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Tailoring three point functions and integrability from weak to strong coupling

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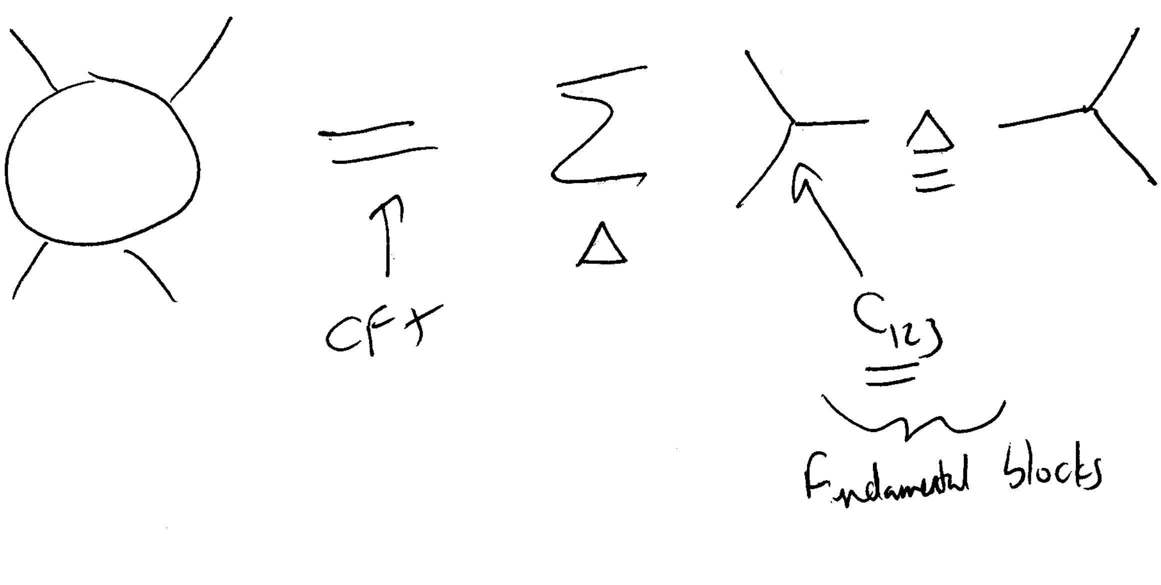
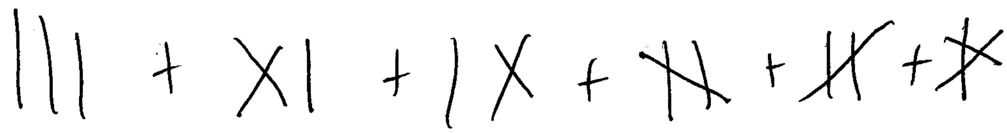
- compute C_{123} in perturbation theory
- Classical limit
- Match with strong coupling + new strong coupling predictions



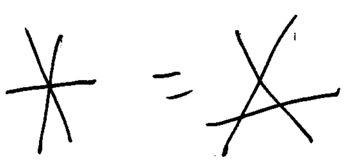
C_{123} (?)

$$O_{(2)} = \sum_{n_1<n_2} \mathrm{Tr}(Z\cdots Z \underset{\substack{\uparrow \\ n_1}}{X} Z\cdots Z \underset{\substack{\uparrow \\ n_2}}{X} Z\cdots)\left[e^{ip_1n_1+ip_2n_2} + S(p_1p_2)e^{ip_1n_2+ip_2n_1}\right]$$

