

The Role of Gravity During Preheating

Tom Giblin February 5, 2020 From Inflation to the Hot Big Bang KITP; University of California, Santa Barbara

1511.01105, 1511.01106, 1608.04403, 1907.10601 (among others) work published with Chi Tian, James Mertens Glenn Starkman, and Avery Tishue

Work With

• For the Late Universe (not preheating)





Jim Mertens York/PI

Glenn Starkman Case Western



Chi Tian Case Western

For the Early Universe (preheating)



My group at Kenyon



related to stuff we're talking about here





Gwyneth Phillips '20 Rand Burnette '21

Schrödinger-Poisson Systems

Allegra Fass '21 Modified Gravity and Compact Objects



Ericka Florio '22



Mary Gerhardinger '22

Preheating, EMDE, Early Dark Energy



General Relativity appears to be one heck of a theory

Gravity

- An example:
 - Two black holes collide*
 - General Relativity predicts** a signal
 - We measure the signal***



**Many contributors, this analysis from Simulating Extreme Space-time (not me) **LIGO: Phys. Rev. Lett. 116, 061102 (absolutely not me)

Unfortunately

• No one seemed to tell the Universe

According to General Relativity here's what happened (mathematically speaking)

The Universe today is a combination of Matter and Radiation (mostly matter)

The Universe cools enough to be transparent

Because matter dilutes slower than radiation, the earlier Universe was more radiation than matter

In the distant past, the Universe was very very dense

mathematically speaking, the model ends (begins) with a zero-volume, t=0.

According to Concordance Cosmology here's what happened (mathematically speaking)

Dark Energy Dominated Universe (expansion of the universes seems to be accelerating)

The Universe today is a combination of Matter and
Radiation (mostly matter)

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In the distant past, the Universe was very very dense

Inflation? Ekpyrosis? Bubbles? Gnomes?

The Main Point: Gravity is Non-Linear

- Being "non-linear" is more that just "not being small"
- We like to *separate scales* when doing physics problems (e.g. what happens here, stays here)
 - Non-linear physics can mix up scales power transferred between scales through cascades or inverse-cascades

Sometimes things that look like Perturbations



Takao Itami https://www.cradle-cfd.com/

Anas Maaz https://www.quora.com/



The Main Question: For the Universe, does it matter?

Averaging

 Generally a Hubble Volume is taken to be the region over which we do averaging — we all agree that different Hubble patches could have different expansion rates (causality, right?)

$$H^{-3} \approx (4000 \,\mathrm{Mpc})^3$$

- Yet there is structure at (just) smaller scales
 - Galaxy Clusters $\sim 1-10\,{
 m Mpc}$
 - Inter-Cluster Distances $~\sim 50\,{
 m Mpc}$

Scales at Reheating

 Generally a Hubble Volume is taken to be the region over which we do averaging — we all agree that different Hubble patches could have different expansion rates (causality, right?)

$$H^{-1} \propto \mathcal{O}(1) \times \frac{m_{\mathrm{pl}}^2}{m^2 \phi_0^2} = \mathcal{O}(1) \times m^{-1}$$

- YET: we talk about things at scales around this
 - Oscillons
 - Tachyonic/Parametric Resonance

$$\left. \right\} k \propto \mathcal{O}(1) \times m^{-1}$$

Can non-linear physics help explain the great mysteries of the Universe?

We want to investigate

GABE: Scalar Fields (Gravitational Waves) Isotropic and Homogeneous evolution







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GABE: Scalar Fields (Gravitational Waves) Isotropic and Homogeneous evolution

t = 276.1m

NewtGABE: Newtonian Gravity Scalar Fields Newtonian Potential + back reaction

 $ds^{2} = -(1+2\Phi)dt^{2} + a^{2}(t)(1-2\Phi)\delta_{ij}dx^{i}dx^{j}$

Our first go

Perturbation theory, of course!

 $ds^{2} = -(1+2\Phi) dt^{2} + a^{2} (1-2\Phi) \left[dx^{2} + dy^{2} + dz^{2} \right],$

- Which gives us an equation of motion for the field $\ddot{\phi} = -3H\dot{\phi} + 4\dot{\phi}\dot{\Phi} + (1+4\Phi)\frac{\nabla^2\phi}{a^2} - (1+2\Phi)\frac{\partial V}{\partial\phi}$
- and an equation to satisfy for the Newtonian potential

$$-3\hat{H}\tilde{\Phi}^{2}\dot{\Phi}^{3}a^{3}H\tilde{\Phi}(\dot{\tilde{\Phi}}^{+}+\frac{4\pi a^{2}}{H_{2}^{0}})2\Phi\rho_{0}^{+}h_{2}^{+}\delta\phi).$$

Other things to look for?

 $\delta \rho / \rho$



 Gravitational back reaction

 Preheating produces gravitational inhomogenei ties

Or Oscillons....

Non-topological structures that come from (slightly open) inflationary potentials





We want to investigate

GABE: Scalar Fields (Gravitational Waves) Isotropic and Homogeneous evolution

NewtGABE: Newtonian Gravity Scalar Fields Newtonian Potential + back reaction

GabeREL: Scalar Fields/Fluids Full Numerical Relativity

What you would like to do

Write down the most general form of the metric,

	g_{00}	g_{01}	g_{02}	g_{03})
a =	g_{01}	g_{11}	g_{12}	g_{13}
$g\mu\nu$ —	g_{02}	g_{12}	g_{22}	g_{23}
	g_{03}	g_{13}	g_{23}	g_{33} /

Plug it into Einstein's Equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

 Solve the system of second order differential equations (subject to your gauge-constraints)



What can we do?

- You can do a little better by making gauge choices that reduce the number of parameters or (re)parameterize so that you have nice equations for.. some.. of them...
- Even then they are extremely difficult to numerically stabilize

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Numerical Relativity and Compact Binaries

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Abstract

Numerical relativity is the most promising tool for theoretically modeling the inspiral and coalescence of neutron star and black hole binaries, which, in turn, are among the most promising sources of gravitational radiation for future detection by gravitational wave observatories. In this article we review numerical relativity approaches to modeling compact binaries. Starting with a brief introduction to the 3+1 decomposition of Einstein's equations, we discuss important components of numerical relativity, including the initial data problem, reformulations of Einstein's equations, coordinate conditions, and strategies for locating and handling black holes on numerical grids. We focus on those approaches which currently seem most relevant for the compact binary problem. We then outline how these methods are used to model binary neutron stars and black holes, and review the current status of inspiral and coalescence simulations.

