Phenomenology of Fermion Production During Inflation

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Based on:
P. Adshead, L. Pearce, M. Peloso, M. Roberts, L. Sorbo, JCAP 1910 (2019) no.10, 018
arXiv:1904.10483
P. Adshead, L. Pearce, M. Peloso, M. Roberts, L. Sorbo, JCAP 1806 (2018) no.06, 020
arXiv:1803.04501

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- Source metric perturbations as well

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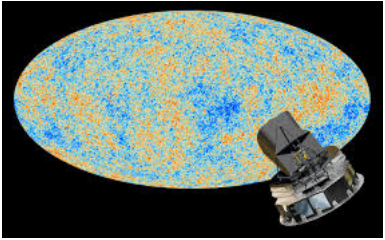
- CMB power spectrum measurements $\leftrightarrow$ two-point correlation function $\langle \delta\phi^2 \rangle$



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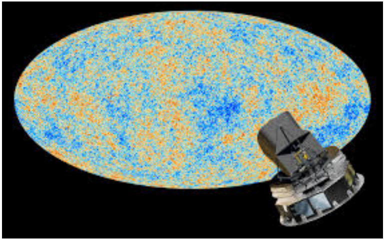

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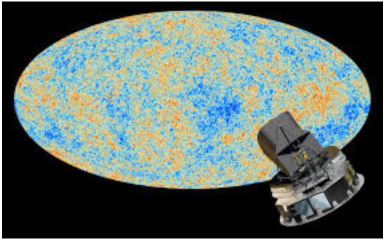
- Tensor-to-scalar ratio $\leftrightarrow$ two-point function $\langle \gamma\gamma \rangle$



Specific Model

K. Freese, et. al.
Phys. Rev. Lett. 65 (1990) 3233

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W. D. Garretson, et. al.,
Phys. Rev. D46 (1992) 5346-5351
hep-ph/9209238

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P. Adshead & E. I. Sfakianakis
JCAP 1511 (2015) 021
Phys.Rev.Lett. 116 (2016) no.9, 091301

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- What about fermions?

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- **But in axion inflation, we have a new scale: $\partial_t \phi / f$...**

Our Set-Up

Lagrangian

$$\mathcal{L} = \frac{1}{2}a^2\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - a^4 V(\phi) + \bar{Y}\left[i\gamma^\mu\partial_\mu - ma\right]Y - \frac{1}{f}\partial_\mu\phi\,\bar{Y}\gamma^\mu\gamma^5 Y$$

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Coupling between them

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Notation: $\mu \equiv m/H$, $\xi \equiv (\partial_t\phi_0)/2Hf$,

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Start in vacuum state at time $t = 0$

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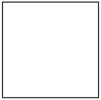
As inflaton VEV evolves, creation and annihilation operators evolve into a superposition of $t = 0$ creation and annihilation operators



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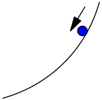

$$a(t) \sim \alpha a(0) + \beta a^\dagger(0)$$

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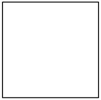
At a later time, the “vacuum” state is no longer empty!



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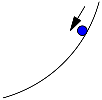

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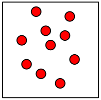

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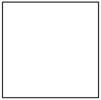

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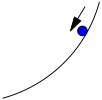

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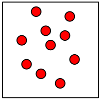

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Time-dependent creation & annihilation operators found by diagonalizing the free (quadratic) Hamiltonian

Original Basis

- Elements of diagonal free fermionic Hamiltonian:

$$\omega_r = k\sqrt{\frac{\mu^2}{x^2} + \left(1 + r\frac{2\xi}{x}\right)^2}$$

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- *Expansion is not under perturbative control for small fermion mass*

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Coupling explicitly decouples as $m \to 0$

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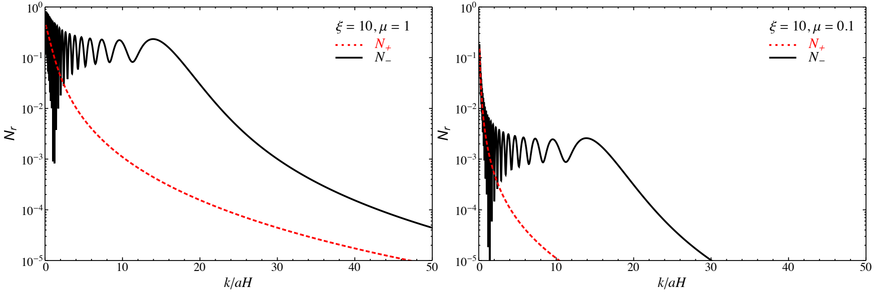
- No problems with validity of perturbative expansion

