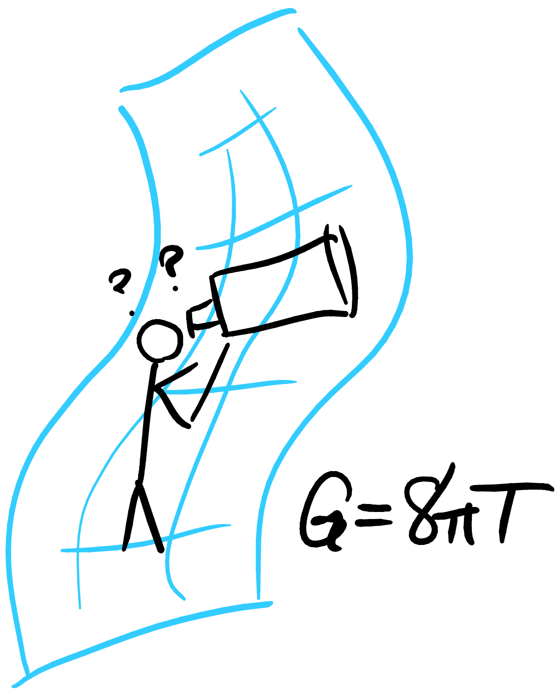
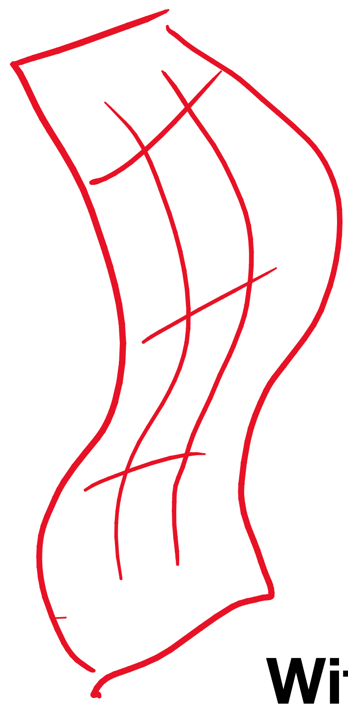
Brane World Cosmology From Symmetry Principles





Jay Hubisz
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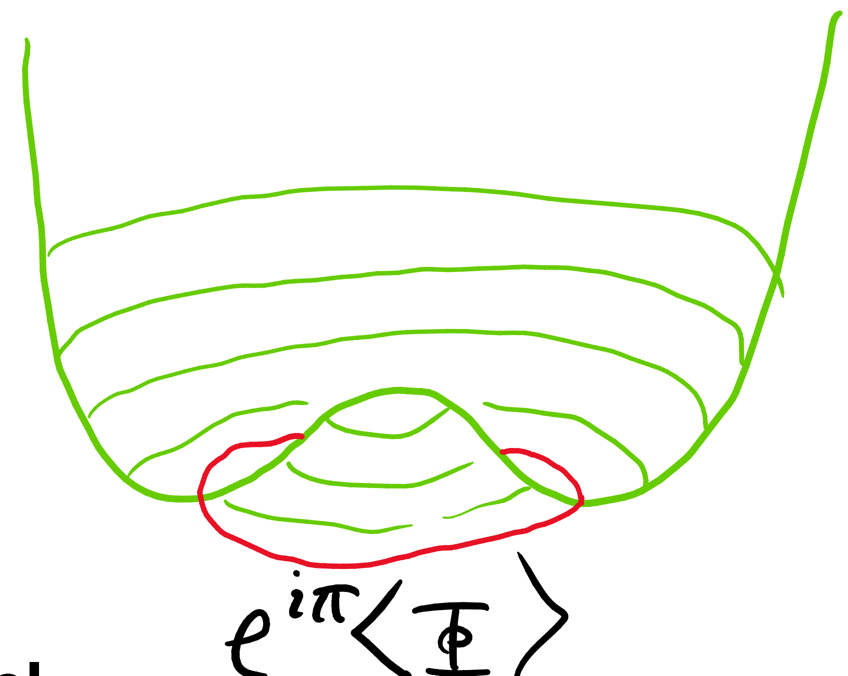
Feb. 5 2020

work in progress
With Rigo, Csáki, Ismail, Eröncel



Inflationary Reheating Meets Particle Physics Frontier
KITP
Santa Barbara

Brane Worlds and Problems of Particle Physics

- Extra dimensions large and small have proven to be rich troves for model building and phenomenology
  - many interesting ways to address
    - hierarchy problem
    - flavor
    - SUSY breaking
- New viewpoints on cosmological constant problem

Additionally, novel cosmological behaviors in presence of instability or macroscopic modulus displacement

Examples: Matter/Radiation Cosmologies from Rolling Branes

- The motion of branes in systems with no static and stable solution can have interesting consequences
  - RS 2 brane model - mistuned tensions
    - solutions include 4D ρ_r, ρ_Λ cosmologies

