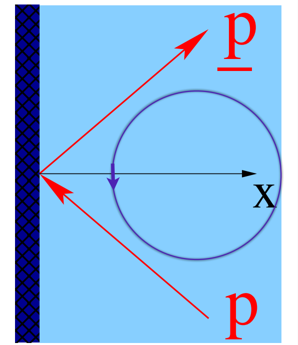

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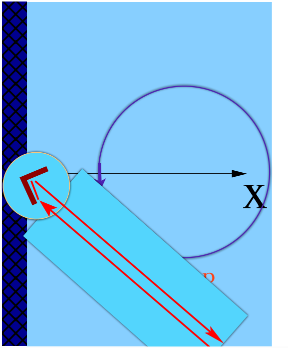
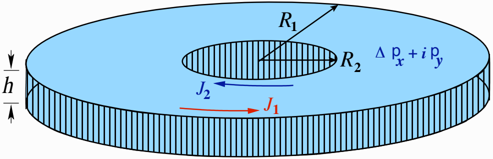


Zero-Energy Fermions for all $\mathbf{p}$:

$$g^{\mathrm{R}}(\mathbf{p},\varepsilon;x) = \frac{\pi\Delta}{\varepsilon + i\gamma}\, e^{-2\Delta x/v_x}$$

$\rightsquigarrow$ Edge Current: $J = 0$

Engineered Edges of a Toroidal Cavity



- Sheet Current: $J = f \times \frac{1}{4}\, n\, \hbar$
- Non-Specular Surfaces

  $0 \leq f \leq 1$

Incomplete Screening of Counter-Propagating Currents

$$L_z = (N/2)\,\hbar \times \left(\frac{f_1 - r\,f_2}{1 - r}\right)$$

Scaling of L_z with $r = (R_2/R_1)^2$ $\quad 0 < r < 1$

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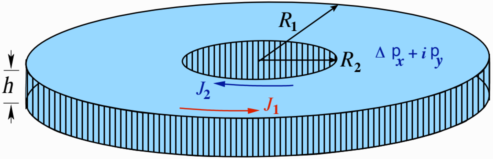

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Scaling of L_z with $r = (R_2/R_1)^2 \qquad 0 < r < 1$

- $f_1 = 1,\ f_2 = 0$

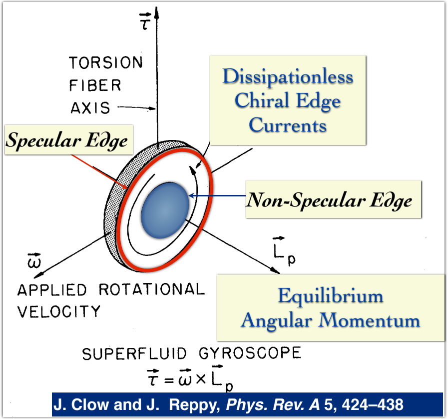
  $$L_z = (N/2)\,\hbar \times \left(\frac{1}{1 - r}\right) \gg (N/2)\,\hbar$$

- $f_1 = 0,\ f_2 = 1$

  $$L_z = (N/2)\,\hbar \times \left(\frac{-r}{1 - r}\right) \ll -(N/2)\,\hbar$$

- Strong violations of the McClure-Takagi Result

Gyroscopic Experiment to Measure of $L_z(T)$



Signatures of Chiral Edge States

- Power Law for $T \lesssim 0.5T_c$

$L_z \approx (N/2)\hbar\,(1 - c\,(T/\Delta)^2\,)$

Toroidal Geometry with Engineered Surfaces

- Incomplete Screening

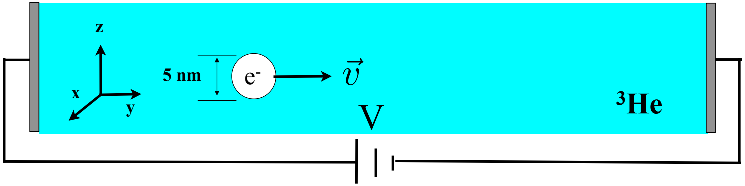
$L_z > (N/2)\hbar$ or
$L_z < -(N/2)\hbar$

