Intertwining fractionalization and broken symmetry: theories for the onset of magnetism in disordered and clean metals

Return of the Intertwined:
New Developments in Correlated Materials -
Online Reunion Conference
KITP Santa Barbara
July 29, 2020

Subir Sachdev

Talk online: sachdev.physics.harvard.edu

PHYSICS
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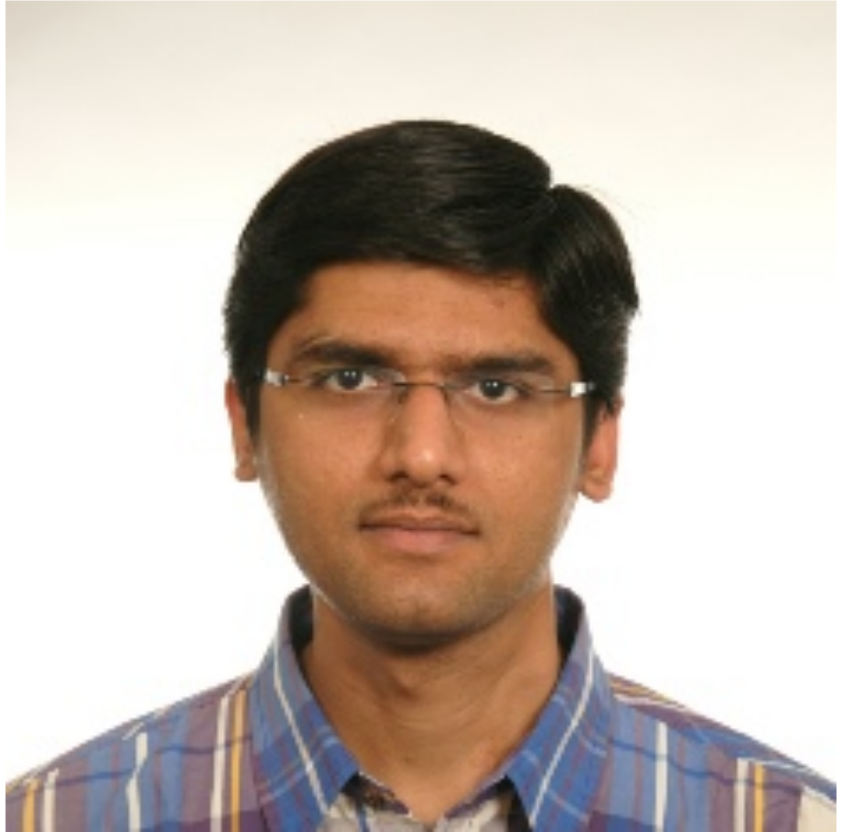
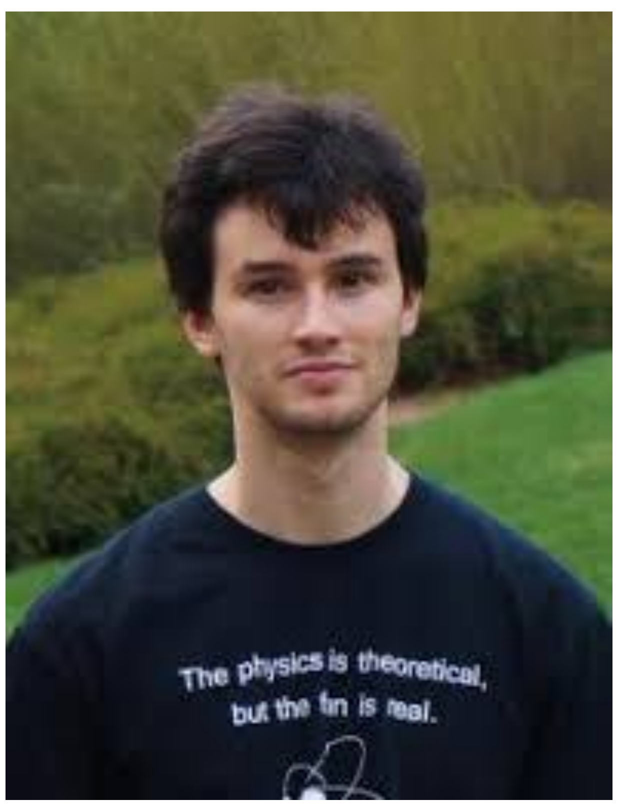
10, 021033 (2020)



Darshan Joshi

Henry Shackleton

Chenyuan Li



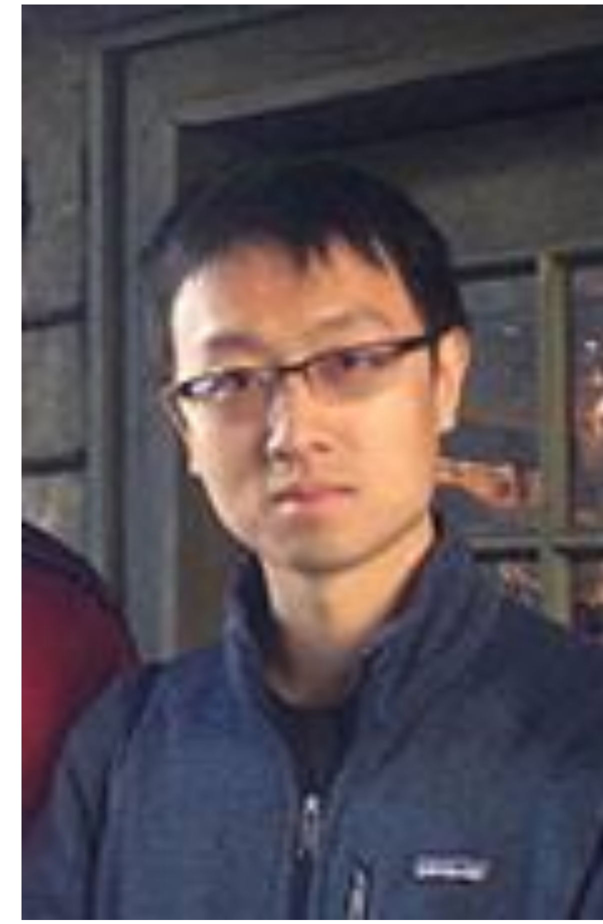
Grigory Tarnopolsky

Alexander Wietek

Antoine Georges

Haoyu Guo

Annals of Physics, **418**, 168202 (2020)



Yingfeu Gu

Yahui Zhang

Physical Review Research, **2**, 023172 (2020)

arXiv:2006.01140

1. All-to-all random Hubbard and *t*-*J* models

   Numerical results

2. Random *J* model (insulator)

   RG analysis and exact exponent

3. Random t-*J* model (metals)

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4. Non-random t-*J* model (metals)

   Ancilla qubits and ghost Fermi surfaces

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Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c^\dagger_{i\alpha} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} c^\dagger_{i\alpha} \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad n_{i\alpha} = c^\dagger_{i\alpha} c_{i\alpha},$$

$$J_{ij} \text{ random}, \overline{J_{ij}} = 0, \overline{J^2_{ij}} = J^2$$
$$t_{ij} \text{ random}, \overline{t_{ij}} = 0, \overline{t^2_{ij}} = t^2$$

$$U_H > 0 \text{ non-random}$$

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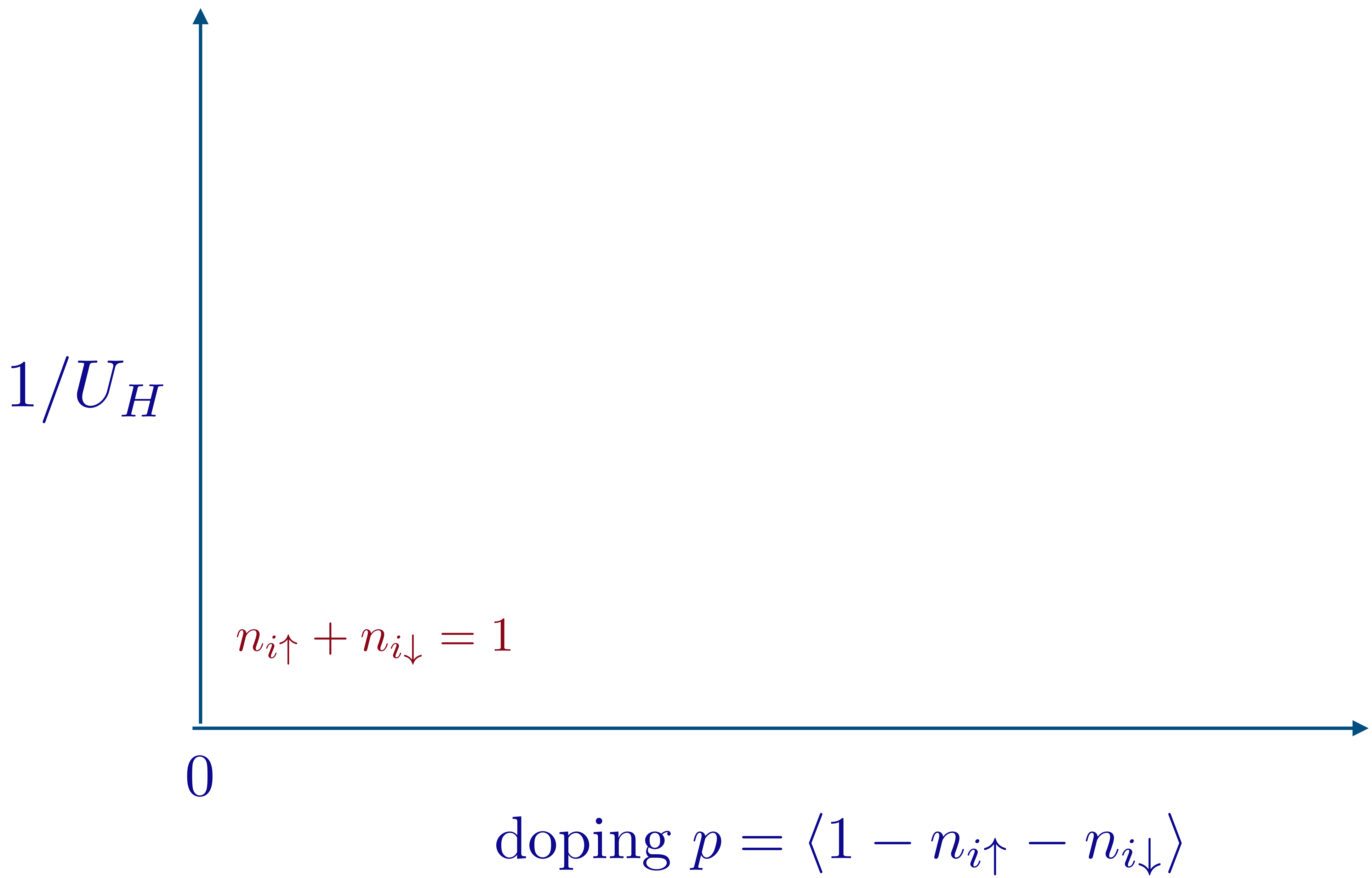

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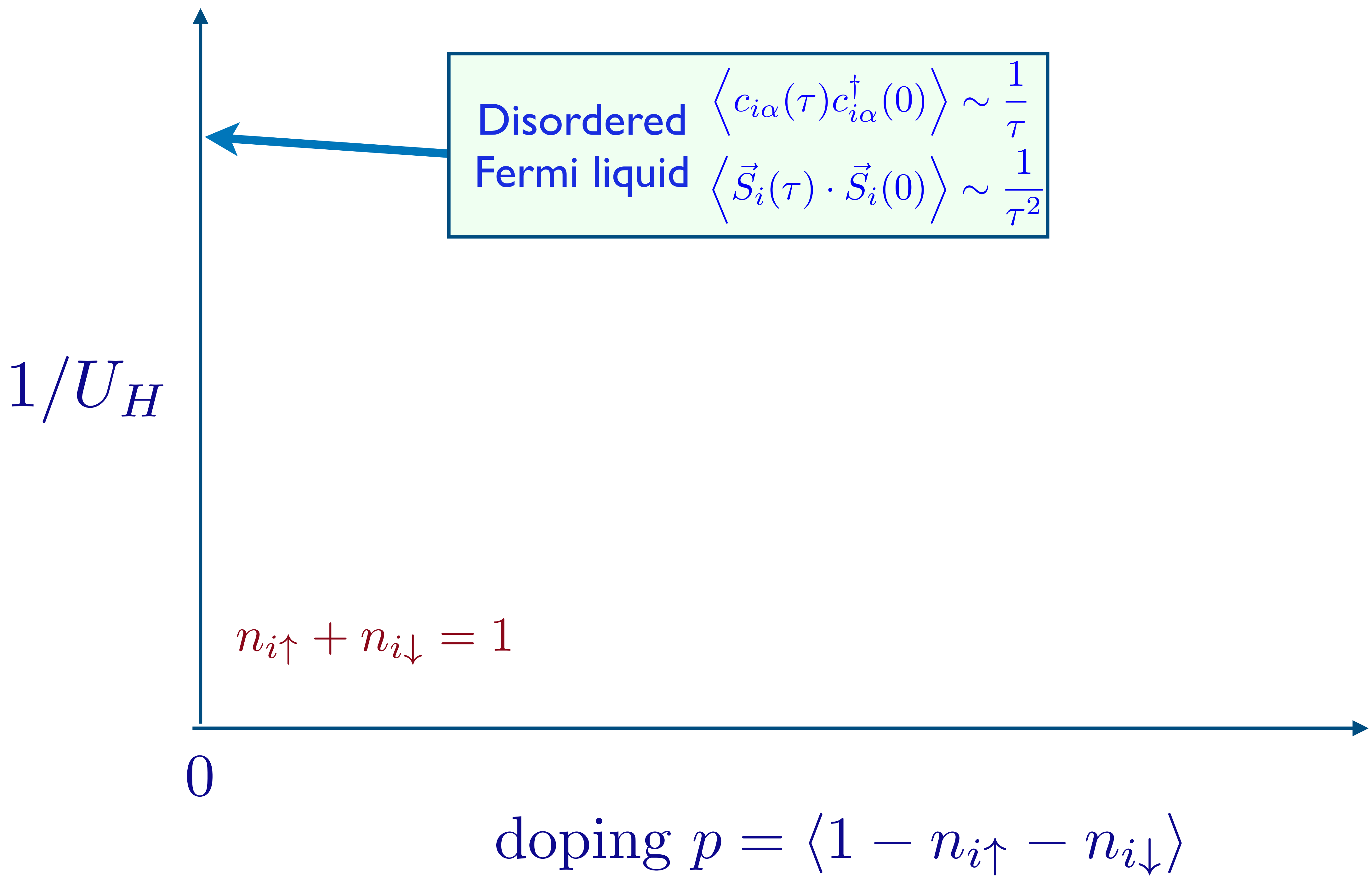

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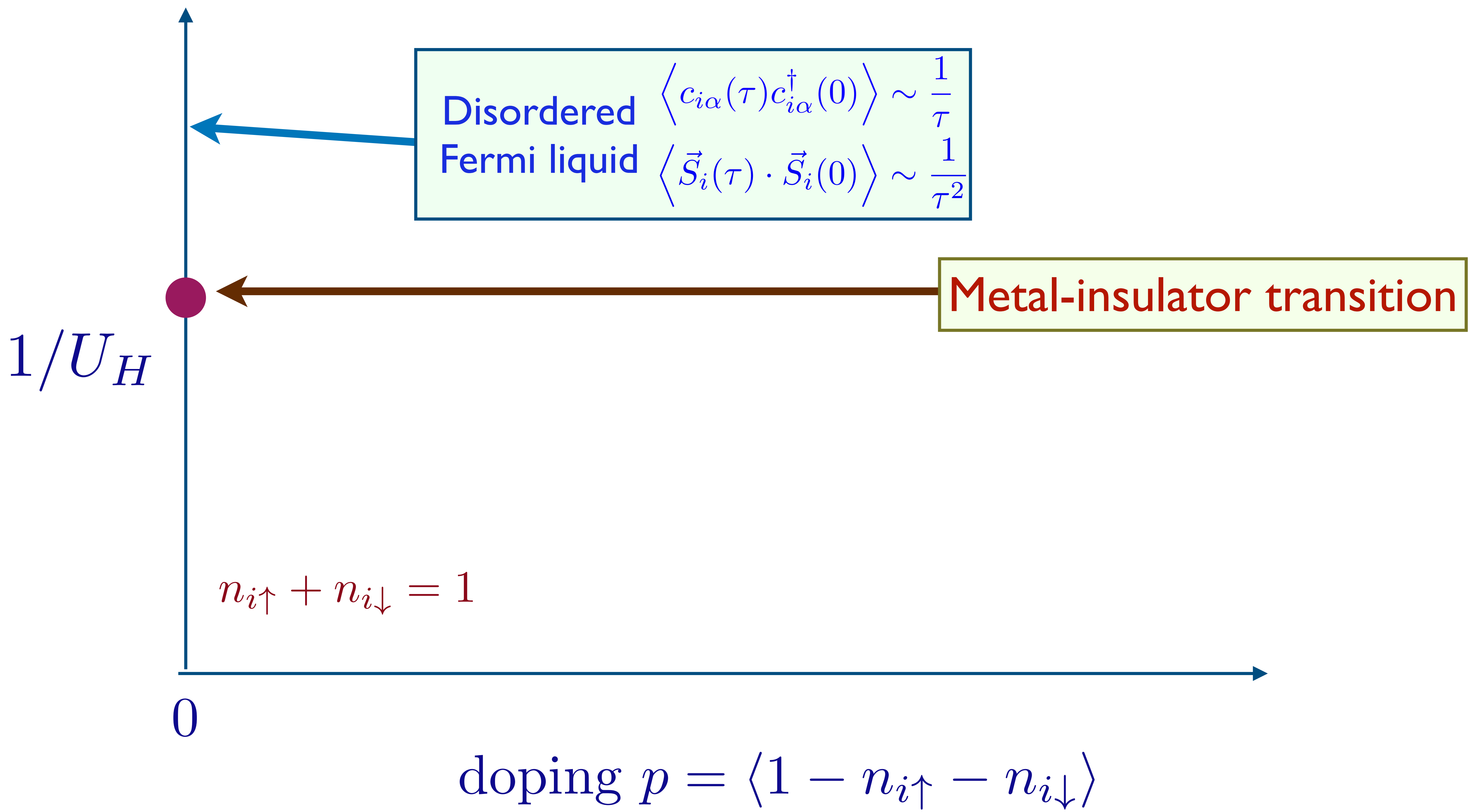