New Basis

Total number

$$\int d^3k\, N(k) \approx 52 H^3 \mu^2 \xi^2, \qquad 1, \mu \ll \xi$$

can be large when new scale $\xi \gg 1$!



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Interactions

$$\begin{aligned} H_{\rm int} \supset & -\frac{2am}{f}\int d^3x\,\bar{\psi}\left[\sin\left(2r\,\frac{\phi_0}{f}\right) + i\,\gamma^5\cos\left(2\,\frac{\phi_0}{f}\right)\right]\psi\,\delta\phi & \text{cubic} \\ & -\frac{2am}{f^2}\int d^3x\,\bar{\psi}\left[\cos\left(2\,\frac{\phi_0}{f}\right) - i\,\gamma^5\sin\left(2\,\frac{\phi_0}{f}\right)\right]\psi\,\delta\phi^2 & \text{quartic} \\ & + \ldots \end{aligned}$$

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Fermions

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- Solutions are sums of Whittaker functions:

$$s_r(x) = e^{-\pi r\xi}\,W_{\frac{1}{2}+2ir\xi,\,i\sqrt{\mu^2+4\xi^2}}(-2ix)$$

$$d_r(x) = -i\,\mu\,e^{-\pi r\xi}\,W_{-\frac{1}{2}+2ir\xi,\,i\sqrt{\mu^2+4\xi^2}}(-2ix)$$

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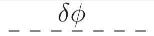

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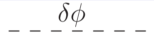

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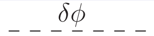

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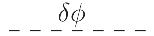

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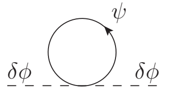

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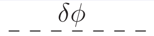

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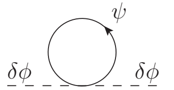

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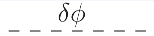

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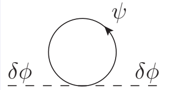

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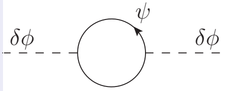

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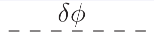

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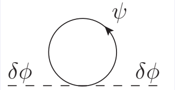

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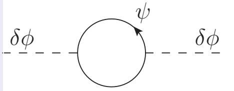

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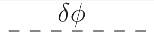

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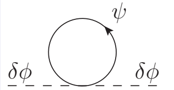

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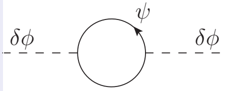

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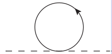
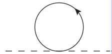
Evaluate with approximations
Subdominant

Analytic Results



Renormalization

Analytic Results



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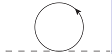

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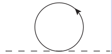

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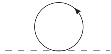

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- Concretely: Subtract off adiabatic limit mode-by-mode

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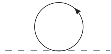

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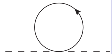

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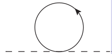

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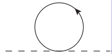

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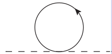

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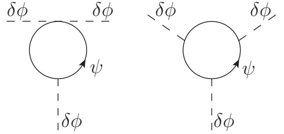
Non-Gaussianity

Three-Point Function

- To leading order ($\sim 1/f^3$):



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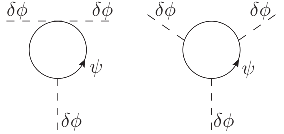

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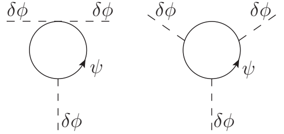

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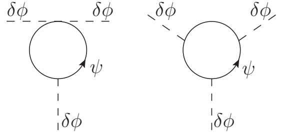

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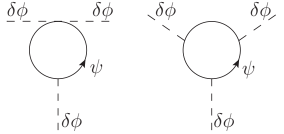

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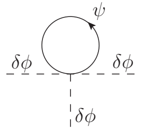