Direct Signature of Edge Currents

Detection of Broken *Time-Reversal* Symmetry of Cooper pairs in Superfluid ^{3}He-A

Hiroki Ikegami, Yasumasa Tsutsumi, Kimitoshi Kono, Science 341, 59-62 (2013)

RIKEN, Japan



Electron Mobility:

$$\vec{v} = \widehat{\mu} \cdot \vec{E}$$

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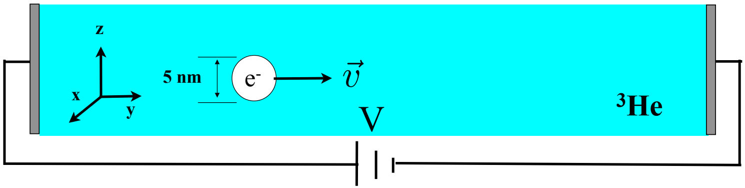

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$$\vec{v} = \widehat{\mu} \cdot \vec{E}$$

B-phase Mobility

$$\widehat{\mu} = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Isotropic
Fully Gapped

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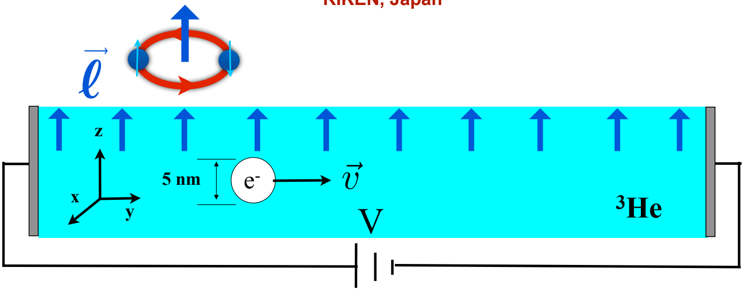

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$$\widehat{\mu} = \mu \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Isotropic
Fully Gapped

A-phase Mobility

$$\widehat{\mu} = \begin{pmatrix} \mu_\perp & \mu_{xy} & 0 \\ -\mu_{xy} & \mu_\perp & 0 \\ 0 & 0 & \mu_\parallel \end{pmatrix}$$

Anisotropic
Transverse Force

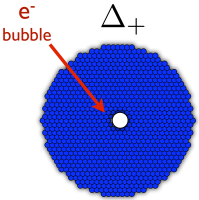
$\vec{\ell} = +\hat{\mathbf{z}}$

Structure of an Ion embedded in ^{3}He-A

$\hbar/p_f \ll R \lesssim \xi_0$

JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

$(p_x + ip_y)$



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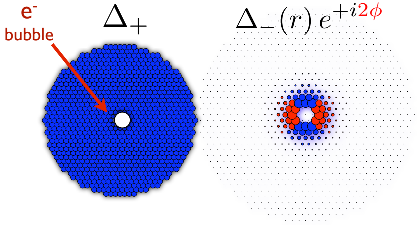
Δ_+ $\Delta_-(r)\, e^{+i2\phi}$



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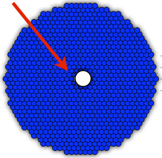
e$^-$ bubble

Δ_+



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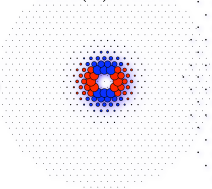
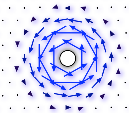


Chiral Currents



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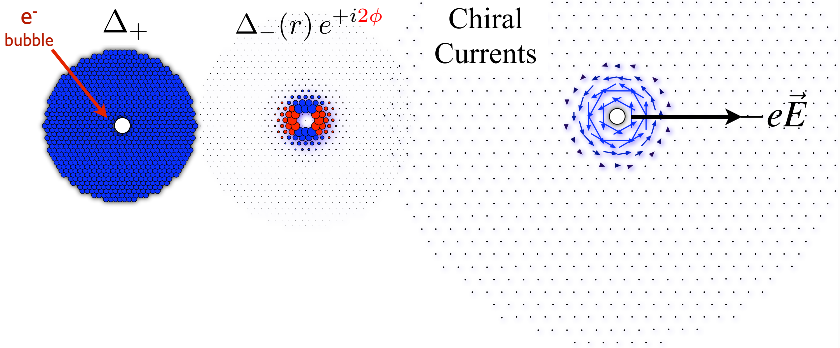

