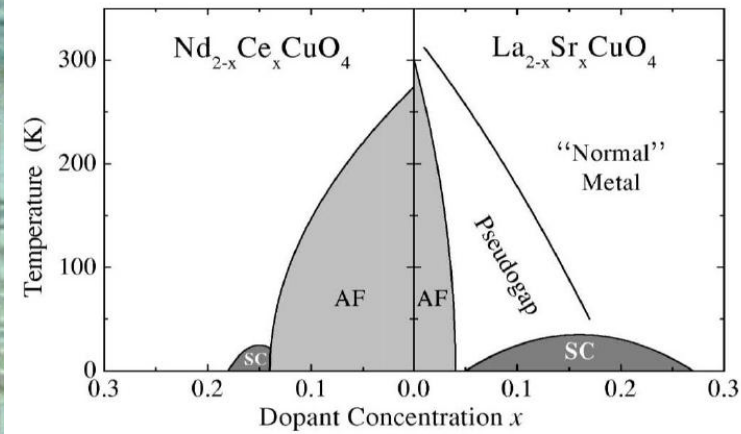


A New look at the Pseudogap Phase in the Cuprates.

Patrick Lee

MIT



Common themes:

1. Competing order.
2. superconducting fluctuations.
3. Spin gap: RVB.

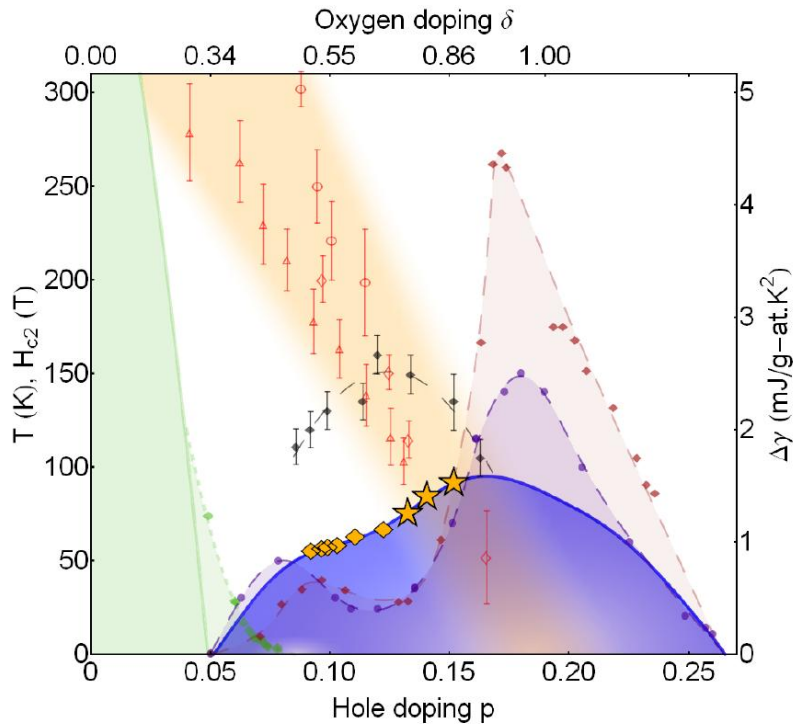
What is the elephant?

My answer:

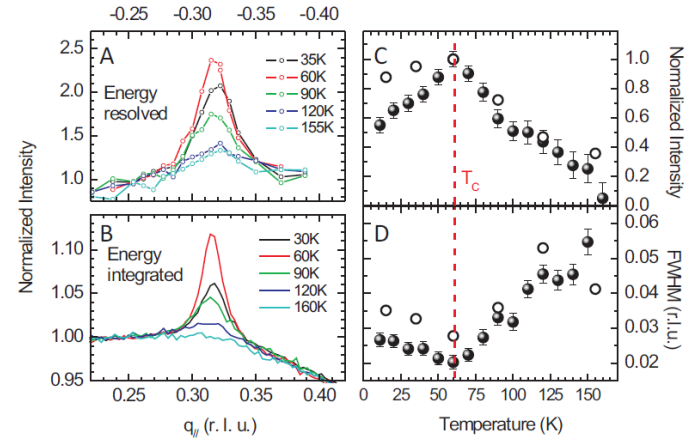
All of the above!

Competing order is a novel kind of fluctuating superconductor which has its microscopic origin in spin-charge separation.

Recent experiments show the appearance of charge order in the pseudo-gap phase.



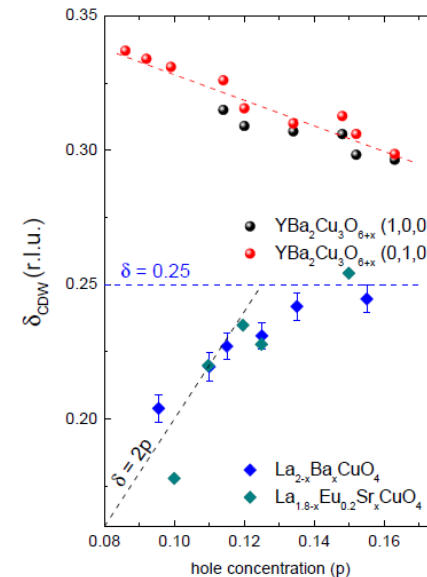
Charge density wave.
Medium range and quasi-static.
Competes with SC.



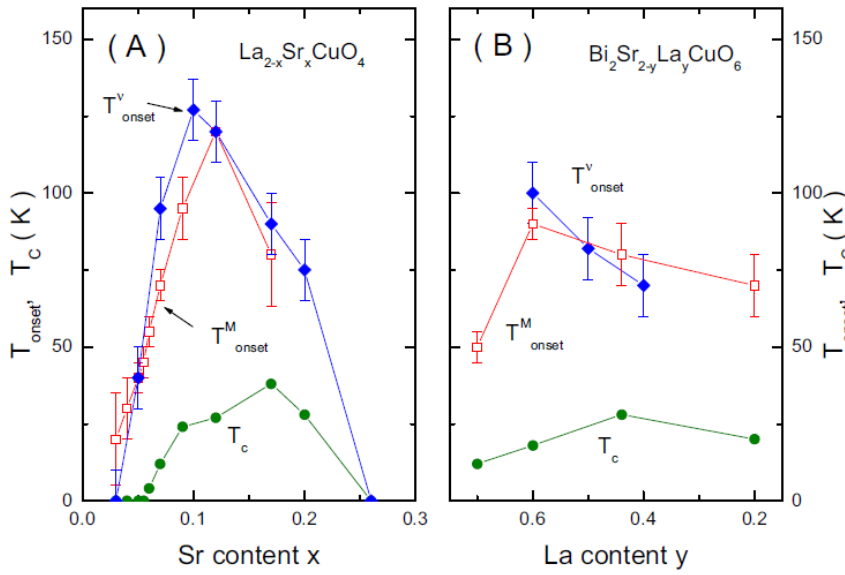
Ghiringhelli et al **Science** 337,821(2012)

Distinct from stripe in that the wave-vector decreases with increasing doping. (as seen by Hudson earlier in STM). Also spin fluctuation is at a different wave-vector.

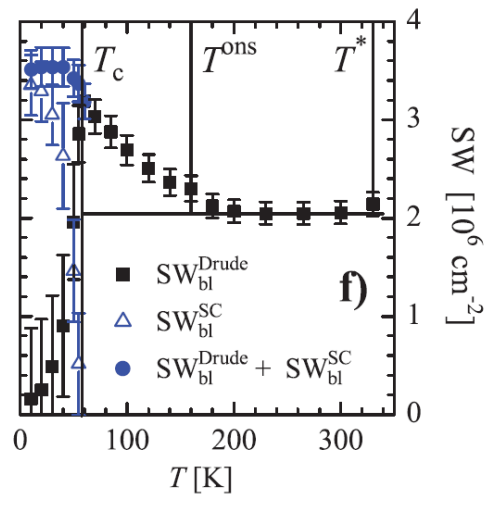
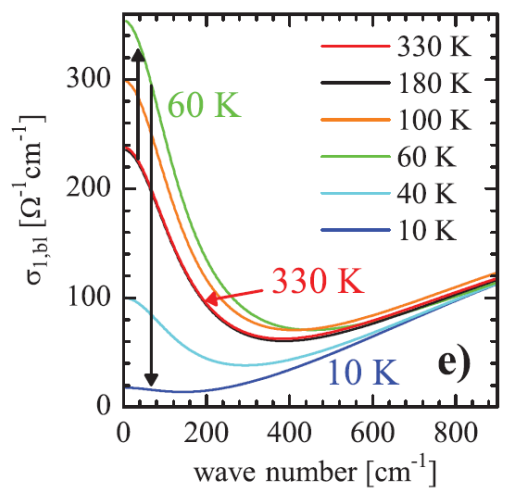
I will not discuss intra unit cell order, such as orbital current.



On the other hand, there is evidence for fluctuation SC much above T_c .

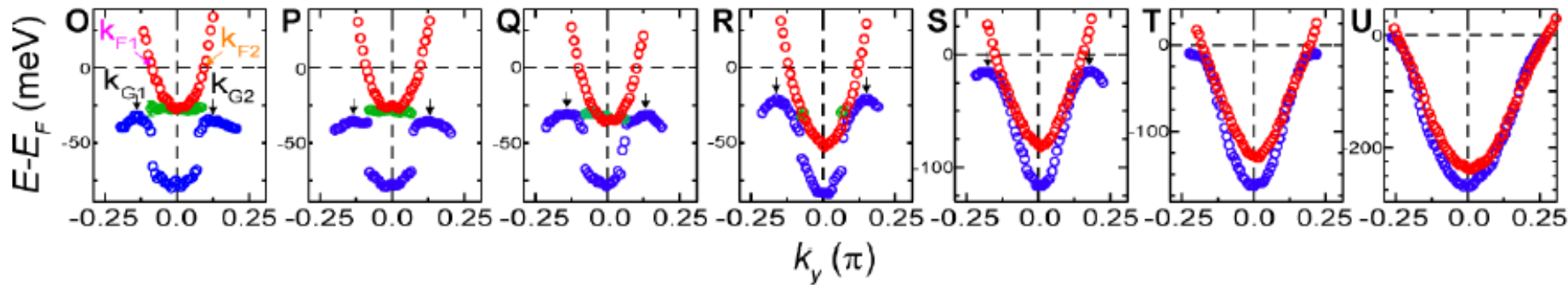


Lu Li et al PRB 2010 observes fluctuation diamagnetism.