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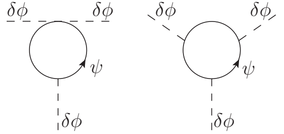

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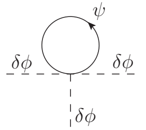

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Results

- Sourced power spectrum:
  For $\mu \lesssim 1$, $\xi \gg 1$:
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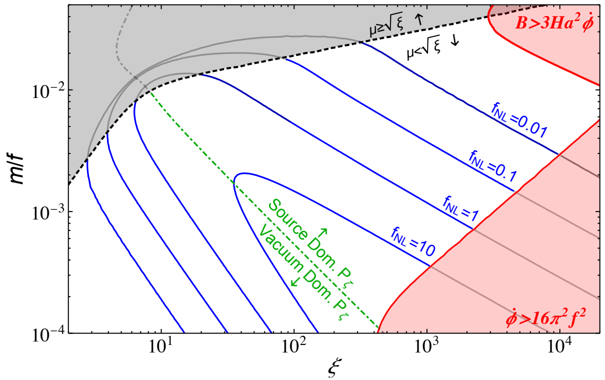
Results



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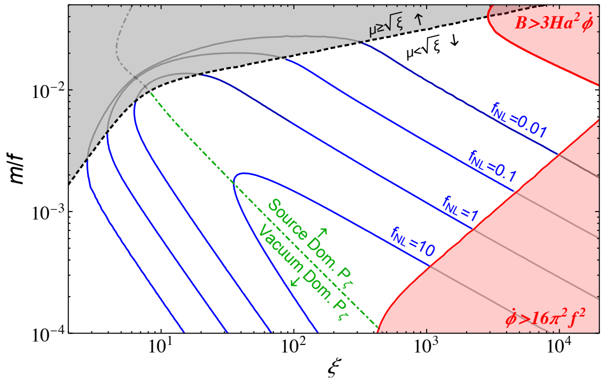

- Red region: EFT for interaction term breaks down or strong backreaction
- Grey region: Other diagrams contribute to f_{NL} ($\mathcal{O}(1)$ uncertainty)

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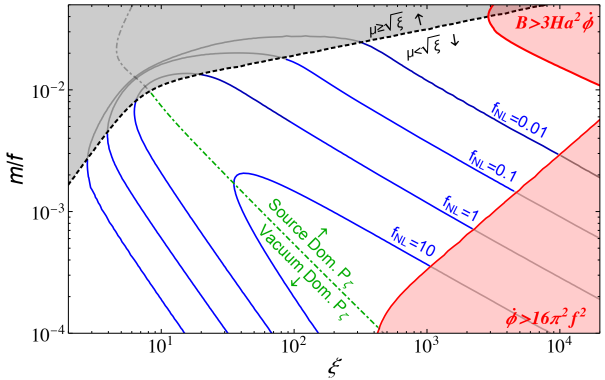
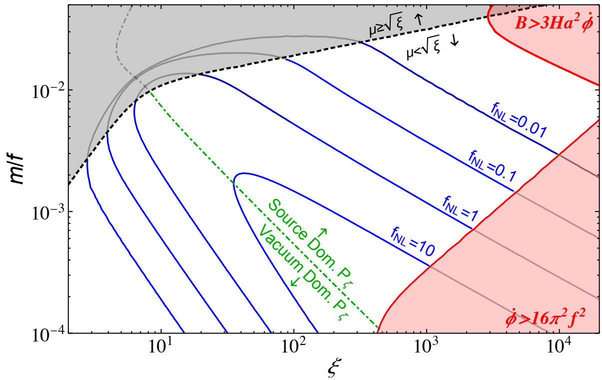


Dot-dashed line:

- Left: Vacuum perturbations dominate power spectrum
- Right: *Sourced perturbations dominate power spectrum*

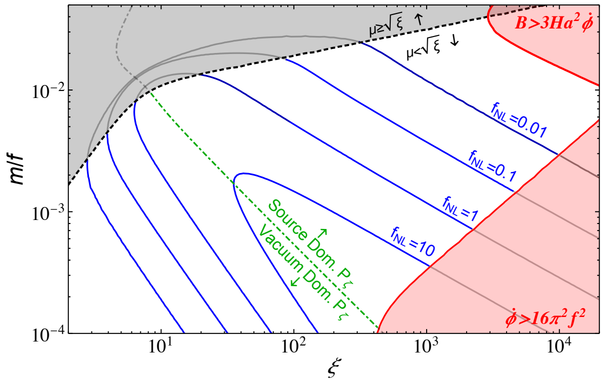
Plank Collaboration
Astron.Astrophys. 594 (2016) A17
arXiv:1502.01592