Key words:

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6

2.1 Foliations of Spacetime

What we have to do...

- Luckily there are a set of new approaches. We use the most common of these: the BSSN formalism.
- It is based on the ADM metric decomposition

$$g_{\mu\nu} = \begin{pmatrix} -\alpha^2 + \gamma_{lk}\beta^l\beta^k & \beta_i \\ \beta_j & \gamma_{ij} \end{pmatrix}$$

• We we introduce more parameters than (minimally) necessary so that the equations are easier to solve

In Cosmology

- We can fix the gauge (we will give up being able to create black holes, as well as some other concessions) to focus on spatial slices
- We can then track the spatial 3-metric

$$\gamma_{ij} = e^{4\phi} \bar{\gamma}_{ij}$$

as well as the extrinsic curvature

$$K_{ij} = e^{4\phi}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$

In Cosmology

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In Cosmology

- We can fix the gauge (we will give up being able to create black holes, as well as some other concessions) to focus on spatial slices
- We Think of this as keeping track of the size of local volumes $\gamma_{ij} = e^{4\phi} \bar{\gamma}_{ij}$

as well as the extrinsic curvature

Think of this as measuring the local expansion rate

$$K_{ij} = e^{4\phi}\bar{A}_{ij} + \frac{1}{3}\gamma_{ij}K$$



 $\partial_t \phi = -\bar{\gamma}_{ij} rac 16\alpha K + \beta^i \partial_i \phi + \frac{1}{6} \partial_i \beta^i$ Importantly $\partial_t \bar{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \tilde{A}_{ij} + \bar{\gamma}_{ik} \partial_j \beta^k$ $+ \bar{\gamma}_{kj}\partial_i\beta^k - \frac{2}{3}\bar{\gamma}_{ij}\partial_K\beta^k$ These variables have well-behaved differential $\partial_t K = \gamma^{ij} D_j D_i \alpha + \alpha \left(\tilde{A}_{ij} \tilde{A}_{ij} u + \frac{1}{3} K^2 \right)$ equations and are a complete description of $\begin{array}{l} +4\pi\alpha(\rho+S)+\beta^{i}\partial_{i}K \\ -\frac{\partial_{t}\tilde{A}_{ij}}{\partial_{t}\tilde{A}_{ij}}=e^{-4\psi}\left(-\overline{D}_{j}\overline{D}_{i}\alpha+A^{i}(R_{ij}-8\pi S_{ij})\right) \\ \mathcal{A}_{t}\tilde{A}_{ij}=e^{-4\psi}\left(-\overline{D}_{j}\overline{D}_{i}\alpha+A^{i}(R_{ij}-8\pi S_{ij})\right) \\$ $+\alpha \left(K\tilde{A}_{ij} - 8 \tilde{M}_{ll} \dot{A}_{j} \dot{B}_{j} + \beta \delta_{kij} \tilde{A}_{jj} \phi \right) + \beta^{j} \partial_{j} \bar{\Gamma}^{i} simplifications$ $+\tilde{A}_{ik}\partial_{j}\beta^{k}+\bar{R}_{k}^{j}\partial_{j}\beta^{i}k+\frac{2}{3}\bar{A}_{ij}^{i}\partial_{jk}\beta^{j}+\frac{1}{3}\bar{\gamma}^{li}\partial_{l}\partial_{j}\beta^{j}$ $+ \bar{\gamma}^{lj} \partial_i \partial_l \beta^i$

 $\partial_t \bar{\gamma}_{ij} = -2\alpha \tilde{A}_{ij} + \beta^k \partial_k \tilde{A}_i$ The POWER is in the redundancy of the equations of motion $\partial_t \partial_t \bar{\Gamma}^{ij} = D_j D_2 \tilde{A}^{ij} \partial_j \left(\tilde{\Gamma}^{i}_{j\bar{k}} \tilde{A}^{kj} \right) - \frac{2}{3} \bar{\gamma}^{ij} \partial_j K^{ki} \text{ and } are a$ $+4\pi \omega \, 8\pi ar{\gamma}^{ij} S^+_i + 6 ilde{A}^{ij} \partial_i \phi) + eta^j \partial_j ar{f R}^i$ without dimensional

$\partial_t \phi = -\bar{\gamma}_{ij} rac 16\alpha K + \beta^i \partial_i \phi + \frac{1}{\epsilon} \partial_i \beta^i$



 $+A_{ik}\partial_{j}\beta^{\kappa}+A_{kj}\partial_{i}\beta^{\kappa}-\frac{1}{3}A_{ij}\partial_{k}\beta^{\kappa}$

 $h(e) = \frac{1}{1 + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[\phi[t, x, y, z], t] + (D[sqrtg0 * (gupper0[[1, 1]] * D[sqrtg0 * (gupper0[[1, 1]] * (D[sqrtg0 * (gupper0[[1, 1]] * (D[sqrtg0 * (gupper0[[1, 1]] * (gupper0[[1, 1]) * (gupper0[[1, 1]$ gupper0[[1, 4]] * $D[\phi[t, x, y, z], z]$, t]) == 0;

 $\gamma_{xx}[x, y, z] (\gamma_{yz}[x, y, z]^2 - \gamma_{yy}[x, y, z] \gamma_{zz}[x, y, z]))^2 +$

 $\gamma_{xy}[x, y, z]^{2} \gamma_{zz}[x, y, z] +$

 $\alpha^{(1,0,0,0)}$ [t, x, y, z] $\phi^{(1,0,0,0)}$ [t, x, y, z]

 α [t, x, y, z]³

 $\phi^{(2,0,0,0)}[t, x, y, z]$

 α [t, x, y, z]²

gupper0[[1, 2]] * $D[\phi[t, x, y, z], x]$ + gupper0[[1, 3]] * $D[\phi[t, x, y, z], y]$ +

