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Closed strings in bosonic open string field theory (OSFT)

$\ni \Psi$

OSFT $\quad D6(A_0, Q_0, K)$

$$S = \langle \Psi, Q_0 \Psi \rangle + \frac{1}{3} \langle \Psi, \Psi * \Psi \rangle$$

physical open string states $\sim coh(Q_0)$

" " g.t. $\quad Q_0$ - exact

<u>claim</u> in OSFT $\exists$ 2nd nilpot op d_H, $d_H^2 = 0$, s.t.

1) {physical closed states} $\simeq coh(d_H)$
2) closed string gauge transformations (g.t.) d_H exact
3) $coh(d_H)$ is open string background independent

w/ N. Mueller

cf. • $H_*(\Lambda_0 M)$, $HH^*(\Omega(M), d, \wedge)$ chen

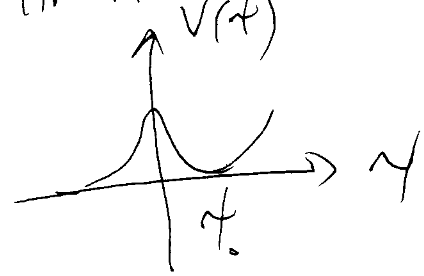
• TCFT $\sim (CY - A_\infty) = \mathcal{A}$, $HH^*(\mathcal{A}) \simeq$ closed states in TCFT

(Lazaroiu, Kapustin, Costello ...

Applications

A) $d_H(Q_0, *) \longrightarrow$ OSFT encodes complete information about closed strings

B) $\rightarrow$ construct $\underbrace{\text{OCSFT}}_{\text{open-closed}}$ in the tachyon vacuum at linearized level



4) finite deformations

$(A_{closed}) \xrightarrow{F} C(TA_0)$ tensor algebra

maps $TA_0 \to TA_0$

F an L_∞?

OCHA (Kajiura-Stasheff)

5) QOCHA (w/ K. Münster)

$(S,S) + \hbar \Delta S = 0$

Interpretation?

closed string

$$S = S_0 + \sum_{n=1}^{\infty} C^\phi(\underbrace{\psi,\dots,\psi}_{n})$$

$$C^\phi_M(\psi_1,\dots,\psi_m) = \int_{T_m} ds_1 \cdots ds_{m-1}$$

$\psi_M(s_M)$, etc.

$$\sum_i |\psi_i^*| = M$$

$$\psi_i = \psi$$

$$\mathrm{Hom}(A^{\otimes M}, \mathbb{C}) \ni C^\phi_M : A_0^{\otimes M} \to \mathbb{C}$$

ψ

$$= \int_{T_M} ds_1 \cdots ds_{M-1} \langle \phi, b^{(1)} \cdots b^{(M-1)} \psi(s_1) \cdots \psi_M(s_M) \rangle$$

$$\psi = c_0|0\rangle \cancel{+} \alpha^n_{-1}(\dots)$$

phys closed states $\quad Q|\phi\rangle = 0 \quad (b_0^- = (b_0 - \bar{b}_0))|\phi\rangle = 0$

$$C^\phi_M(\psi_2, \psi_3, \dots, \psi_m, \psi_1) = (-1)^{M-1+\cdots} C^\phi_M(\psi_1, \dots, \psi_m) \qquad (*)$$

$$C^{\phi}_{M} = CC^{M} = \{ \mathrm{Hom}(A_0^{\otimes M} \to \mathbb{C}) \mid * \}$$

$$\{ Q, b_{-1} \} = L_1 \sim \partial_s$$

$$C_M^{\cancel{\ } Q\phi}(\chi_1, \ldots, \chi_M)$$

$$= C_M^{\phi}(*\chi_1, \chi_2, \ldots, \chi_M) + \sum_{i=1}^{M-1} (-1)^i C_{M-1}^{\phi}(\chi_1 \cdots \chi_i * \chi_{i+1} \cdots \chi_M)$$

$$+ (-1)^{M-1} \sum_{i=1}^{M} (-1)^{\chi_1 + \cdots + \chi_{i-1}} C_M^{\phi}(\chi_1 \cdots (Q\chi_i) \cdots \chi_M)$$