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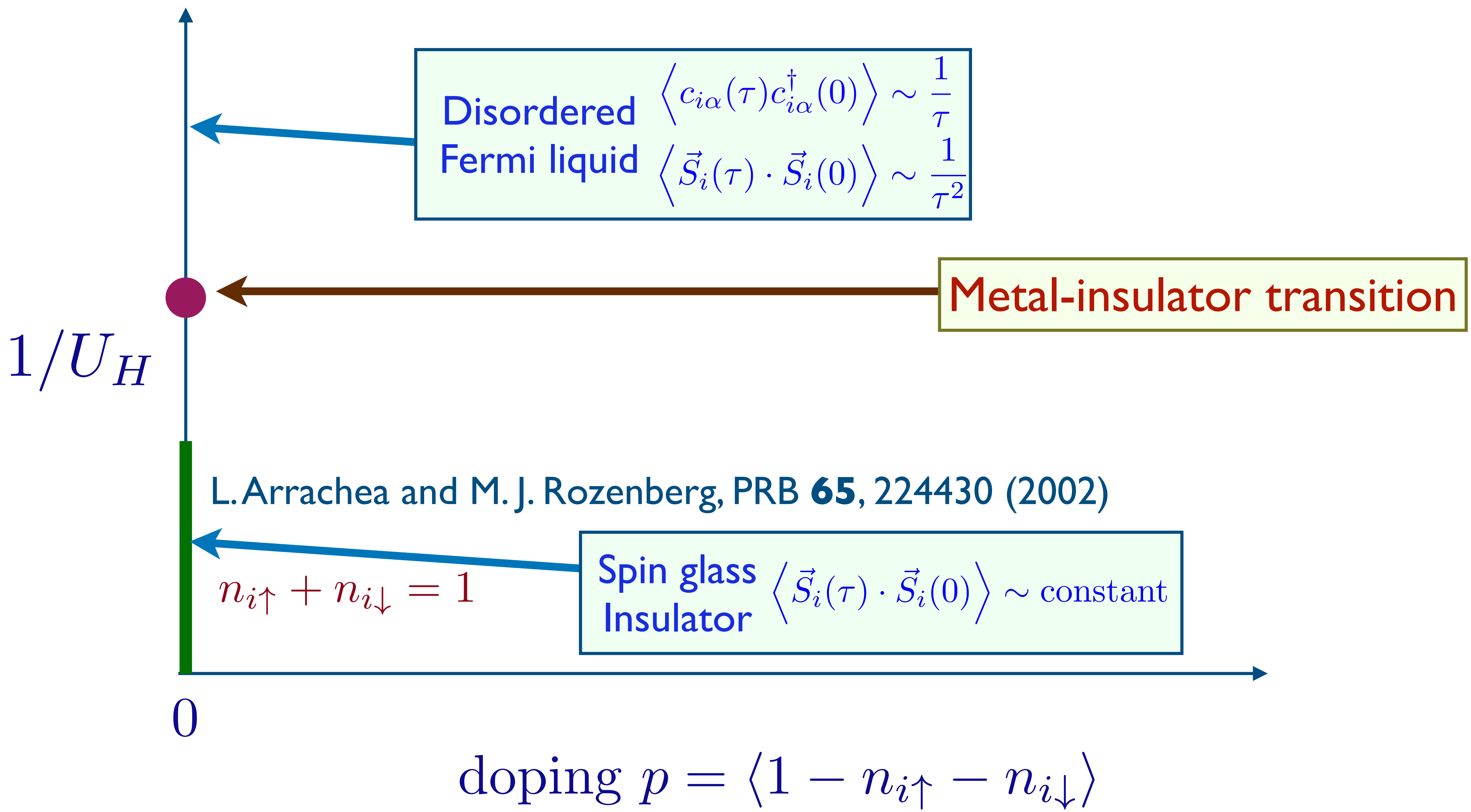

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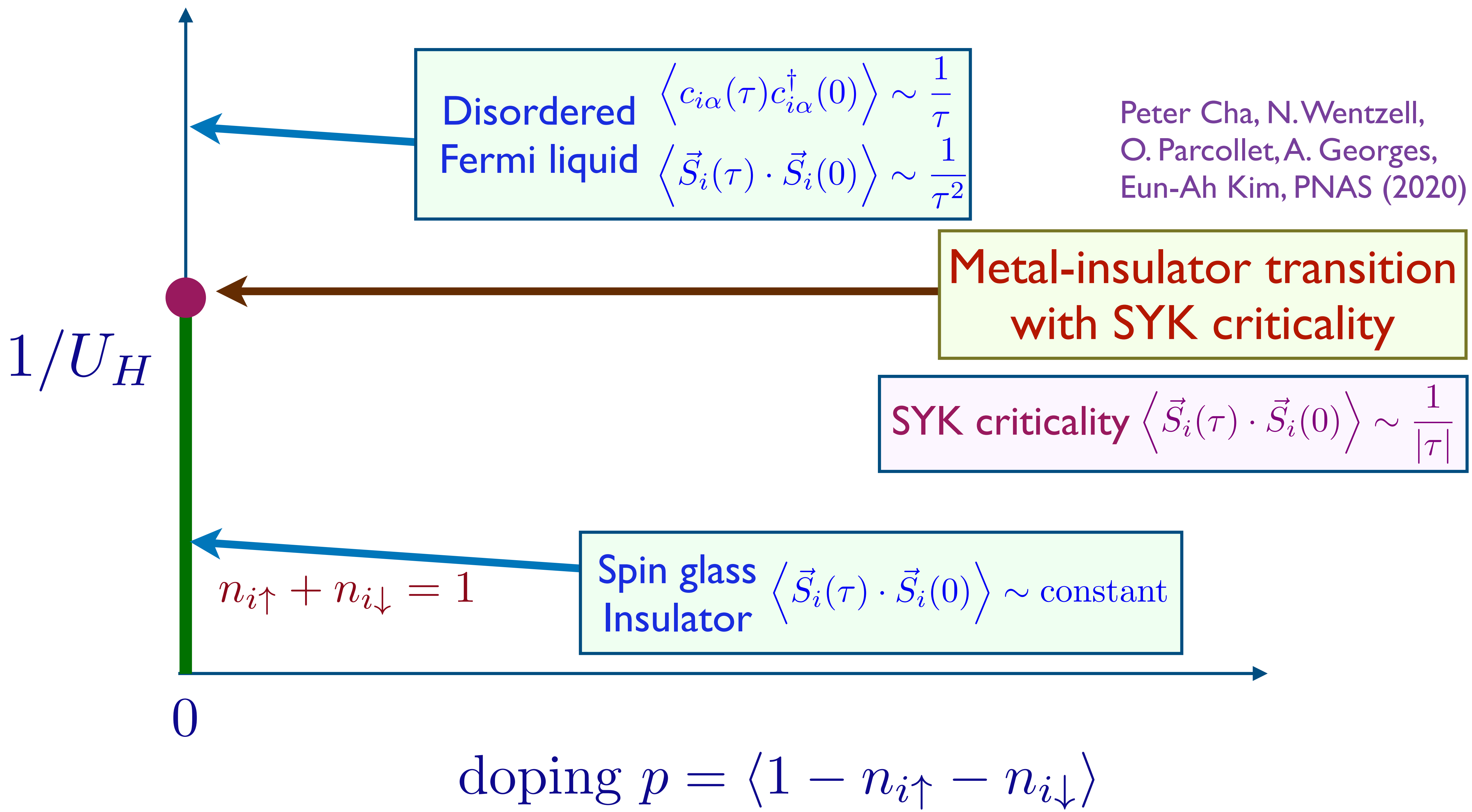
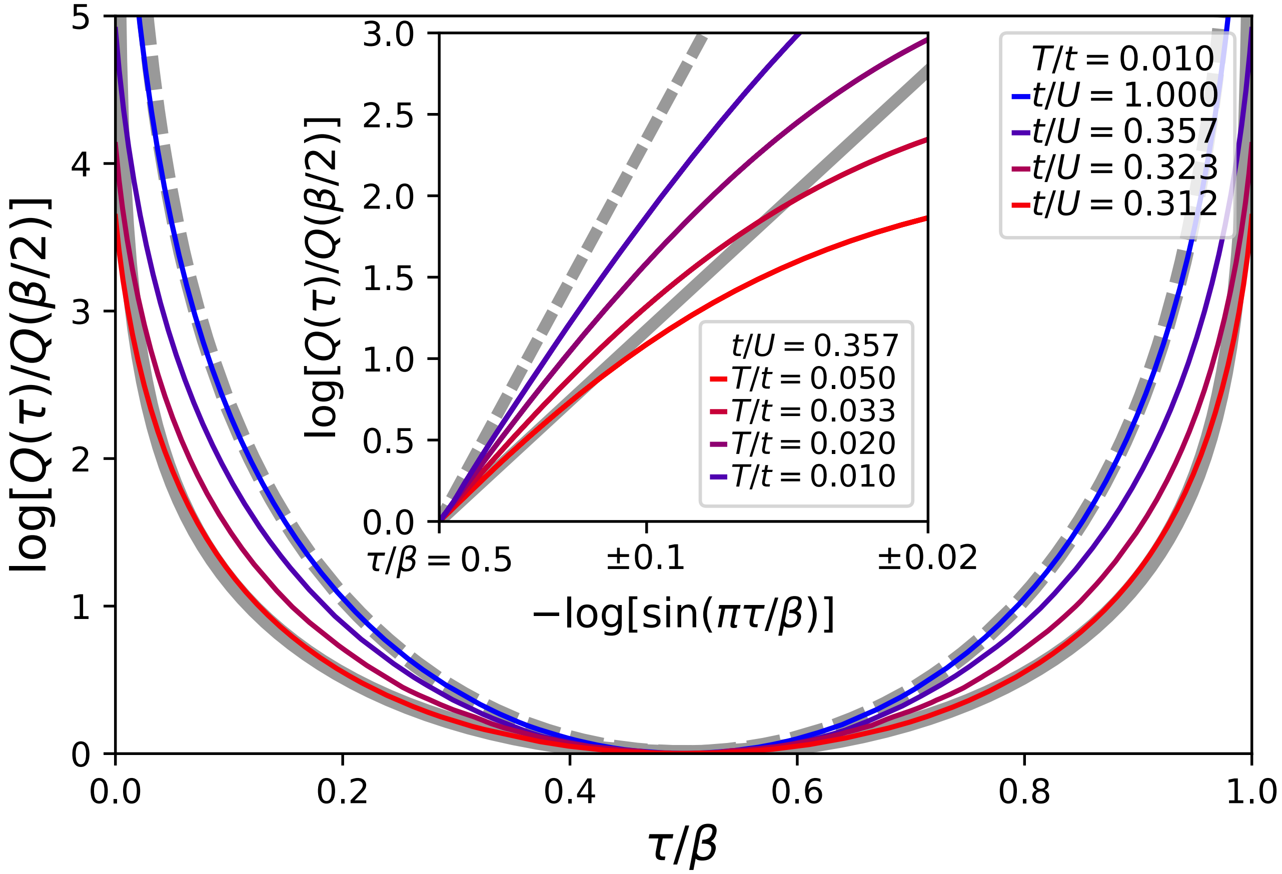
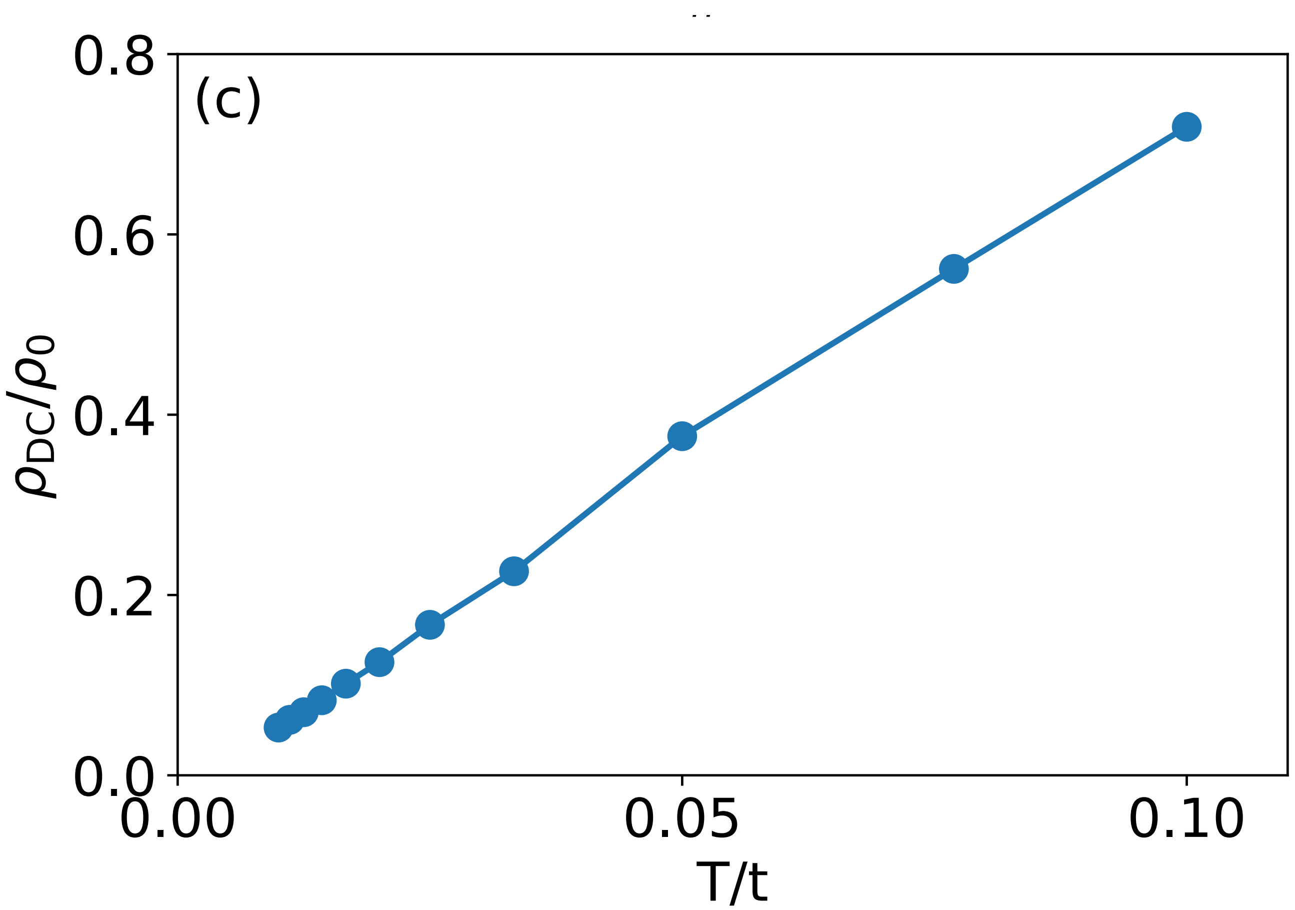


Linear resistivity and Sachdev-Ye-Kitaev (SYK) spin liquid behavior in a quantum critical metal with spin-1/2 fermions





Critical spin correlations:

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}$$

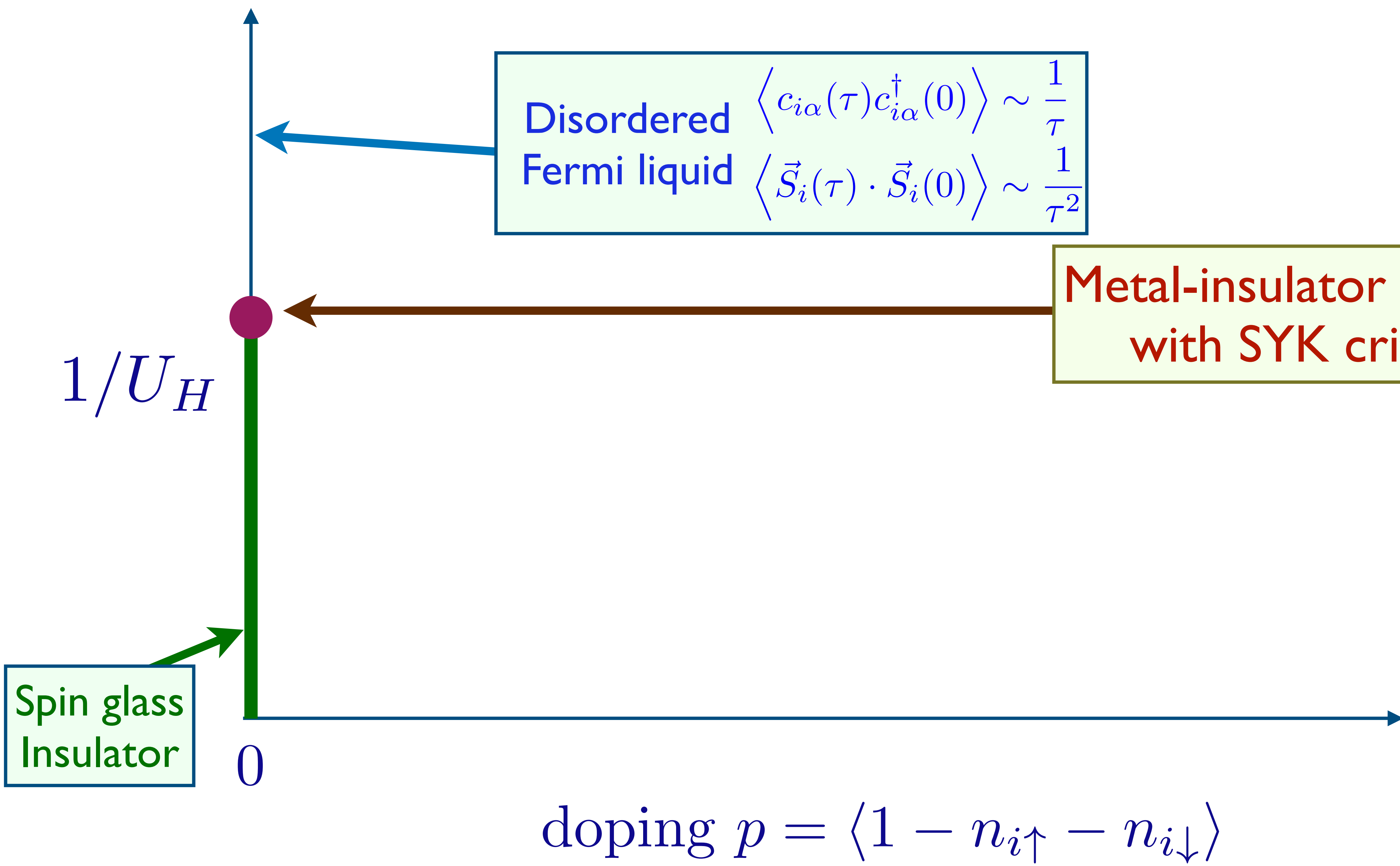
Resisitivity $\rho \sim T$ to the lowest T at the critical point

Onset of insulating gap and spin glass order co-incide (intertwine).

Peter Cha, N. Wentzell, O. Parcollet, A. Georges, Eun-Ah Kim, PNAS (2020)

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



Random t-J-U_H model

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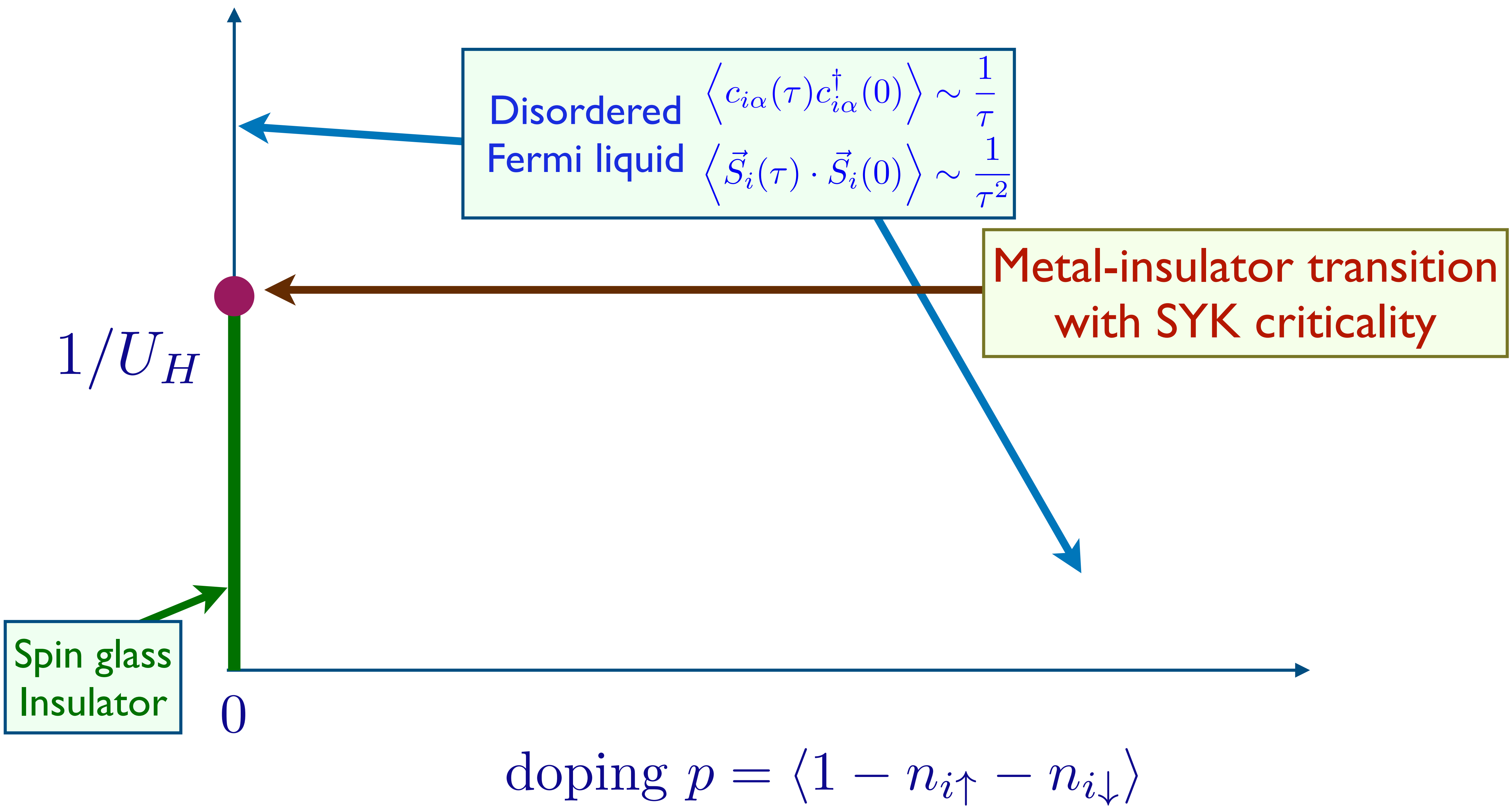

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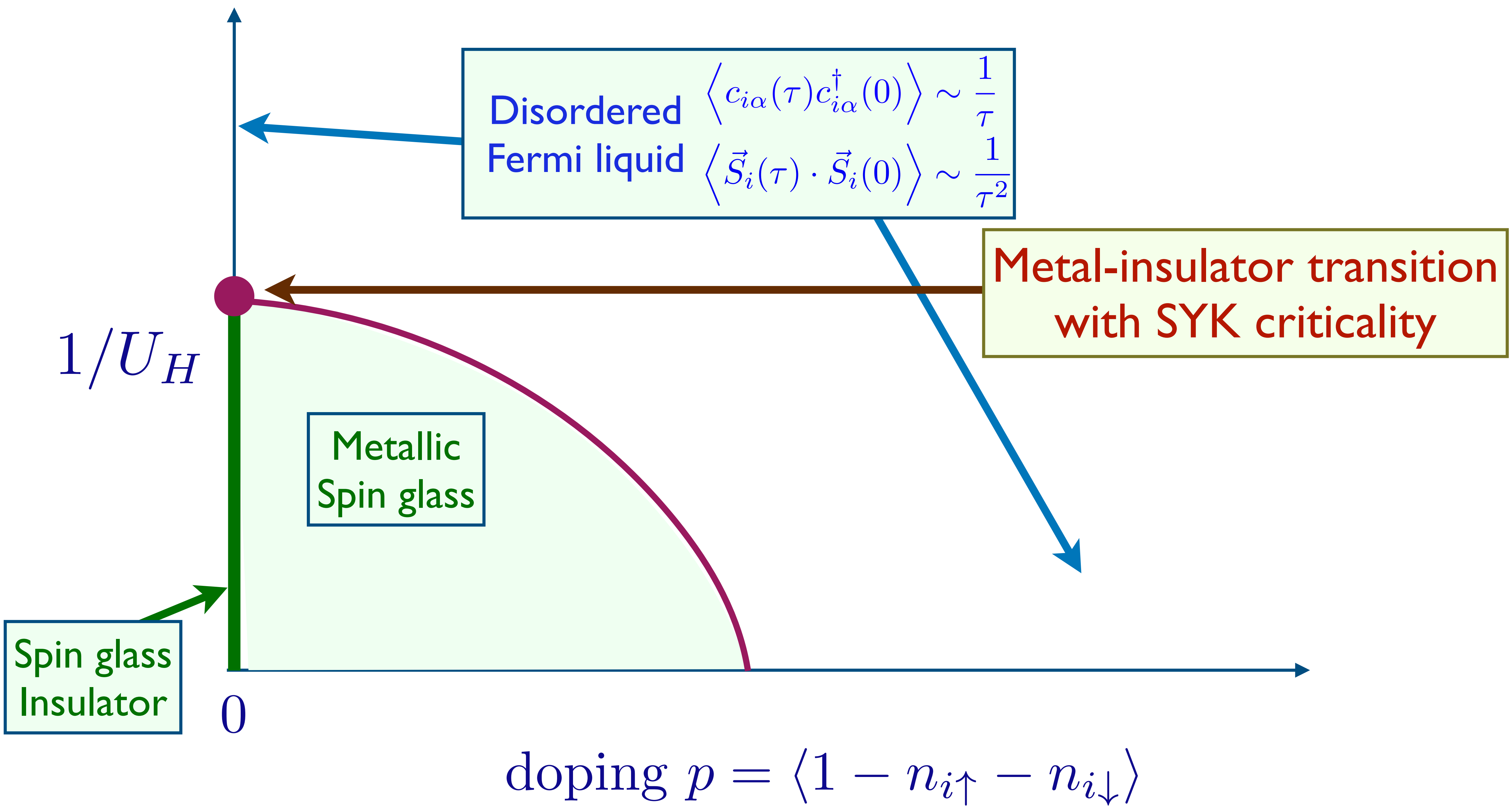

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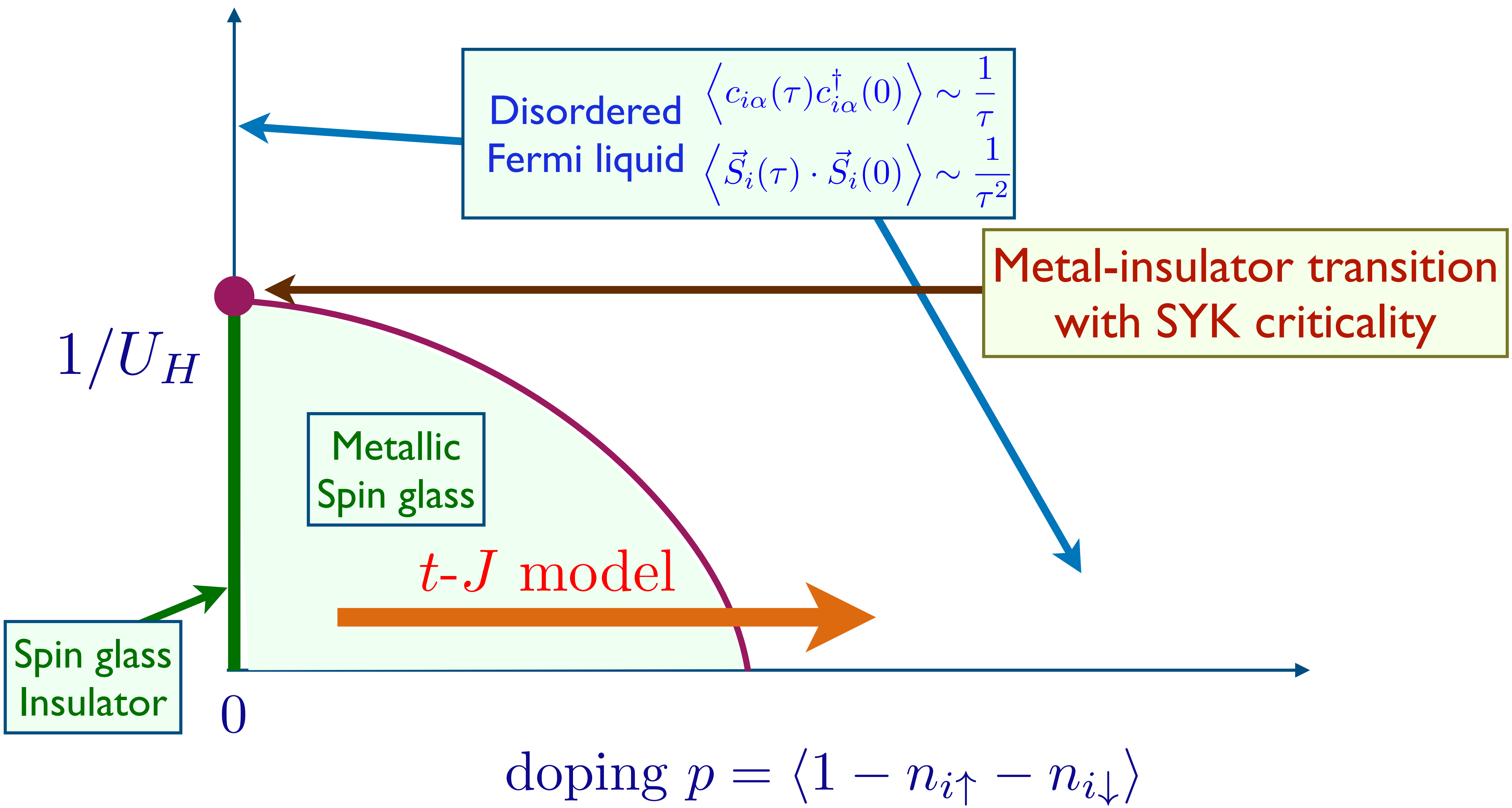

Random *t-J* model $(U_H \to \infty)$

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^\dagger c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

We consider the hole-doped case, with no double occupancy.