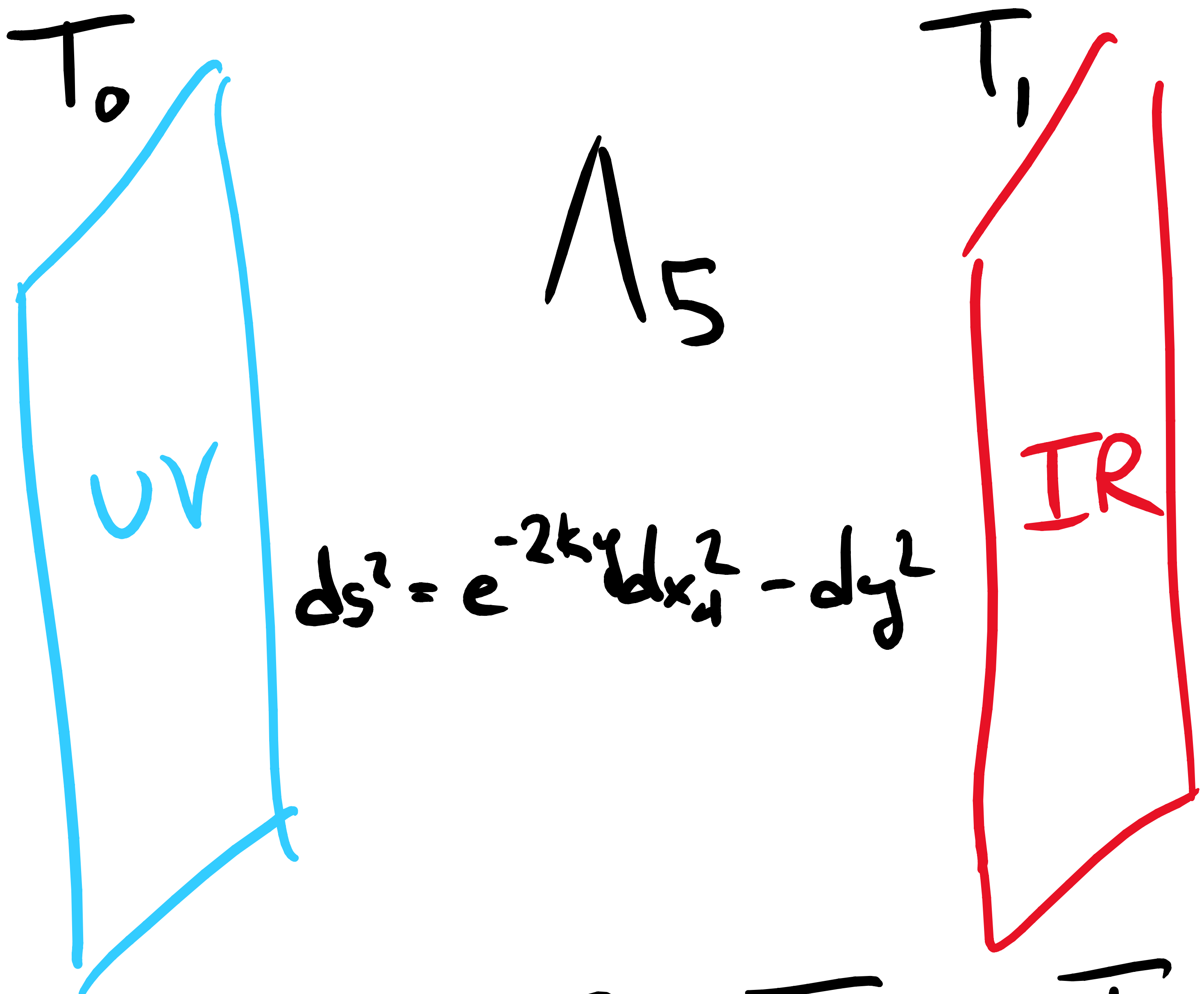
E.g. Csáki, Graesser, Kolda, Terning hep-ph/9906513
Kaloper hep-th/9905210

  - Sen's Tachyon condensation: $D - \bar{D}$ brane collapse/annihilation

e.g. Sen hep-th/0203265

    - rolling tachyon stress-energy approaches matter domination from

Randall-Sundrum 2-Brane



Static solution if $T_1 = -T_0 = \dfrac{\Lambda_5}{k}$

FINE TUNED

Time Dependent Solutions

Using EE's, can simplify metric -
depends only on one function, $a(y, t)$:

$$ds^2 = \left(\frac{\dot{a}}{\dot{a}_0}\right)^2 dt^2 - a^2 dx_3^2 - dy^2$$

$a_0(t)$ **is the scale factor observed on the UV brane,** $a(y = 0, t)$

t is proper time for UV brane observer

Can solve y-dependence:

$$a^2 = \Lambda_+(t)e^{2y} + \Lambda_-(t)e^{-2y} - \frac{1}{2}\dot{a}_0^2$$

And relate Λ's

$$\Lambda_+ = \left[\left(\frac{\dot{a}_0}{2}\right)^4 + \lambda\right]\frac{1}{\Lambda_-}$$