$\delta : CC^{M-1} \to CC^{M}$ coboundary op

$Q : CC^{M} \to CC^{M}$

$$[\delta, Q] = 0 \,, \quad \delta^2 = 0$$

$$d_H = (\delta - (-1)^M Q) : CC^* \to CC^*$$

$$C_*^{Q\phi} = d_H C_*^{\phi}$$

$$\Rightarrow \left\{ \mathrm{coh}(Q_c) \subset \mathrm{coh}(d_H) \right.$$

Other d_H exact elements $\qquad Q\Psi_0 = 0$

(α) infinitesimal shifts of open string background $\quad \Psi = \Psi_0 + \delta\Psi$

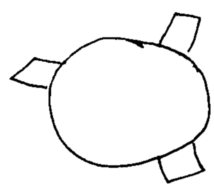
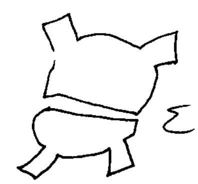
$\rightsquigarrow S_0 + C_2$

$$C_0(\Psi_1\Psi_2) = \langle \Psi_0, \Psi_1 * \Psi_2 \rangle + (-1)^{\Psi_1\Psi_2} \langle \Psi_0, \Psi_2 * \Psi_1 \rangle$$

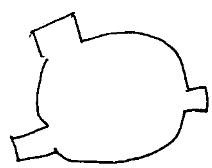
$$= \delta(\langle \Psi_0, \Psi_1 \rangle)$$

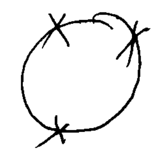
$$=: \delta D_1 \;/\; (QD_1) = 0$$

$\Rightarrow d_H$ exact

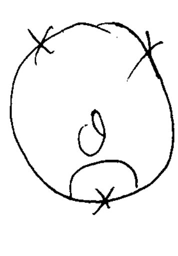
(β)  + 

$+ O(\varepsilon^2)$

C_3  $\qquad C_4$

$C_3 = (Q_0 D_3), \quad C_4 = (\delta D_3) \qquad D_3 =$ 

d_H exact

(γ) 

+ cyclic

$\mathrm{coh}(d_H) = \mathrm{coh}(Q_c)$ ✓

background independence

$$Q_0 \Psi_0 + \Psi_0 * \Psi_0 = 0$$

$$\Psi \to \Psi_0 + \Psi$$

$$Q \to \tilde{Q}, \quad \tilde{Q}\Psi = Q\Psi + \Psi_0 * \Psi + \Psi * \Psi_0$$

$$h^{-1} d_H(\tilde{Q}, *) h = d_H(Q, *)$$

Extensions

BV equation $\quad (S,S) = 0.$

$$TA_0 = \bigoplus_n A_0^{\otimes n}$$

$$SA_c = \sum_n A_c^{\wedge n}$$

$\mathrm{coder}(TA) : TA \to TA$

$\mathrm{coder}(SA) : SA \to SA$

open string BV equation $\longleftrightarrow$ $M \in \mathrm{coder}(TA_0)$, $M^2 = 0$

open vertices

closed string BV equation $\longleftrightarrow$ $L \in \mathrm{coder}(SA)$, $L^2 = 0$

$\longleftrightarrow$ L_∞

OCHA $\qquad (SA, L) \xrightarrow{F} (\mathrm{coder}(TA_o), M)$,

$FL = MF$.

unknown if F is an iso.

Solution of closed $\quad \underset{\hat{A_C}}{\phi} \quad L(\underset{\hat{SA_c}}{e^{\phi}}) = 0 \quad$ (Maurer-Cartan elements)

QOCHA: $\qquad (S, S) = \hbar \Delta S$

L modified to

$L = \sum \hbar^g L^g + \hbar \underbrace{D(\omega_c^{-1})}_{\Delta}$

$\Phi = \sum_{g,n} \hbar^{gn} \phi^{gn} \qquad \phi^{00} = \phi_{clas}$

$L(e^{\Phi}) = 0$. $\qquad$ interpretation?