# Superfluids and the Cosmological Constant Problem

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*1805.05937 (JCAP)*

“The cosmological constant problem is the unwanted child of two pillars of twentieth century physics: quantum field theory and general relativity.”

Tony Padilla

# The CC problem in a nutshell

- The energy of the vacuum gravitates as a cosmological constant: $T_{\mu\nu} \sim \frac{\rho_{\rm vac}}{M_{\rm Pl}^2} g_{\mu\nu}$
- Massive fields: $\rho_{\rm vac} \sim m^4$
- Electron vacuum energy alone would lead to de Sitter horizon ~ $10^6$ km
  - Pauli: the radius of the world "nicht einmal bis zum Mond reichen würde" (would not even reach the Moon!)

# Where is the vacuum energy?

- The problem gets worse with more particle species:

$$\frac{\rho_{\rm vac,electron}}{\rho_{\rm vac,obs}} \sim 10^{32}$$

$$\frac{\rho_{\rm vac,SM}}{\rho_{\rm vac,obs}} \sim 10^{54}$$

$$\frac{\rho_{\rm vac,Planck}}{\rho_{\rm vac,obs}} \sim 10^{121}$$

- "The worst theoretical prediction in the history of physics!"

# Cosmological constant problems old and new

- Can split into two (logically distinct?) CC problems:
  - "Old problem": why does an enormous vacuum energy not gravitate?
  - "New problem": why is there some residual acceleration anyway?
- Often treated separately! Solve one while ignoring the other
- This talk: focus on the **old problem**

# Some approaches to the old problem

- Anthropics: if $\Lambda$ were bigger, we wouldn't be around to remark on it
    - ¯_(ツ)_/¯
    - Could follow from string landscape + eternal inflation
    - Dimopoulos: danger of "premature application"

# Some approaches to the old problem

- Modifications of gravity: leave $\Lambda$ alone, but change how it gravitates
    - Degravitation: weaken gravitational response to long-wavelength sources
    - Self-tuning: introduce new field(s) which dynamically counteract $\Lambda$

# Self-tuning and our modest goal

- We will set a modest goal: field equations solved by Minkowski for arbitrary $\Lambda$
- Necessary but not sufficient condition for solving old CC problem
  - Other criteria: UV insensitivity, radiative stability, no pathologies, agreement with experiments, reproduce observed cosmological history, etc.!
  - We'll address some but not all of these

# Weinberg's famous no-go theorem

- Weinberg (1988); see also Padilla review 1502.05296

- Self-tuning runs into a famous obstruction due to Weinberg

- Assume some fields $\phi^A$ "eat up" vacuum energy,

$$T^{\phi}_{\mu\nu} = -T^{\Lambda}_{\mu\nu}$$

- Assume *Poincaré-invariant* vacua,

$$\phi^A = \text{const}, \qquad g_{\mu\nu} = \eta_{\mu\nu}$$

# Weinberg's famous no-go theorem: two possibilities

- Fine-tuning: only cancels out one specific value of $\Lambda$
- Scaling symmetry: Particle masses also vanish; not physical

# Evading Weinberg's theorem

- Any no-go theorem has assumptions, pointing the way forward
- Common approach: *break Poincaré invariance*

$$\Phi^A = \Phi^A(x^\mu), \quad g_{\mu\nu} = \eta_{\mu\nu}$$

- *e.g.*, in cosmology, we might have fields with time dependence
- To get flat space, fields must be accompanied by derivatives
- To leading order in EFT, one derivative per field

# Warmup: one scalar

- Can we degravitate with a single scalar?
    - **No.**
    - Why?
- At leading order in derivatives, most general action is

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 P(X)\right], \quad X \equiv (\partial\Phi)^2$$

- NB: this is the EFT of a zero-temperature superfluid

- The total stress tensor is

$$\frac{1}{M_{\rm Pl}^2}T_{\mu\nu} = \frac{1}{2}m^2\left(Pg_{\mu\nu} - 2P_X\partial_\mu\Phi\partial_\nu\Phi\right) - \Lambda g_{\mu\nu}, \quad P_X \equiv \frac{\partial P}{\partial X}$$

- In order to have flat solutions for arbitrary Λ, this must vanish:

1. $P_X = 0$

2. $m^2 P(X) = 2\Lambda$

- The latter requires *fine-tuning*.