 $\phi^{\,(\theta\,,\,\theta\,,\,1\,,\,\theta)}\,[\,t\,,\,x\,,\,y\,,\,z\,]\,+\,\left(\gamma_{yy}^{\,(\theta\,,\,\theta\,,\,1)}\,\left[\,x\,,\,y\,,\,z\,\right]\,-2\,\gamma_{yz}^{\,(\theta\,,\,1\,,\,\theta)}\,\left[\,x\,,\,y\,,\,z\,\right]\,\right)$ $\phi^{(0,1,0,0)} [t, x, y, z] - 4 \gamma_{yz} [x, y, z]^2 \phi^{(0,1,0,1)} [t, x, y, z] +$ $\gamma_{vz}[x, y, z] (2 (\gamma_{xv}^{(0,0,1)}[x, y, z] + \gamma_{xz}^{(0,1,0)}[x, y, z] + \gamma_{vz}^{(1,0,0)}[x, y, z])$ $\phi^{(0,0,0,1)}$ [t, x, y, z] + $(2\gamma_{xz}^{(0,0,1)}$ [x, y, z] + $\gamma_{zz}^{(1,0,0)}$ [x, y, z]) $\phi^{(0,0,1,0)}[t, x, y, z] + (2\gamma_{yz}^{(0,0,1)}[x, y, z] + \gamma_{zz}^{(0,1,0)}[x, y, z])$ $\phi^{(0,1,0,0)}$ [t, x, y, z] - 4 γ_{zz} [x, y, z] $\phi^{(0,1,1,0)}$ [t, x, y, z]))) + $\gamma_{xy}[x, y, z] (\gamma_{xx}[x, y, z] (\gamma_{zz}[x, y, z] (\gamma_{yy}^{(0,1,0)}[x, y, z] \phi^{(0,0,0,1)}[t, x, y, z] +$ $(\gamma_{yy}^{(0,0,1)}[x, y, z] + 2\gamma_{yz}^{(0,1,0)}[x, y, z]) \phi^{(0,0,1,0)}[t, x, y, z]) +$ $12 \gamma_{yz} [x, y, z]^2 \phi^{(0,0,1,1)} [t, x, y, z] - 4 \gamma_{yz} [x, y, z]$ $(\gamma_{yy}^{(0,0,1)} [x, y, z] \phi^{(0,0,0,1)} [t, x, y, z] + \gamma_{zz}^{(0,1,0)} [x, y, z]$ $\phi^{(0,0,1,0)}$ [t, x, y, z] + γ_{zz} [x, y, z] $\phi^{(0,0,2,0)}$ [t, x, y, z]) + $\gamma_{yz}[x, y, z] (-4\gamma_{zz}[x, y, z] (\gamma_{xx}^{(0,1,0)}[x, y, z] \phi^{(0,0,1,0)}[t, x, y, z] +$ $\gamma_{vv}^{(1,0,0)}[x, y, z] \phi^{(0,1,0,0)}[t, x, y, z] + \gamma_{vz}[x, y, z]$ $((3 \gamma_{xx}^{(0,1,0)} [x, y, z] - 2 \gamma_{xy}^{(1,0,0)} [x, y, z]) \phi^{(0,0,0,1)} [t, x, y, z] +$ $(3 \gamma_{xx}^{(0,0,1)} [x, y, z] - 2 \gamma_{xz}^{(1,0,0)} [x, y, z]) \phi^{(0,0,1,0)} [t, x, y, z] 2\left(\gamma_{xy}^{(\theta,\theta,1)}[x, y, z] + \gamma_{xz}^{(\theta,1,\theta)}[x, y, z] - 3\gamma_{yz}^{(1,\theta,\theta)}[x, y, z]\right)$ $\phi^{\,(\theta,1,\theta,\theta)}\,[\,t,\,x,\,y,\,z\,]\,\big)\,+4\,\gamma_{yz}\,[\,x,\,y,\,z\,]^{\,2}\,\phi^{\,(\theta,2,\theta,\theta)}\,[\,t,\,x,\,y,\,z\,]\,\big)\,+$ $\gamma_{yy}\left[x, \, y, \, z\right] \, \left(\gamma_{xx}\left[x, \, y, \, z\right] \, \left(\left(2 \, \gamma_{yz}^{\, (\theta, \theta, 1)}\left[x, \, y, \, z\right] + \gamma_{zz}^{\, (\theta, 1, \theta)}\left[x, \, y, \, z\right]\right)\right)$ $\phi^{(0,0,0,1)} [t, x, y, z] + \gamma_{zz}^{(0,0,1)} [x, y, z] \phi^{(0,0,1,0)} [t, x, y, z] - \phi^{(0,0,1,0)} [t, y, y] - \phi^{(0,0,1,0)} [t, y, y] - \phi^{(0,0,1,0)} [t, y, y$ $4 \gamma_{zz} [x, y, z] \phi^{(0,0,1,1)} [t, x, y, z] + \gamma_{zz} [x, y, z]$ $((\gamma_{xx}^{(0,1,0)}[x, y, z] + 2\gamma_{xy}^{(1,0,0)}[x, y, z])\phi^{(0,0,0,1)}[t, x, y, z] +$ $(\gamma_{xx}^{(0,0,1)}[x, y, z] + 2\gamma_{xz}^{(1,0,0)}[x, y, z]) \phi^{(0,0,1,0)}[t, x, y, z] +$ $2 \left(\gamma_{xy}^{(0,0,1)} [x, y, z] + \gamma_{xz}^{(0,1,0)} [x, y, z] + \gamma_{yz}^{(1,0,0)} [x, y, z] \right) \phi^{(0,1,0,0)} [x, y, z]$ t, x, y, z]) - 4 γ_{VZ} [x, y, z] ($\gamma_{xx}^{(0,0,1)}$ [x, y, z] $\phi^{(0,0,0,1)}$ [t, x, y, z] + $\gamma_{xx}[x, y, z] \phi^{(0,0,0,2)}[t, x, y, z] + \gamma_{zz}^{(1,0,0)}[x, y, z]$ $\phi^{(0,1,0,0)}[t, x, y, z] + \gamma_{zz}[x, y, z] \phi^{(0,2,0,0)}[t, x, y, z])))))/$ $\left(2\left(\gamma_{xz}[x, y, z]^{2}\gamma_{yy}[x, y, z] - 2\gamma_{xy}[x, y, z]\gamma_{xz}[x, y, z]\gamma_{yz}[x, y, z] + \right.\right.$

y, z] -2 $(\gamma_{xy}^{(0,0,1)}[x, y, z] - \gamma_{xz}^{(0,1,0)}[x, y, z] + \gamma_{yz}^{(1,0,0)}[x, y, z])$