Dot-dashed line:

- Left: Vacuum perturbations dominate power spectrum
- Right: *Sourced perturbations dominate power spectrum*
- *Consistent with Planck non-gaussianity limit* $(f_{NL} = -4 \pm 43)$

Results



Unlike bosonic production:

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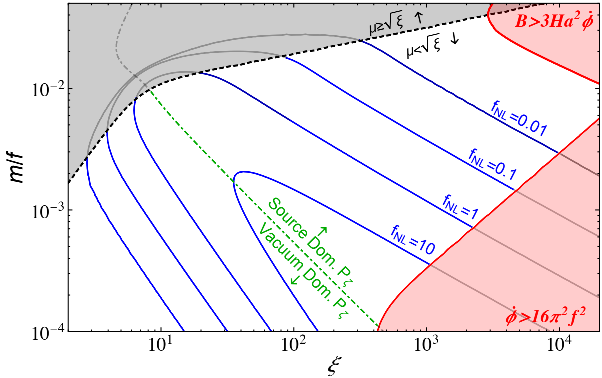

Unlike bosonic production:

- Exponential growth of a single mode → large non-gaussianity

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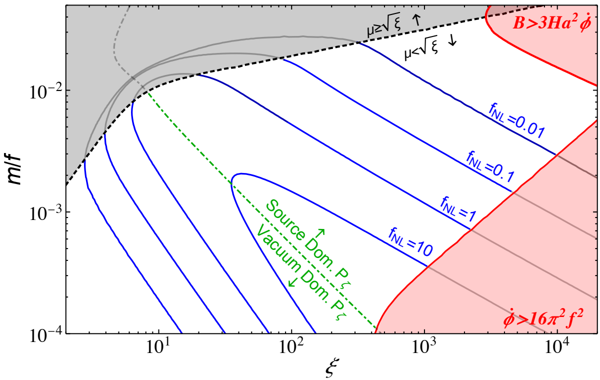

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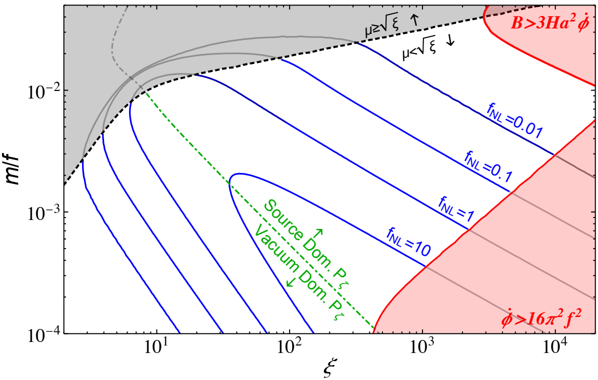

Consider fixing μ and increasing ξ:

- At first, populate more modes with non-gaussian perturbations $\rightarrow$ increasing f_{NL}

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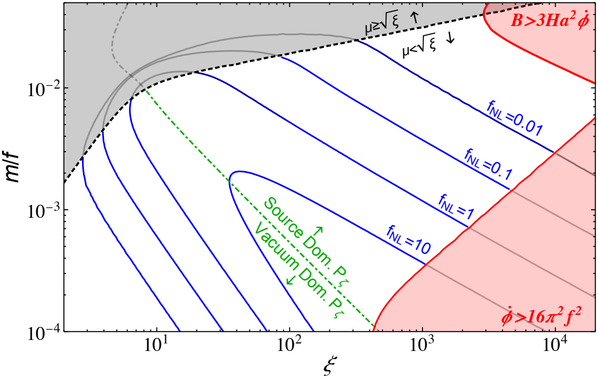