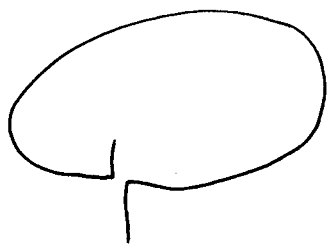
$$O_{(3)} = \sum_{n_1<n_2<n_3}$$



$$S(p_1p_2p_3) = SSS$$



$\{p_i\}$



$$e^{ip_iL}\prod_{j\neq i} S(p_i,p_j) = 1$$

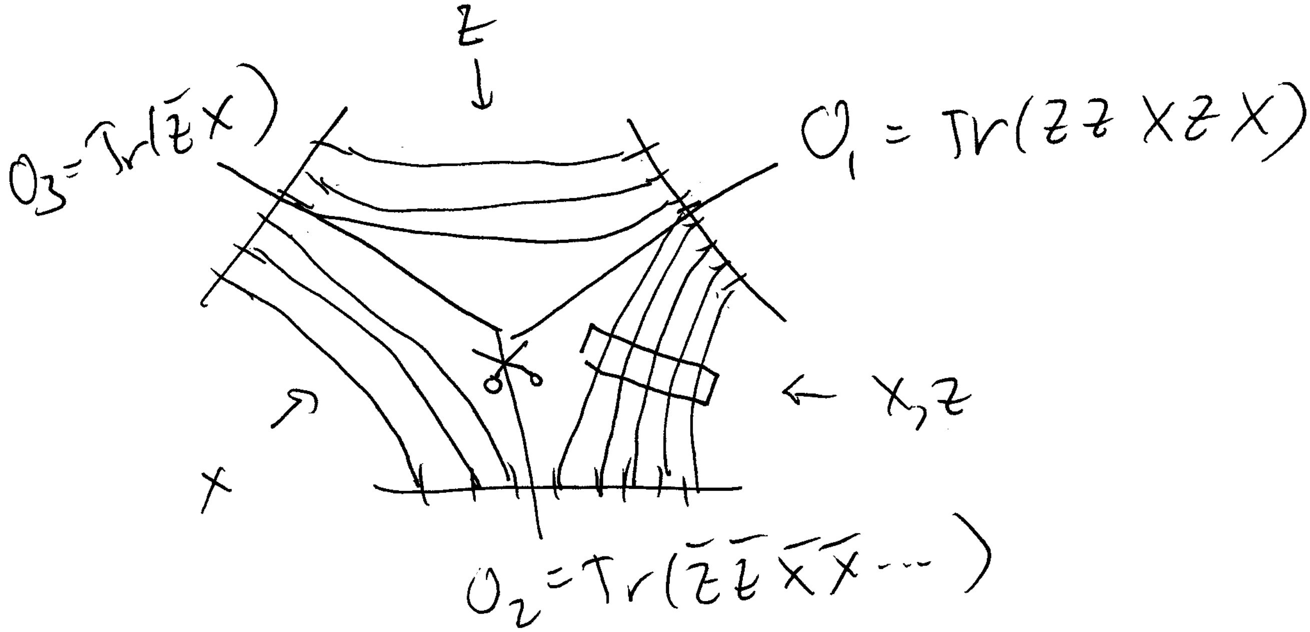
n

$$e^{2\pi i \phi_i\{p\}}$$

$$C_{123}(\lambda, \{p_i\}, \{q_i\}, \{h_i\})$$

$$C_{123}(\lambda, \{0\}, \{0\}, \{0\}) = C_{BPS}$$

$$C_{123} = C_{123}^{(0)} + \lambda C^{(1)} + \lambda^2 C^{(2)} + \cdots$$



$$|\psi\rangle = \sum |\psi\rangle_L \otimes |\psi\rangle_R$$

2. Overlap
3. Normalize the operators O_i

$$\det\left|\frac{\partial \phi_i}{\partial p_j}\right|$$

$$C_{123}^{\bullet\bullet\circ} = \frac{1}{B_1 B_2 B_3} \sum_{\substack{\alpha \cup \bar{\alpha} = \{u\} \\ |\alpha| = \ell_{13}}} e_{L_1}^{\bar{\alpha}} f^{\alpha\bar{\alpha}} \mathcal{A}(\ell_{13}|\bar{\alpha}) \mathcal{C}(\ell_{12}|\alpha, \{v\})$$

$$B_a = \frac{g^{\{v - i/2\}} f_<^{\{v\}\{v\}}}{\sqrt{g^{\{v + i/2\}} g^{\{v - i/2\}}}} \sqrt{\frac{f_>^{\{v\}\{v\}} \det \varphi_{\{v\}}^{i}}{f_>^{\{v^*\}\{v\}} L_a}} \binom{L_a}{N_a}$$

$$\mathcal{A}(\ell|\{u\}) = \sum_{\alpha \cup \bar{\alpha} = \{u\}} (-1)^{|\alpha|} f^{\alpha\bar{\alpha}} / e_\ell^{\alpha}$$

$$\mathcal{C}(\ell|\{u\}, \{v\}) \equiv \sum_{\substack{\alpha \cup \bar{\alpha} = \{u\} \\ \beta \cup \bar{\beta} = \{v\}}} g_<^{\{\ \}\{\ \}} g_>^{\{v\}\{v\}} (-1)^{P_\alpha + P_\beta} e_\ell^{\bar{\beta}} e_\ell^{\alpha}$$