$$\alpha = \uparrow, \downarrow, \quad \{c_{i\alpha}, c_{j\beta}^\dagger\} = \delta_{ij}\delta_{\alpha\beta}, \quad \{c_{i\alpha}, c_{j\beta}\} = 0$$

$$\vec{S}_i = \frac{1}{2} c_{i\alpha}^\dagger \vec{\sigma}_{\alpha\beta} c_{i\beta}, \quad \sum_\alpha c_{i\alpha}^\dagger c_{i\alpha} \leq 1, \quad \frac{1}{N}\sum_{i\alpha} c_{i\alpha}^\dagger c_{i\alpha} = 1 - p$$

$$J_{ij} \text{ random}, \ \overline{J_{ij}} = 0, \ \overline{J_{ij}^2} = J^2$$

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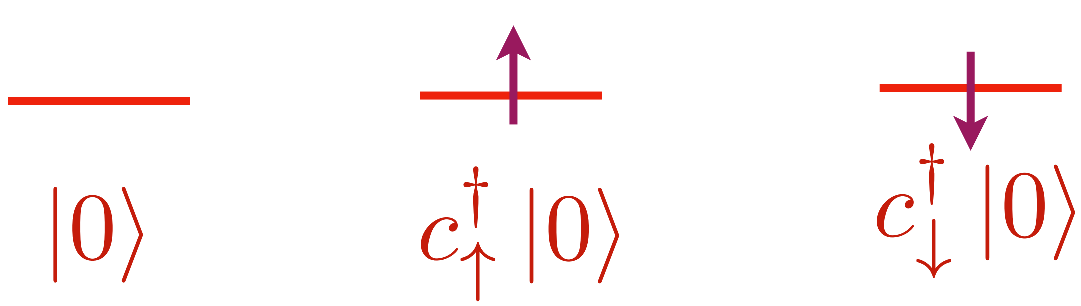
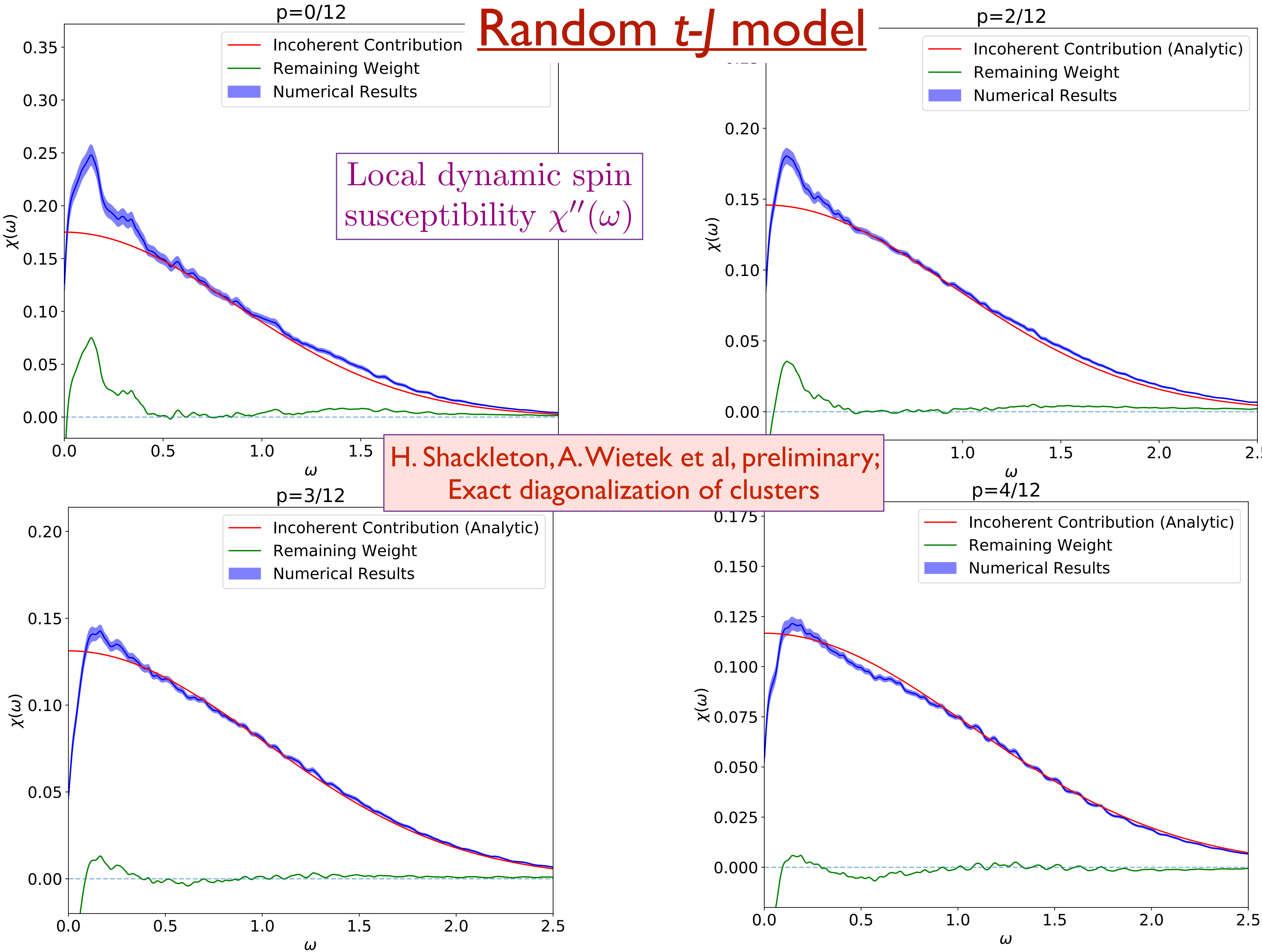


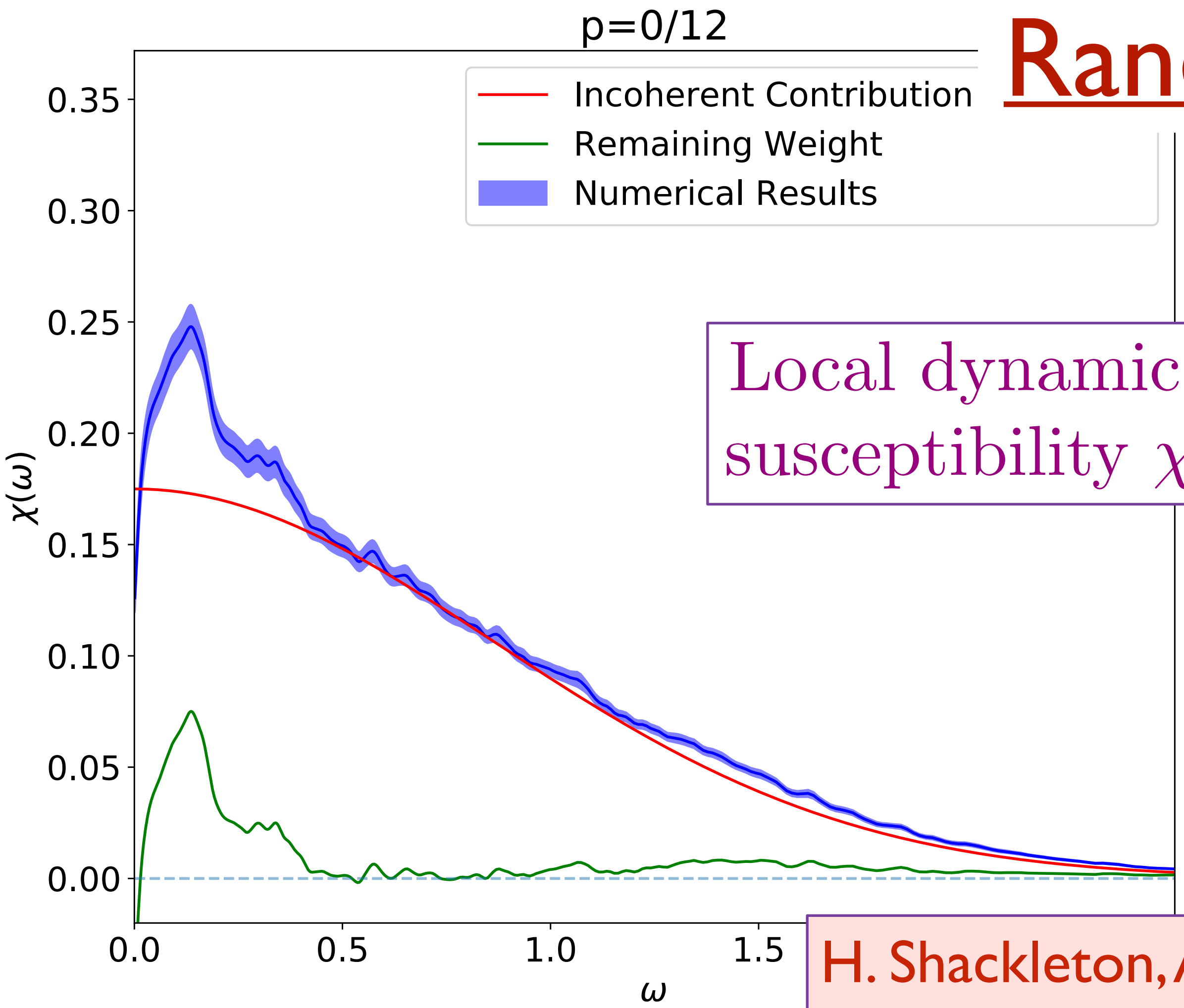
Random *t-J* model

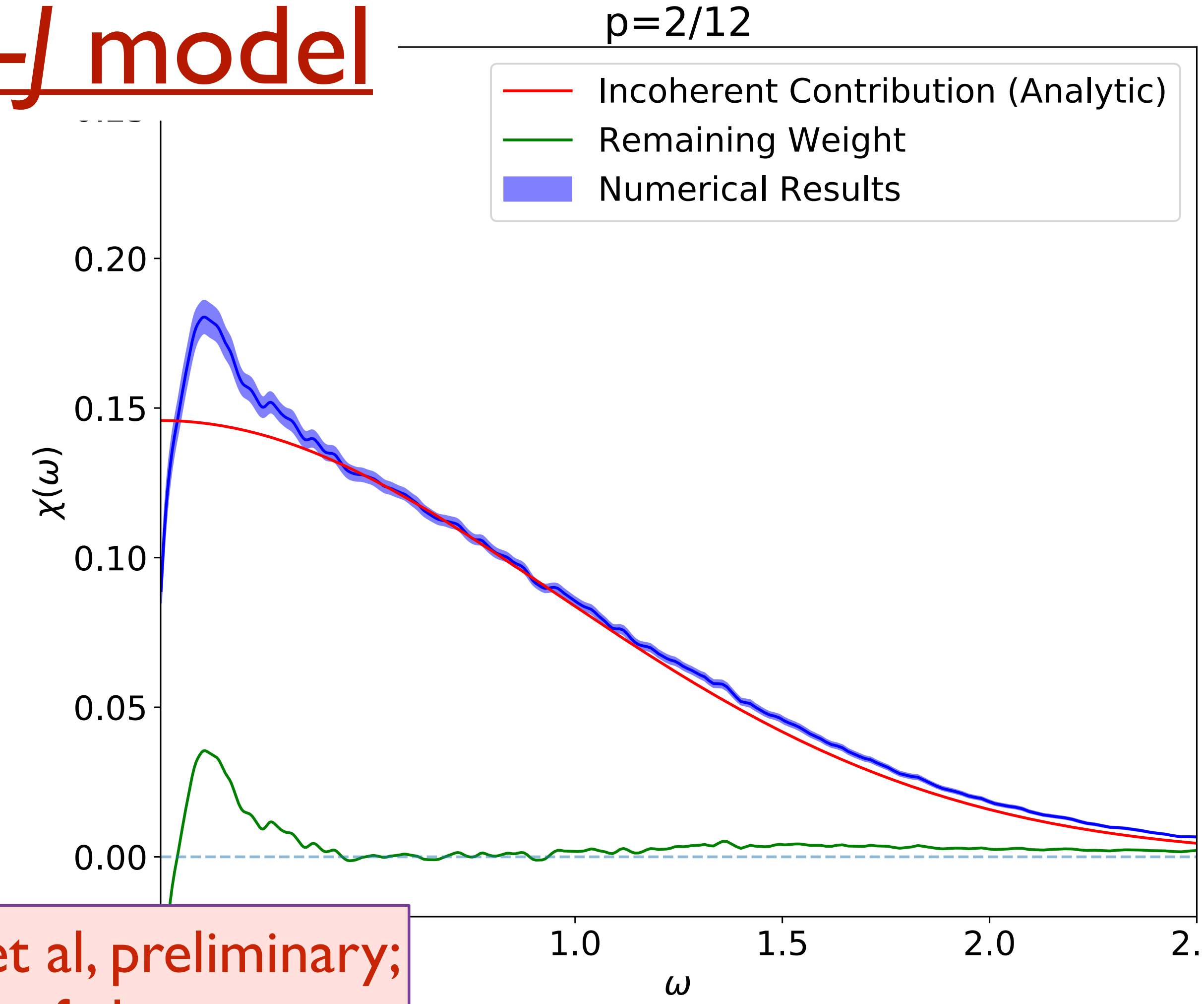


Local dynamic spin susceptibility $\chi''(\omega)$

H. Shackleton, A. Wietek et al, preliminary;
Exact diagonalization of clusters

Random *t-J* model





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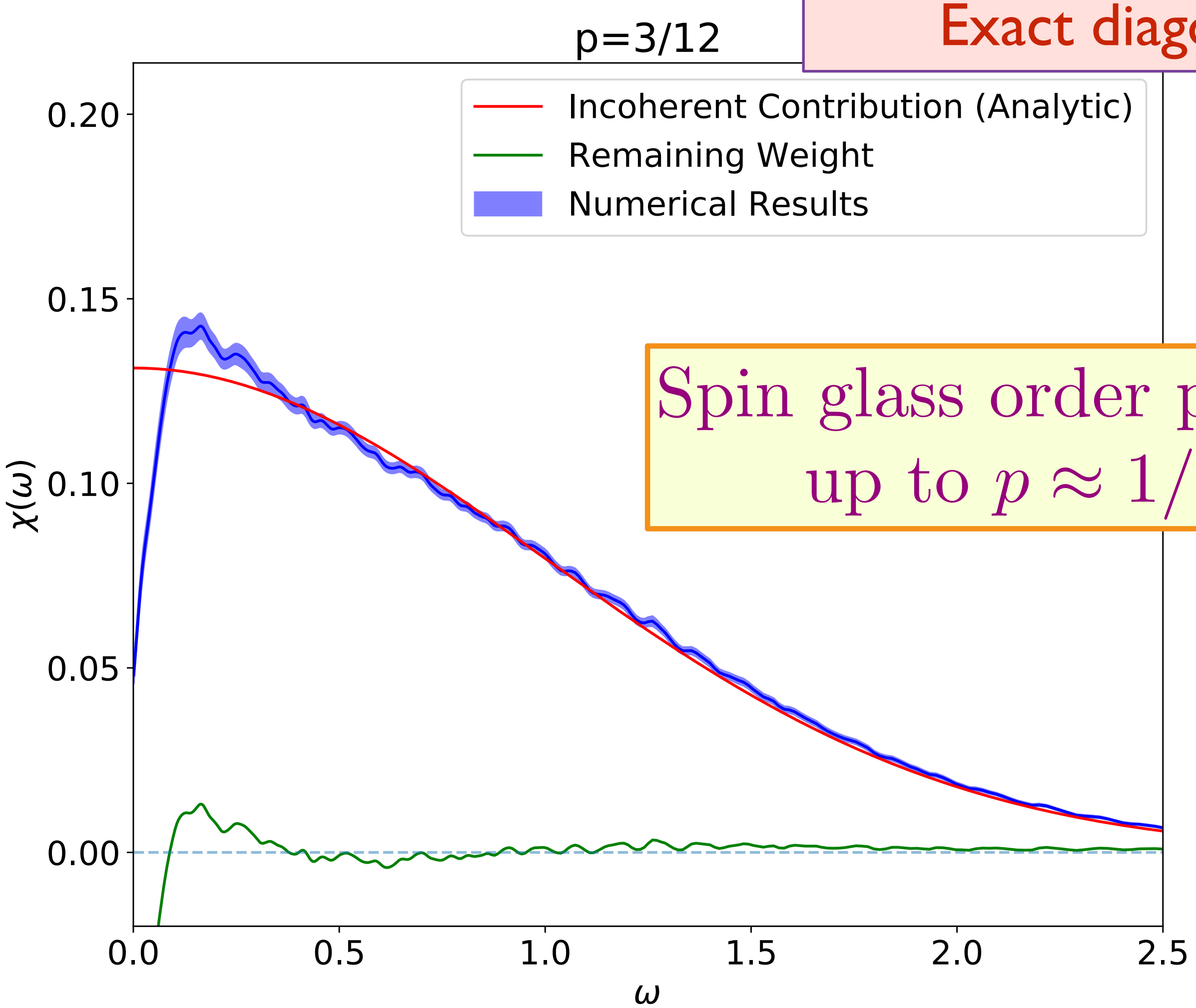

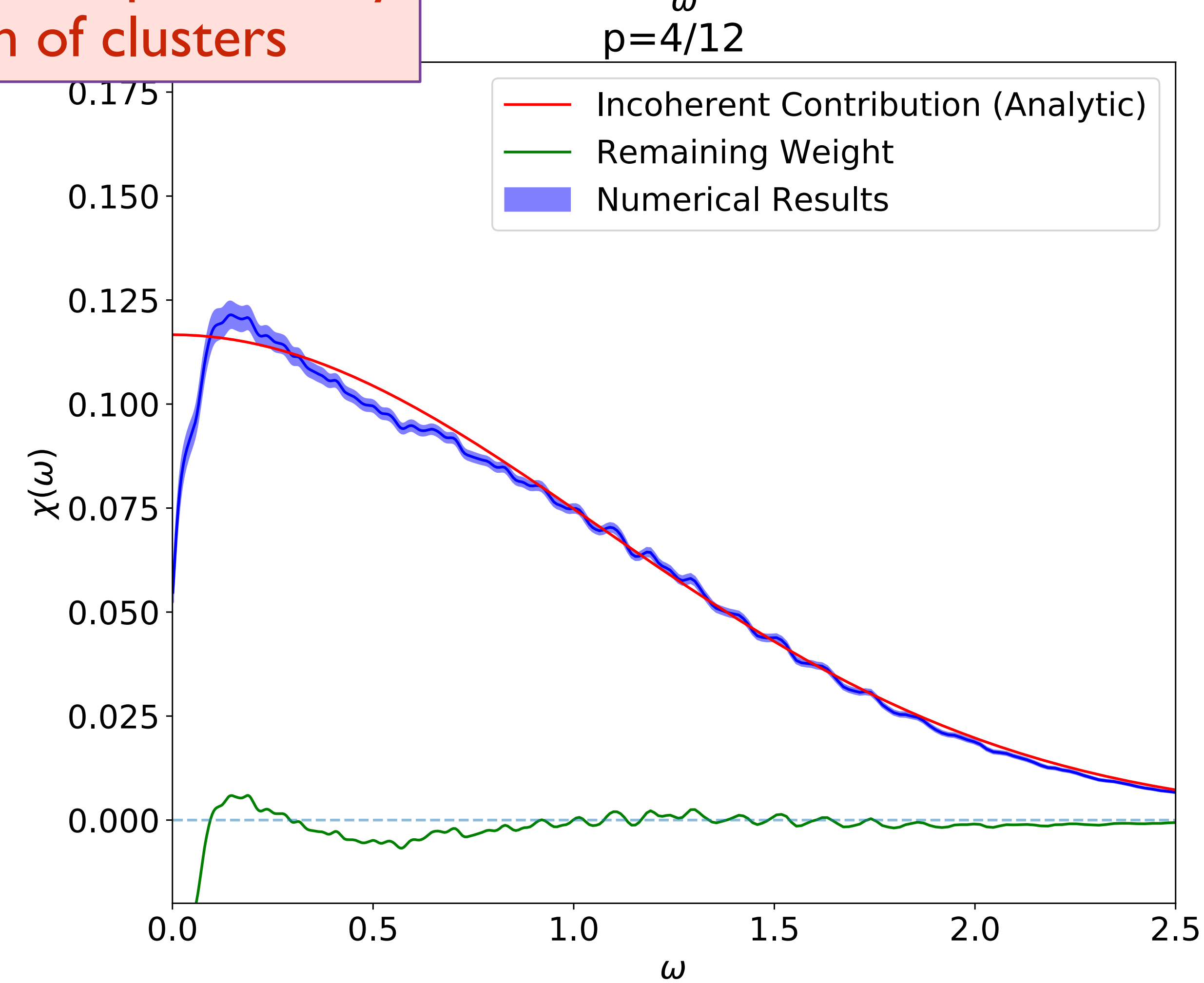

Spin glass order present up to $p \approx 1/3$.