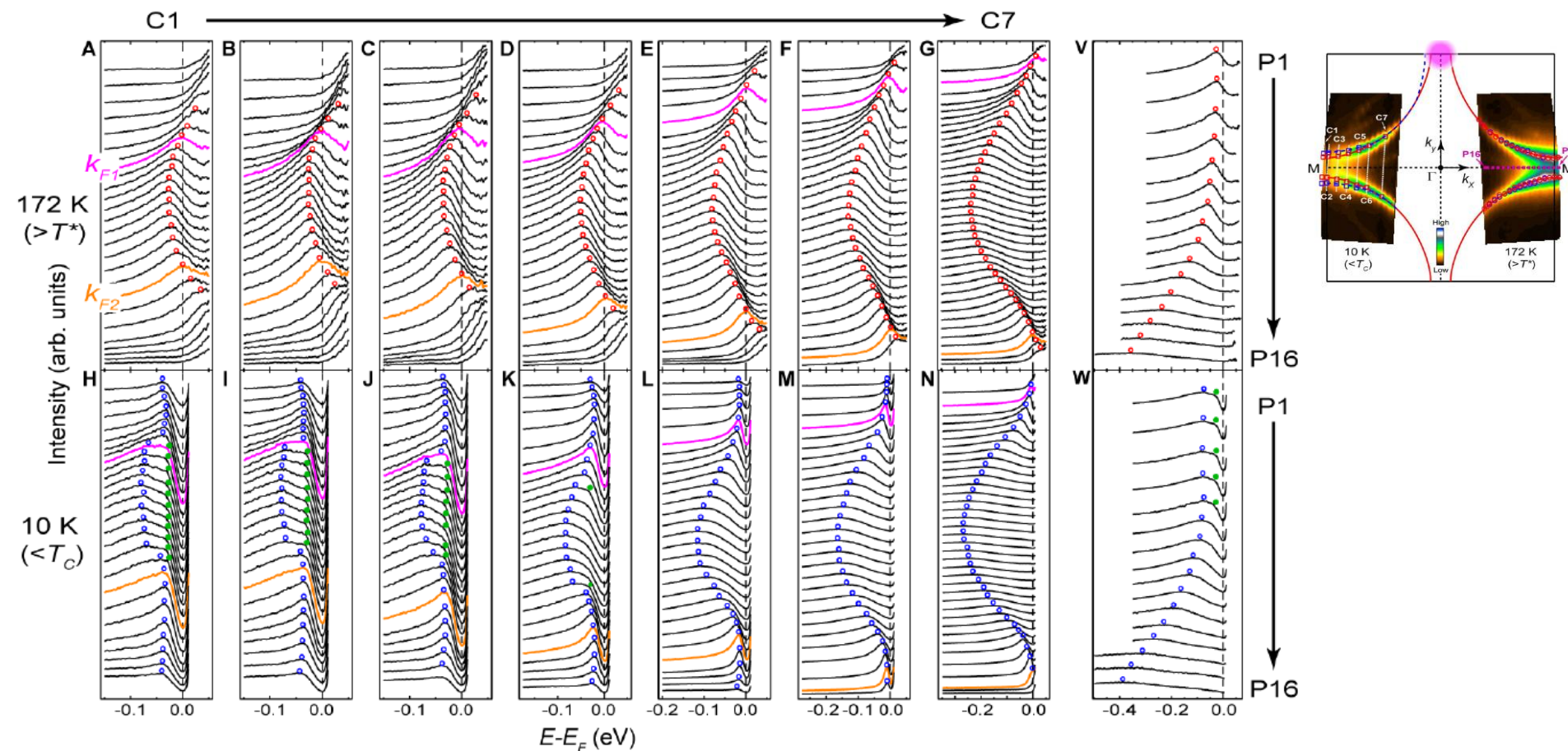


Dubrovka...C. Bernhard, PRL 2011 found that the Drude weight for conductivity between members of bi-layers in YBCO to **increase** with decreasing T starting at 180K. For gap due to charge order, we expect decrease.

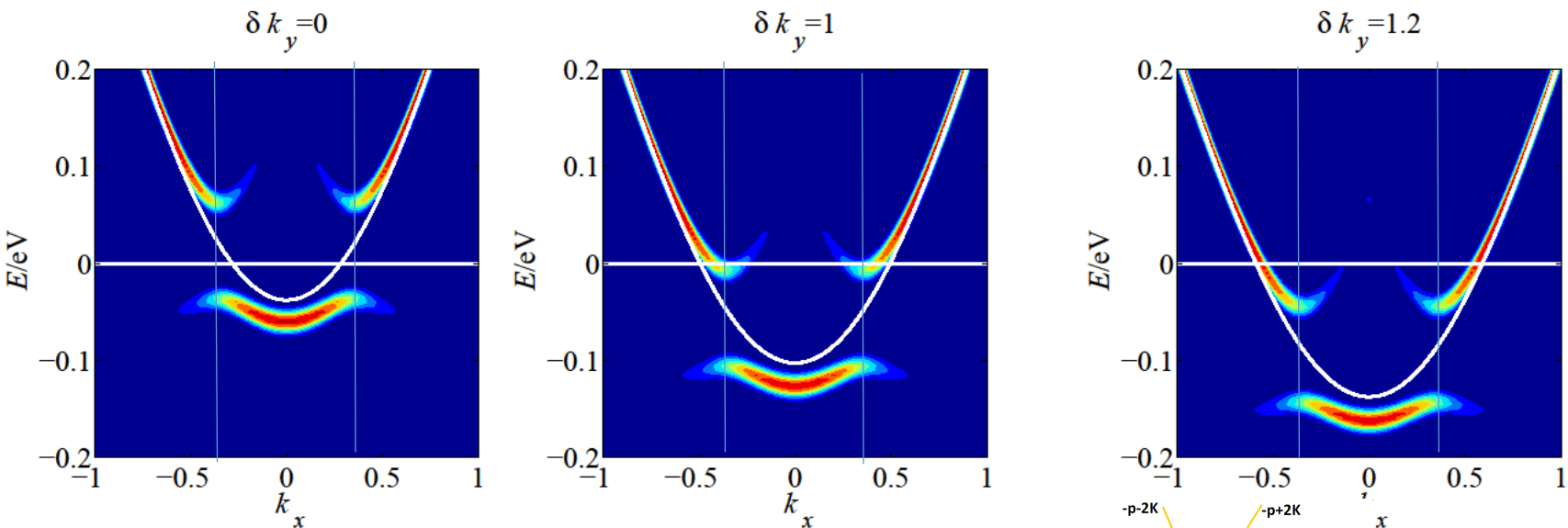
ARPES on single layer Bi2201, R-H He et al Science 2011.



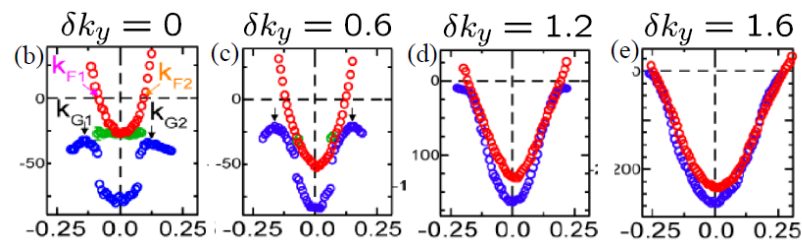
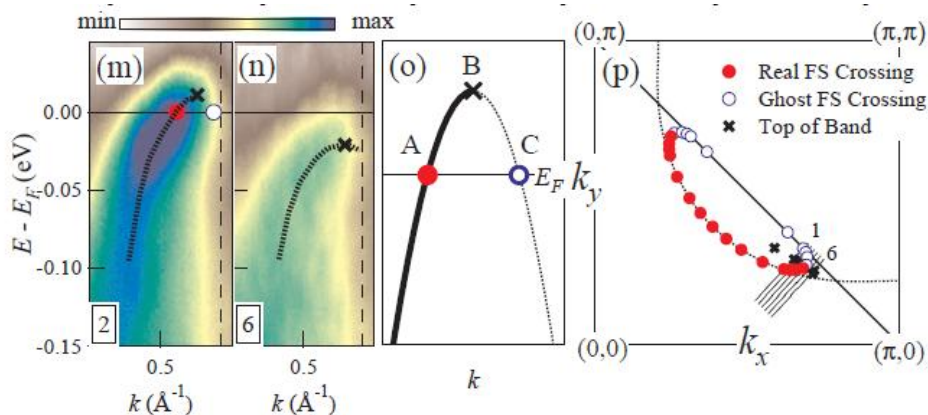
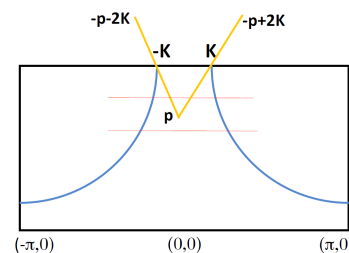
Concluded that competing order is not pairing, but CDW does not work either.



Why CDW mean field theory cannot explain ARPES data?



Choose CDW wave-vector = 1.2 Q,
 where $Q=2K$ is the FS spanning vector.
 Location of gap is fixed and the Fermi
 level crossing is formed by states
 coming down in energy.