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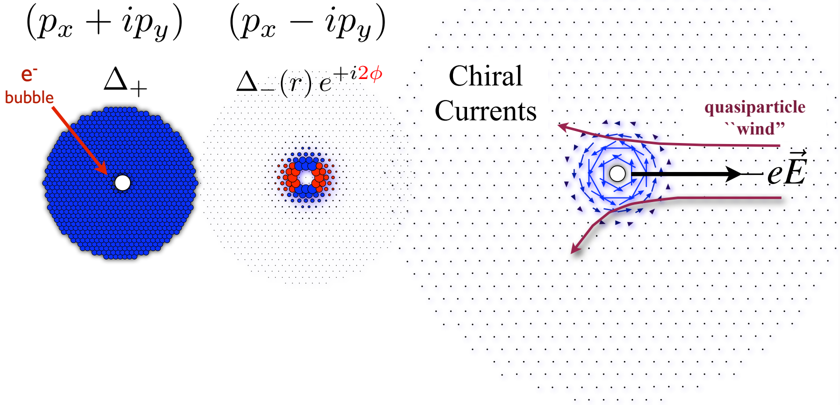

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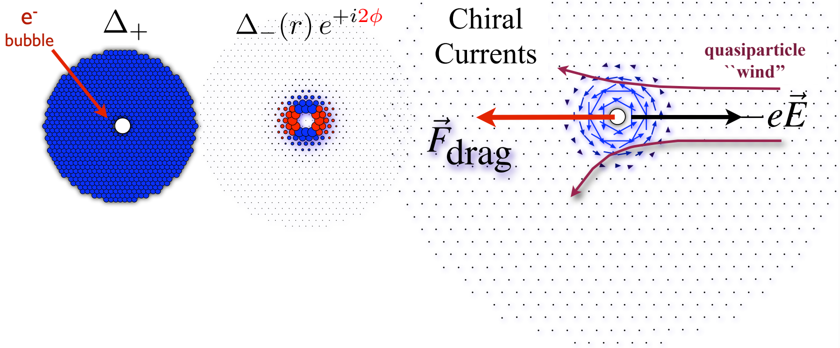

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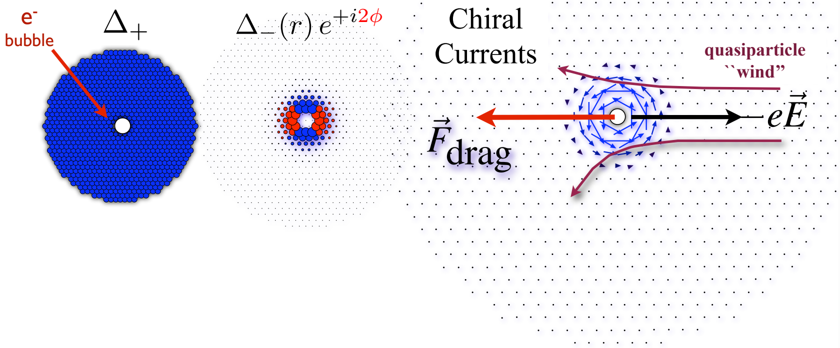

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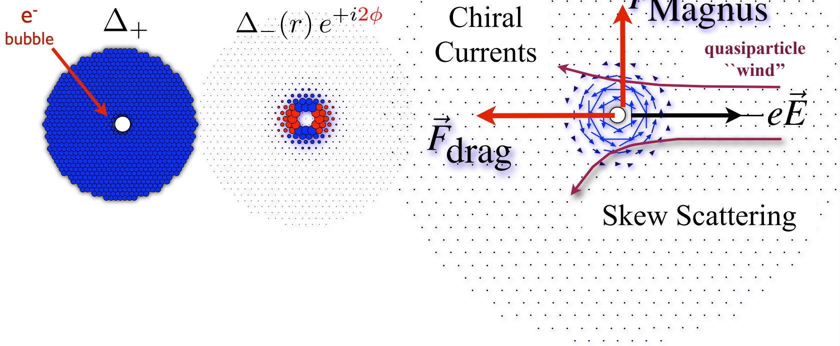
JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