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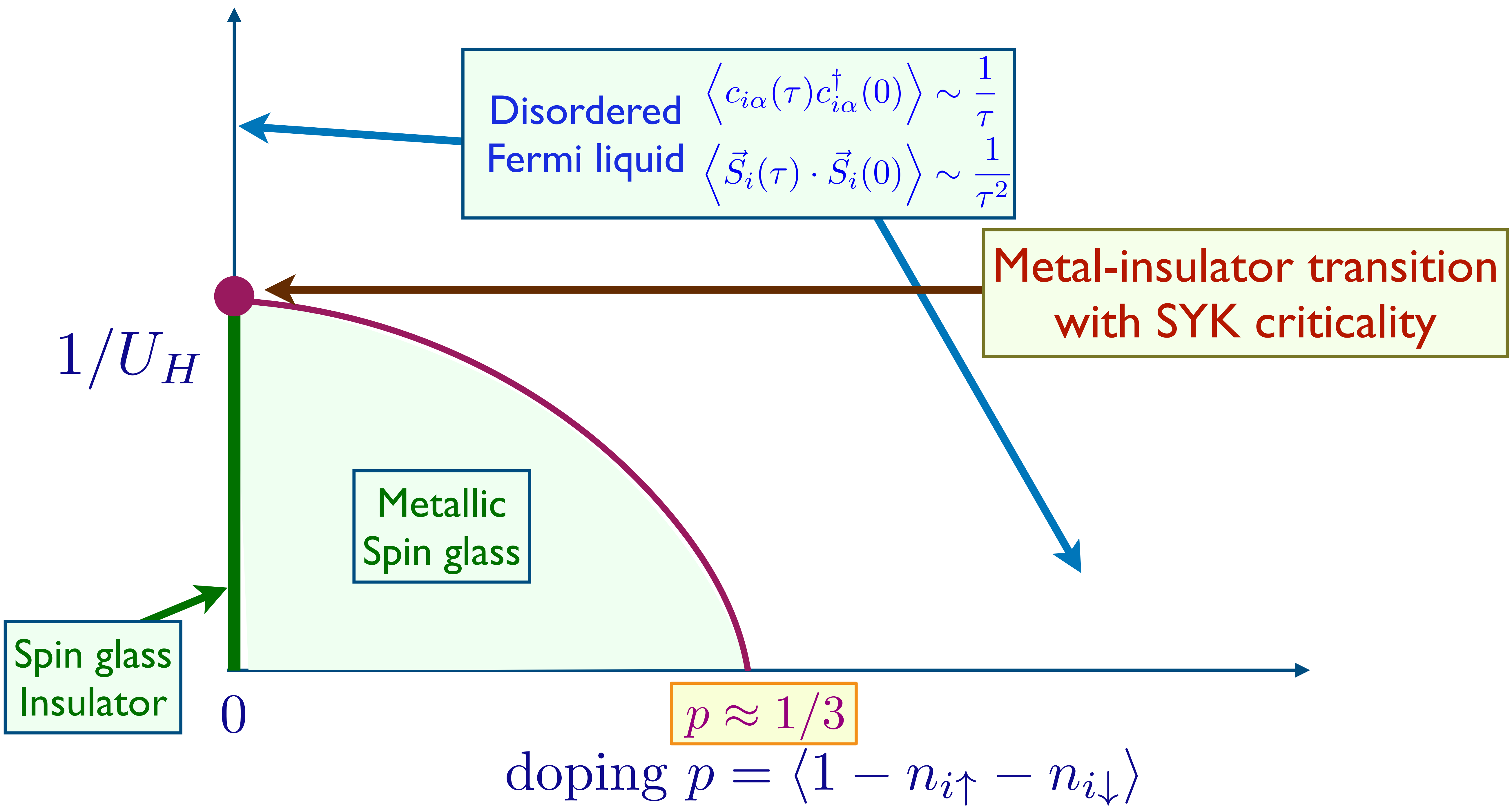
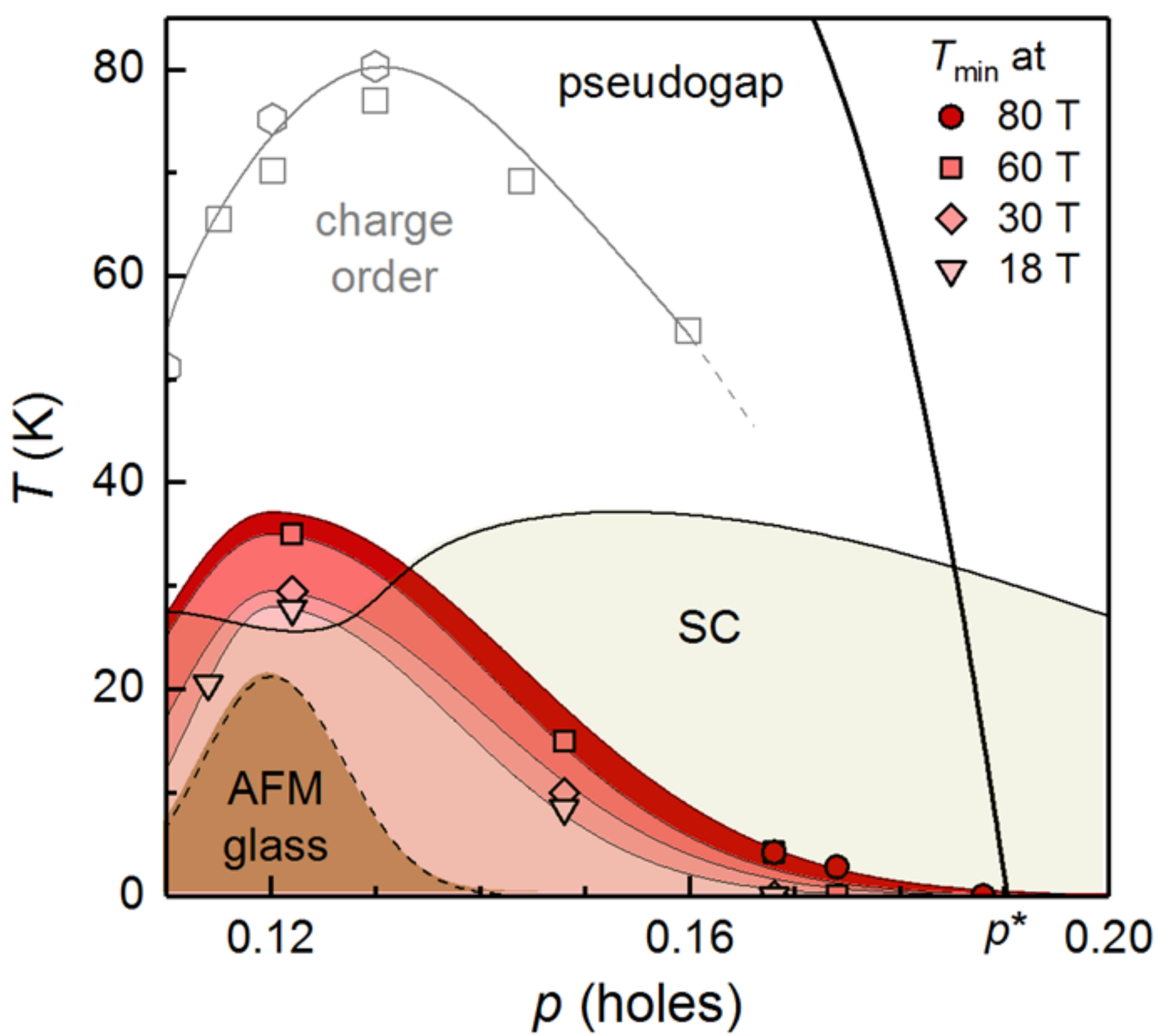


Hidden magnetism at the pseudogap critical point of a high temperature superconductor

Nature Physics doi: 10.1038/s41567-020-0950-5

Mehdi Frachet[1]†, Igor Vinograd[1]†, Rui Zhou[1,2], Siham Benhabib[1], Shangfei Wu[1], Hadrien Mayaffre[1], Steffen Krämer[1], Sanath K. Ramakrishna[3], Arneil P. Reyes[3], Jérôme Debray[4], Tohru Kurosawa[5], Naoki Momono[6], Migaku Oda[5], Seiki Komiya[7], Shimpei Ono[7], Masafumi Horio[8], Johan Chang[8], Cyril Proust[1], David LeBoeuf[1*], Marc-Henri Julien[1*]



Quasi-static magnetism in the pseudogap state of $La_{2-x}Sr_xCuO_4$. Temperature – doping phase diagram representing T_{min}, the temperature of the minimum in the sound velocity, at different fields. Since superconductivity precludes the observation of T_{min} in zero-field, the dashed line (brown area) represents the extrapolated $T_{min}(B=0)$. While not exactly equal to the freezing temperature T_f (see Fig. 2), T_{min} is closely tied to T_f and so is expected to have the same doping dependence, including a peak around $p = 0.12$ in zero/low fields (ref. 2). Onset temperatures of charge order are from ref. 33 (squares) and 35 (hexagons).

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$$H = \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij} \, \vec{S}_i \cdot \vec{S}_j$$

$$\alpha = \uparrow, \downarrow, \quad \vec{S}_i = \frac{1}{2} f_{i\alpha}^{\dagger} \vec{\sigma}_{\alpha\beta} f_{i\beta}, \quad \sum_{\alpha} f_{i\alpha}^{\dagger} f_{i\alpha} = 1$$

$$J_{ij} \text{ random}, \; \overline{J_{ij}} = 0, \; \overline{J_{ij}^2} = J^2$$

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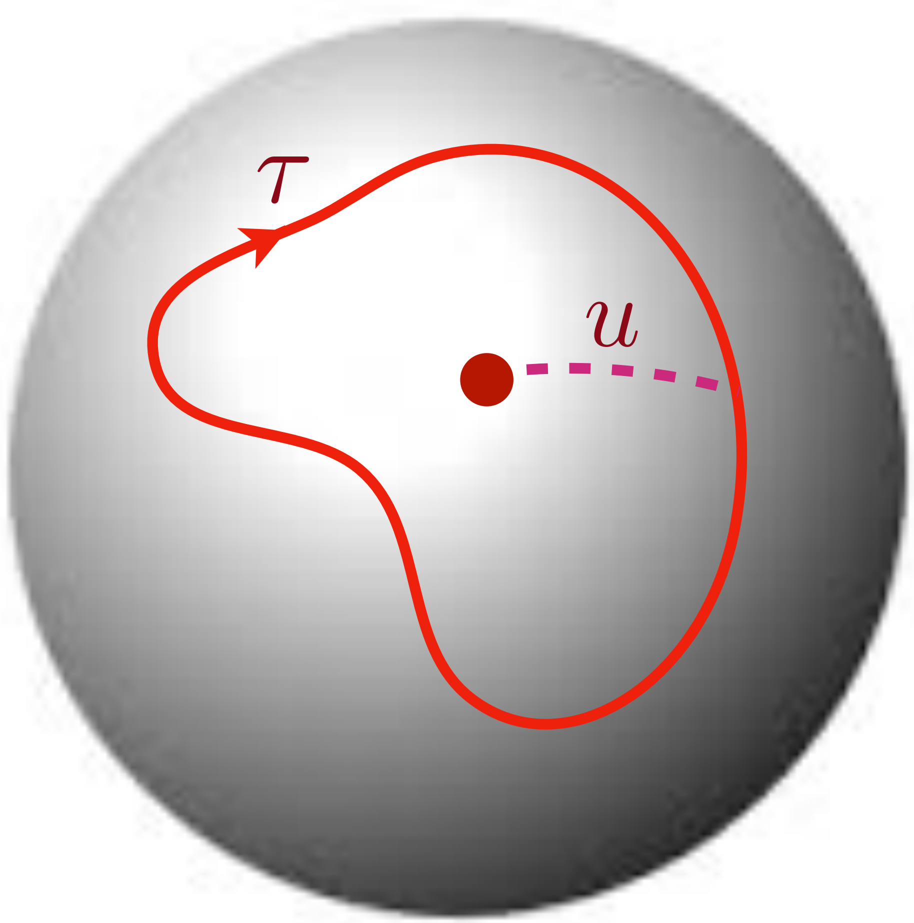
$$\mathcal{Z} = \int \mathcal{D}\vec{S}(\tau)\delta(\vec{S}^2 - 1)e^{-\mathcal{S}_B - \mathcal{S}_J}$$

$$\mathcal{S}_B = \frac{i}{2}\int_0^1 du \int d\tau \vec{S} \cdot \left(\frac{\partial \vec{S}}{\partial \tau} \times \frac{\partial \vec{S}}{\partial u} \right)$$

$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') \,.$$



S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

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$$\mathcal{S}_J = -\frac{J^2}{2}\int d\tau d\tau' Q(\tau - \tau')\vec{S}(\tau) \cdot \vec{S}(\tau') .$$

From this action we compute

$$\overline{Q}(\tau - \tau') = \frac{1}{3}\left\langle \vec{S}(\tau) \cdot \vec{S}(\tau') \right\rangle_{\mathcal{Z}}$$

and then impose the self-consistency condition

$$Q(\tau) = \overline{Q}(\tau).$$

S. Sachdev and J. Ye, PRL **70**, 3339 (1993)

Random J model (insulator):RG

Assume a power-law decay

$$Q(\tau) = \gamma^2 \frac{1}{|\tau|^\alpha} \, .$$

- The conjugate field to the spin acquires a wavefunction renormalization
- Because of the quantized Berry phase (Wess-Zumion-Witten) term, the renormalization of the coupling γ is given only by the wavefunction renormalization

M. Vojta, C. Buragohain, and S. Sachdev, PRB **61**, 15152 (2000)
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- The β-function of γ can by computed order-by-order in $\epsilon = 2 - \alpha$
- There is an attractive fixed point at $\gamma = \gamma^* = \mathcal{O}(\sqrt{\epsilon})$.
- We can prove that at this fixed point $\overline{Q}(\tau) \sim 1/|\tau|^{2-\alpha}$ to all orders in ϵ.

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- The β-function of γ can by computed order-by-order in $\epsilon = 2 - \alpha$
- There is an attractive fixed point at $\gamma = \gamma^* = \mathcal{O}(\sqrt{\epsilon})$.
- We can prove that at this fixed point $\overline{Q}(\tau) \sim 1/|\tau|^{2-\alpha}$ to all orders in ϵ.
- The self-consistency condition therefore yields

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|}\,.$$

to all orders in ϵ.

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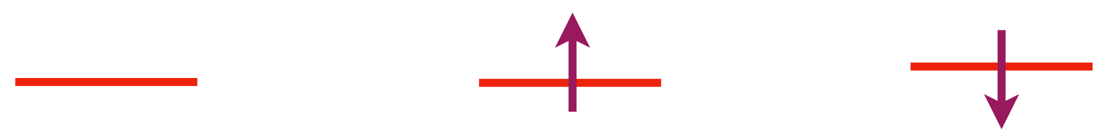
Each site has 3 states which we map to the '*superspin*' space of a boson b (the holon) and a fermion f_α (the spinon):



$b^\dagger \left|v\right\rangle$ $\quad$ $f_\uparrow^\dagger \left|v\right\rangle$ $\quad$ $f_\downarrow^\dagger \left|v\right\rangle$

$$\begin{aligned} c_\alpha &= f_\alpha b^\dagger \\ \vec{S} &= \frac{1}{2} f_\alpha^\dagger \sigma_{\alpha\beta} f_\beta \\ f_\alpha^\dagger f_\alpha + b^\dagger b &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad b \rightarrow b e^{i\phi}\,, \quad f_\alpha \rightarrow f_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(1|2) superspin space.