Bi2212 UD65 measured at 140K, from
 H-B Yang et al, PRL 2011.
 Data shows clearly that the gap is
above the Fermi level crossing.

t-J model: the simplest model which describes
the strong correlation physics of the Mott insulator

$$H = \sum_{\langle i,j \rangle} J \left(\mathbf{S}_i \cdot \mathbf{S}_j - \frac{1}{4} n_i n_j \right) - \sum_{ij} t \left(c_{i\sigma}^\dagger c_{j\sigma} + \text{H.c.} \right)$$

No double occupation allowed!

Enforce by decomposing electron into fermion and boson

$$c_{i\sigma}^\dagger = f_{i\sigma}^\dagger b_i$$

$$f_{i\uparrow}^\dagger f_{i\uparrow} + f_{i\downarrow}^\dagger f_{i\downarrow} + b_i^\dagger b_i = 1.$$

Enforce constraint with Lagrange multiplier λ

$$S_i \cdot S_j = -\frac{1}{4} f_{i\alpha}^\dagger f_{j\alpha} \underbrace{\langle f_{j\beta}^\dagger f_{i\beta} \rangle}_{\chi_{ij}} - \frac{1}{4} (f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger - f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger) \underbrace{\langle f_{j\downarrow} f_{i\uparrow} - f_{j\uparrow} f_{i\downarrow} \rangle}_{\Delta_{ij}} + \frac{1}{4} (f_{i\alpha}^\dagger f_{i\alpha})$$

$$Z = \int d\chi d\lambda df df^\dagger e^{-S}$$

$$S = \int d\tau \left[\sum_i f_{i\alpha}^\dagger \partial_\tau f_{i\alpha} + i\lambda_i (f_{i\alpha}^\dagger f_{i\alpha} - 1) \right. \\ \left. \sum_{ij} 2J |\chi_{ij}|^2 + J (\chi_{ij} f_{j\alpha}^\dagger f_{i\alpha} + h.c.) \right]$$

The phase of χ_{ij} becomes a compact gauge field a_{ij} on link ij and $i\lambda$ becomes the time component.

Compact U(1) gauge field coupled to fermions.

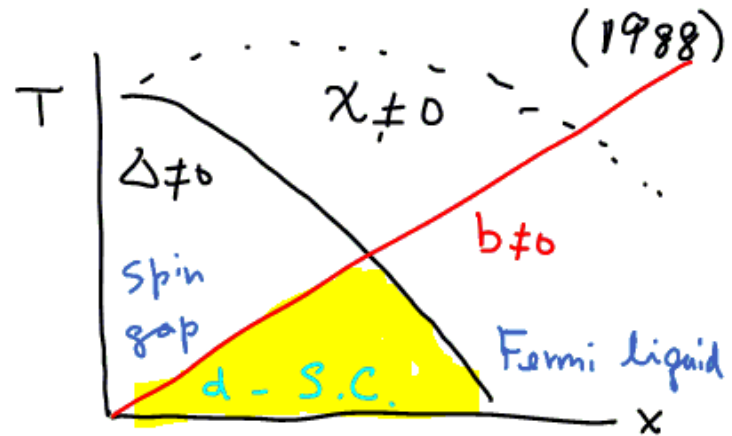
$$\begin{aligned}
\mathbf{S}_i \cdot \mathbf{S}_j &= -\frac{1}{4} \langle f_{i\sigma}^\dagger f_{j\sigma} \rangle f_{j\beta}^\dagger f_{i\beta} \\
&- \frac{1}{4} \left(f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger - f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger \right) \langle (f_{j\downarrow} f_{i\uparrow} - f_{j\uparrow} f_{i\downarrow}) \rangle \\
&+ \frac{1}{4} \left(f_{i\alpha}^\dagger f_{i\alpha} \right).
\end{aligned}$$

Mean field theory (Kotliar and Liu, 1988) predicted d-SC and spin gap

RVB decoupling:

$$\chi_{ij} = \langle f_{i\alpha}^\dagger f_{j\alpha} \rangle$$

$$\Delta_{ij} = \langle f_{i\uparrow}^\dagger f_{j\downarrow}^\dagger - f_{i\downarrow}^\dagger f_{j\uparrow}^\dagger \rangle$$

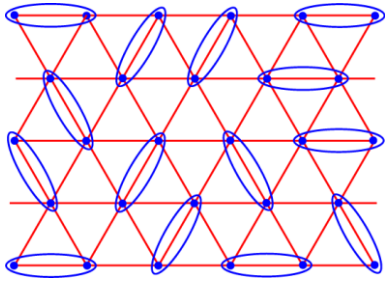


Pseudo-gap is described by spinon pairing in the mean field theory.

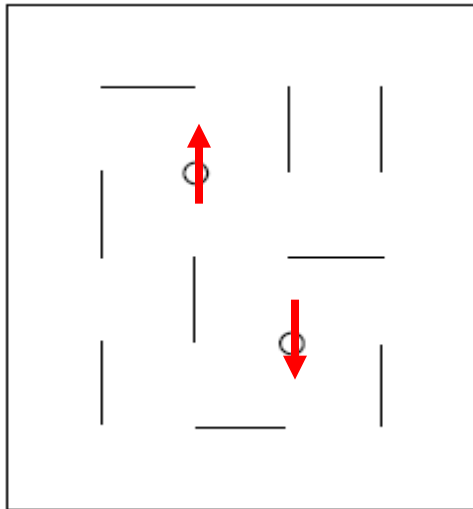
Our new proposal is that d wave pairing is initially usurped by another pairing state.

Consider the “simpler” problem of undoped Mott insulator.

Spin liquid: destruction of Neel order due to quantum fluctuations.



Spin liquid is more than the absence of Neel order.



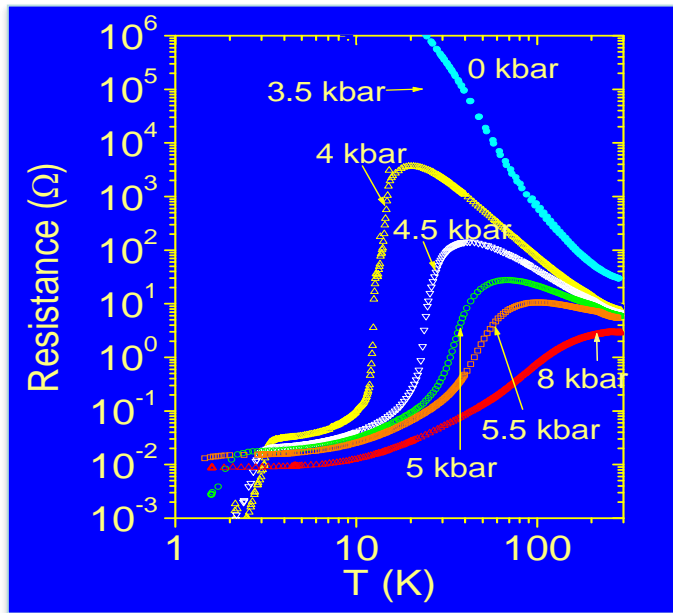
In 1973 Anderson proposed a spin liquid ground state (RVB) for the triangular lattice Heisenberg model. It is a linear superposition of singlet pairs. (not restricted to nearest neighbor.)

New emergent property of spin liquid: Excitations are spin $\frac{1}{2}$ particles (called spinons), as opposed to spin 1 magnons in AF. These spinons may even form a Fermi sea.

Emergent gauge field. (U(1), Z₂, etc .)

Topological order (X. G. Wen) in case of gapped spin liquid:
ground state degeneracy,
entanglement entropy.

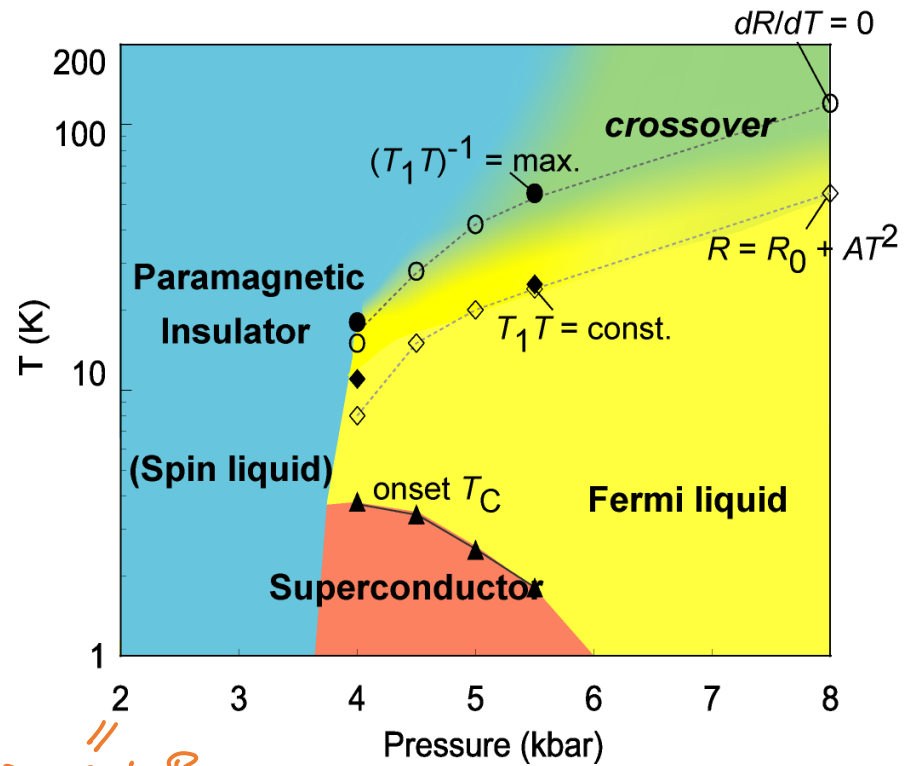
40 frustrating years later, we may finally have several examples of spin liquid in higher than 1 dimension!



$$t'/t=1.06$$

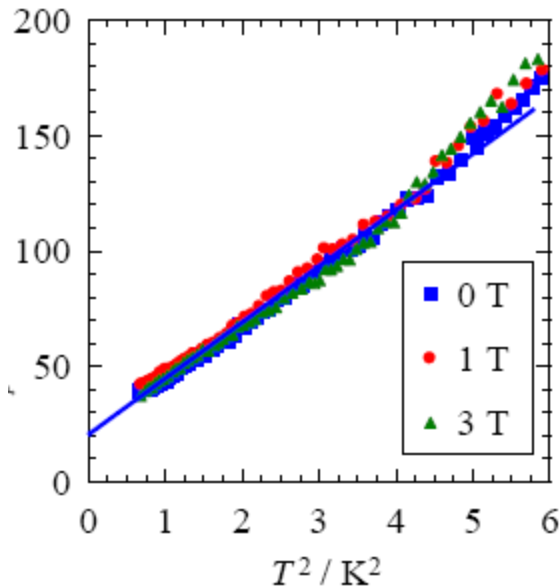
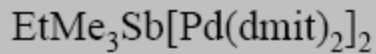
No AF order down to 35mK.
 $J=250\text{K}$.