Integration constant

Impose UV brane metric junction condition and consistency relation

Radiation as an integration constant

Hubble on the UV brane:

$$H^2 = \left(\frac{\dot{a}_0}{a_0}\right)^2 = \frac{4\lambda}{a_0^4} + \delta_0(2+\delta_0)$$

$$\frac{\kappa^2}{6k} T_0 \equiv 1 + \delta_0$$

Without adding "stuff" beyond cosmological constants/tensions, system of branes has a big-bang

So far, we have ignored the IR brane
its motion determined by IR brane metric junction conditions

$$\frac{\frac{\dot{R}}{n}\frac{\dot{a}}{an} + \frac{a'}{a}}{\sqrt{1 - \left(\frac{\dot{R}}{n}\right)^2}} = \frac{\kappa^2}{6} T_1$$

$n = \dot{a}/\dot{a}_0$ **and metric functions evaluated at y=R(t)**

sqrt factor sets speed limit

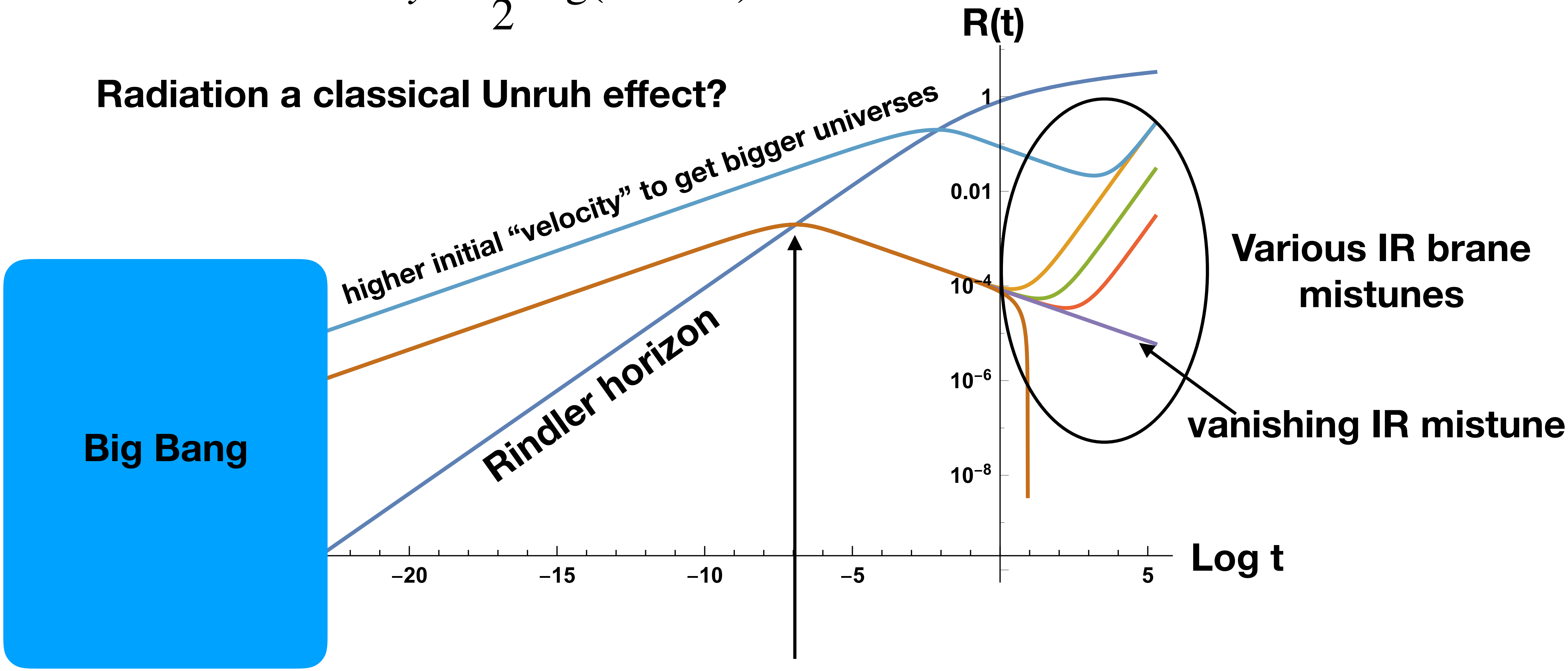
Big bang singularity = overlap of the branes, IR brane moving near the speed of light

Example behavior

Some R(t) solutions when $\delta_0 = 0$:

Rindler Horizon at $y = \frac{1}{2}\log(1 + 4kt)$

Radiation a classical Unruh effect?