BSSN_scalars.nb 7

 $\gamma_{zz}^{(1,\theta,\theta)}\left[x, y, z\right] \phi^{(\theta,1,\theta,\theta)}\left[t, x, y, z\right] + \gamma_{zz}\left[x, y, z\right] \phi^{(\theta,2,\theta,\theta)}\left[t, x, y, z\right] \right) \right) - \phi^{(\theta,1,\theta,\theta)}\left[t, x, y, z\right] = 0$ $\gamma_{xy}[x, y, z]^{2} (\gamma_{zz}[x, y, z] (\gamma_{xx}[x, y, z] ((\gamma_{yy}^{(0,0,1)}[x, y, z] - 2\gamma_{yz}^{(0,1,0)}[x, y, z]))$ $\phi^{\,(\theta,\theta,\theta,1)}\,[\,t,\,x,\,y,\,z\,]\,+\,\left(-2\,\gamma_{yz}^{\,(\theta,\theta,1)}\,[\,x,\,y,\,z\,]\,+\,\gamma_{zz}^{\,(\theta,1,\theta)}\,[\,x,\,y,\,z\,]\,\right)$ $\phi^{(0,0,1,0)}$ [t, x, y, z] + 2 γ_{zz} [x, y, z] $\phi^{(0,0,2,0)}$ [t, x, y, z] + $2 \gamma_{zz}[x, y, z] (\gamma_{xx}^{(0,1,0)}[x, y, z] \phi^{(0,0,1,0)}[t, x, y, z] +$ $\gamma_{yy}^{(1,0,0)}[x, y, z] \phi^{(0,1,0,0)}[t, x, y, z]) + \gamma_{yz}[x, y, z] (\gamma_{xx}[x, y, z])$ $((2 \gamma_{yz}^{(0,0,1)} [x, y, z] + \gamma_{zz}^{(0,1,0)} [x, y, z]) \phi^{(0,0,0,1)} [t, x, y, z] + \gamma_{zz}^{(0,0,1)} [t, z, y, z] + \gamma_{zz}^{(0,0,1)} [z, z, y, z] + \gamma_$ x, y, z] $\phi^{(0,0,1,0)}$ [t, x, y, z] -4 γ_{zz} [x, y, z] $\phi^{(0,0,1,1)}$ [t, x, y, z]) - $2\gamma_{zz}[x, y, z] (\gamma_{xx}^{(0,1,0)}[x, y, z] \phi^{(0,0,0,1)}[t, x, y, z] + \gamma_{xx}^{(0,0,1)}[x, y, z] + \gamma_{xx}^{(0,0$ y, z] $\phi^{(0,0,1,0)}$ [t, x, y, z] + 2 $\gamma_{VZ}^{(1,0,0)}$ [x, y, z] $\phi^{(0,1,0,0)}$ [t, x, y, z])) + $\gamma_{yz}[x, y, z]^{2} \left(\left(\gamma_{xx}^{(0,0,1)}[x, y, z] + 2\gamma_{xz}^{(1,0,0)}[x, y, z] \right) \phi^{(0,0,0,1)}[t, x, y, z] - \right) \right) = 0$ $2\gamma_{xx}[x, y, z] \phi^{(0,0,0,2)}[t, x, y, z] + (2\gamma_{xz}^{(0,0,1)}[x, y, z] + \gamma_{zz}^{(1,0,0)}[x, y, z])$

 $2\gamma_{yy}[x, y, z]\gamma_{yz}[x, y, z]^{2}\gamma_{zz}[x, y, z]\gamma_{xx}^{(1,0,0)}[x, y, z]\phi$ $\gamma_{yy}[x, y, z]^{2} \gamma_{zz}[x, y, z]^{2} \gamma_{xx}^{(1,0,0)}[x, y, z] \phi^{(0,1,0,0)}[t, x]$ $\gamma_{xx}\,[\,x\,,\,y\,,\,z\,]\,\,\gamma_{yz}\,[\,x\,,\,y\,,\,z\,]^{\,2}\,\gamma_{zz}\,[\,x\,,\,y\,,\,z\,]\,\,\gamma_{yy}^{\,(1\,,\,\theta\,,\,\theta)}\,[\,x\,,\,y\,,\,z\,]\,\,\phi^{\,(6)}$ $\gamma_{xx}[x, y, z] \gamma_{yy}[x, y, z] \gamma_{zz}[x, y, z]^{2} \gamma_{yy}^{(1,0,0)}[x, y, z] \phi^{(6)}$ $2\gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]^{3} \gamma_{yz}^{(1,0,0)}[x, y, z] \phi^{(0,1,0,0)}[t, x]$ $\gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z] \gamma_{zz}[x, y, z] \gamma_{yz}^{(1,0,0)}[x, y, z] \phi^{(1,0,0)}[x, y] \phi^{(1,0,0)}[x, y] \phi^$ x,y,z]φ^{(€} $\phi^{(0,0,0,1)}$ [t, x, y x, y, z] $\phi^{(0,1,0)}$ Yzz [x, y, z] (Yzz [x,] $\phi^{(0,0,1,0)}$ [t, γyy [x, y, z] (2 (γ $\phi^{(0,0,0,1)}$ [t, $\phi^{(0,0,1,0)}$ [t. $\phi^{(0,1,0,0)}$ [t, $2 \gamma_{xx}[x, y, z] (\gamma_{yz}[x, y, z]^2)$ $\gamma_{xz}[x, y, z]^2 (\gamma_{xx}[x, y, z] \gamma_{yz})$ $\gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]$ γ_{yz}[x, y, z]² γ_{xx}^(0,1,0) [x] $2\gamma_{xx}[x, y, z]\gamma_{zz}[x, y, z]$ 2 Y_{xx} [x, y, z] Y_{yz} [x, y, z $2 \gamma_{yz} [x, y, z]^2 \gamma_{xv}^{(1,0,0)}$ [$2 \gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]$ $\gamma_{xy} [x, y, z]^2 ((\gamma_{yy}^{(0,0,1)})$ (2 yyz^(0,0,1) [x, y, z] 2 (4 γ_{yz} [x, y, z] $\phi^{(0,6)}$ $\gamma_{yz} \left[x \, , \, y \, , \, z \, \right]^2 \, \left(2 \, \gamma_{xy} \,^{(\theta, 1, \theta)} \right.$ γ_{xy}[x, y, z] (γ_{zz}[x, y, z $\phi^{(0,0,1,0)}$ [t, x, y γ_{yz}[x, y, z] ((-2 γ_{xy} x, y, z] - 2 (_{Yx} $\phi^{\,(0\,,0\,,1\,,0)}\,[\,t\,,\,x\,,\,y\,,\,\ldots\,,\,\ldots\,,\,y\,y$ $\phi^{(0,1,0,0)}[t, x, y, z] - 8 \gamma_{yz}[x,$ $\gamma_{yy}[x, y, z] (2 \gamma_{yz}[x, y, z] \gamma_{xx}^{(0,1,0)}[x, y, z])$ $2 \gamma_{xy}[x, y, z]^2 \phi^{(0,0,0,2)}[t, x, y, z] +$ $\phi^{(0,0,1,0)}$ [t, x, y, z] - γ_{zz} [x, y, z] γ_{zz} $2\,\gamma_{zz}\,[\,x\,,\,y\,,\,z\,]\,\,\gamma_{xy}^{\,\,(1\,,\,\theta\,,\,\theta)}\,[\,x\,,\,y\,,\,z\,]\,\,\phi^{\,(\theta\,,\,\theta}$ $\gamma_{xx}[x, y, z] ((\gamma_{yy}^{(0,0,1)}[x, y, z] - 2))$ $(-2 \gamma_{yz}^{(0,0,1)} [x, y, z] + \gamma_{zz}^{(0,1,0)} [$ $4 \gamma_{yz} [x, y, z] \phi^{(0,0,1,1)} [t, x, y, z]$ $2 \gamma_{zz}[x, y, z] \gamma_{xy}^{(0,1,0)}[x, y, z] \phi^{(0,1)}$ $\gamma_{zz} \left[x \, , \, y \, , \, z \right] \gamma_{yy}^{(1, \, \theta \, , \, \theta)} \left[\, x \, , \, y \, , \, z \right] \, \phi^{(\theta \, , \, 1 \, , \, \theta}$ $4 \gamma_{yz}[x, y, z] \gamma_{yz}^{(1,0,0)}[x, y, z] \phi^{(0,1)}$ $2 \gamma_{xy}[x, y, z] (2 \gamma_{xy}^{(0,0,1)}[x, y, z] \phi$ z] $\phi^{(0,0,1,0)}$ [t, x, y, z] + $\gamma_{zz}^{(0,0,1,0)}$ $6 \gamma_{yz} [x, y, z] \phi^{(0,1,0,1)} [t, x, y, z]$ $2 \gamma_{yz}[x, y, z]^2 \phi^{(0,2,0,0)}[t, x, y, z]$ $(\gamma_{xx}^{(0,0,1)} [x, y, z] \phi^{(0,0,0,1)} [t, x, y, z]$