$$h^{\beta\alpha} h^{\bar{\alpha}\beta} h^{\beta\bar{\beta}} h^{\bar{\alpha}\alpha} \det(f^{\alpha\beta}) \det(f^{\bar{\alpha}\bar{\beta}})$$

$$f_<^{\{u\}\{u\}} \equiv \prod_{\substack{u_i, u_j \in \{u\} \\ i < j}} \left(1 + \frac{i}{u_i - u_j}\right), \quad e^{i\varphi_j} \equiv \left(\frac{u_j + \frac{i}{2}}{u_j - \frac{i}{2}}\right)^L \prod_{k \neq j} \frac{u_j - u_k - i}{u_j - u_k + i}$$

$$e^{ip} \equiv \frac{u+i/2}{u-i/2} \longrightarrow \frac{x(u+i/2)}{x(u-i/2)}$$

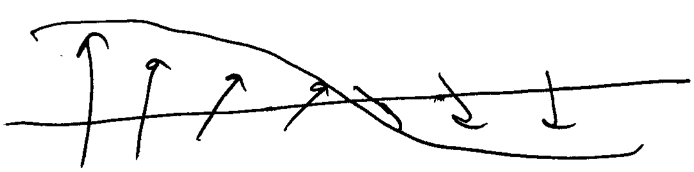
$$\frac{\sqrt{\lambda}}{2\pi}\, x(u) = u + \sqrt{u^2 - \frac{\lambda}{4\pi^2}}$$

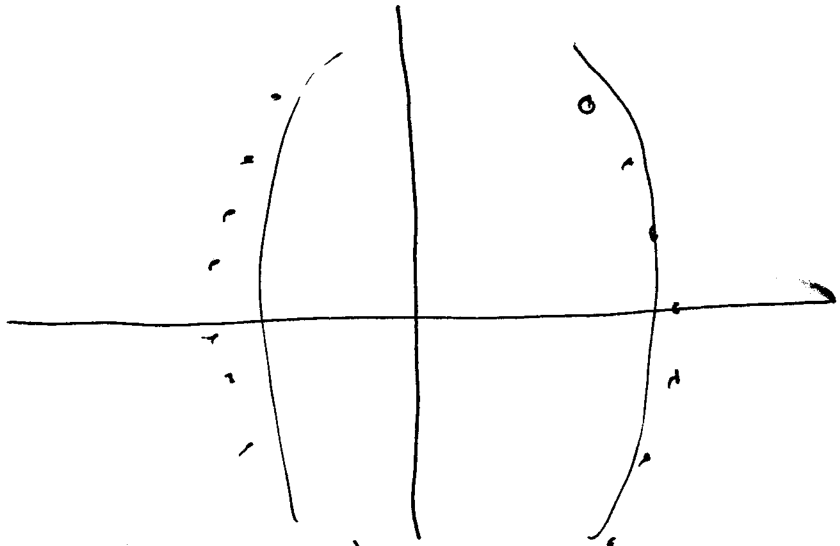
Frolov–Tseytlin limit

$$\left.\begin{array}{l} L \to \infty \\ N \to \infty \end{array}\right\} \quad \frac{N}{L} \text{ fixed}$$

$$p_i \sim \frac{1}{L}, \qquad E \simeq \sum p_i^2 \sim \frac{1}{L}$$

$$u_{i+1} - u_i \sim \frac{1}{L}.$$





$\angle u = \frac{1}{2}\omega t + \frac{\theta}{2}$

$\tilde{\lambda} = \frac{\lambda}{J^2}$

$$|\Psi\rangle = \sum (N!\ \text{terms}) \simeq \underbrace{|\vec{u}\rangle_1 \otimes |\vec{u}\rangle_2 \otimes \cdots}_{=|\varphi\rangle}$$