IR brane “comes into view” for our UV brane observer

Darkness from Brane-worlds: Radiation without Radiation

- Dark Radiation From RS brane world:
    - Mis-tuning the UV or IR brane tensions can require big-bang cosmology without addition of matter (only cosmological constants)
    - radiation term in Hubble evolution appears as integration constant
    - Observer on UV brane sees Hubble law for radiation
- Radiation seems Unruh-like - Rindler style horizon in bulk recedes from UV brane with cosmological evolution
    - however it appears classically from bulk gravity

The RS Radion

- Dynamics of the RS radion are well understood at the level of linear perturbation theory
  - expand RS metric to first order in perturbations, solve EE's and identify spectrum
    - 4D massless graviton, massless scalar
      (gains mass if radius is stabilized), massive tower of KK gravitons
- Ill-suited to macroscopic motion of branes
  - all higher derivative/self interactions truncated
- **Linearized theory completely fails to capture physics**

Simplest Dilaton Theory

AdS/CFT relates the 5D radion to the dilaton of a spontaneously broken 4D CFT

4D conformally invariant actions:

$$g_{\mu\nu} = e^{2\tau(x)}\eta_{\mu\nu}$$

flat space rescaled by coordinate dependent conformal factor

Build actions consistent with general covariance:

E.g. $S = -\int d^4x\sqrt{g}\left(\Lambda + \frac{1}{12}R\right)$ **+ higher derivatives**

Expanding, performing a field redefinition, get:

$$S = \int d^4x \frac{1}{2}(\partial\phi)^2 - \lambda\phi^4$$

no higher derivatives = no "speed limit"

Radion is NOT a simple realization of the dilaton - 5D causality requires specific *pattern* of higher derivative operators (constraints on interacting CFTs)

Baumann, Green, and Hartman 1906.10226

The Truncated Radion

Linearization of 5D gravity theory plus calculation of classical static effective potential gives:

$$S = \int d^4x \frac{1}{2}(\partial\phi)^2 - \lambda\phi^4$$

IR brane mistune

Might couple this minimally to gravity, and then could seemingly use this as a (now non-viable) theory for the inflaton

Actual cosmology is not even inflationary - instead is epoch of radiation domination

Any finite order truncation of theory would miss this

Nambu-Goto Brane Action

- Action of a thin co-dim 1 brane in a spacetime parametrized by embedding functions $X^M(\xi^\mu)$, where ξ are coordinates on world-volume of brane:

$$S_{\text{brane}} = \int d^d\xi T\sqrt{-\det g^{\text{ind}}}$$

$$g^{\text{ind}}_{\mu\nu} = \eta_{MN}\frac{\partial X^M}{\partial \xi^\mu}\frac{\partial X^N}{\partial \xi^\nu}$$

Choose coordinates, $g^{\text{ind}}_{\mu\nu} \approx \eta_{MN} + \partial_M\phi\partial_N\phi$ where ϕ captures motion of brane

Sqrt encodes causality limit on classical motion of brane

Single massless degree of freedom living on the brane - Goldstone mode

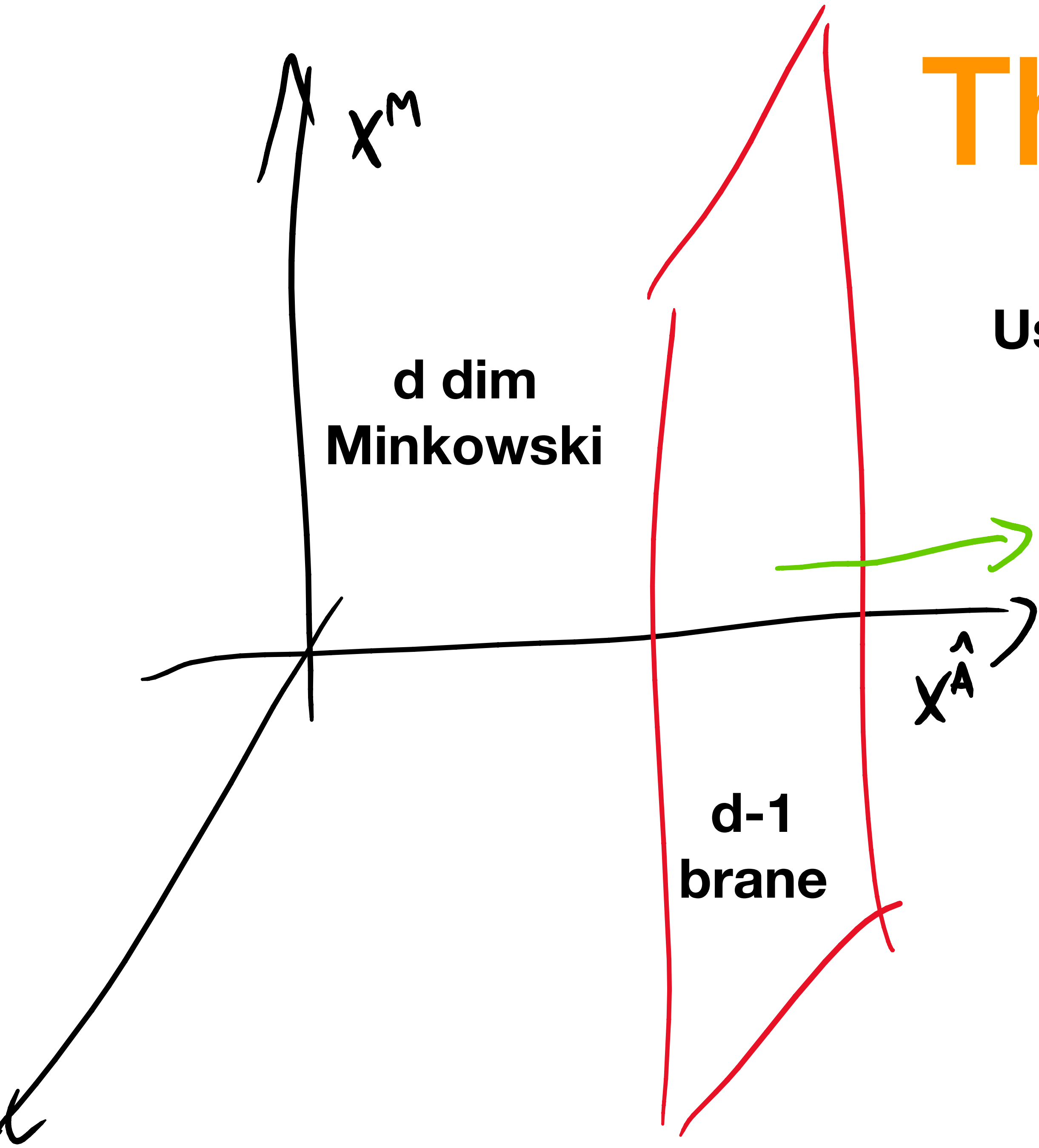
Radion action should be similar - IR brane can't move faster than c