 $(-\gamma_{vv})^{(0,1,0)}[x, y, z] \phi^{(0,0,1,0)}[t, x, y, z] + 2\gamma_{vv}[x, y, z] \phi^{(0,1,0)}[t, x, y, z] + 2\gamma_{vv}[x, y, z] \phi^{(0,1,0)}[t, x, y, z] \phi^{(0,1,0)}[t, x, y, z] + 2\gamma_{vv}[x, y, z] \phi^{(0,1,0)}[t, x, y, z] \phi^{(0,1,0)}[t, x, y, z] + 2\gamma_{vv}[x, y, z] \phi^{(0,1,0)}[t, x, y, z] \phi^{(0,1,0)}[t, y, z] \phi^{(0,1,0)}[t, y, y,$

 $2\gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]^{3} \gamma_{xy}^{(0,0,1)}[x, y, z] \phi^{(0,1,0,0)}[t, x]$

 $\gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z] \gamma_{zz}[x, y, z] \gamma_{xy}^{(0,0,1)}[x, y, z] \phi^{(0,0,1)}[x, y, z] \gamma_{yz}[x, y, z] \gamma_{yz}[x,$

 $2\gamma_{xx}[x, y, z]\gamma_{yy}[x, y, z]\gamma_{yz}[x, y, z]^{2}\gamma_{xz}^{(0,0,1)}[x, y, z]\phi$

 $2\,\gamma_{xx}[\,x,\,y,\,z]\,\,\gamma_{yy}[\,x,\,y,\,z\,]^{\,2}\,\gamma_{zz}[\,x,\,y,\,z\,]\,\,\gamma_{xz}^{\,(\theta,\,\theta,\,1)}\,[\,x,\,y,\,z\,]\,\,\phi$

 $2\,\gamma_{xx}[\,x,\,y,\,z\,]\,\gamma_{yz}[\,x,\,y,\,z\,]^{\,2}\,\gamma_{zz}[\,x,\,y,\,z\,]\,\gamma_{xy}^{\,(\theta,\,1,\,\theta)}\,[\,x,\,y,\,z\,]\,\phi$

 $2\,\gamma_{xx}[\,x,\,y,\,z\,]\,\,\gamma_{yy}[\,x,\,y,\,z\,]\,\,\gamma_{zz}[\,x,\,y,\,z\,]^{\,2}\,\gamma_{xy}\,^{(\theta,\,1,\,\theta)}\,[\,x,\,y,\,z\,]\,\,\phi$ $2\gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]^{3} \gamma_{xz}^{(0,1,0)}[x, y, z] \phi^{(0,1,0,0)}[t, x]$ $2\gamma_{xx}[x, y, z]\gamma_{yy}[x, y, z]\gamma_{yz}[x, y, z]\gamma_{zz}[x, y, z]\gamma_{xz}^{(0,1,6)}$ $\gamma_{xx}[x,]$ $\gamma_{xy}[x,$ γzz γ_{xz}[x, γyz γуу $\gamma_{xy}[x, :$ Ϋ́xx $2\gamma_{xx}[x, y, z]^{2}\gamma_{yz}[x, y, z]\gamma_{zz}[x, y, z]$ $2\gamma_{xx}[x, y, z]^{2}\gamma_{yz}[x, y, z]^{2}\gamma_{zz}^{(0,1,0)}$ $\gamma_{xx}[x, y, z]^{2} \gamma_{yy}[x, y, z] \gamma_{zz}[x, y, z]$ $2\gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]^{2} \gamma_{zz}[x, y, z]$ $2 \gamma_{xx}[x, y, z] \gamma_{yy}[x, y, z] \gamma_{zz}[x, y, z]$ $2 \gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]^{3} \gamma_{xz}^{(1,0,0)}[:$ $2 \gamma_{xx}[x, y, z] \gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z]$ $\phi^{(0,0,1,0)}$ [t, x, y, z] + 4 γ_{xx} [x, y, z] $4 \gamma_{xx}[x, y, z]^2 \gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z]$ $2 \gamma_{xx}[x, y, z]^{2} \gamma_{yz}[x, y, z]^{2} \gamma_{zz}[x, y, z]^{2}$ $2\gamma_{xx}[x, y, z]^{2}\gamma_{yy}[x, y, z]\gamma_{zz}[x, y, z]$