200 MPa



Q2D spin liquid
 $\kappa\text{-Cu}_2(\text{CN})_3$

Evidence for Fermionic spinons
 from linear T specific heat and
 thermo-conductivity.



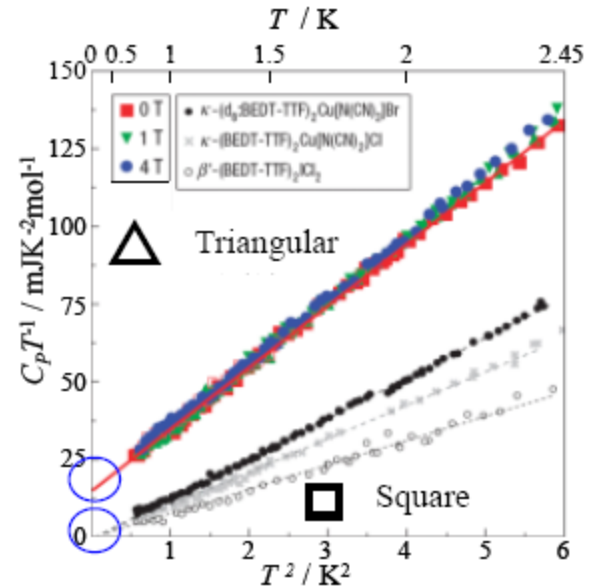
$$C_p T^{-1} = \gamma + \beta T^2$$

γ \rightarrow T -linear term
 β \rightarrow Debye term

$\gamma \approx 20$ (0T), 24 (3T) $\text{mJK}^{-2}\text{mol}^{-1}$
 $\beta \approx 24 \text{ mJK}^{-4}\text{mol}^{-1}$

Existence of T -linear term

Difference of Band-width in the spinon excitations

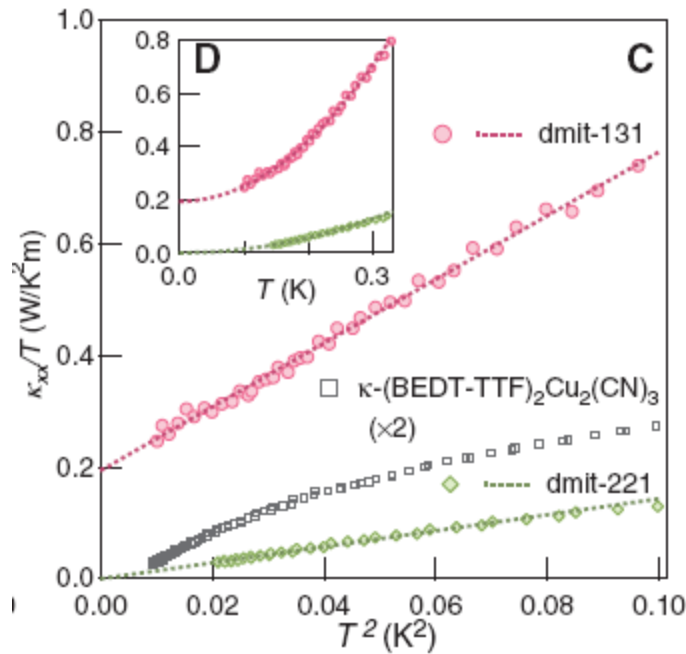


$\gamma = 15 \pm 5 \text{ mJK}^{-2}\text{mol}^{-1}$
 $\beta = 23.6 \text{ mJK}^{-4}\text{mol}^{-1}$

Gapless excitations

Spin Liquid is realized in
 EtMe₃Sb[Pd(dmit)₂]₂

Thermal conductivity of dmit salts.



$$\kappa_{xx}^{\text{spin}} = C_s v_s l_s / 3$$

mean free path l_s reaches 500 inter-spin spacing.

M. Yamashita et al, Science 328, 1246 (2010)

However, ET salt seems to develop a small gap below 0.2 K.

Evidence for phase transition at 6K in ET.

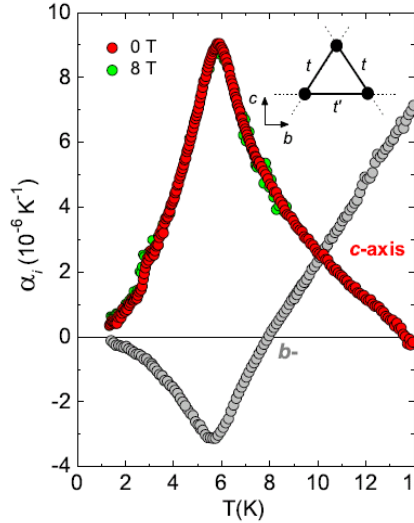
Spinon pairing?

U(1) breaks down to Z2 spin liquid. The gauge field is gapped.

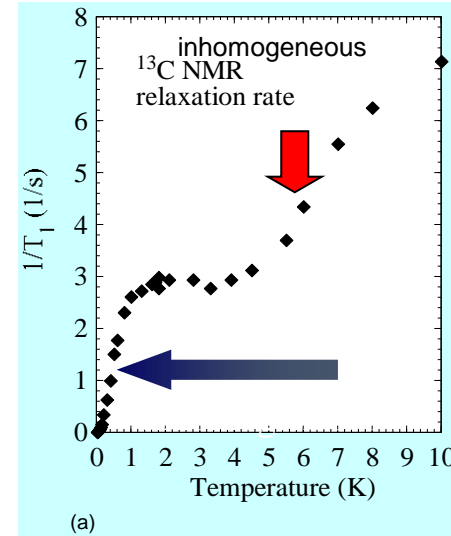
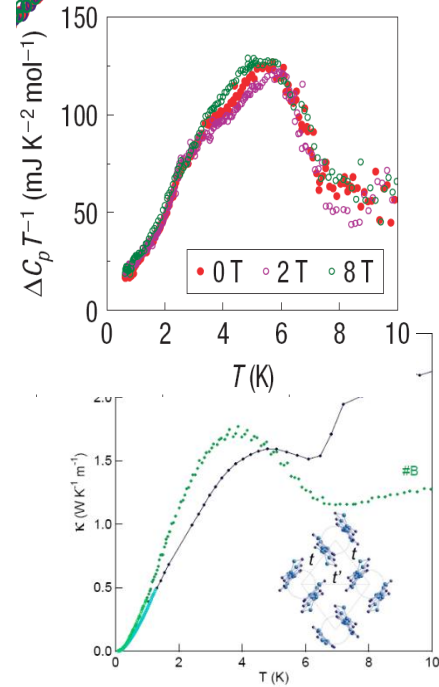
What kind of pairing?

One candidate is d wave pairing. With disorder the node is smeared and gives finite density of states. κ/T is universal constant (independent on impurity conc.) However, singlet pairing seems ruled out by smooth behavior of spin susceptibility up to 30T.

More exotic pairing? Amperian pairing, Sung-Sik Lee, PL, T. Senthil. (PRL 2007).



Thermal expansion coefficient
Manna et al., *PRL* **104** (2010) 016403

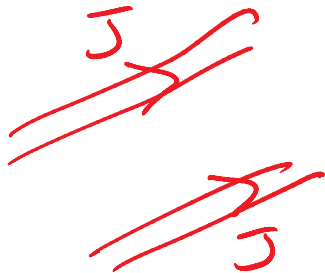
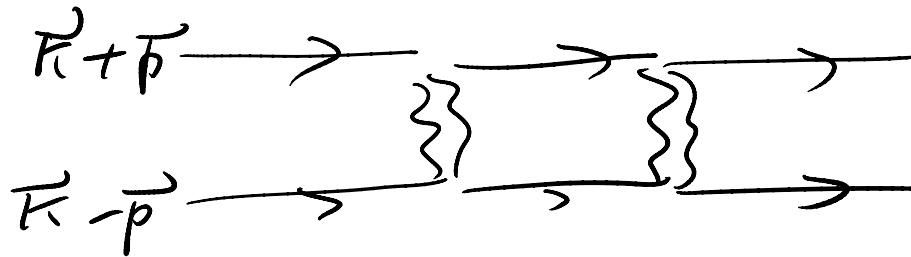


NMR Relaxation rate
Shimizu et al., *PRB* **70** (2006) 060510

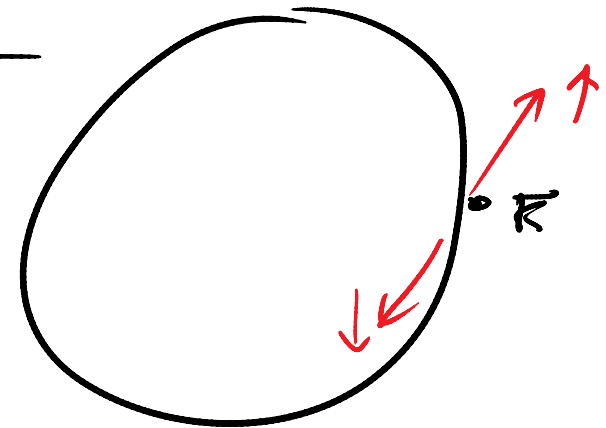
Transvers Gauge field propagator (gauge magnetic field fluctuation) is singular.