(Saw this in the dark radiation analysis)

Nambu-Goto From Symmetry Principles

- Embedding of a (d-1) dimensional brane into d-dimensional space endowed with Poincaré algebra, ISO(d-1,1)
  - brane spontaneously breaks subset of translations and rotations/boosts: ISO(d-1,1) → ISO(d-2,1)
  - low energy action should be described by coset construction, Goldstone bosons parametrize coset G/H

The Counting of The Goldstones



Usual counting rules don't apply in same way:

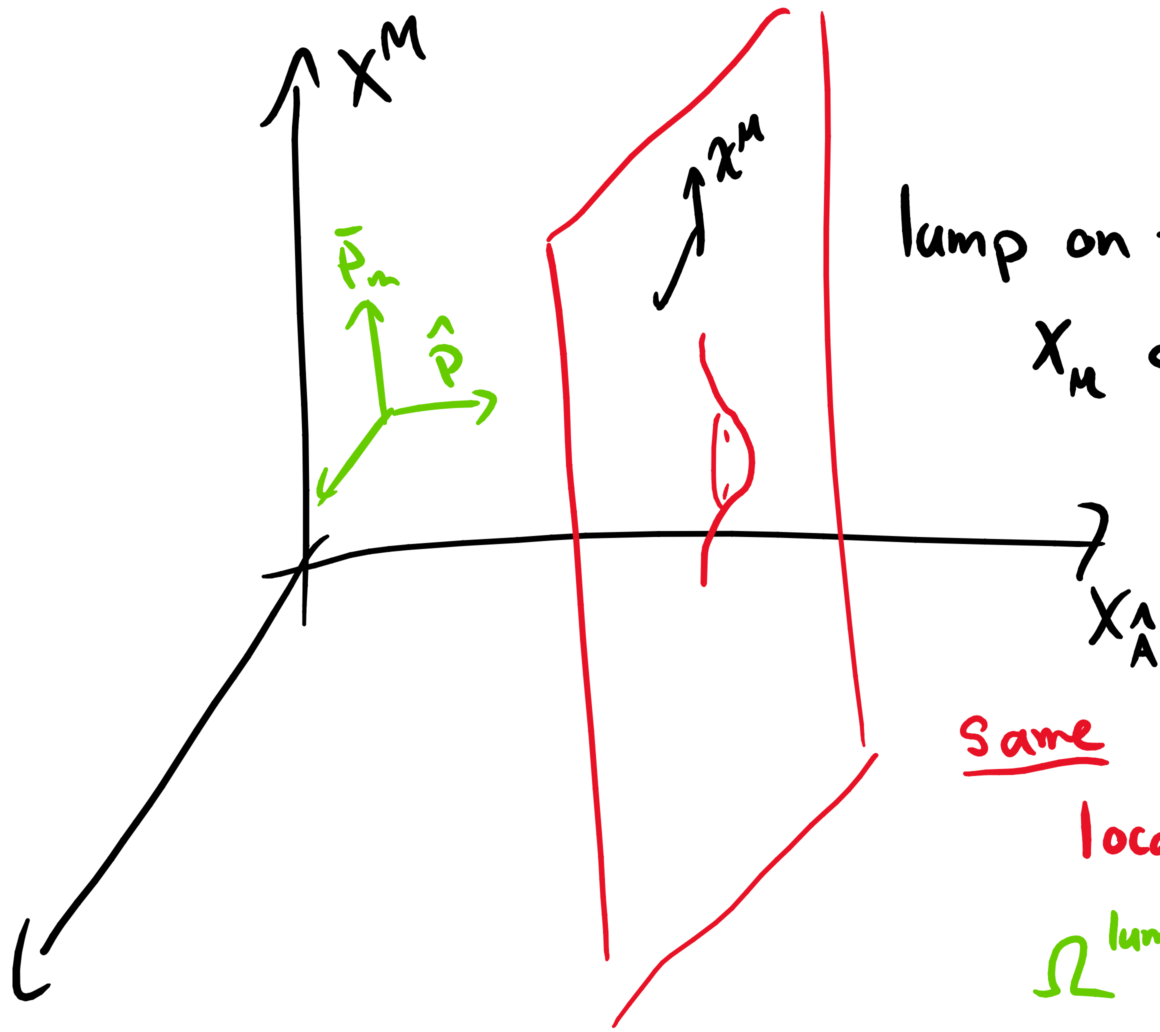
$$\left\{ \begin{array}{l} P_{\hat{A}} \\ M_{\hat{A}N} \end{array} \right. \text{ spontaneously broken generators}$$