gupper0[[1, 3]] $* D[\phi[t, x, y, 2]$ t] + D[sqrtg0 * (gupper0[[2, 1]] * D gupper0[[2, 2]] $*D[\phi[t, x, y, 2]$ gupper0[[2, 4]] $* D[\phi[t, x, y, 2]$ $(gupper0[[3, 1]] * D[\phi[t, x, y, z]]$ gupper0[[3, 3]] $* D[\phi[t, x, y, 2]$ y + D[sqrtg0 * (gupper0[[4, 1]] * D gupper0[[4, 2]] $* D[\phi[t, x, y, 2]]$ gupper0[[4, 4]] $*D[\phi[t, x, y, 2]$ [α[t, x, y (2√ Out[=]= (α[**t**, x, Yxy [] γxx [] $\sqrt{(-\gamma_x)}$ (- Y_{xx} [x, y Yxx $\gamma_{xx}[x,$ Yxx 2 Y_{xx} [x $\gamma_{xx}[x]$ large output 2 Y xx [X $\gamma_{xx}[x,$ In[=]:= FullSimplify $\gamma_{xx}[x,$

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 $(gupper0[[1, 1]] * D[\phi[t, x, y, z]]$

 $ln[*]:= eom = \frac{1}{sartg0} * (D[sartg0 *$

4 BSSN_scalars.nb

 $\gamma_{xx} [x, y, z] \left(\left(\gamma_{yy}^{(0,0,1)} [x, y, z] - 2 \right) \right)$ $\left(-2\gamma_{yz}^{(0,0,1)}[x, y, z] + \gamma_{zz}^{(0,1,0)}\right)$ $4 \gamma_{yz} [x, y, z] \phi^{(0,0,1,1)} [t, x, y, z]$ $2 \gamma_{zz}[x, y, z] \gamma_{xy}^{(0,1,0)}[x, y, z] \phi^{(0,1)}$ $\gamma_{zz}\,[\,x\,,\,y\,,\,z\,]\,\,\gamma_{yy}\,^{(1,\,\theta\,,\,\theta)}\,[\,x\,,\,y\,,\,z\,]\,\,\phi^{\,(\theta\,,\,1\,,\,\theta}$ 4 γ_{yz} [x, y, z] γ_{yz} (1,0,0) [x, y, z] ϕ (0,1 $2\,\gamma_{x\,y}\,[\,x\,,\,y\,,\,z\,]~\left(2\,\gamma_{x\,y}^{\,(\theta\,,\,\theta\,,\,1)}\,[\,x\,,\,y\,,\,z\,]~\phi^{\,+}\right.$ z] $\phi^{(0,0,1,0)}$ [t, x, y, z] + $\gamma_{zz}^{(0,-)}$ $6 \gamma_{yz} [x, y, z] \phi^{(0,1,0,1)} [t, x, y, z]$ $2 \gamma_{yz}[x, y, z]^2 \phi^{(0,2,0,0)}[t, x, y, z]$ $(\gamma_{xx}^{(0,0,1)}[x, y, z] \phi^{(0,0,0,1)}[t, x, y, z]$

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y, z] $\phi^{(6)}$ ^{,0,0)} [t, x y,z] $\phi^{(}$

-1 -4 (G

 $h(e) = \text{teom} = \frac{1}{(1 + 1)^2} \star (D[\text{sqrtg0} \star (\text{gupper0}[[1, 1]] \star D[\phi[t, x, y, z], t] + 1)]$ sqrtg0 gupper0[[1, 2]] * $D[\phi[t, x, y, z], x]$ + gupper0[[1, 3]] * $D[\phi[t, x, y, z], y]$ + gupper0[[1, 4]] * $D[\phi[t, x, y, z], z]$) = 0;

¹⁾ [t, x, y, z] + (, y, z]) + **z** 1 z]))+ ,y,z]+ x, y, z] + x,y,z]-,y,z]) y,z])+ y, z]) к,у, z] y,z]+ <, y, z] + y, z]) φ^(θ,1,θ,θ) Γ (0,1) [t, x, y, z] + z]))))// Million The The New York Eimes **ISRAELIS TO PUSH EFFORT FOR PEACE**

 $\textbf{y, z]} - 2 \, \left(\gamma_{xy}^{\,(\theta, \theta, 1)} \, [\, \textbf{x, y, z} \,] - \gamma_{xz}^{\,\,(\theta, 1, \theta)} \, [\, \textbf{x, y, z} \,] + \gamma_{yz}^{\,(1, \theta, \theta)} \, [\, \textbf{x, y, z} \,] \right)$ (0,1,0) [x, y, z]

BSSN_scalars.nb 7

y,z]+ ^(1,0,0) [x, y, z]) [x,y,z])

[x,y,z])