- *e.g.*, ghost condensate, $P(X) = X + \lambda X^2/2$

  - $P_X = 0$ solved by $X = -\lambda^{-1}$. But $m^2 P = 2\Lambda$ only if λ is carefully tuned against Λ!

$$\lambda = -\frac{m^2}{4\Lambda}$$

# Warmer up: Four scalars

- Fine-tuning not a problem with four scalars $\Phi^A$

- Consider a simple (and trivially wrong) model:

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda - m^2\eta_{AB}\partial_\mu\Phi^A\partial^\mu\Phi^B\right]$$

- The stress tensor no longer has an inhomogeneous term when $\Phi^A \sim x^A$:

$$\frac{1}{M_{\rm Pl}^2}T_{\mu\nu} = \frac{1}{2}m^2\left(-\eta_{AB}\partial_\alpha\Phi^A\partial^\alpha\Phi^B g_{\mu\nu} + 2\eta_{AB}\partial_\mu\Phi^A\partial_\nu\Phi^B\right) - \Lambda g_{\mu\nu}$$

- This admits flat solutions $\Phi^A = \alpha x^A$, with

$$\alpha = \frac{\sqrt{-\Lambda}}{m}$$

- Degravitates any negative $\Lambda$ without fine-tuning!
  - Key: instead of tuning theory parameters against $\Lambda$, tune *integration constant* $\alpha$ to $\Lambda$
  - Tuning is achieved *dynamically*

- Fatal problem: $\Phi^0$ is a **ghost**

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda - m^2\eta_{AB}\partial_\mu\Phi^A\partial^\mu\Phi^B\right]$$

$$= \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2(\partial\Phi^0)^2 - m^2\sum_{i=1}^{3}(\partial\Phi^i)^2\right]$$

- Direct result of internal Lorentz symmetry, which is what we used to remove inhomogeneous term!
  - Blessing and a curse
- Can we self-tune without a ghost?

# Ghosts and massive gravity

- The ghost is easy to understand if we recognize this is a theory of **massive gravity**
- Why? Consider adding to GR a non-derivative interaction

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda - m^2 g^{\mu\nu}\eta_{\mu\nu}\right]$$

- This breaks diff invariance due to $\eta_{\mu\nu}$. Can restore diffs by introducing Stückelberg fields $\Phi^A$,

$$\eta_{\mu\nu} \to \eta_{AB}\partial_\mu\Phi^A\partial_\nu\Phi^B$$

- And we recover the action discussed in the previous slides

# Massive gravity and degravitation

- This connects to an old and venerable story:

  Massive graviton
  → finite range of gravity
  → gravity acts as "high-pass filter" screening out sources with wavelengths >> $m^{-1}$

  - $\Lambda$ is infinite-wavelength source!

- Massive gravity at linear level: Fierz-Pauli (1939) $h_{\mu\nu}h^{\mu\nu} - (h^{\mu}{}_{\mu})^2$

  - Other linear mass terms have ghosts, just like the example we discussed

# Massive gravity and degravitation

- Non-linear: ghost re-emerges! (Boulware-Deser, 1972)
- Unique non-linear, ghost-free, Lorentz-invariant massive gravity: de Rham-Gabadadze-Tolley (dRGT, 2010)
- Ghost-free massive gravity cannot degravitate large $\Lambda$ without violating solar system tests of GR (1010.1780)
- This means we cannot use a Lorentz-invariant theory,

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 U(\partial_\mu \Phi^A, \eta_{AB}, \varepsilon_{ABCD})\right]$$

# Is Lorentz invariance too strong a requirement?

- For cosmology, we only need SO(3), not SO(3,1)
- Idea: break internal boosts

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 U(\partial_\mu \Phi^0, \partial_\mu \Phi^i, \delta_{ij}, \varepsilon_{ijk})\right]$$

- Aim: use newfound freedom to avoid ghosts (and other pathologies) while retaining degravitation
- Look for: for physically sensible degravitating models with

$$\Phi^0 = \alpha t, \quad \Phi^i = \beta x^i \quad \text{such that} \quad T^{\Phi}_{\mu\nu} = M_{\rm Pl}^2 \Lambda g_{\mu\nu}$$

# Interpreting our theory

- Complementary physical interpretations of this type of theory:

    1. Lorentz-violating massive gravity

    2. Low-energy EFT of self-gravitating fluid

- Difference hinges only on coordinate choice

# Lorentz-violating massive gravity as a fluid EFT

- Consider the fields $\Phi^A = \Phi^A(t,x)$ to be comoving (Lagrangian) coordinates of a fluid

- Fluid rest frame is a coordinate system in which $\Phi^A = \alpha x^A$

- EFT describing excitations of fluid is a derivative expansion in $\Phi^A$ obeying any relevant symmetries

# Building blocks and symmetry

- At leading order in derivatives, action is built out of

$$C^{AB} \equiv g^{\mu\nu}\partial_\mu\Phi^A\partial_\nu\Phi^B \quad \Longrightarrow \quad \mathscr{L} = U(C^{00}, C^{0i}, C^{ij})$$

- Choice of operators determines symmetry-breaking pattern and hence fluid, e.g.,
  - Solids: $\mathscr{L} = U(C^{ij})$
  - Zero-temperature superfluids: $\mathscr{L} = U(C^{00})$
  - Finite-temperature superfluids:

$$\mathscr{L} = U(C^{00}, \det C^{ij}, \det C^{AB})$$

# The importance of coordinates

- If we move to the fluid rest frame, $\Phi^A = \alpha x^A$, then we recover the Lorentz-violating massive gravity picture:

$$U(C^{00}, C^{0i}, C^{ij}) \xrightarrow{\Phi^A=\alpha x^A} U(h_{00}, h_{0i}, h_{ij})$$