Random t-J model (metal)

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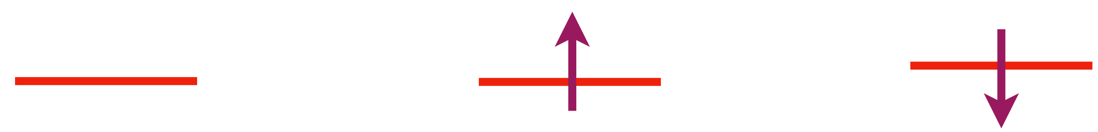
Each site has 3 states which we map to the '*superspin*' space of a fermion $\mathfrak{f}$ (the holon) and a boson $\mathfrak{b}_\alpha$ (the spinon):



$\mathfrak{f}^\dagger \left|v\right\rangle$ $\qquad$ $\mathfrak{b}_\uparrow^\dagger \left|v\right\rangle$ $\qquad$ $\mathfrak{b}_\downarrow^\dagger \left|v\right\rangle$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \end{aligned}$$

U(1) gauge invariance, $\quad \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi}$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random t-J model (metal)

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

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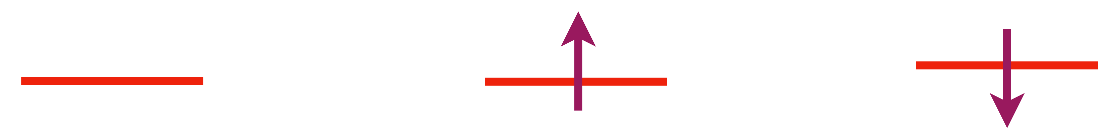

$$\mathfrak{f}^\dagger \left|v\right\rangle \qquad \mathfrak{b}_\uparrow^\dagger \left|v\right\rangle \qquad \mathfrak{b}_\downarrow^\dagger \left|v\right\rangle$$

$$\mathrm{SU}(1|2) \equiv \mathrm{SU}(2|1)$$

$$\begin{aligned} c_\alpha &= \mathfrak{b}_\alpha \mathfrak{f}^\dagger \\ \vec{S} &= \frac{1}{2} \mathfrak{b}_\alpha^\dagger \sigma_{\alpha\beta} \mathfrak{b}_\beta \\ \mathfrak{b}_\alpha^\dagger \mathfrak{b}_\alpha + \mathfrak{f}^\dagger \mathfrak{f} &= 1 \\ \text{U(1) gauge invariance,} \quad & \mathfrak{f} \to \mathfrak{f} e^{i\phi}\,, \quad \mathfrak{b}_\alpha \to \mathfrak{b}_\alpha e^{i\phi} \end{aligned}$$

The physical electron (c_α) and spin ($\vec{S}$) operators are rotations in this SU(2|1) superspin space.

Random t-J model (metal)

$$\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}\mathcal{P}(\tau) e^{-\mathcal{S}_B - \mathcal{S}_{tJ}} \\
\mathcal{S}_B &= i \int_0^1 du \int d\tau \, \mathrm{Tr} \left(\mathcal{P} \partial_\tau \mathcal{P} \partial_u \mathcal{P} \right) \\
\mathcal{S}_{tJ} &= \int d\tau d\tau' \, \mathrm{Tr} \left(\mathcal{P}(\tau) \mathcal{Q}(\tau - \tau') \mathcal{P}(\tau') \right) \\
&\quad + \int d\tau \mathrm{Tr} \left(s_0 \, \mathcal{P}(\tau) \right) \, .
\end{aligned}$$

Path integral over a superspin $\mathcal{P}(\tau)$ with a self-consistent self-interaction $\mathcal{Q}(\tau)$ and a 'Zeeman superfield' s_0.

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J model (metal)

$$
\begin{aligned}
\mathcal{Z} &= \int \mathcal{D}f_\alpha(\tau)\mathcal{D}b(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}} \\
\mathcal{S}_B &= \int d\tau \left[f_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)f_\alpha(\tau) + b^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)b(\tau) - i\lambda\right] \\
\mathcal{S}_{tJ} &= \int d\tau\, s_0 f_\alpha^\dagger(\tau)f_\alpha(\tau) + t^2\int d\tau d\tau' R(\tau-\tau')c_\alpha^\dagger(\tau)c_\alpha(\tau') \\
&\quad -\frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau')\,.
\end{aligned}
$$

SU(1|2) theory

From this action we determined the correlators

$$
\begin{aligned}
\overline{R}(\tau-\tau') &= -\left\langle c_\alpha(\tau)c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}} \\
\overline{Q}(\tau-\tau') &= \frac{1}{3}\left\langle \vec{S}(\tau)\cdot\vec{S}(\tau')\right\rangle_{\mathcal{Z}}
\end{aligned}
$$

and finally impose the self-consistency conditions

$$
R(\tau) = \overline{R}(\tau) \quad , \quad Q(\tau) = \overline{Q}(\tau).
$$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

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$$\mathcal{Z} = \int \mathcal{D}\mathfrak{b}_\alpha(\tau)\mathcal{D}\mathfrak{f}(\tau)\mathcal{D}\lambda(\tau)e^{-\mathcal{S}_B-\mathcal{S}_{tJ}}$$

$$\mathcal{S}_B = \int d\tau \left[\mathfrak{b}_\alpha^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{b}_\alpha(\tau)+\mathfrak{f}^\dagger(\tau)\left(\frac{\partial}{\partial\tau}+i\lambda\right)\mathfrak{f}(\tau)-i\lambda\right]$$

$$\mathcal{S}_{tJ} = \int d\tau\, s_0\mathfrak{b}_\alpha^\dagger(\tau)\mathfrak{b}_\alpha(\tau)+t^2\int d\tau d\tau' R(\tau-\tau')c_\alpha^\dagger(\tau)c_\alpha(\tau') - \frac{J^2}{2}\int d\tau d\tau' Q(\tau-\tau')\vec{S}(\tau)\cdot\vec{S}(\tau').$$

From this action we determined the correlators

SU(2|1) theory

$$\overline{R}(\tau-\tau') = -\left\langle c_\alpha(\tau)c_\alpha^\dagger(\tau')\right\rangle_{\mathcal{Z}}$$

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$$R(\tau)=\overline{R}(\tau) \quad , \quad Q(\tau)=\overline{Q}(\tau).$$

D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J model (metal):RG

- The RG analysis is very similar to that for the J model, except that the SU(2) spin is replaced by a SU(1|2) $\cong$ SU(2|1) superspin.

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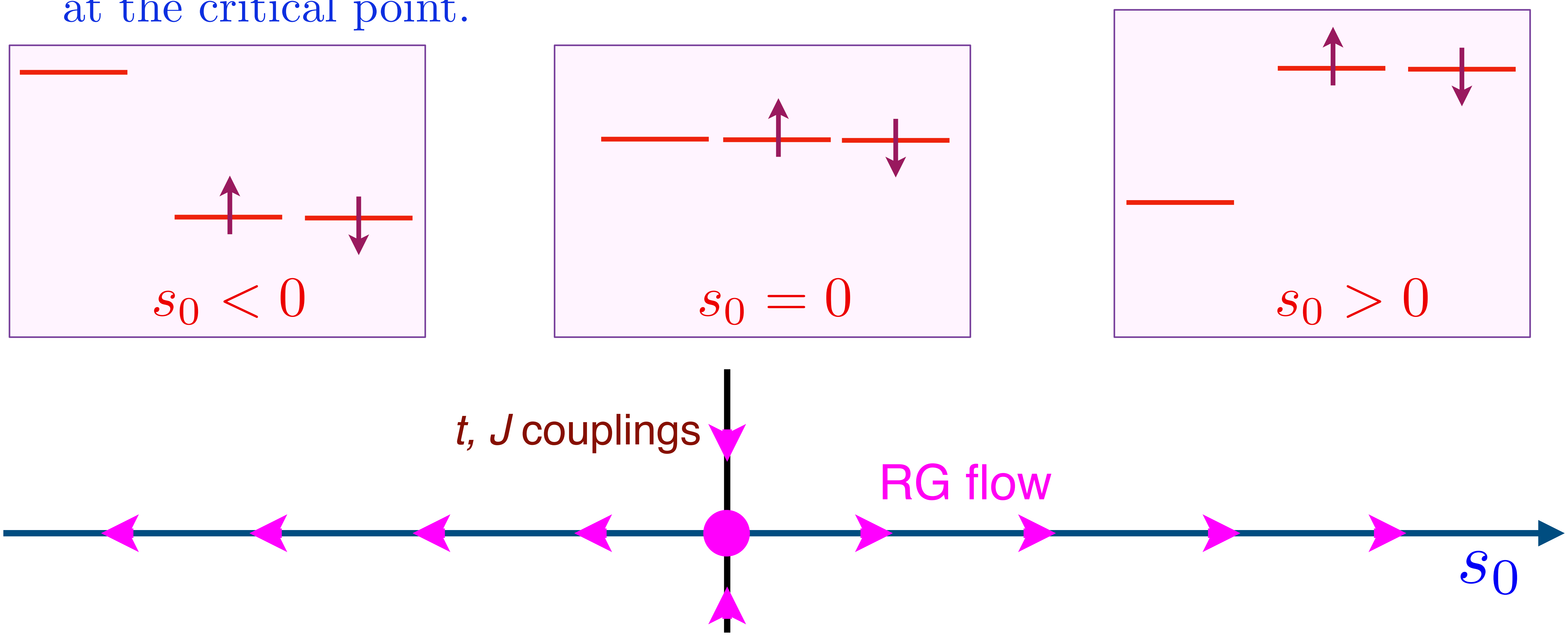

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- The Wess-Zumino-Witten term in superspace now ensures the exact exponents at the fixed point

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \quad , \quad \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau} \,.$$

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Square of spinon correlator

$$\left\langle \vec{S}(\tau) \cdot \vec{S}(0) \right\rangle \sim \frac{1}{|\tau|} \quad , \quad \left\langle c_\alpha(\tau) c_\alpha^\dagger(0) \right\rangle \sim \frac{1}{\tau} .$$

Product of spinon and holon correlators

- These exponents do not have a quasiparticle interpretation. However, they can be understood (in a large M limit of a model with SU(M) symmetry) by *fractionalization* of the electron into a spinon and holon, each of which decay as $1/\sqrt{\tau}$.

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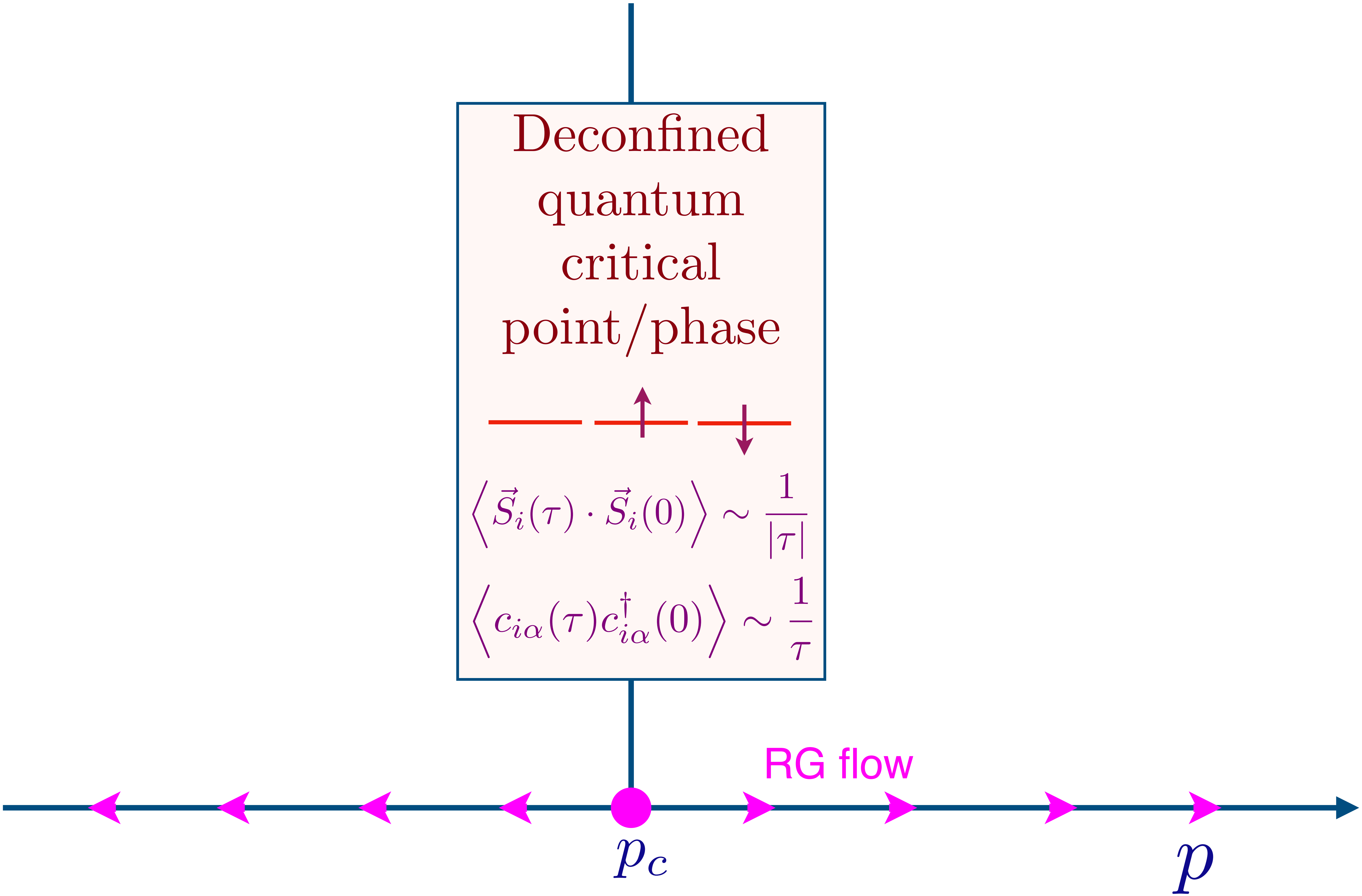
t-J phase diagram: RG using *either* SU(2|1) or SU(1|2)



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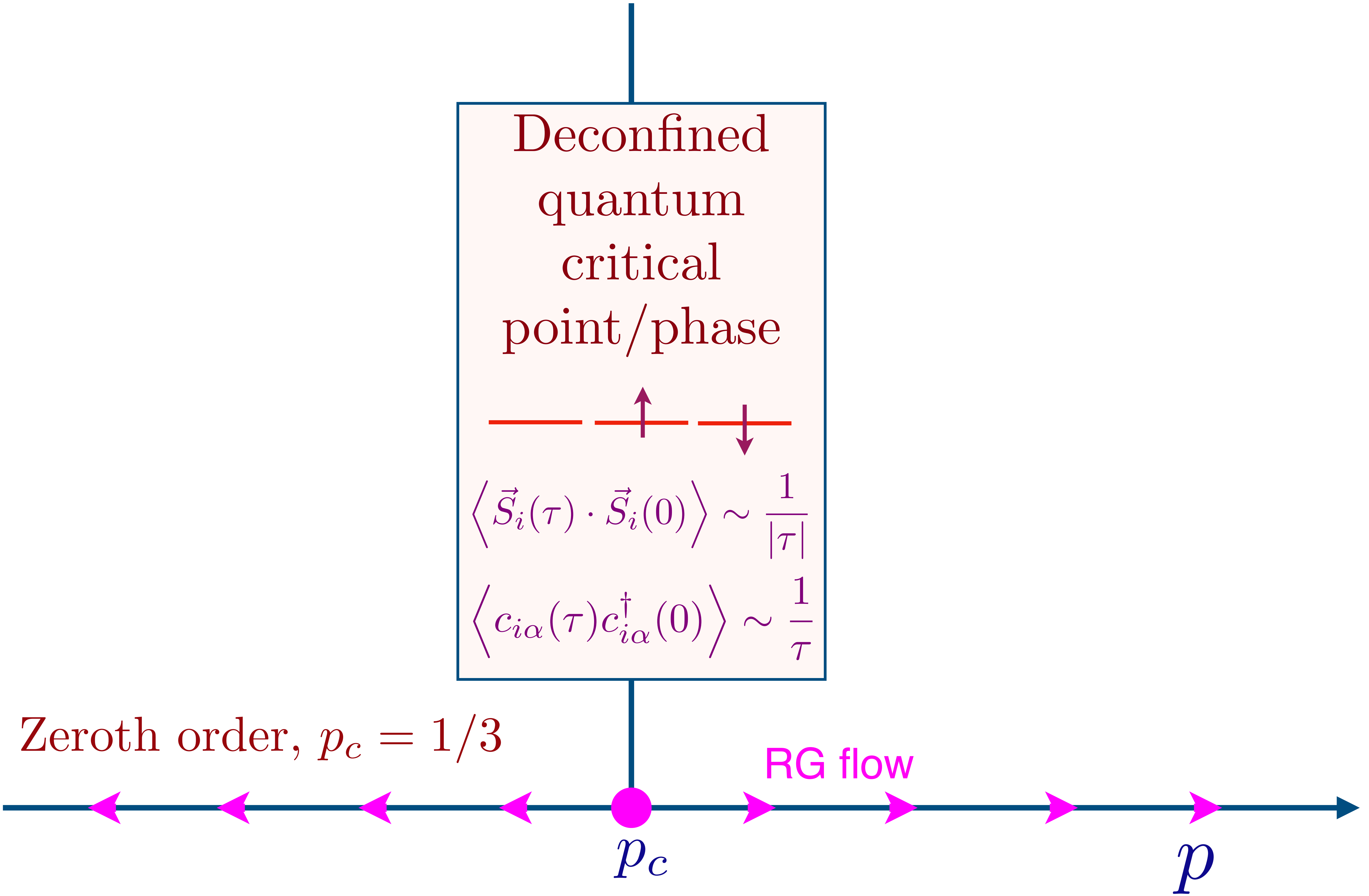

