BO and AGT - Alexey Litvinov

(Benjamin - Ono)

1. Alba, Fateev, Tasnopolsky
2. Fateev } unpublished
3. Tasnopolsky

- $V_t + 2VV_x + HV_{xx} = 0$

$$x \in [0, 2\pi]$$

$$H\,F(x) = P\left[\frac{1}{2\pi}\int_0^{2\pi} F(y)\cot\left(\frac{y-x}{2}\right)dy\right]$$

- "Miura" transformation $\qquad P_\pm = \frac{1}{2}\left(1 \mp H\right)$

$$(*) \qquad V = \lambda(e^{\omega}-1) + P_+\bar{\omega}_x$$

$$\left(\lambda e^{\omega} + iP\frac{\partial}{\partial x}\right)\left[\omega_t + 2V\omega_x - i\omega_x^2 + H\omega_{xx}\right] = 0$$

LAX:

$$\left(i\frac{\partial}{\partial x} + \lambda + V\right)\Phi^+ = \lambda\Phi^-$$

$$\Phi^\pm_t - 2\lambda\Phi^\pm_x - i\Phi^\pm_{xx} \mp 2\Phi^\pm P_\pm V_x = 0$$

$$\omega = \omega^+ + \omega^-$$

$$\Phi^+ = e^{-\omega^+} \qquad \Phi^- = e^{\omega^-}$$

• BO_n $\qquad D_n \Phi^+ = \lambda \Phi^-$

$BO_n \rightsquigarrow$ AGT for $U(N)$ linear quivers

BO_2 : $\quad V, u$

$$[\partial - iV + \lambda^2 + u]\,\Phi^+ = \lambda^2 \Phi^-$$

$$\Phi^\pm_+ - i\lambda \Phi^\pm_x - \frac{i}{2}\Phi^\pm_{xx} \mp 2\Phi^\pm P_\pm V_x = 0$$

$$\downarrow$$

$$\begin{cases} u_t + Vu_x + 2uV_x + \frac{1}{2}V_{xxx} = 0 \\ V_t + \frac{u_x}{2} + HV_{xx} + VV_x = 0 \end{cases}$$

IM (integrals of motion)

$$I_k = \frac{1}{2\pi}\int G_k\, dx$$

$$\partial_t I_k = 0, \qquad k\text{-spin}$$

$$G_1 = V \qquad G_2 = u^2 + V$$

$$G_3 = uV + VDV + \frac{1}{3}V^3$$

$$D = H\frac{\partial}{\partial x} \xrightarrow{FT} -|k|$$

$$u_t = \{ I_2, u \}$$

$$v_t = \{ I_t, v \}$$

$$\{ v(x), v(y) \} = \frac{1}{2} \delta'(x-y)$$

$$\{ u(x), u(y) \} = (u(x) + u(y)) \delta'(x-y) + \frac{1}{2} \delta'''(x-y)$$

$$\{ v, u \} = 0$$

$c = 1$ model

$$\{ I_k, I_\ell \} = 0$$

<u>Quantize</u>

CFT on a cylinder

$$\mathcal{A} = Vir_c \otimes^{*} Heis$$

$$T(x), \quad J(x) = \cancel{\delta} \, \delta \Phi(x)$$

$$\mathcal{U}(\mathcal{A}) \quad : \quad I_k = \frac{1}{2\pi} \int G_{k+1} \, dx$$

$$G_{k+1}(T, J, \partial, D) \qquad K\text{-Spin}$$

2. $[I_k, I_\ell] = 0$

3. $c \to \infty \qquad T \to -\frac{c}{6} u \qquad J \to -i \sqrt{\frac{c}{6}} \, V$

$$[\, , \,] \rightsquigarrow \frac{-i\pi c}{?} \{ \, , \, \}$$

Notations

- $c = 1 + 6Q^2, \quad Q = b + \frac{1}{b}$
- $V^L_\alpha \longrightarrow \Delta(\alpha) = \alpha(Q - \alpha)$

$$\alpha = \frac{Q}{2} + P, \qquad \Delta = \frac{Q^2}{4} - P^2$$

- $V_\alpha = V^L_\alpha \mathcal{V}_\alpha$

$$\mathcal{V}_\alpha = e^{2(\alpha - Q)\Phi_-} e^{2\alpha \Phi_+}$$

$$\Phi_\pm = \sum \frac{a_n}{n} e^{-inx}$$

$$|P\rangle = L_n|P\rangle = a_n|P\rangle = 0 \qquad n > 0$$

$$L_0|P\rangle = \left(\frac{Q^2}{4} - P^2\right)|P\rangle$$

$$|P\rangle_{\vec{\lambda}} \qquad\qquad \mathbb{I}_k |P\rangle_{\vec{\lambda}} = h^{(k)}_{\vec{\lambda}}(P)\, |P\rangle_{\vec{\lambda}}$$