Naively expect multiple Goldstone modes

$1 + (d-1) = d$ broken generators

(only 1 massless Goldstone boson)

Making brane bumps:



lump on the brane created by X_μ dependent translations

$$\Omega^{lump} = \exp[i\xi(x)\, P_{\hat{A}}]$$

same lump can be created by local (broken) rotations / boosts

$$[P, X] \sim X'$$

$$\Omega^{lump} = \exp[i Y^m(x) \bar{P}_m] \exp[i \Phi^N(x^\mu)\, M_{\hat{A}N}]$$

we should thus be unsurprised that naïve counting of Goldstone bosons gets "wrong" answer

In spontaneous breaking of spacetime symmetries, counting (and spectrum) of Goldstone modes is modified

Removal of modes - constraint relating Φ^N to derivatives of ξ

Aside: Not always a redundancy - brane (order parameter) not necessarily a scalar

Hikaka, Noumi, Shiu 1412.5601

CCWZ

Callan, Coleman, Wess, Zumino taught us how to build phenomenological lagrangians:

Low energy theory non-linearly realizes the broken symmetries via Goldstones

$$g = e^{i\Pi^{\hat{a}}T^{\hat{a}}}$$

Formalism to build G-invariant Lagrangians:

find objects that transform nicely under $g_0 \in G$, and make invariants

Maurer-Cartan form:

$$\Omega_\mu \equiv g^{-1}\partial_\mu g = \Omega_\mu^{\hat{a}}T^{\hat{a}} + \Omega_\mu^a T^a$$

Takes values on the algebra of G

break into components along broken/unbroken directions

Broken part transforms linearly, trace/contract

unbroken part transforms non-linearly, like gauge field:

invariant couplings of NGB's to light matter fields in reps of H

CCWZ and the brane

Delacrétaz, Endlich, Monin, Penco, Riva 1405.7384

A general (but with redundancies) representative of the coset

$$g = e^{iY^M(x)P_M} e^{i\Phi^N(x)M_{\hat{A}N}}$$

broken AND unbroken translations

Expand out the MC form: $\Omega_\mu = i\partial_\mu Y^A \Lambda_A^B P_B + \frac{i}{2}(\Lambda^{-1})_C^A \partial_\mu \Lambda^{CB} M_{AB}$

Φ dep. local lorentz transform

Can express MC form in manner where geometric interpretation clear:

$$\Omega_\mu = ie_\mu^N \left[\bar{P}_N + \nabla_N \xi \hat{P} + \nabla_N \Phi^M M_{\hat{A}M}\right] + iA_\mu^{MN} M_{MN}$$

Set to zero - removes redundancy

(relates Φ's and derivatives of ξ- "Inverse Higgs Constraints)

Can create invariant action using the "vielbein" $e \sim \partial Y$

Simplest Brane Action

Reproduces Nambu-Goto action:

$$S = -T\int d^d x \det e = -T\int d^d x \sqrt{-\det \partial_\mu Y^M \partial_\nu Y_M}$$

From symmetry considerations, have built up the action for a brane that automatically includes bulk symmetry constraints (Causality)

Perturbation theory: relations between S-matrix elements for goldstone (branon) scattering

Relativistic constraints arose from structure of Maurer-Cartan form: vielbein - Goldstone dependent volume element

Also: gives a formalism for adding higher derivative interactions (extrinsic curvature), couplings to matter fields on the brane, all with corresponding geometric structures

Would like to see this applied to realistic brane-worlds, RS type models know radion must be analog of this

Brane in global AdS

Isometries of AdS match 4D conformal algebra

4D Poincaré + D, K_μ

A "more natural" basis for AdS brane coset:

R **is AdS radius**

Might like $R \to \infty$ **to reproduce usual basis for ISO(1,4) - Poincaré algebra of 5D Mink.**

$$\hat{D} = \frac{D}{R} \qquad \hat{K}_\mu = \frac{K_\mu}{R} - 2RP_\mu$$

Important for parametrization of the coset:

$$g_{\text{brane}} = e^{x^\mu P_\mu} e^{\eta(x)\hat{D}} e^{\zeta^\mu(x)\hat{K}_\mu} \qquad g_{\text{dilaton}} = e^{y^\mu P_\mu} e^{\Phi(x)D} e^{\Sigma^\mu(x)K_\mu}$$

Non-linear map (field redefinition) from $x, \eta, \zeta \to y, \Phi, \Sigma$

Translates between AdS brane action and boundary dilaton action

Simplest AdS Action

P_μ **component of MC form transforms as vierbein**

Minimal invariant encodes causality :

$$S = \int d^4x e^{-4m\eta} \left(\det E - 1\right) = \int d^4x e^{-4m\eta} \left(\sqrt{1 - \frac{1}{2} e^{2m\eta} (\partial\eta)^2} - 1\right)$$

What is this on the CFT (dilaton) side?

Bellucci, Ivanov, Krivonos hep-th/0206126

$$S_{\text{dilaton}} = \frac{1}{4} \int d^4x e^{-4R\Phi} \frac{(\partial\Phi)^2}{1 - 1/8 e^{2R\Phi} (\partial\Phi)^2} \det \left(\delta^\mu_\nu + \frac{1}{2} D_\mu \Omega^\nu\right)$$

A non-minimal dilaton EFT - many higher derivative interactions

Similarly, simplest dilaton theory has very complicated AdS image (additional extrinsic curvature terms)

Organization of coset important - picture of order parameter

Instabilities of branes: Have we classified all phenomena?

- DGP gravity - interesting infrared modification of GR
  - plagued by ghosts (e.g. radion instability)
- Unstabilized (or unstabilizable) configurations
  - e.g. ~AdS tachyons: JH, Rigo, Eröncel 2018
- What are the resolutions of generic brane-world instabilities?

Lessons learned from examples:

relativistic motion of branes is crucial ingredient
picture of order parameter crucial (branes vs vevs)
gravity is important (and still missing from CCWZ)

Everything is Goldstones

Radion with 4D gravity arises from two branes coupled to 5D gravity

Formalism must give not just the brane, but also its mixing and interactions with bulk metric fluctuations

(Radion interaction with both IR brane and UV brane degrees of freedom)

Gravity from cosets:

e.g. Goon, Hinterbichler, Joyce, Trodden 1412.6098

Gauge bosons and gravitons can be viewed as NGB's of infinite number of non-linearly realized space-time global symmetries

Try on KK theory (extra dimension compactified on S_1) - Graviton, Gauge Field, Dilaton

5D Kaluza Klein Theory:

$$
\begin{aligned}
P_A^{n\mu_1\ldots\mu_k} &= e^{iny/R} x^{\mu_1}\ldots x^{\mu_k} P_A \\
Q_A^{\mu_1\ldots\mu_k} &= y x^{\mu_1}\ldots x^{\mu_k} P_A \\
J_{AB}^{n\mu_1\ldots\mu_k} &= e^{iny/R} x^{\mu_1}\ldots x^{\mu_k} J_{AB} \\
K_{AB}^{\mu_1\ldots\mu_k} &= y x^{\mu_1}\ldots x^{\mu_k} J_{AB}.
\end{aligned}
$$

Infinite goldstones before removing redundancies (inverse Higgs constraints)

$$
\begin{aligned}
\tilde{g} = {} & e^{x^\mu P_\mu} e^{y P_y} e^{\phi_n^A P_A^n} e^{\phi_{n\mu}^A P_A^{n\mu}} e^{\phi_{n\mu\nu}^A P_A^{n\mu}} e^{\zeta^A Q_A} e^{\zeta_\mu^A Q_A^\mu} e^{\zeta_{\mu\nu}^A Q_A^{\mu\nu}} \\
& \times e^{\frac{1}{2}\theta_{n\mu}^{AB} J_{AB}^{n\mu}} e^{\frac{1}{2}\theta_{n\mu\nu}^{AB} J_{AB}^{n\mu\nu}} e^{\frac{1}{2}\tau^{AB} K_{AB}} e^{\frac{1}{2}\tau_\mu^{AB} K_{AB}^\mu} e^{\frac{1}{2}\tau_{\mu\nu}^{AB} K_{AB}^{\mu\nu}}
\end{aligned}
$$

KK modes

Dilatations of S_1

Maurer-Cartan:

Lots of components - lots of redundancies, but not hard to pick out terms of interest:

$$\Omega^A_Q = \mathrm{d}\zeta^A - \mathrm{d}x^\mu\,\zeta^A_\mu + (\mathrm{d}\phi^B - \mathrm{d}x^\mu\,\phi^B_\mu)\tau^A_B + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta^B_\mu)\tau^A_B c_q - \mathrm{d}y\,\zeta^B\tau^A_B$$

$$\begin{aligned}\Omega^A_{Q\mu} &= \mathrm{d}\zeta^A_\mu - 2\,\mathrm{d}x^\nu\,\zeta^A_{\mu\nu} + (\mathrm{d}\phi^B_\mu - 2\,\mathrm{d}x^\nu\,\phi^B_{\mu\nu})\tau^A_B + (\mathrm{d}\phi^B - \mathrm{d}x^\mu\,\phi^B_\mu)\tau^A_{B\mu} + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta^B_\mu)\theta^A_{B\mu}\\ &\quad + (\mathrm{d}\zeta^B_\mu - 2\,\mathrm{d}x^\nu\,\zeta^B_{\mu\nu})\tau^A_B c_q + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta^B_\mu)\tau^A_{B\mu}c_q - \mathrm{d}y\,(\zeta^B\tau^A_{B\mu} + \zeta^B_\mu\tau^A_B)\end{aligned}$$

Will be the vielbein:

The dilatation

$$\Omega^A = \mathrm{d}\phi^A - \mathrm{d}x^\mu\,\phi^A_\mu + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta^B_\mu)\tau^A_B c_0 - \mathrm{d}y\,\zeta^A$$

$$\begin{aligned}\Omega^A_\mu &= \mathrm{d}\phi^A_\mu - 2\,\mathrm{d}x^\nu\,\phi^A_{\mu\nu} + (\mathrm{d}\phi^B - \mathrm{d}x^\mu\,\phi^B_\mu)\theta^A_{B\mu} + (\mathrm{d}\zeta^B_\mu - 2\,\mathrm{d}x^\nu\,\zeta^B_{\mu\nu})\tau^A_B c_0\\ &\quad + (\mathrm{d}\zeta^B - \mathrm{d}x^\mu\,\zeta^B_\mu)\tau^A_{B\mu}c_0 - \mathrm{d}y\,\zeta^A_\mu\end{aligned}$$

The gauge field

$$\Omega^{AB}_K = \mathrm{d}\tau^{AB} - \mathrm{d}x^\mu\,\tau^{AB}_\mu - \mathrm{d}x^\mu\,\theta^{[A|C}_\mu\tau^{|B]}_C + \mathrm{d}\tau^{[A|C}\,\tau^{|B]}_C c_q - \mathrm{d}y\,\tau^{[A|C}\tau^{|B]}_C$$

$$\begin{aligned}\Omega^{AB}_{K\mu} &= \mathrm{d}\tau^{AB}_\mu - 2\,\mathrm{d}x^\nu\,\tau^{AB}_{\mu\nu} + (\mathrm{d}\theta^{[A|C}_\mu - 2\,\mathrm{d}x^\nu\,\theta^{[A|C)}_{\mu\nu})\tau^{|B]}_C - \mathrm{d}x^\nu\,\theta^{[A|C}_\nu\tau^{|B]}_{C\mu}\\ &\quad + (\mathrm{d}\tau^{[A|C} - \mathrm{d}x^\nu\,\tau^{[A|C)}_\nu)\tau^{|B]}_{C\mu}c_q - \mathrm{d}y\,\tau^{[A|C}\tau^{|B]}_{C\mu}\end{aligned}$$

$$\Omega^{AB} = -\,\mathrm{d}x^\mu\,\theta^{AB}_\mu + \mathrm{d}\tau^{[A|C}\,\tau^{|B]}_C c_0 - \mathrm{d}y\,\tau^{AB}$$

$$\Omega^{AB}_\mu = \mathrm{d}\theta^{AB}_\mu - 2\,\mathrm{d}x^\nu\,\theta^{AB}_{\mu\nu} - \mathrm{d}x^\nu\,\theta^{[A|C}_\nu\theta^{|B}_{C\mu} + (\mathrm{d}\tau^{[A|C} - \mathrm{d}x^\nu\,\tau^{[A|C)}_\nu)\tau^{|B]}_{C\mu}c_0 - \mathrm{d}y\,\tau^{AB}_\mu.$$

Messy - but much of it can be ignored if interested in (e.g. gravitons, KK modes)

det e makes the sqrt

Volume factor from funfbein will contain kinetics for the dilaton and gauge field

DBI from pure gravity

envisioned in: Maxfield, Sethi 1612.00427

Outlook

- Brane worlds are fascinating models for cosmology
  - In most interesting regimes of models: relativistic motion of branes coupled to gravity
  - truncations of EFT fail to capture basic physics
  - Many interesting models with instabilities to drive these cosmologies
- Coset constructions potentially offer path to relevant physics without specifying details of symmetry breaking (soft vs hard wall, e.g.)
  - An art to getting physics right for spacetime symmetry breaking
    - inverse higgs constraints: non-scalar order parameters
    - basis choice for different order parameters shapes building blocks for EFT
- Goal is to further develop toolkit for “chiral lagrangians” for extra dimensional physics/cosmology
  - expand on and classify zoo, turn tools onto “pathological” models (ghosts and or tachyons)