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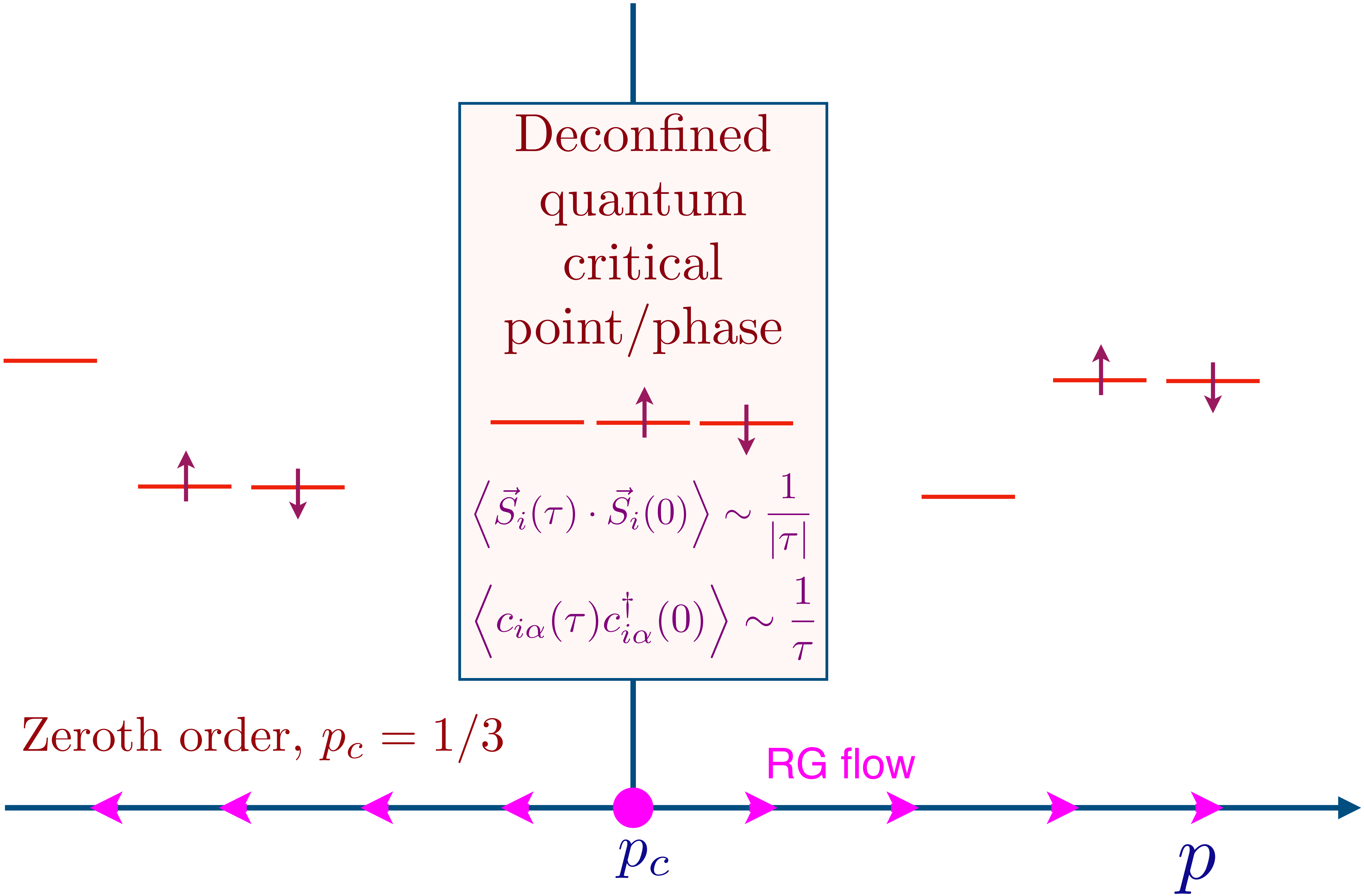

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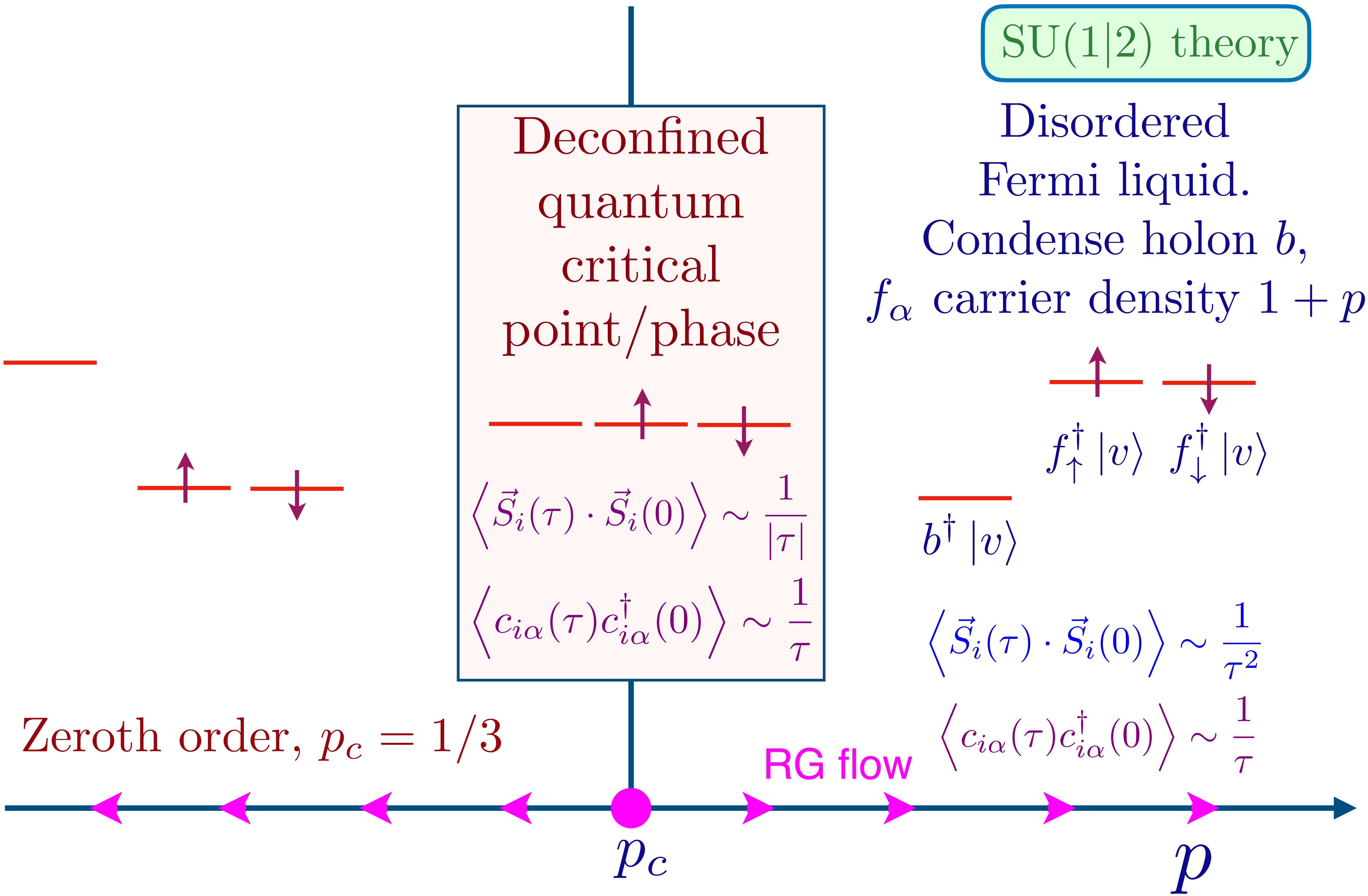

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t-J phase diagram: RG using *either* SU(2|1) or SU(1|2)

SU(2|1) theory

Metallic spin glass.
Condense spinon $\mathfrak{b}_\alpha$,
$\mathfrak{f}$ carrier density p

$\mathfrak{f}^\dagger |v\rangle$

$\mathfrak{b}_\uparrow^\dagger |v\rangle$ $\mathfrak{b}_\downarrow^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \text{constant}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

Deconfined quantum critical point/phase

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$

SU(1|2) theory

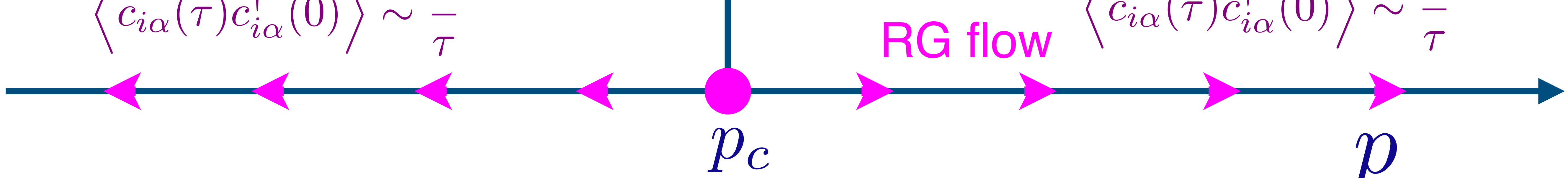
Disordered Fermi liquid.
Condense holon b,
f_α carrier density $1+p$

$f_\uparrow^\dagger |v\rangle$ $f_\downarrow^\dagger |v\rangle$

$b^\dagger |v\rangle$

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{\tau^2}$$

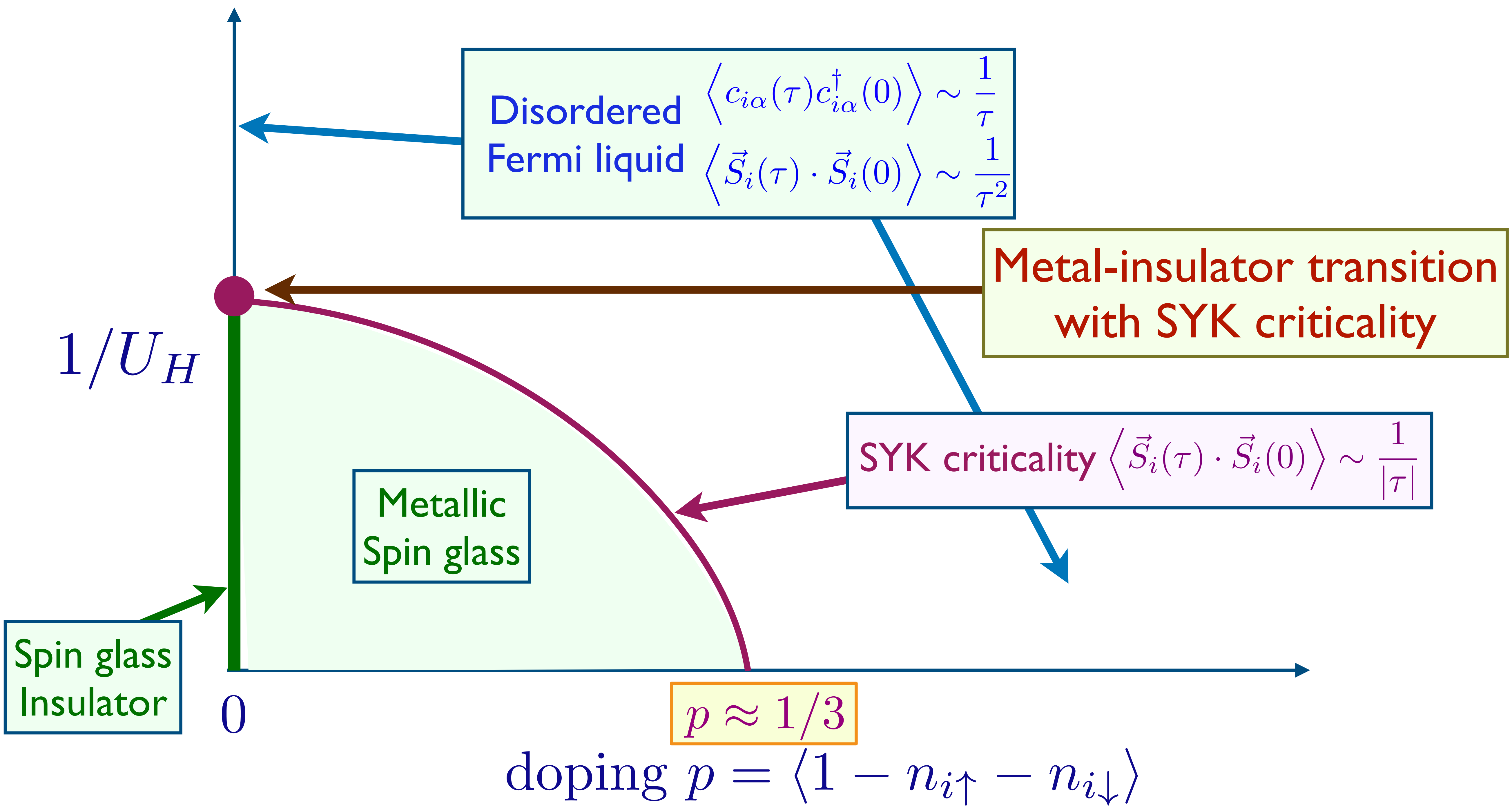
$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^\dagger(0) \right\rangle \sim \frac{1}{\tau}$$



D. Joshi, Chenyuan Li, G. Tarnopolsky, A. Georges, S. Sachdev, PRX **10**, 021033 (2020)

Random t-J-U_H model

$$H = -\frac{1}{\sqrt{N}} \sum_{i,j=1}^{N} t_{ij}\, c_{i\alpha}^{\dagger} c_{j\alpha} + \frac{1}{\sqrt{N}} \sum_{i<j=1}^{N} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + U_H \sum_{i=1}^{N} n_{i\uparrow} n_{i\downarrow}$$



At the critical point/phase of the t-J model, the Fermi liquid-like behavior of the electron Green's function

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \right\rangle \sim \frac{1}{\tau}$$

leads to a non-zero *residual resitivity*, $\rho(0) \neq 0$.

However, the critical state is *not* a Fermi liquid, as indicated by the slow decay of the spin correlations

$$\left\langle \vec{S}_i(\tau) \cdot \vec{S}_i(0) \right\rangle \sim \frac{1}{|\tau|}$$

Moreover, in a Fermi liquid, we expect $\rho(T) - \rho(0) \sim T^2$, which also does not hold here.

Time reparameterization soft mode

The dominant corrections to the SL(2,R) invariant critical Green's function can arise from the time reparameterization soft mode, and these take the form

$$\left\langle c_{i\alpha}(\tau) c_{i\alpha}^{\dagger}(0) \right\rangle \sim \frac{\pi T}{\sin(\pi T \tau)} \left(1 + \alpha_G \frac{T}{J} \Phi_{\text{non-conformal}}(T\tau) \right)$$

where $\Phi_{\text{non-conformal}}(T\tau)$ is a computable (in the large M limit) scaling function, and α_G is universally proportional to the co-efficient α_S of the Schwarizan action for the time reparameterization mode.

J. Maldacena and D. Stanford, PRD **94**, 106002 (2016)
A. Kitaev and J. Suh, JHEP 183 (2018)
Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

Time reparameterization soft mode

Computing the resistivity from this Green's function via the Kubo formula, we find

$$\rho(T) = \rho(0)\left(1 + 8\alpha_G \frac{T}{J} + \dots\right)$$

Haoyu Guo, Yingfei Guo, S. Sachdev, Annals of Physics **418**, 168202 (2020)

1. All-to-all random Hubbard and *t-J* models
Numerical results

2. Random *J* model (insulator)
RG analysis and exact exponent

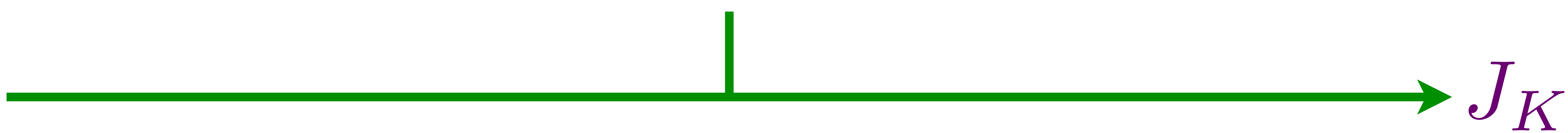
3. Random t-*J* model (metals)
RG analysis and exact exponents

4. Non-random t-*J* model (metals)
Ancilla qubits and ghost Fermi surfaces

Carrier density transitions in Kondo lattice models

Kondo lattice of f electron spins coupled
to a conduction band of c electrons.