$\vec{\lambda} = (\lambda_1, \lambda_2)$
pair of Young diagrams

$$|P\rangle_{\vec{\lambda}} = \left((-L_{-1})^{|\vec{\lambda}|} + \quad \right) |P\rangle$$

$|\vec{\lambda}|$ - number of boxes in $\vec{\lambda}$

Statement

$$\vec{n}\,\frac{\langle P'|V_\alpha|P\rangle_{\vec\lambda}}{\langle P'|V_\alpha|P\rangle} = Z_{b,f}(\ldots)$$

bifund

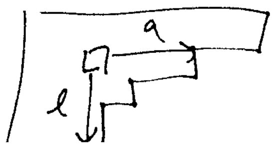
$$Z_{bf}(\alpha; P, \vec\lambda; P', \vec\mu) =$$

$$\prod_i \prod_{s\in\lambda_i} \left(Q - E_{\lambda_i,\mu_j}(P_i - P'_j|s) - \alpha\right)$$

$$\times \prod_{t\in\mu_j} \left(E_{\mu_j,\lambda_i}(P'_j - P_i|t) - \alpha\right)$$

$$\vec P = (P, -P)$$

$$E_{\lambda,\mu}(P|s) = P - b\,l_\mu(s) + b^{-1}(a_\lambda(s)+1)$$



$BO_n \rightsquigarrow$ AGT for $U(n)$

$$L_n = \sum_{k \neq 0, n} c_n c_{n-k} + i(nQ - 2P) c_n$$

$$L_0 = \ldots$$

$$I_2 = \frac{1}{2\pi} \int :\left(T J - iQ J D J + \frac{1}{3} J^3 \right): dx$$

$$\alpha_k = \frac{1}{2}(a_k - c_k) \qquad \beta_k = \frac{1}{2}(a_k + c_k)$$

$$I_2 = \hat{I}_2(P, \alpha) + \hat{I}_2(-P, \beta) + \hat{I}_{int}(\alpha, \beta)$$

$$\hat{I}_2(P, \alpha) = 4iP \sum \alpha_{-k} \alpha_k + 2Q \sum k \alpha_{-k} \alpha_k$$

$$+ \frac{4}{3} \sum_{i+j+k=0} \alpha_i \alpha_j \alpha_k$$

$$\hat{I}_{int}(\alpha, \beta) = 4iQ \sum_{k>0} k \, \alpha_{-k} \beta_k$$

$$|\alpha\rangle \qquad I_2 \sim \hat{I}_2$$

$$\alpha_{-k} = \frac{-ib}{2} P_k(x)$$

$$\alpha_k = \frac{i}{2b} k \frac{\partial}{\partial P_k}(x)$$

$$P_k(x) = \sum x_i^k$$

$$\frac{\partial}{\partial P_k} = \frac{\partial}{\partial x_i}$$

$$I_1 = 4\sum \alpha_{-k}\alpha_k \simeq \sum x_i \frac{\partial}{\partial x_i}$$

$$\hat{I}_2 \sim \sum \left(x_i \frac{\partial}{\partial x_i}\right)^2 + g\sum_{i<j} \frac{x_i}{x_i - x_j}\left(x_i \frac{\partial}{\partial x_i} - x_j \frac{\partial}{\partial x_j}\right)$$

$$g = -\beta^2$$

Jack polynomials

$$\hat{I}_2 J^{1/g}_{\lambda}(x) = (\quad) J^{1/g}_{\lambda}(x)$$

$$|P\rangle_{(\lambda,\phi)} = \Sigma_\lambda(P)\,\hat{J}^{1/g}_{\lambda}(x)\,|P\rangle$$

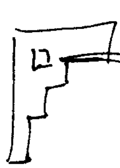
$${}_{(\mu,\phi)}\langle P'|V_\alpha|P\rangle_{(\lambda,\phi)} = Z$$

$$\alpha_n + P + P' + nb = 0 \qquad n = 0, 1, \ldots$$

Selberg integral w/ 2 Jack polynomials

Recursion formula

$$|P\rangle_{(\lambda,\Phi)} = X^{(P)}_{(\lambda,\Phi)}\,|P\rangle$$



(m,n) $\qquad P = P_{m,n} = -\frac{mb}{2} - \frac{nb^{-1}}{2}$

$$X_{(\lambda,\Phi)}(P_{m,n}) = (-1)^{mn} X_{(\lambda_1,\lambda_2)}(P_{m,-n}) \times D_{m,n}$$

$$|X_{mn}\rangle = D_{m,n}|P_{m,n}\rangle$$

$$L_n|X_{mn}\rangle = 0 \qquad n > 0$$