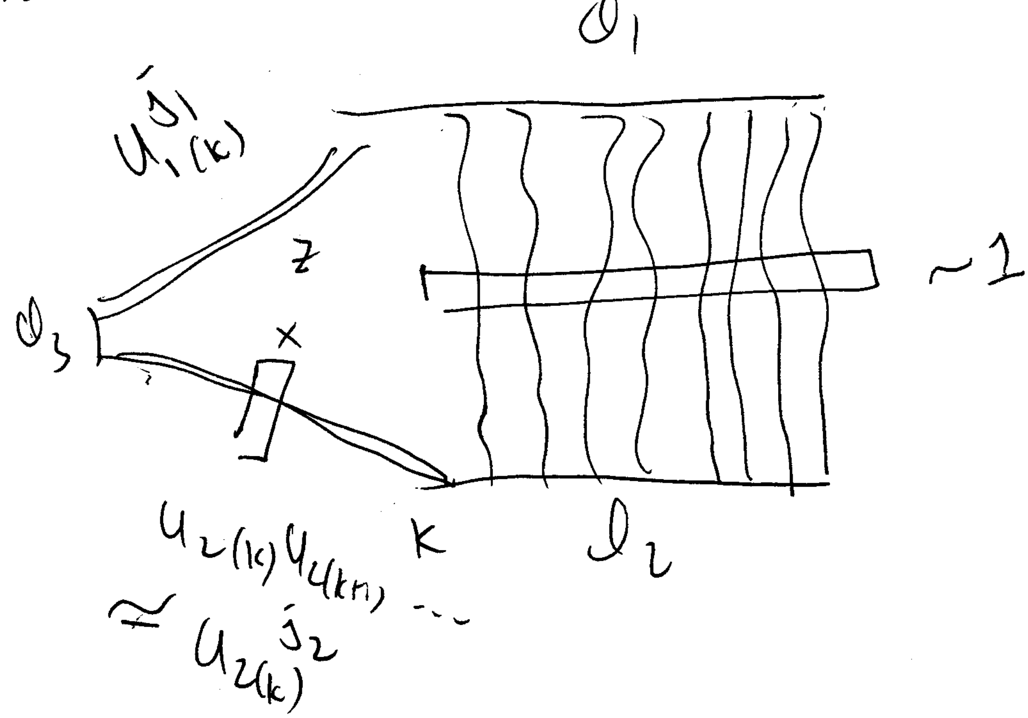
$$|\vec{u}\rangle = \underset{\uparrow}{u_1|z\rangle} + \underset{\downarrow}{u_2|x\rangle} = \cos\theta_i\,|\uparrow\rangle + e^{i\varphi_i}\sin\theta_i\,|\downarrow\rangle$$

$u \cdot \bar{u} = 1$

$$\frac{\langle \psi | O | \psi \rangle}{\langle \psi | \psi \rangle} = \frac{\langle \varphi | O | \varphi \rangle}{\langle \varphi | \varphi \rangle} \left(1 + O\left(\tfrac{1}{L}\right) \right)$$

$\mathcal{O}_2 = \mathcal{O}_1^\dagger$ classical

$\mathcal{O}_3$ small BPS

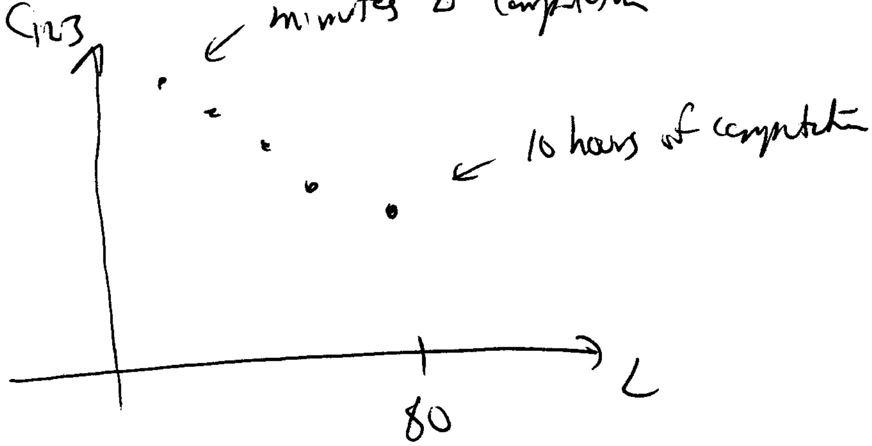


$$C_{123} = \# \int_0^{2\pi} d\sigma \; u_2^{j_2}(\sigma)\, \bar{u}_1^{j_1}(\sigma)$$