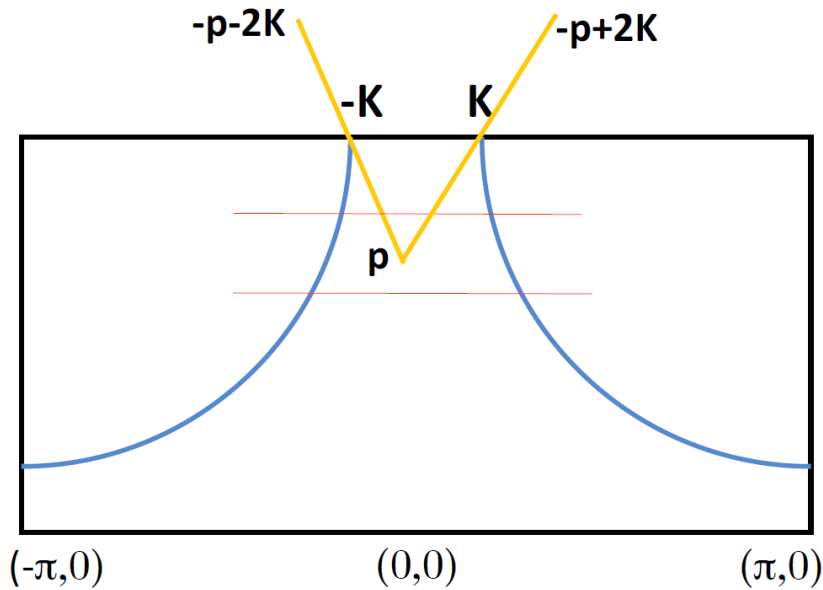


Amperean Pairing. (Proposed by Sung-Sik Lee, PAL and T. Senthil, PRL 2007))
 Not suppressed by Zeeman field.



Ampere effect
 wires attract.





$$S_{int} = -\frac{1}{2v\beta} \sum_{p_1 p_2, q} D(q) (\mathbf{v}_{p_1} \times \hat{q}) \cdot (\mathbf{v}_{p_2} \times \hat{q}) f_{p_1+q, \sigma}^\dagger f_{p_2-q, \sigma'}^\dagger f_{p_2, \sigma'} f_{p_1, \sigma}$$

These considerations give us a microscopic motivation.

We will proceed **phenomenologically** from this point on. We assume mean field PDW order and compare with experiment.

$$S^{\text{MF}} = \Delta_{2\mathbf{K}}^*(\mathbf{k}) f_{\mathbf{k}\uparrow} f_{-\mathbf{k}+2\mathbf{K}\downarrow} + c.c. + \Delta_{-2\mathbf{K}}^*(\mathbf{k}) f_{\mathbf{k}\uparrow} f_{-\mathbf{k}-2\mathbf{K}\downarrow} + c.c.$$

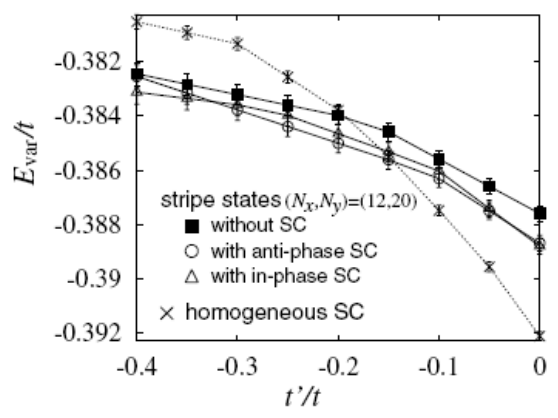
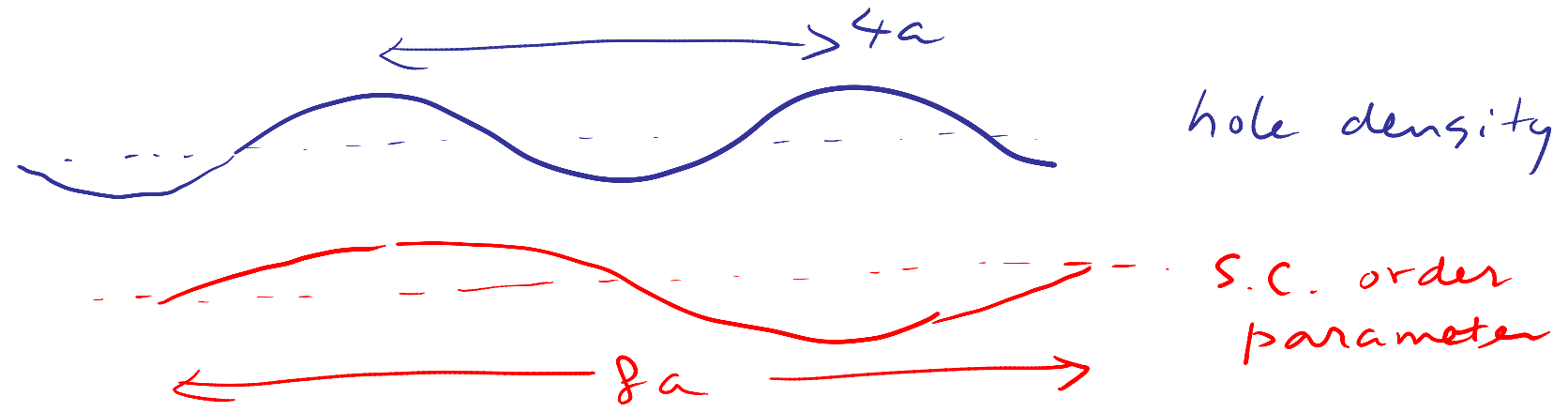
Pair carries total momentum $Q=2K$, due to Umklapp.

Pair density wave (PDW). Earlier example is LOFF state, but that still pairs state on the opposite side of FS.

Another example is stripe-PDW.

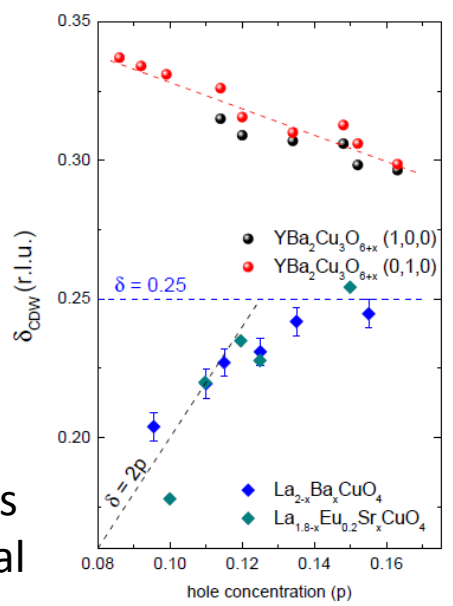
Tranquada's group discovered a novel form of 2D superconductivity in LBCO which onset at 40K, much above T_c .

Berg et al interpret this as anti-phase SC.



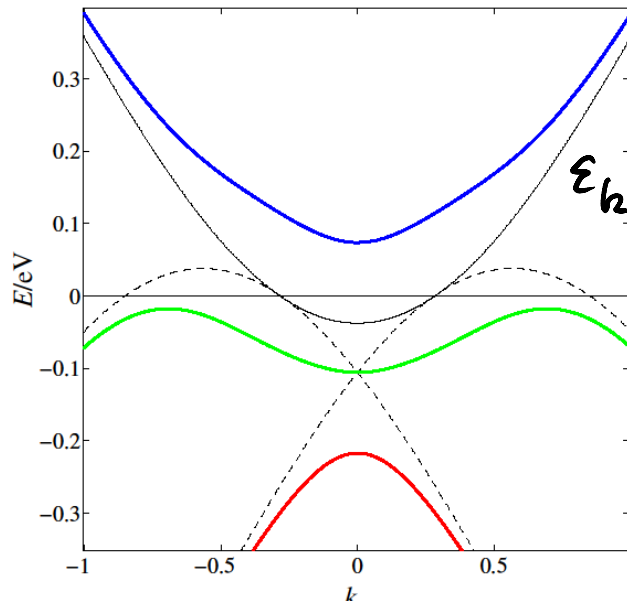
Himeda, Kato and Ogata, PRL 2002.

The relative stability of anti-phase SC was predicted 10 years earlier by Himeda et al using projected fermionic wavefunctions.