$(p_x + ip_y)$ $(p_x - ip_y)$



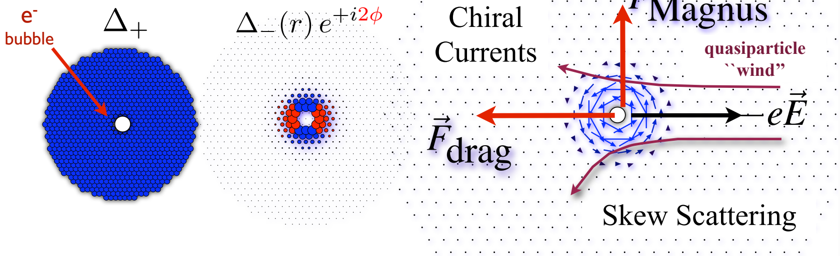
$\vec{\ell} = +\hat{\mathbf{z}}$

Structure of an Ion embedded in ^{3}He-A

$\hbar/p_f \ll R \lesssim \xi_0$

JAS & M. Eschrig, New J. Phys. 11, 075008 (2009)

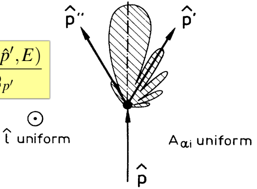
$(p_x + ip_y)$ $(p_x - ip_y)$



$R \approx \text{Å}$ ➡ M.Vuorio and D. Rainer, J. Phys. C Sol. State 10 3093(1977)

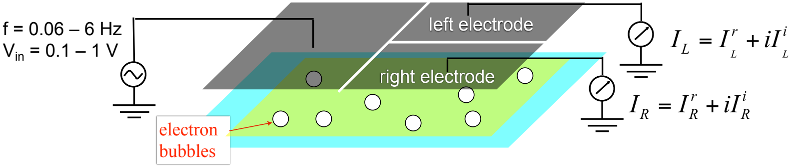
$$e(\mu^{-1})_{ij} = n\, p_f \int \frac{d\Omega_p}{4\pi} \int \frac{d\Omega_{p'}}{4\pi} \int_{-\infty}^{+\infty} dE \left(-\frac{\partial f}{\partial E}\right) (\Delta\hat{p})_i (\Delta\hat{p})_j \frac{\partial\sigma(\hat{p},\hat{p}',E)}{\partial\Omega_{p'}}$$

$$\vec{v} = \left[\mu_\parallel \left(\hat{\ell}\cdot\vec{E}\right)\hat{\ell} + \mu_\perp\, \hat{\ell}\times\left(\hat{\ell}\times\vec{E}\right) + \mu_{xy}\, \hat{\ell}\times\vec{E}\right]$$



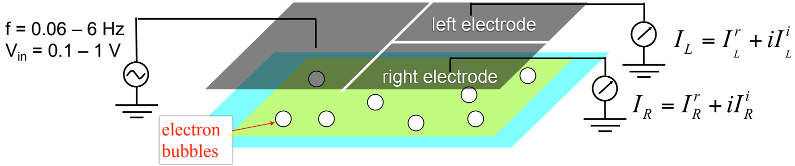
R. Salmelin, M. Salomaa, V. Mineev, Phys. Rev. Lett. 63, 868 (1989)

Measurement of the Transverse e⁻ mobility in Superfluid ³He Films



H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

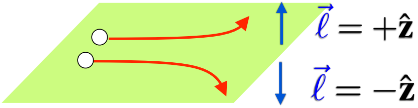
Measurement of the Transverse e$^-$ mobility in Superfluid ^{3}He Films