, z]))) +

 $\gamma_{zz}^{(1,0,0)}[x, y, z] \phi^{(0,1,0,0)}[t, x, y, z] + \gamma_{zz}[x, y, z] \phi^{(0,2,0,0)}[t, x, y, z]) - \phi^{(0,2,0,0)}[t, x, y, z]$ $\gamma_{xy}[x, y, z]^{2} \left(\gamma_{zz}[x, y, z] \left(\gamma_{xx}[x, y, z] \left(\left(\gamma_{yy}^{(\theta, \theta, 1)}[x, y, z] - 2\gamma_{yz}^{(\theta, 1, \theta)}[x, y, z] \right) \right) \right)\right)$ $\phi^{(0,0,0,1)} [t, x, y, z] + (-2 \gamma_{yz}^{(0,0,1)} [x, y, z] + \gamma_{zz}^{(0,1,0)} [x, y, z])$ $\phi^{\,(\theta,\,\theta,\,1,\,\theta)}\,[\,t,\,x,\,y,\,z\,]\,+2\,\gamma_{zz}\,[\,x,\,y,\,z\,]\,\phi^{\,(\theta,\,\theta,\,2,\,\theta)}\,[\,t,\,x,\,y,\,z\,]\,\big)\,+$ $2 \gamma_{zz}[x, y, z] (\gamma_{xx}^{(0,1,0)}[x, y, z] \phi^{(0,0,1,0)}[t, x, y, z] +$ $\gamma_{yy}^{(1,0,0)}[x, y, z] \phi^{(0,1,0,0)}[t, x, y, z]) + \gamma_{yz}[x, y, z] (\gamma_{xx}[x, y, z])$ $\left(\left(2\,\gamma_{yz}^{(\theta,\theta,1)}\left[x,\,y,\,z\right]+\gamma_{zz}^{(\theta,1,\theta)}\left[x,\,y,\,z\right]\right)\,\phi^{(\theta,\theta,\theta,1)}\left[t,\,x,\,y,\,z\right]+\gamma_{zz}^{(\theta,\theta,1)}\left[t,\,x,\,y,\,z\right]$ x, y, z] $\phi^{(0,0,1,0)}$ [t, x, y, z] -4 γ_{zz} [x, y, z] $\phi^{(0,0,1,1)}$ [t, x, y, z]) - $2\gamma_{zz}[x, y, z] (\gamma_{xx}{}^{(0,1,0)}[x, y, z] \phi^{(0,0,0,1)}[t, x, y, z] + \gamma_{xx}{}^{(0,0,1)}[x, y, z] + \gamma_{xx}{}^{(0,0,1)}[x$ y, z] $\phi^{(0,0,1,0)}$ [t, x, y, z] + 2 $\gamma_{yz}^{(1,0,0)}$ [x, y, z] $\phi^{(0,1,0,0)}$ [t, x, y, z])) + $\gamma_{yz}[x, y, z]^{2} \left(\left(\gamma_{xx}^{(\theta, \theta, 1)}[x, y, z] + 2\gamma_{xz}^{(1, \theta, \theta)}[x, y, z] \right) \phi^{(\theta, \theta, \theta, 1)}[t, x, y, z] - \right) \right) = 0$ $2 \gamma_{xx}[x, y, z] \phi^{(0,0,0,2)}[t, x, y, z] + (2 \gamma_{xz}^{(0,0,1)}[x, y, z] + \gamma_{zz}^{(1,0,0)}[x, y, z])$

 $(-\gamma_{yy}^{(0,1,0)}[x, y, z] \phi^{(0,0,1,0)}[t, x, y, z] + 2\gamma_{yy}[x, y, z]$ $z] \phi^{(0)}$

 $2\gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]^{3} \gamma_{xy}^{(0,0,1)}[x, y, z] \phi^{(0,1,0,0)}[t, x]$ $\gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z] \gamma_{zz}[x, y, z] \gamma_{xy}^{(0,0,1)}[x, y, z] \phi^{(1)}$ $2\gamma_{xx}[x, y, z] \gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z]^{2} \gamma_{xz}^{(0,0,1)}[x, y, z] \phi$ $2\,\gamma_{xx}[\,x,\,y,\,z\,]\,\,\gamma_{yy}[\,x,\,y,\,z\,]^{\,2}\,\gamma_{zz}[\,x,\,y,\,z\,]\,\,\gamma_{xz}^{\,(\theta,\,\theta,\,1)}[\,x,\,y,\,z\,]\,\,\phi$ $2\,\gamma_{xx}[\,x,\,y,\,z\,]\,\,\gamma_{yz}[\,x,\,y,\,z\,]^{\,2}\,\gamma_{zz}[\,x,\,y,\,z\,]\,\,\gamma_{xy}^{\,\,(\theta,\,1,\,\theta)}\,[\,x,\,y,\,z\,]\,\,\phi$ $2\,\gamma_{xx}[\,x,\,y,\,z\,]\,\,\gamma_{yy}[\,x,\,y,\,z\,]\,\,\gamma_{zz}[\,x,\,y,\,z\,]^{\,2}\,\gamma_{xy}^{\,\,(\theta,\,1,\,\theta)}\,[\,x,\,y,\,z\,]\,\,\phi$ $2\,\gamma_{xx}[\,x,\,y,\,z\,]\,\,\gamma_{yz}[\,x,\,y,\,z\,]^{\,3}\,\gamma_{xz}^{\,(\theta,\,1,\,\theta)}\,[\,x,\,y,\,z\,]\,\,\phi^{\,(\theta,\,1,\,\theta,\,\theta)}\,[\,t,\,x,\,\theta,\,z\,]$

2

 $\gamma_{xx} [x, y, z]^2 \gamma_{yy} [x, y, z] \gamma_{zz} [x, y, z]$ $2 \gamma_{xx}[x, y, z] \gamma_{yz}[x, y, z]^2 \gamma_{zz}[x, y, z]^2$ $2\,\gamma_{xx}[\,x\,,\,y\,,\,z\,]\,\,\gamma_{yy}[\,x\,,\,y\,,\,z\,]\,\,\gamma_{zz}[\,x\,,\,y\,,\,z\,$ $2\,\gamma_{xx}[\,x\,,\,y\,,\,z\,]\,\,\gamma_{yz}\,[\,x\,,\,y\,,\,z\,]^{\,3}\,\gamma_{xz}\,{}^{(1,\,\theta\,,\,\theta)}\,[:$ $2 \gamma_{xx}[x, y, z] \gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z]$ $\phi^{(0,0,1,0)}$ [t, x, y, z] + 4 γ_{xx} [x, y, z] $4 \gamma_{xx}[x, y, z]^{2} \gamma_{yy}[x, y, z] \gamma_{yz}[x, y, z]$ $2 \gamma_{xx}[x, y, z]^{2} \gamma_{yz}[x, y, z]^{2} \gamma_{zz}[x, y, z]^{2}$ $2\gamma_{xx}[x, y, z]^{2}\gamma_{yy}[x, y, z]\gamma_{zz}[x, y, z]$