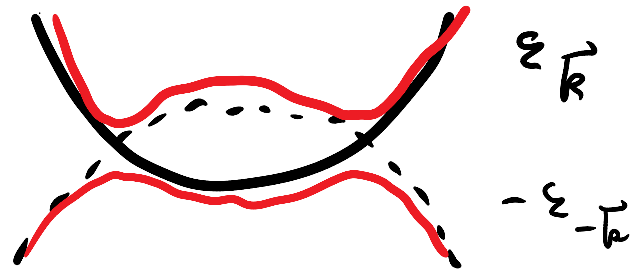


Similar phenomenology and Landau theory, but stripe-PDW is driven by SDW has been discussed only near 1/8 filling whereas the pair momentum in our case is $2k_F$ and spin fluctuation is at an unrelated wavevector.

$$k_y = \pi$$



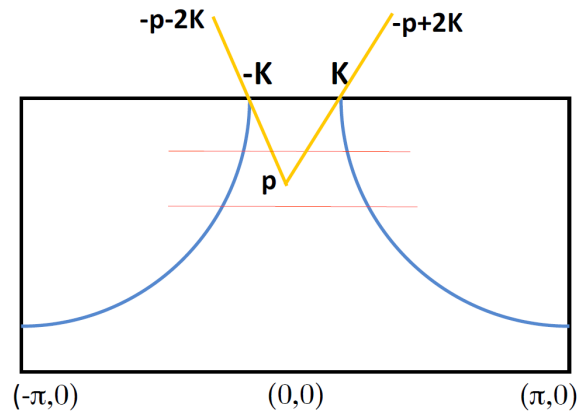
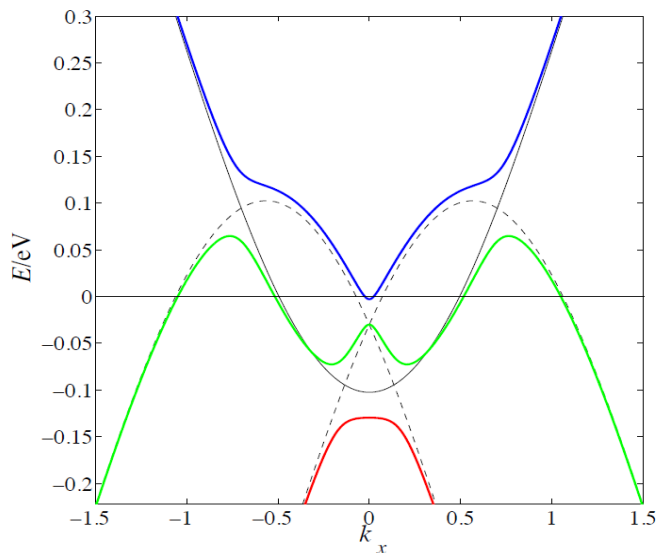
$$-\xi_{-k+Q}$$



Spectrum not particle-hole symmetric!

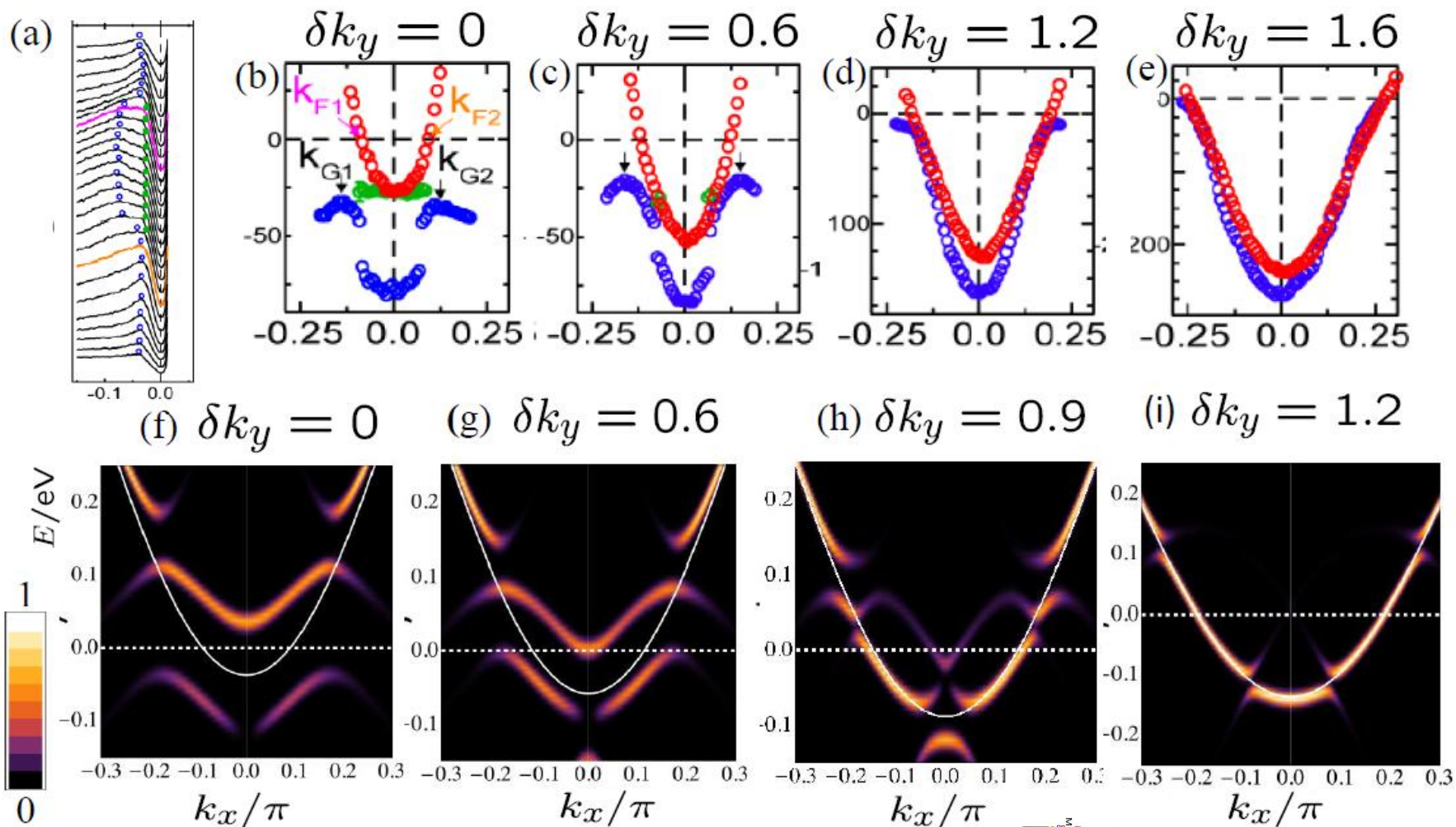
$$E_{k\uparrow}^{\pm} = \frac{1}{2} (\xi_k - \xi_{-k+Q}) \pm \sqrt{\frac{1}{4} (\xi_k + \xi_{-k+Q})^2 + |\Delta_Q|^2}$$

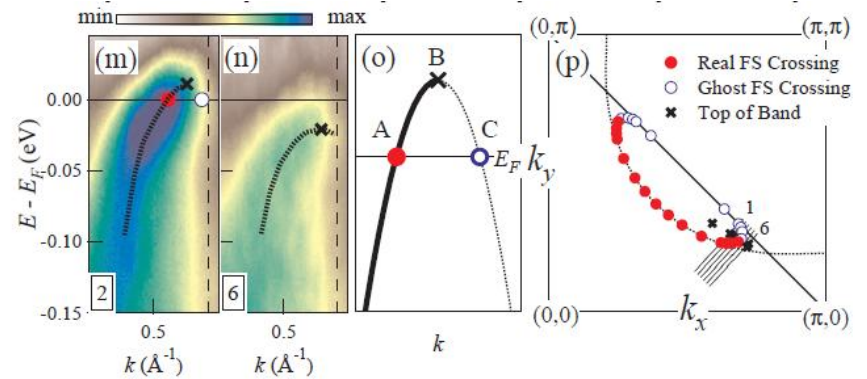
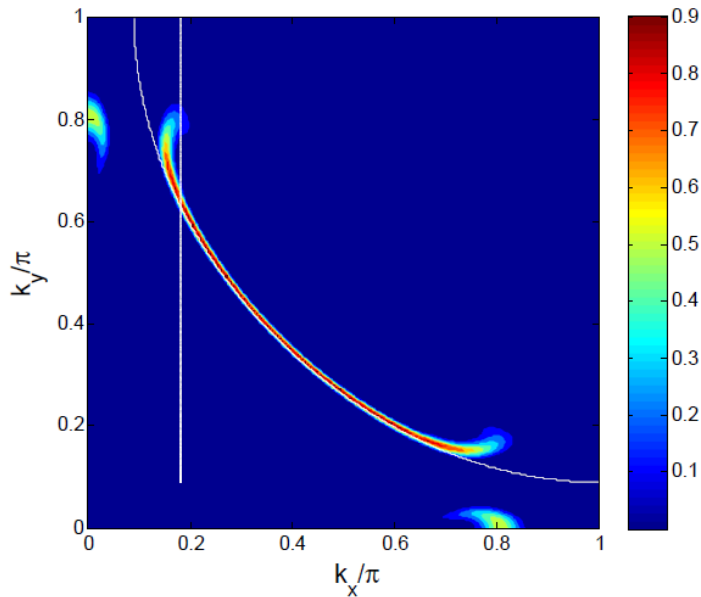
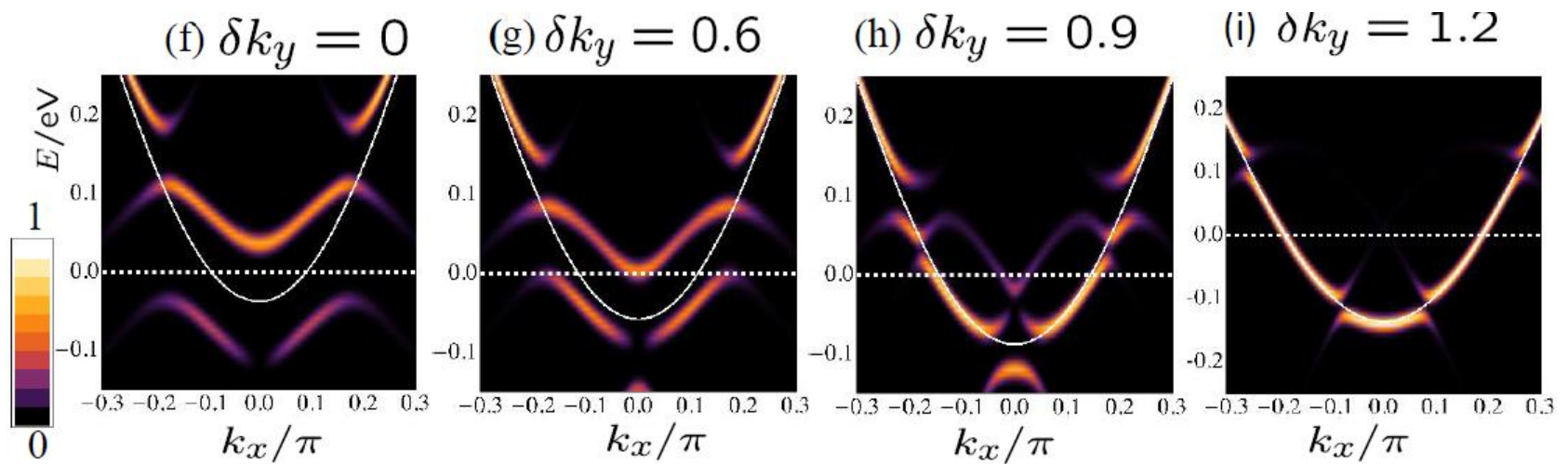
$$k_y = \pi - 1$$



Forms Fermi arc.