Kondo-breakdown or 'selective Mott' transition



T. Senthil, M. Vojta, and S. Sachdev, PRB **69**, 035111 (2004)

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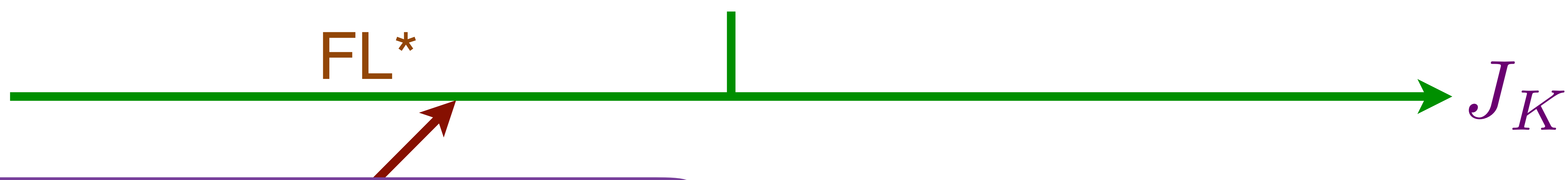

Small Fermi surface

$$|\Phi\rangle = |\text{Spin liquid insulator of } f\rangle \otimes |\text{Slater determinant of } c\rangle$$

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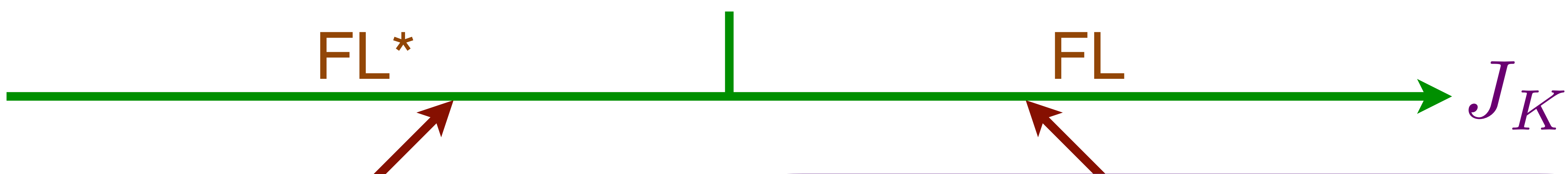

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Large Fermi surface

$$|\Phi\rangle = [\text{Projection onto one } f \text{ per site}] \bowtie |\text{Slater determinant of } (c, f)\rangle$$

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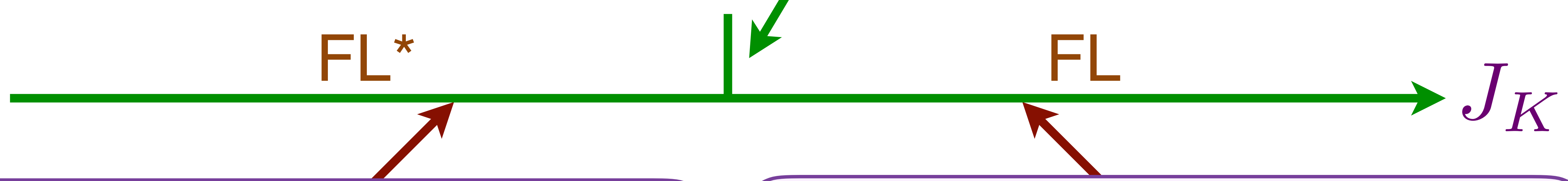
U(1) gauge theory of a
'hybridization-Higgs' boson
which condenses on the
'Large Fermi surface' side.



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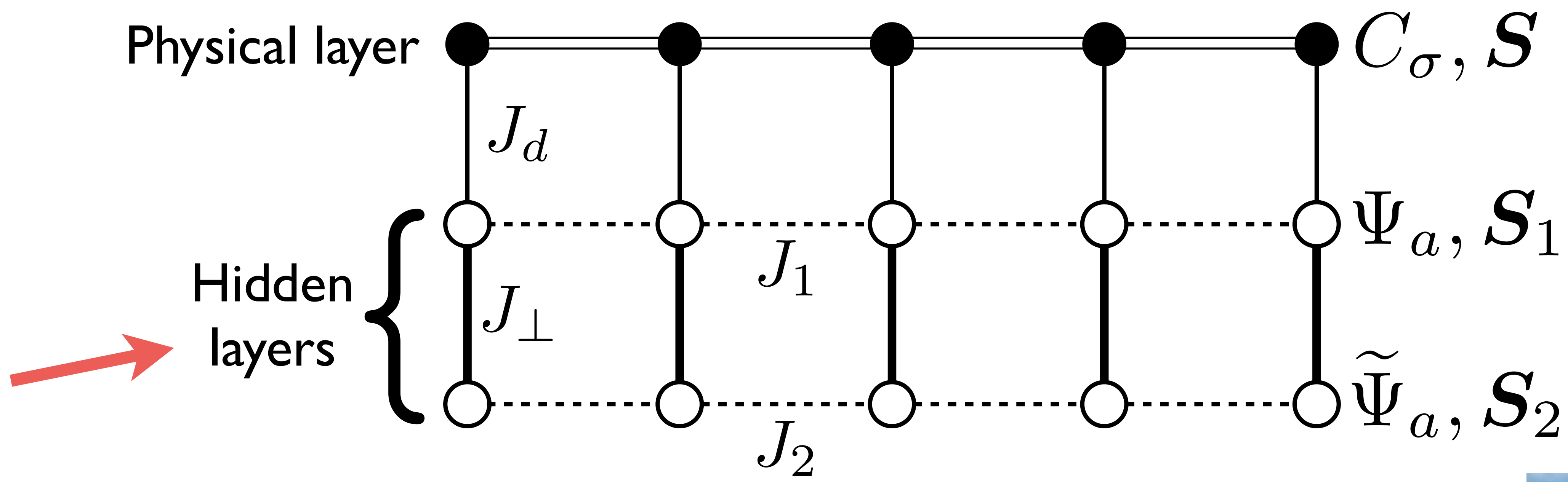
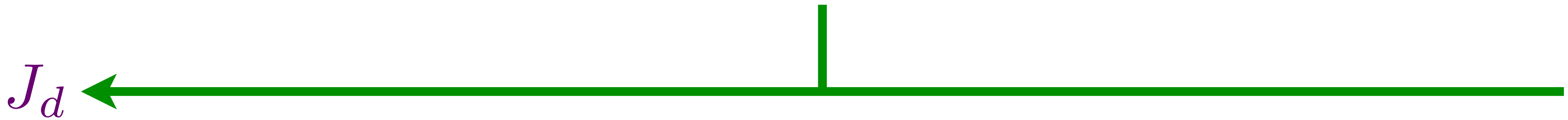
Carrier density transitions in a one-band model

Ancilla qubits

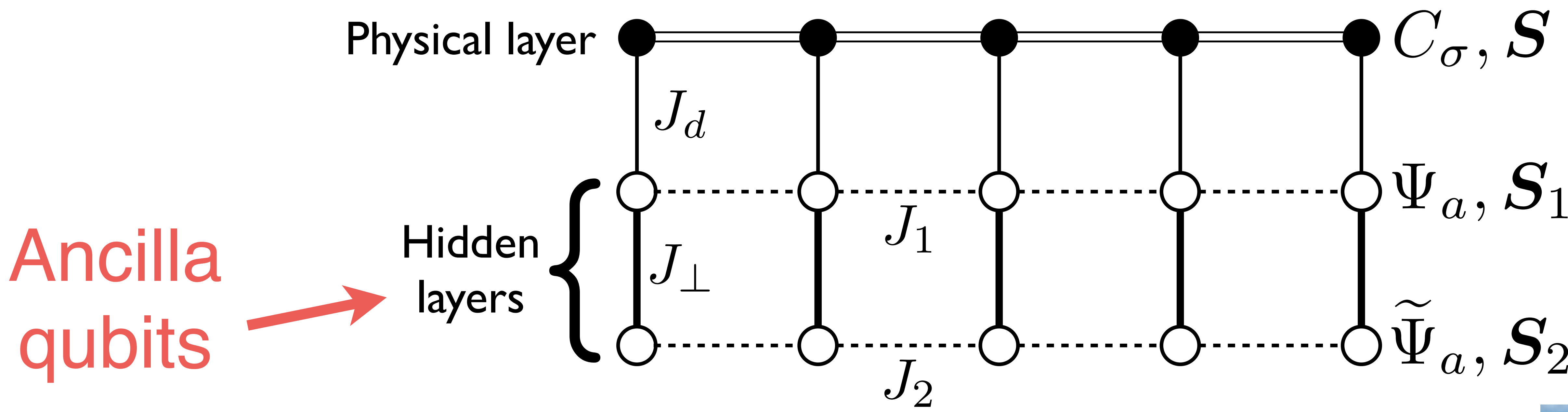


Yahui Zhang



Ya-Hui Zhang, S. Sachdev, PRR **2**, 023172 (2020); arXiv:2006.01140.

Carrier density transitions in a one-band model



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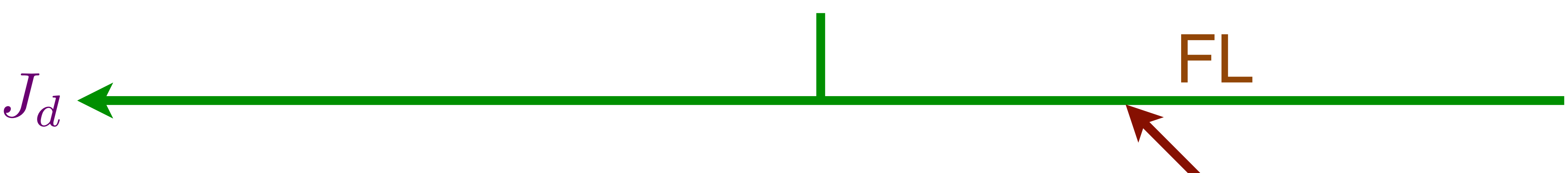

Large Fermi surface

$$|\Phi\rangle = \left|\text{Rung singlets of } \Psi, \widetilde{\Psi}\right\rangle \otimes \left|\text{Slater determinant of } C\right\rangle$$

Ya-Hui Zhang, S. Sachdev, PRR **2**, 023172 (2020); arXiv:2006.01140.

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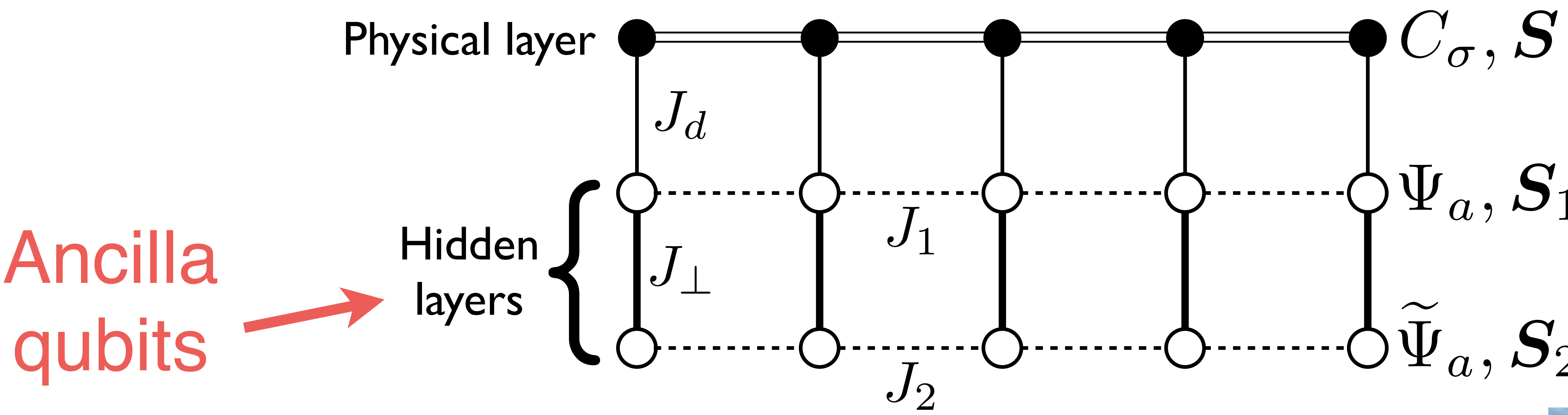

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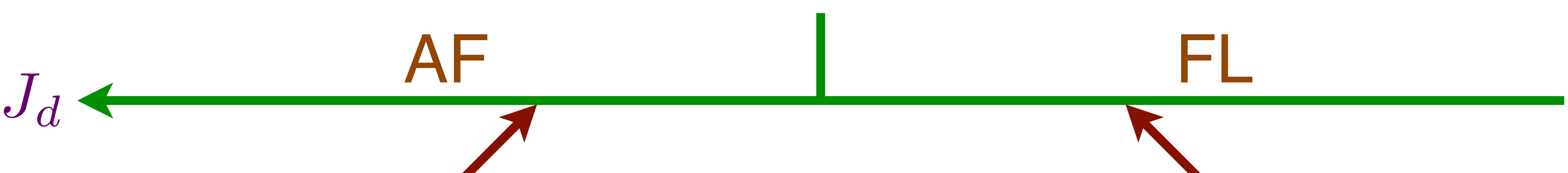

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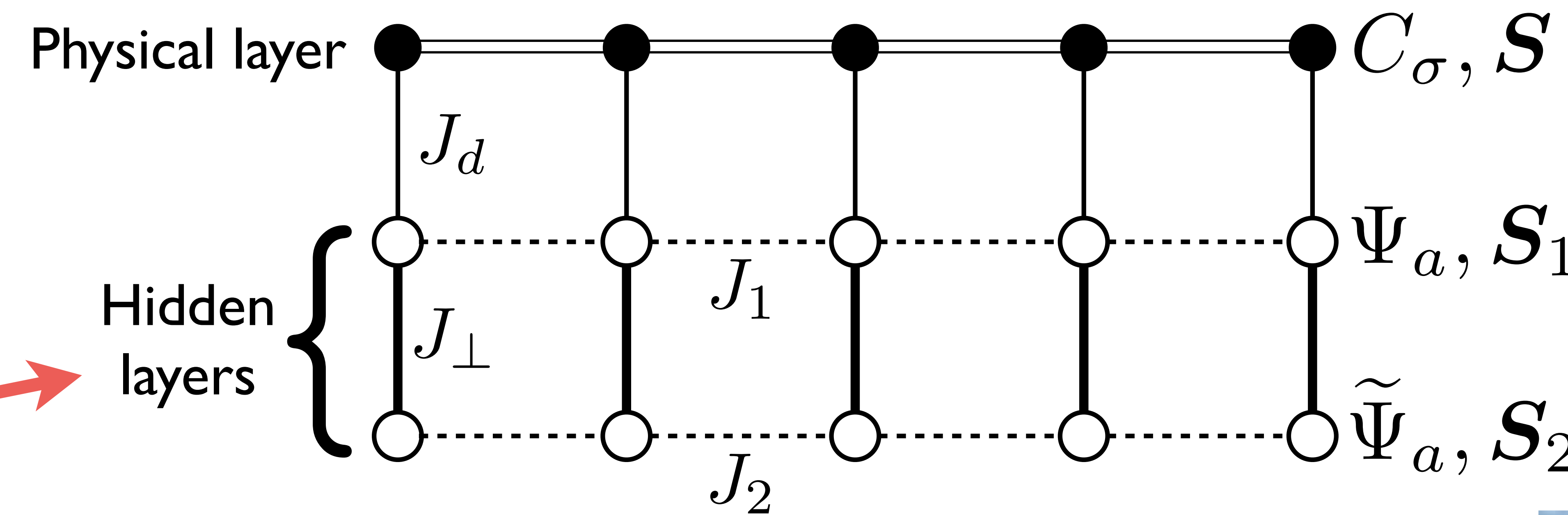
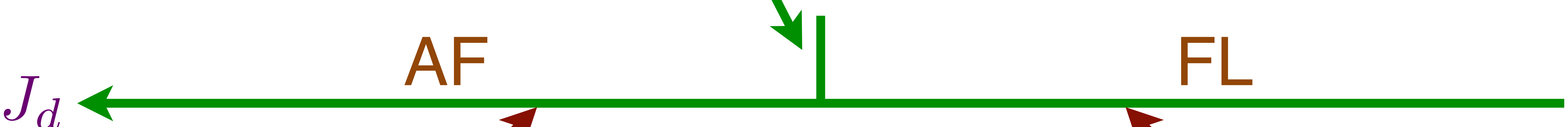


$(U(1) \times U(1))/Z_2$ gauge theory of a ghost Fermi surface and a 'hybridization-Higgs' boson which condenses on the 'Small Fermi surface' side.

Yahui Zhang



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