$\uparrow$

$\frac{2\pi k}{L}$

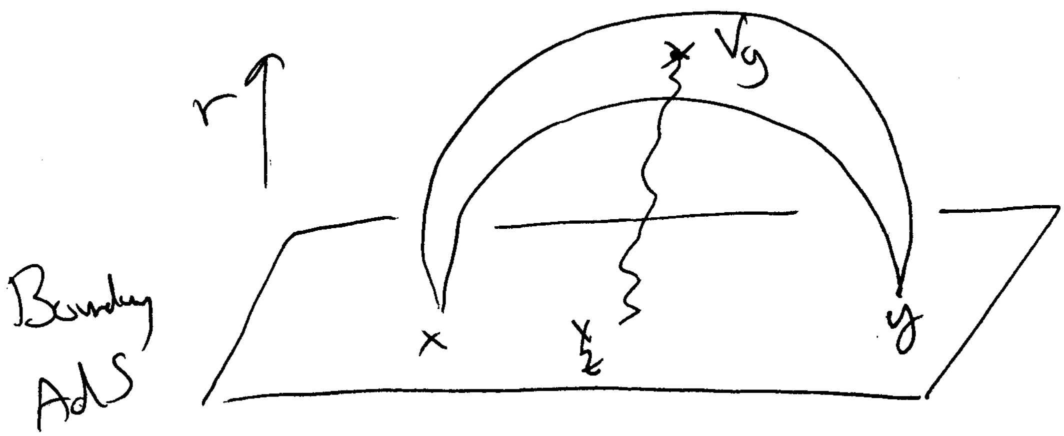
$\frac{N}{L} = \frac{1}{5}$



interpolate from data: $C_{123} = a_0 + \frac{a_1}{L} + \frac{a_2}{L^2} + \cdots$

Then plug in $L = 50000$

get a match up to 10^{-6}.



$$C_{123}^{\circ\circ\circ} = \# \int_{-\infty}^{\infty} \int d\tau_E \int_0^{2\pi} d\sigma \underbrace{u_1^{\,j_1}\, \overline{u_2}^{\,j_2}}_{Y_{BPS}(\vec{u})} \frac{1}{\cosh^{j_1+j_2}(\kappa\tau_E)} \left[\frac{2\kappa^2}{\cosh^2(\kappa\tau_E)} - \kappa^2 - 2\,\partial_a \overline{u} \cdot \partial^a u\right]$$

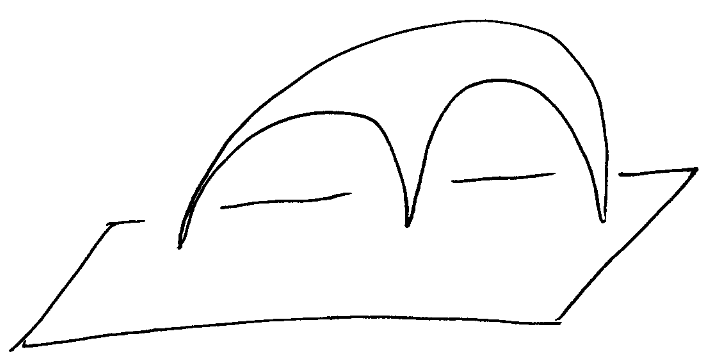
$\sqrt{\lambda}\,\kappa = \text{Energy} \simeq J$

$\vec{u} = (X_1 + i X_2, X_3 + i X_4, X_5 + i X_6) \in S^5$

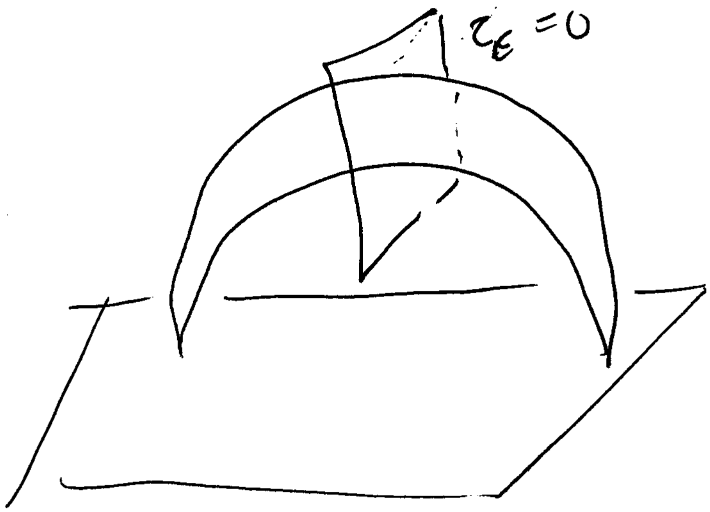
$$q(u) \equiv \frac{l}{u} - \int_{UC_x} dv \, \frac{\rho(v)}{u-v}$$