Proceed phenomenologically: assume a reasonable form of $\Delta_Q(k)$.
 Self consistent mean field solution has been found by Maksym Serbyn.





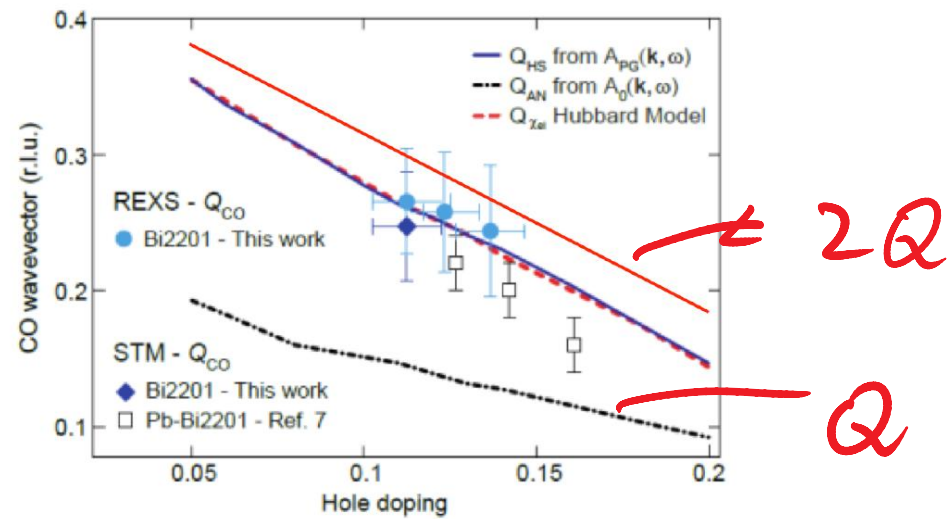
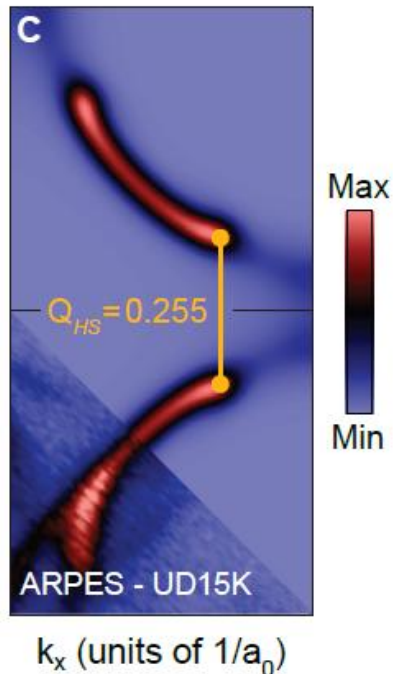
Bi2202 UD65 measured at 140K,
 from H-B Yang et al, PRL 2011.

Yields Fermi arcs and the correct gap closing from below at the mean field level.

$$P_{2Q} = \sum_{k,\sigma} c_{k-2Q,\sigma}^\dagger c_{k\sigma} \quad \langle P_{2Q} \rangle \neq 0$$

$$F = \Delta_Q^* \Delta_{-Q} P_{2Q}$$

Subsidiary order. Prefer bi-directional PDW and CDW, ie checkerboard.



Comin et al, cond-mat 2013.

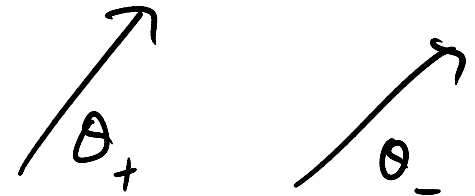
$$\Delta_{\pm Q_2} = |\Delta| e^{i\theta_{\pm Q_2}}$$

$$\alpha = x, y$$

$$\rho_{2Q_2} = |\rho| e^{i\phi_2}$$

$$\Delta_Q^* \Delta_{-Q} \rho_{2Q} \Rightarrow \cos(\theta_{Q_\alpha} - \theta_{-Q_\alpha} - \phi_\alpha)$$

The PDW state has small superfluid density and is subject to large phase fluctuations due to vortex pair creation. (BKT) We assume PDW has short range order only in most of the phase diagram.



It is possible to have strong fluctuation in total phase and weak fluctuation in relative phase. Then a (short range) order PDW and a **long range ordered CDW** is possible. In practice CDW order will be limited by disorder.

Handwritten notes:
 (ϕ_1, ϕ_2) breaks inversion.
 (ϕ_1, ϕ_2) stacking
 $(-\phi, -\phi)$

Berg, Fradkin and Kivelson, Nature Physics (2009)

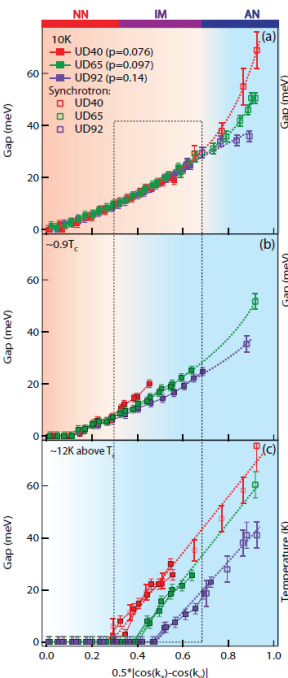
The PDW state is a competing phase to d-wave with higher energy, but it has higher entropy due to the Fermi arcs, and therefore has lower Free energy than d-wave above some finite T.

At T_c , d-wave SC appears and co-exists with PDW. This explains the hybrid gap structure.

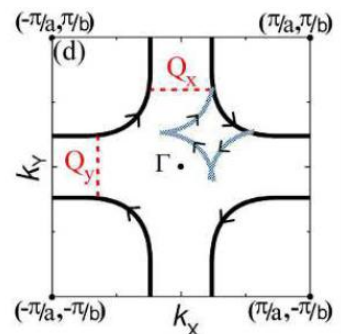
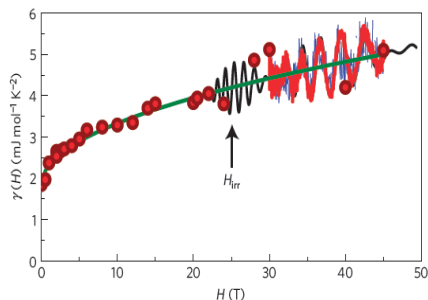
The PDW state is the normal state that sits in the vortex core. If energy cost is low, core size is large. The cores overlap in a relatively low magnetic field.

The “normal” state above “ H_{c2} ” is the short range ordered Amperian PDW state.

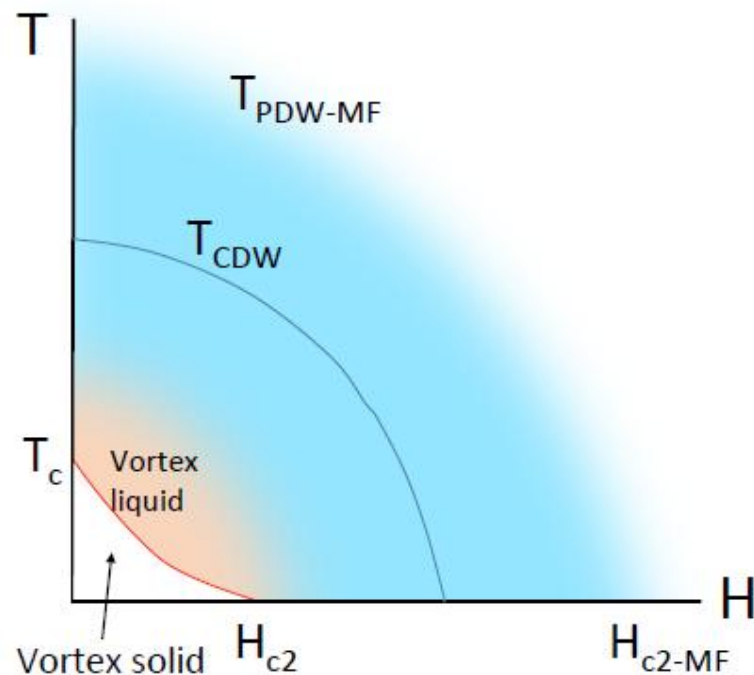
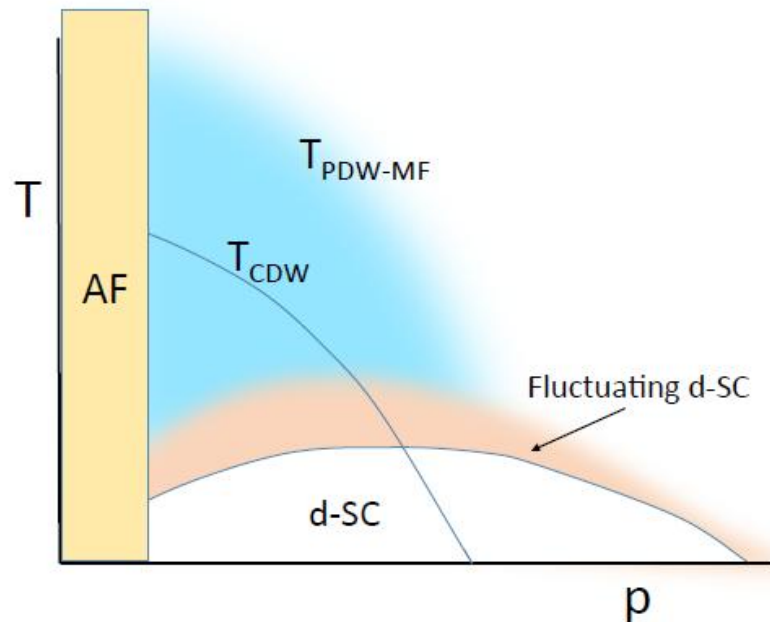
This explains why CDW is enhanced by magnetic field and the low “ H_{c2} ”. Quantum oscillation is in the short range ordered PDW state.