 $\gamma_{xx}[x]$ γ_{xz}[x, 2 Yxx [X $\gamma_{xx}[x,$ $\gamma_{xx}[x,$ $\gamma_{xx}[x]$ γyz γ_{xx}⁽⁰, $\gamma_{xx}[x,$ Yvv 2 γ_{xx} [x $\gamma_{xx}[x,$ 2 Y xx [X 2 γ_{xx} [x $\gamma_{xy}[x,$ Yzz[X 2 γ_{xx} [x 2 γ_{xx} [x 2 Y_{xx} [x Yxx 2 γ_{xx} [x (-Yzz $\gamma_{xx}[x,$ γ_{yy}[x $\gamma_{xx}[x,$ 2 Yxx [X $\gamma_{xx}[x]$ $\gamma_{xx}[x,$ $\gamma_{xx}[x,$ $\gamma_{xx}[x,$ $2\gamma_{xx}[x, y, z]^{2}\gamma_{yz}[x, y, z]\gamma_{zz}[x, y, z]$ $2 \gamma_{xx} [x, y, z]^2 \gamma_{yz} [x, y, z]^2 \gamma_{zz}^{(0,1,0)}$

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large output

In[#]:= FullSimplify

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 $(gupper0[[1, 1]] * D[\phi[t, x, y, z]]$ gupper0[[1, 3]] $* D[\phi[t, x, y, 2]$ t] + D[sqrtg0 * (gupper0[[2, 1]] * D gupper0[[2, 2]] $*D[\phi[t, x, y, 2]$ gupper0[[2, 4]] $* D[\phi[t, x, y, 2]]$ $(gupper0[[3, 1]] * D[\phi[t, x, y, z]]$ gupper0[[3, 3]] $* D[\phi[t, x, y, 2]$ y] + D[sqrtg0 * (gupper0[[4, 1]] * D 4 BSSN_scalars.nb

"Group Motto"

Our First Test: Preheating



The Inflationary field is coupled to a second "matter" field

Non-linear interactions case the energy to be transferred quickly and violently.





- The process never happens when you take gravity into account: the universe fragments immediately and black holes form. The end of times begin shortly after inflation
- The resonance proce create primordial bla

Delayed Reheating and the Breakdown of Coherent Oscillations

• 1003.3011 Richard Easther, Raphael Flauger, and James B. Gilmore¹

> ¹ Department of Physics, Yale University, New Haven, CT 06520, USA



The process never happens when you take gravity into

Lighting the Dark: The Evolution of the Post-Inflationary Universe

Nathan Musoke,^{1,*} Shaun Hotchkiss,^{1,†} and Richard Easther^{1,‡} ¹Department of Physics, The University of Auckland, Private Bag 92019, Auckland, New Zealand (Dated: September 26, 2019)

In simple inflationary cosmological scenarios the near-exponential growth can be followed by a long period in which the Universe is dominated by the oscillating inflaton condensate. The condensate is initially almost homogeneous, but perturbations grow gravitationally, eventually fragmenting the condensate if it is not disrupted more quickly by resonance or prompt reheating. We show that the gravitational fragmentation of the condensate is well-described by the Schrödinger-Poisson

> show that large overdensities form quickly after the onset ion of this phase of nonlinear dynamics in the very early m of the inflationary power spectrum and the dark matter coupled to the inflaton.

and rtly after

Breakdown tions

mes B. Gilmore 1

¹ Department of Physics, Yale University, New Haven, CT 06520, USA

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Primordial black holes from the preheating instability

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ake gravity into liately and in shortly after

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• The resonance process *itself* is strong enough to create primordial black holes.

- The process never happens when you take gravity into account: the universe fragments immediately and black holes form. The end of times begin shortly after inflation
- The resonar create prime

What do we learn by looking at the Newtonian Potential

For "reasonable" box sizes



How big/small does the Newtonian potential get?



The ratio of the Hubble horizon to the *longest* wavelength in the box

red: 2 m⁻¹, blue: 5 m⁻¹ black: 11 m⁻¹

Can we capture the details?

From perturbation Theory

 $ds^{2} = -(1+2\Phi)dt^{2} + 2a(t)B_{,i}dx^{i}dt$ $+ a^{2}(t)\left[(1-2\Psi)\delta_{ij} + 2\partial_{i}\partial_{j}E\right]dx^{i}dx^{j}$

$$\Phi_B \equiv \Phi - \frac{d}{dt} \left[a^2 \left(\dot{E} - \frac{B}{a} \right) \right]$$
$$\Psi_B \equiv \Psi + Ha^2 \left(\dot{E} - \frac{B}{a} \right)$$

Comparing to BSSN

$$\begin{split} \tilde{E} &= -\frac{1}{k^2} \left[\frac{3}{4} \frac{k_i k_j}{k^2} \frac{\tilde{\gamma}_{ij}}{a^2} - \frac{1}{4} \frac{\tilde{\gamma}_{ii}}{a^2} \right] \\ \tilde{B} &= -\frac{1}{a} \frac{i k_j \tilde{\beta}_j}{k^2} \\ \Phi &= \alpha - 1 \\ \tilde{\Psi} &= \frac{1}{4} \left[\frac{k_i k_j}{k^2} \frac{\tilde{\gamma}_{ij}}{a^2} - \frac{\tilde{\gamma}_{ii}}{a^2} \right] \quad \mathbf{De} \end{split}$$



As we approach larger (box) sizes



red: 2 m⁻¹ blue: 5 m⁻¹ black: 11 m⁻¹ green: 20 m⁻¹

As we approach larger (box) sizes





Black = FLRW, Grey = Perturbative, Blue = BSSN

For the big box



Red: inflaton Perturbative Blue: inflaton BSSN Green: decay field Perturbative Black: decay field BSSN

> The variance of the lapse does not show departures from homogeneity that indicate back hole formation

How do they look



So there's no need to panic

- In these cases: non-linear physics seems to be a friend, not a foe
- But there's still *much* left in parameter space: e.g. we know that collapse will happen

Your Take-home

- The next step in understanding the Universe is hard
 - Physics is non-linear
 - You have to ask the right questions
 - You need to interpret the answers correctly
- We need new, bold, creative, insightful, original and diverse ideas to answer these questions





Comments on Fragmentation

A test with a single field

- A single, massive, scalar in the presence of gravity
- Can we go beyond perturbation theory?



Linearized gravity

BSSN

 $m = 10^{-6} m_{\rm pl}$

Field variances



Linearized gravity

BSSN

Variance of the Newtonian/one of the Bardeen Potentials



BIGGER BOXES



Comments on Oscillons

Oscillons in Monodromy





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No Gravity

Linear Gravity

There is is a slight difference in the TOTAL energy in Oscillons