- This is unitary gauge
- e.g., $X = C^{00} = g^{\mu\nu}\partial_\mu\Phi^0\partial_\nu\Phi^0 \xrightarrow{\Phi^A=\alpha x^A} C^{00} = \alpha^2 g^{00}$
    - $\rightarrow$ Potential for $g^{00}$

# Criteria for degravitation

- **Existence of a Minkowski (degravitating) solution**: equations of motion must be solved by $g = \eta$ for arbitrary $\Lambda$
- **No fine-tuning**: Tune integration constants, not model parameters, against $\Lambda$
- **Massless tensors**: Tensor mass generically is huge, $m \sim O(\Lambda^{1/2})$, unless they are exactly massless. (LIGO: $m < 10^{-22}$ eV)

# Criteria for degravitation

- **No pathologies**: No ghosts, tachyons, gradient instabilities, infinite strong coupling, instantaneous modes

- **UV insensitivity**: Higher-derivative EFT corrections should not introduce new modes at low energy

# Strategy

1. Identify parameter space of Lorentz-violating massive gravity which satisfies these criteria

2. Look for symmetries that protect our parameter choice

3. Determine building blocks for non-linear action

4. Solve cosmological constant problem (incomplete)

# Analysis

- Work in unitary gauge at linear level,

$$\Phi^A = (\alpha t, \beta x^i), \quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

- Most general SO(3)-invariant mass term
  Dubovsky hep-th/0409124

$$\mathscr{L}_{\text{mass}} = \frac{M_{\text{Pl}}^2}{2}\left(m_0^2 h_{00}^2 + 2m_1^2 h_{0i}^2 - m_2^2 h_{ij}^2 + m_3^2 h_{ii}^2 - 2m_4^2 h_{00} h_{ii}\right)$$

- Massless tensors: $m_2 = 0$

- Stability: $m_1 = 0$ (see our paper for gory details! 1805.05937)

# Searching for a symmetry

- With our parameter choice in hand, $m_1 = m_2 = 0$, we need to find a symmetry to protect it
  - Otherwise we're just fine-tuning and not solving anything!
- Several candidate symmetries
  - Most either don't degravitate or are UV-sensitive
  - Again, details in 1805.05937
- Only symmetry that works: **time-dependent, volume-preserving spatial diffeomorphisms**

# Time-dependent, volume-preserving spatial diffeomorphisms

- Scalar language:

$$\Phi^i \to \Psi^i(\Phi^0, \Phi^i) \quad \text{with} \quad \det\left(\partial\Psi^i/\partial\Phi^j\right) = 1$$

- Massive gravity language: break diffs while leaving

$$x^i \to x^i + \xi^i(t, x^j) \quad \text{with} \quad \partial_i \xi^i = 0$$

- Closely related to time-*independent* volume-preserving spatial diffs, which set $m_2 = 0$ and forbid massive tensors

- Adding time dependence further restricts $m_1 = 0$, as needed for stability

# Building blocks of degravitation

- The building blocks invariant under our symmetry,

$$X = (\partial\Phi^0)^2$$

$$Yb = \frac{\det(\partial_\mu\Phi^A)}{\sqrt{-g}}$$

- Our degravitating theory is therefore

$$\frac{\mathscr{L}}{\sqrt{-g}} = \frac{M_{\rm Pl}^2}{2}\left[R - 2\Lambda + m^2 U(X, Yb)\right]$$

- Unique theory that satisfies our criteria

- This describes a finite-temperature superfluid!
  Nicolis 1108.2513

# Degravitating solutions in practice

- Finally, we can see how this all works! Example:

$$U(X, Yb) = \frac{K_1}{2}(X+1)^2 + \frac{K_2}{2}(Yb)^2$$

Ghost condensate plus term quadratic in Yb

- Has degravitating solutions!

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi^0 = t, \quad \Phi^i = \left(-\frac{4\Lambda}{K_2 m^2}\right)^{1/6} x^i$$

- Can degravitate any positive $\Lambda$ for $K_2 < 0$ and vice versa

- No ghost: $K_1 > 0$

# Degravitating solutions in practice

- Another simple example:

$$U(X, Yb) = -X + \gamma XYb - \frac{\lambda}{2}(Yb)^2$$

- Degravitating solutions:

$$g_{\mu\nu} = \eta_{\mu\nu}, \quad \Phi^0 = \sqrt{\frac{2\Lambda}{m^2} - \frac{\lambda}{2\gamma^2}}t, \quad \Phi^i = \left(\frac{2\gamma^2\Lambda}{m^2} - \frac{\lambda}{2}\right)^{-1/6} x^i$$

- Can degravitate any $\Lambda > \lambda m^2/4\gamma^2$

- Ghost-free: $\lambda > 0$

# Summary

- New method for self-tuning $\Lambda$ by breaking Lorentz
  - Circumvent (and extend) Weinberg
- Unique theory: finite-temperature superfluid
- Next step: see whether this cancellation can occur dynamically