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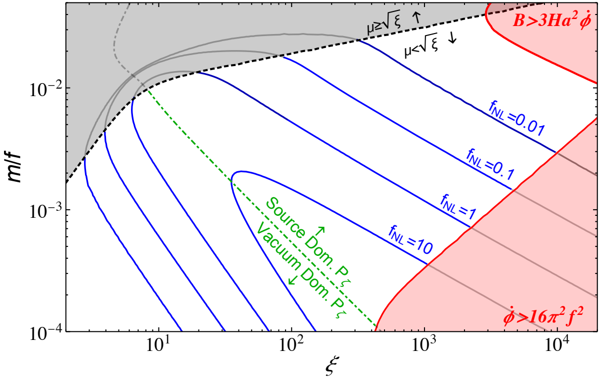

Consider fixing μ and increasing ξ:

- But f_{NL} depends on the *sum* of these modes, which are uncorrelated
- As number of modes increases, sum becomes gaussian (by central limit theorem) $\rightarrow$ decrease in f_{NL}

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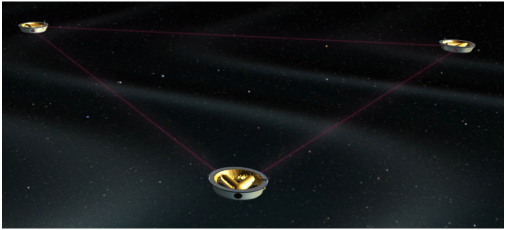

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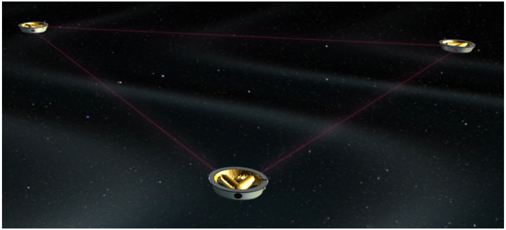
  Perhaps chiral gravitational waves?



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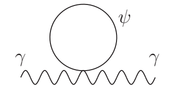

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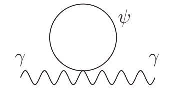

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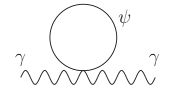

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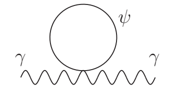

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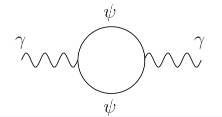

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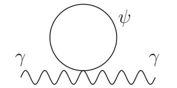

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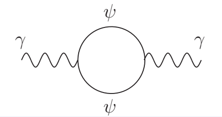

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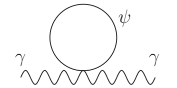

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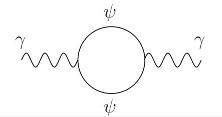

Gravitational Waves

Ratio of sourced tensor power spectrum to vacuum:

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- Tensor-to-scalar ratio *cannot* distinguish this model (with foreseeable observations)

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Thank you! Questions?

Fermions Are Messy!

- Expand fermion in terms of mode functions:

$$\psi = \int \frac{d^3k}{(2\pi)^{\frac{3}{2}}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \sum_{r=\pm} \left[U_{r,\boldsymbol{k}}(\tau)\, a_{r,\boldsymbol{k}} + V_{r,-\boldsymbol{k}}(\tau)\, b^{\dagger}_{r,-\boldsymbol{k}} \right]$$

$$U_{r,\boldsymbol{k}}(\tau) = \frac{1}{\sqrt{2}} \begin{bmatrix} u_{r,k}(\tau)\chi_r(\boldsymbol{k}) \\ r v_{r,k}(\tau)\chi_r(\boldsymbol{k}) \end{bmatrix}, \qquad V_{r,\boldsymbol{k}}(\tau) = \frac{1}{\sqrt{2}} \begin{bmatrix} v^*_{r,k}(\tau)\chi_{-r}(\boldsymbol{k}) \\ -r u^*_{r,k}(\tau)\chi_{-r}(\boldsymbol{k}) \end{bmatrix}$$