Vishik et al, PNAS 2012.



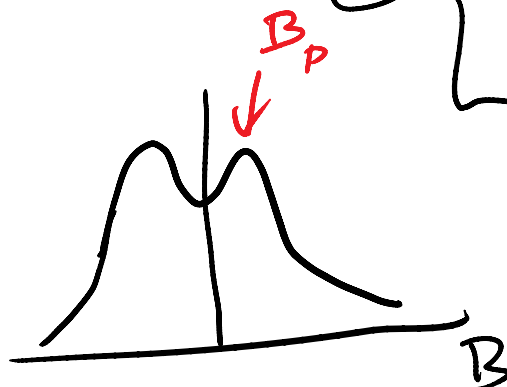
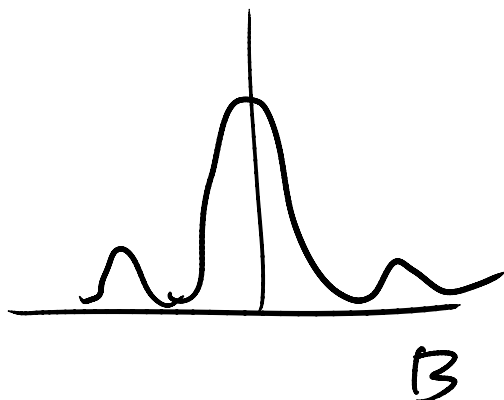
Harrison and Sebastian, PRL 2011.



Schematic phase diagram. PAL, Phys. Rev X4, 031017 (2014)

Direct detection of fluctuating PDW:
 Measure current vs B in a junction
 with HiTc SC.

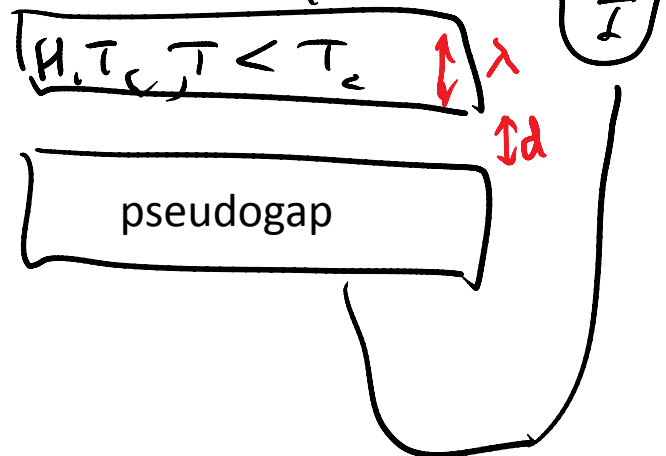
$$q = (2eB/\hbar c)(\lambda + d/2)$$



$T_c < T < T^*$

B

(O)



Peaks at B_p
 which
 corresponds
 to $q = \pm Q$

The same experiment can be done at low temperature with
 conventional SC junction.

<u>$\text{PrBa}_2\text{Cu}_{2.8}\text{Ga}_{0.2}\text{O}_7$ (PBCGO)</u>	underdoped
<u>$\text{NdBa}_2\text{Cu}_3\text{O}_7$ (NdBCO)</u>	insulator
$\text{YBa}_2\text{Cu}_{2.8}\text{Co}_{0.2}\text{O}_7$ (YBCO(Co))	Optimal doped

Indeed, such sandwich
 structure has been built and
 studied in zero field to probe SC
 fluctuations above T_c .

N. Bergeal et al, Nat Phys 4,608 (2008)

CDW is a subsidiary order, and is not responsible for the gap. Is there density waves at wave-vectors other than $2Q_1$ and $2Q_2$?

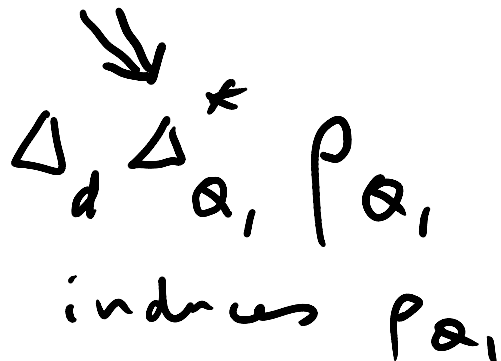
Agterberg and Tsunetsugu, Nat. Phys. 4, 639 (2008).

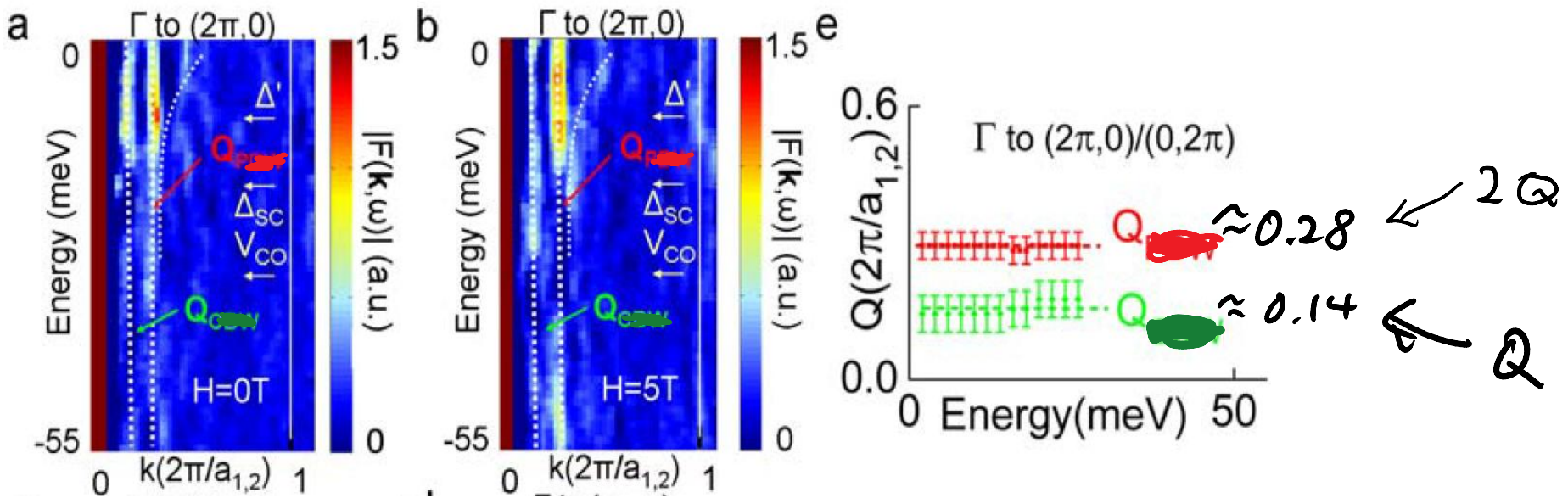
case	$\Delta_{Q_1} \Delta_{-Q_1} \Delta_{Q_2}^* \Delta_{-Q_2}^*$ pins $(\theta_{Q_1} + \theta_{-Q_1}) - (\theta_{Q_2} + \theta_{-Q_2})$	$Q_1 \pm Q_2$	with d wave pairing Q_1 and Q_2
4	0	CDW (not seen)	CDW at Q_1 and Q_2 is possible and strong.
5	π	Orbital magnetization wave, (search by neutron).	CDW at Q_1 and Q_2 is possible and weak.

Does d-wave order induce LRO of PDW? If so, any observable consequences?

$$(\Delta_d^*)^2 (\Delta_{Q_1} \Delta_{-Q_1} + \Delta_{Q_2} \Delta_{-Q_2}) + c.c.$$

pins the phase $\theta_{Q_\alpha} + \theta_{-Q_\alpha}$ (the pinning is much weaker in case 5 than in case 4), and since $\theta_{Q_\alpha} - \theta_{-Q_\alpha}$ is assumed to be locked already, the individual phases θ_{Q_α} will be pinned up to π . Thus, Δ_{Q_α} may have long-range order, in the phase where domain walls with π phase shifts are not important.





N. C. Yeh did STM on YBCO (optimally doped) Int J. Phys. B23,4543(2009) and saw CDW at 2 wave-vectors.

Recently CDW at 0.28 was seen in optimally doped Bi 2202 by RIX.

Conclusion:

The Amperean PDW explains naturally many of the anomalous observations associated with the pseudo-gap. Is this the long sought after “mother state”?

Need experiments!!!

Question:

Can it be stabilized by increasing interlayer coupling to suppress vortex fluctuations? Raise T_c up to T^* ????