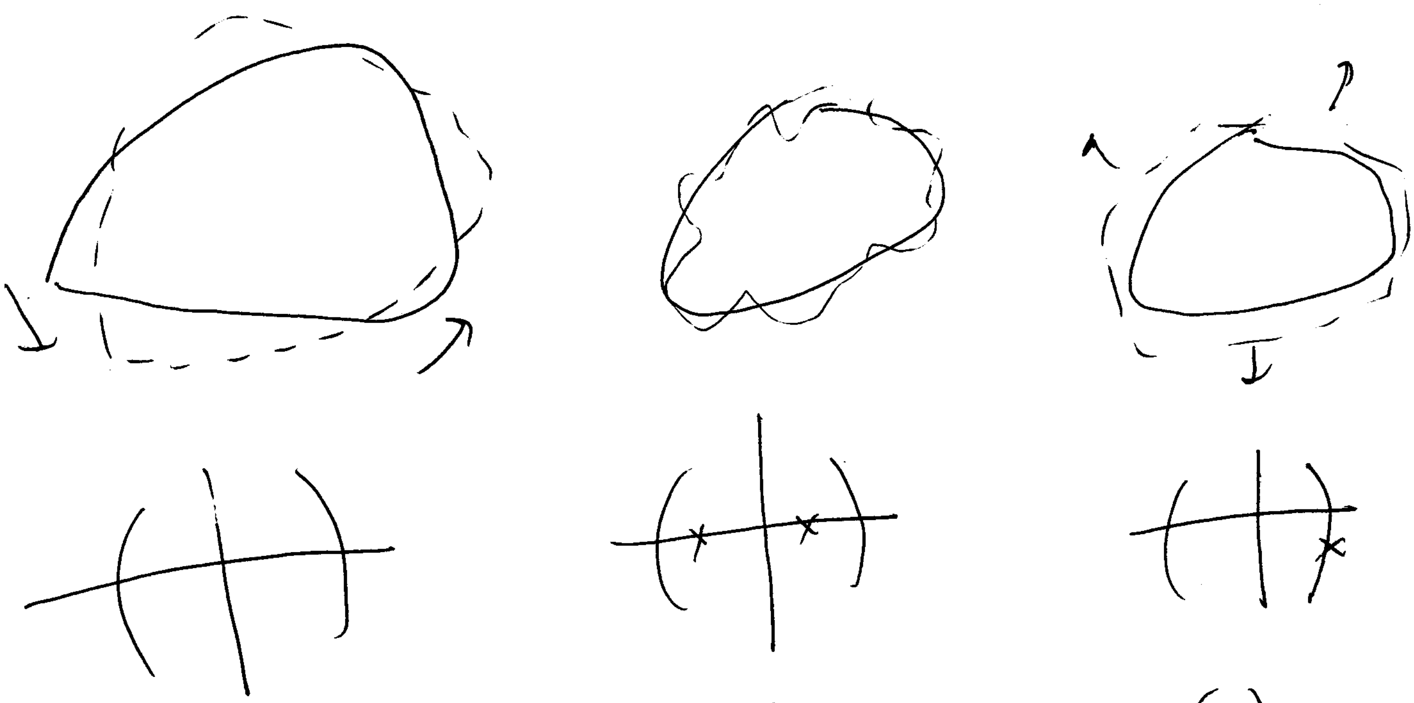
$$\log C_{123} = \log\left(\int d\sigma\, u_1^{\,j_1}\, \overline{u_2}^{\,j_2}\right) \int_0^1 dt \left[\oint_{UC_k} \frac{dy}{2\pi i}\, q \log\left(1 + e^{iqt}\right) - \int_{UC_k} du\, \rho \log\left(2\sinh(\pi t \rho)\right)\right]$$

$\frac{1}{\kappa} = \frac{\sqrt{\lambda}}{J} \ll 1$



$$\bar{U}(\tau_E, \sigma) = e^{ik\tau_E} \vec{u}\left(\frac{\tau_E}{J}, \sigma\right)$$





(A) (B) (C)

$$\phi = A_n \cos((\sigma-\tau)n) \longrightarrow \left(A_n + \delta A_n^{(B)}\right)\cos(n(\sigma-\tau)) + \delta A_n^{(C)} \cos(m(\sigma-\tau))$$

$$n A_n^2 = \hbar N_n$$ (Bohr–A quantization condition)

$$\Rightarrow A_n \sim \sqrt{N_n}$$

$$\Rightarrow \delta A^{(B)} \sim \frac{1}{\sqrt{N_n}}$$

$$\delta A^{(C)} \sim 1$$