- where $(x = -k\tau)$:

$$u_r(x) = \frac{1}{\sqrt{2x}} \left[e^{ir\phi_0(x)/f}\, s_r(x) + \mathrm{e}^{-ir\phi_0(x)/f}\, d_r(x) \right]$$

$$v_r(x) = \frac{1}{\sqrt{2x}} \left[e^{ir\phi_0(x)/f}\, s_r(x) - \mathrm{e}^{-ir\phi_0(x)/f}\, d_r(x) \right]$$

$$s_r(x) = e^{-\pi r \xi}\, W_{\frac{1}{2}+2ir\xi,\, i\sqrt{\mu^2+4\xi^2}}(-2ix)$$

$$d_r(x) = -i\,\mu\, e^{-\pi r\xi}\, W_{-\frac{1}{2}+2ir\xi,\, i\sqrt{\mu^2+4\xi^2}}(-2ix)$$

$W_{\mu,\lambda}(z)$: Whittaker W function

Power Spectrum

- Quartic loop (exact):
  For $\mu \lesssim 1$, $\xi \gg 1$:
  $$\frac{\delta P_\zeta^{\rm quar}(k)}{P_\zeta^{(0)}} = \frac{32\, m^2 \xi^2 \, \log \xi}{3\pi^2 f^2} \, \log(H/k)$$
- Cubic loop (further approximations):
  $$\frac{\delta P_\zeta^{\rm cub}(k)}{P_\zeta^{(0)}} \propto \frac{m^2}{f^2} \, \mu^2 \, \sqrt{\xi} \, |\log(k/H)|$$
- For $\mu \lesssim 1$, $\xi \gg 1$, the quartic contribution dominates the cubic contribution

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  $$\frac{\delta P_\zeta^{\rm quar}(k)}{P_\zeta^{(0)}} = \frac{32\, m^2 \xi^2 \, \log \xi}{3\pi^2 f^2} \log(H/k)$$
- $\log(H/k)$ dependence: Fermion energy density continues sourcing inflaton perturbations, even outside horizon
- Regulated by finite number of e-foldings between when mode left horizon and end of inflation

Non-Gaussianity

- Non-gaussianity parameter:

$$f_{NL}^{eq} = \frac{\frac{160\, H^2 \mu^2 \xi^3}{9 \pi f^2} \log(H/k)}{\left(1 + \frac{32\, H^2\, \mu^2\, \xi^2\, \log \xi}{3\, \pi^2\, f^2} \log(H/k)\right)^2}$$

- The analytic calculation is good for all μ, ξ, but consider regime $\mu \lesssim 1$, $\xi \gg 1$ (other diagrams subdominant & regularization good)
- Fixing observed power spectrum $P_\zeta = 2.2 \times 10^{-9}$ leaves two free parameters: m/f and ξ

Spectral Index

- Slow roll: Assume each mode evolves with constant H, $\dot{\phi}$ & take account of time-dependence when comparing modes
- In regime where sourced contribution dominates power spectrum:

$$n_s - 1 = -3\epsilon - \frac{1}{N} + \frac{2\epsilon - \eta}{\log(\xi)}$$

  N: number of e-foldings
- Get $n_s \approx 0.97$ for reasonable slow roll parameters

Gravitational Waves

Gravitational Wave Power Spectrum

- Dominant quartic contribution:

$$\delta\mathcal{P}_t^{\mathrm{quar}} = \frac{4H^4}{9\pi^3 M_{Pl}^4}\mu^2\xi^3 \ln(H/k)$$

- Chiral contribution ($\lambda = \pm 1$):

$$\delta\mathcal{P}_t^{\mathrm{parity-odd}} = \lambda\frac{H^4}{6\pi M_{Pl}^4}\mu^2\xi^2$$

- Cubic contribution:

$$\delta\mathcal{P}_t^{\mathrm{cub}} \sim \mathcal{O}(0.1)\frac{H^4}{M_{Pl}^4}\mu^2\xi^3$$

- No regime in which quartic dominates cubic

CMB to Correlators

Power Spectrum and f_{NL}

- ζ: Spatial curvature of hypersurfaces of constant energy density
- Power spectrum:

$$P_\zeta = \frac{k^3}{2\pi^2} \langle \zeta\zeta \rangle'$$

- $f_{NL,eq}$:

$$\langle \zeta\zeta\zeta \rangle' = \frac{P_\zeta}{k^6} \cdot \frac{9}{10} (2\pi)^{5/2} \cdot f_{NL,eq}$$

- ζ connected to inflaton perturbations: $\zeta = -H\,\delta\phi/\dot{\phi}_0$