Transverse Force from ***Skew* Scattering**

$$\rightsquigarrow \quad \Delta I = I_R - I_L \neq 0$$

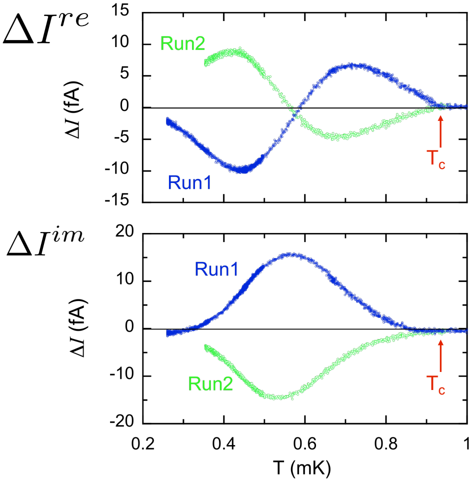
$$\vec{v} = \left[\mu_\perp \vec{E} + \mu_{xy}\, \hat{\ell} \times \vec{E} \right]$$

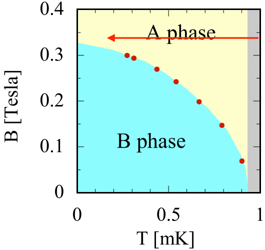


H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Transverse Current in ^{3}He-A

$$\Delta I = I_R - I_L$$





Single Domains:

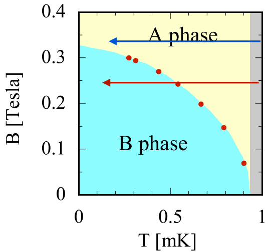
Run 1 $\vec{\ell} = +\hat{\mathbf{z}}$

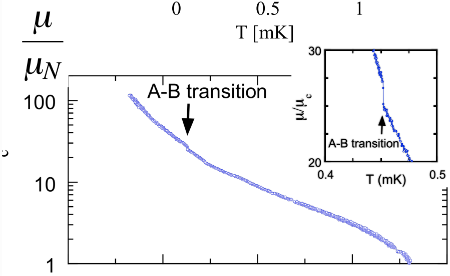
Run 2 $\vec{\ell} = -\hat{\mathbf{z}}$

$$\frac{|I_R - I_L|}{I_R + I_L} \approx 6\,\%$$

H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

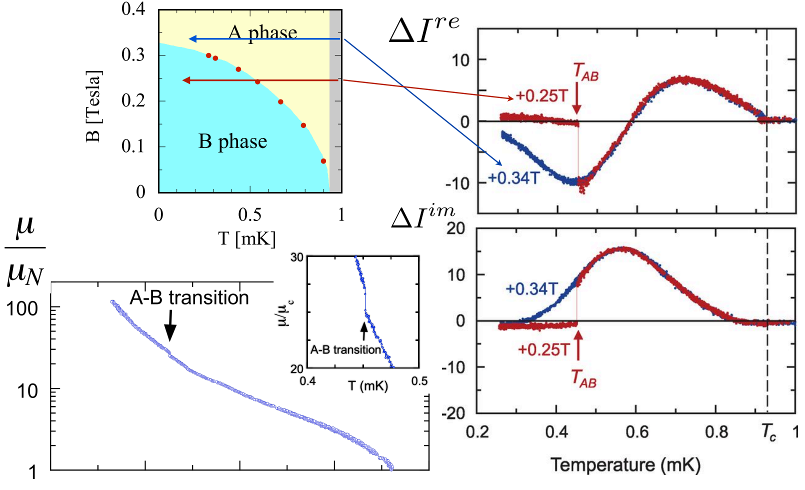
Zero Transverse Current in ^{3}He-B (*T - symmetric phase*)





H. Ikegami, Y. Tsutsumi, K. Kono, Science 341, 59-62 (2013)

Zero Transverse Current in ^{3}He-B (*T - symmetric phase*)



H. Ikegami, Y. Tsutsumi, K. Kono, **Science 341, 59-62 (2013)**

Summary

- Phase Diagram of ^{3}He Films
- Topology of Superfluid ^{3}He-A
- Chiral Edge States in 2D ^{3}He-A
- Edge Currents and $\mathsf{L}_z(T) - L_z(0) \sim -T^2$: $R \gg \xi_0$
- Edge Currents $\leftrightarrow$ Non-Extensive L_z
- Detection of Chiral Currents via e^- Mobility
- Strong Confinement: $D \lesssim 10\xi_0$
- Edge State Interference
- Chiral - Polar Transition
- Broken Translational Symmetry Phase