

**PROBING THE
'PRIMORDIAL DARK AGE'
WITH GRAVITATIONAL WAVES.
IMPLICATIONS FOR REHEATING**

DANIEL G. FIGUEROA
IFIC, Valencia

**INCONSISTENCY OF AN
INFLATIONARY SECTOR
COUPLED ONLY
(minimally) TO GRAVITY**

DANIEL G. FIGUEROA
IFIC, Valencia

TWO PAPERS / IDEAS*

arXiv:1811.04093 (JCAP 1910 (2019) 10, 050)

**Inconsistency of an inflationary sector
coupled only to Einstein gravity**

arXiv:1905.11960 (JCAP 1908 (2019) 011)

**Ability of LIGO & LISA to probe the
Eq. of state of the early Universe**

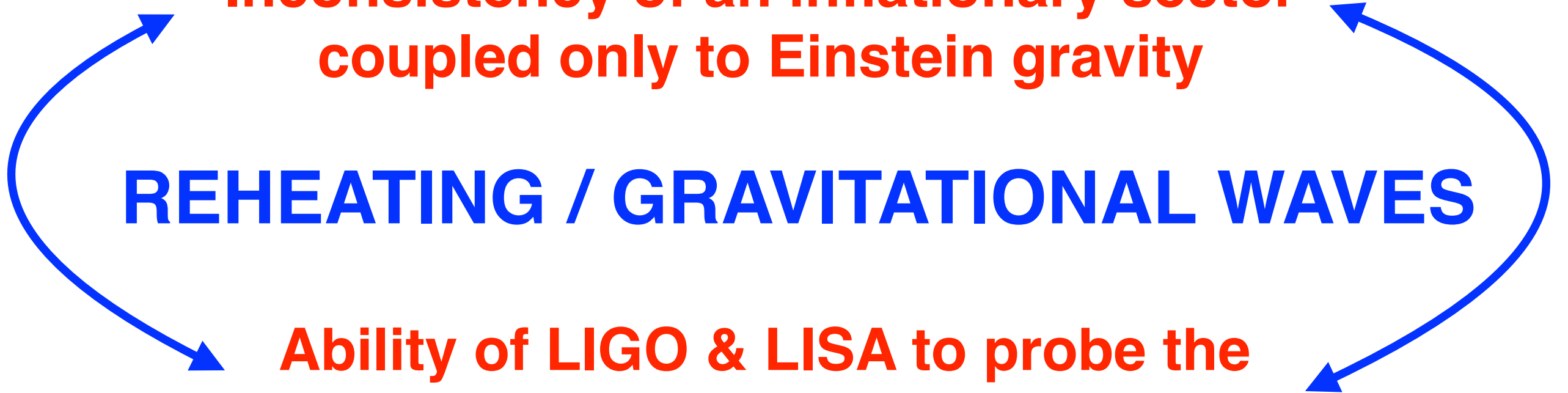
*with E. H. Tanin, MsC EPFL (PhD @ J. Hopkins U.)

TWO PAPERS / IDEAS

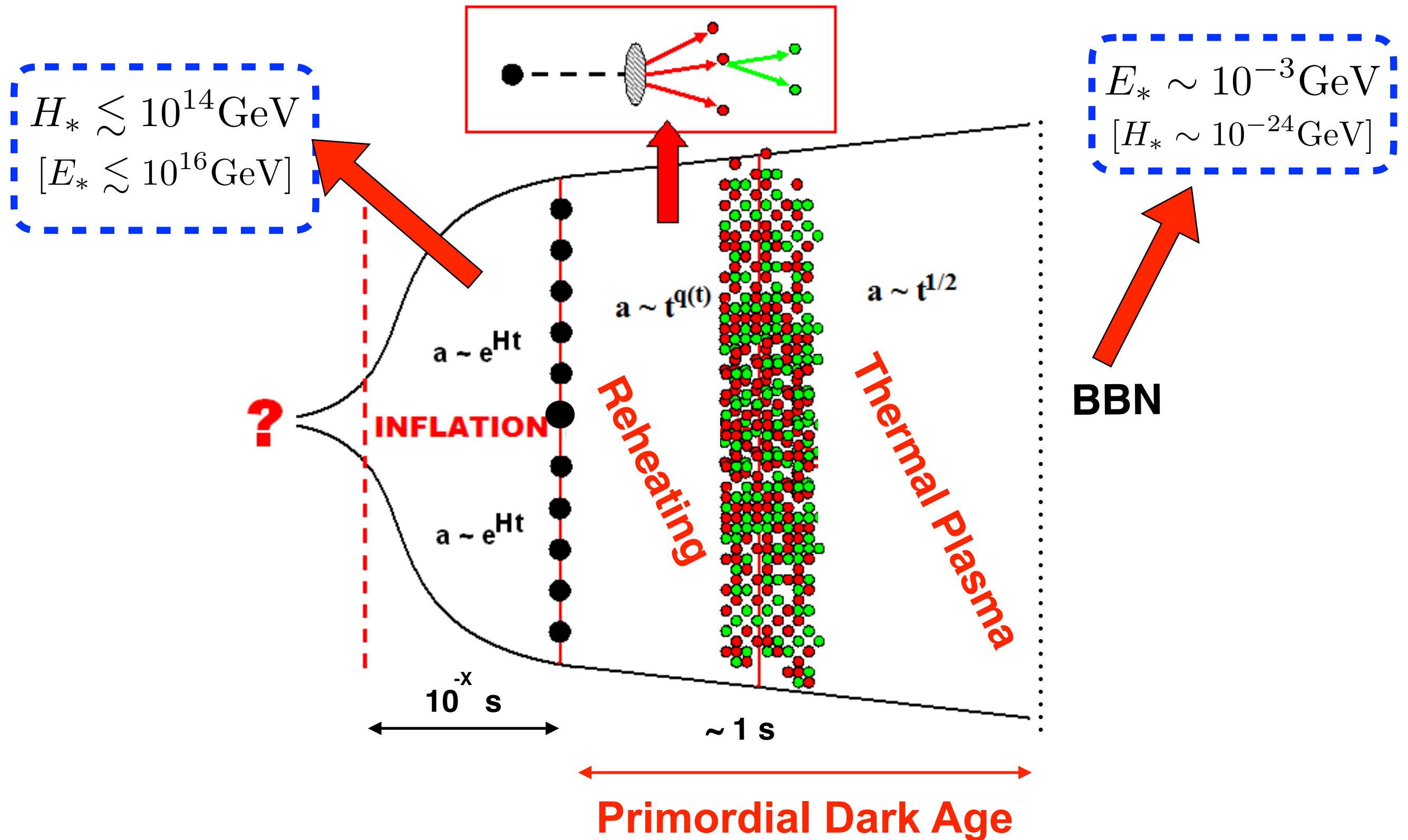
**Inconsistency of an inflationary sector
coupled only to Einstein gravity**

REHEATING / GRAVITATIONAL WAVES

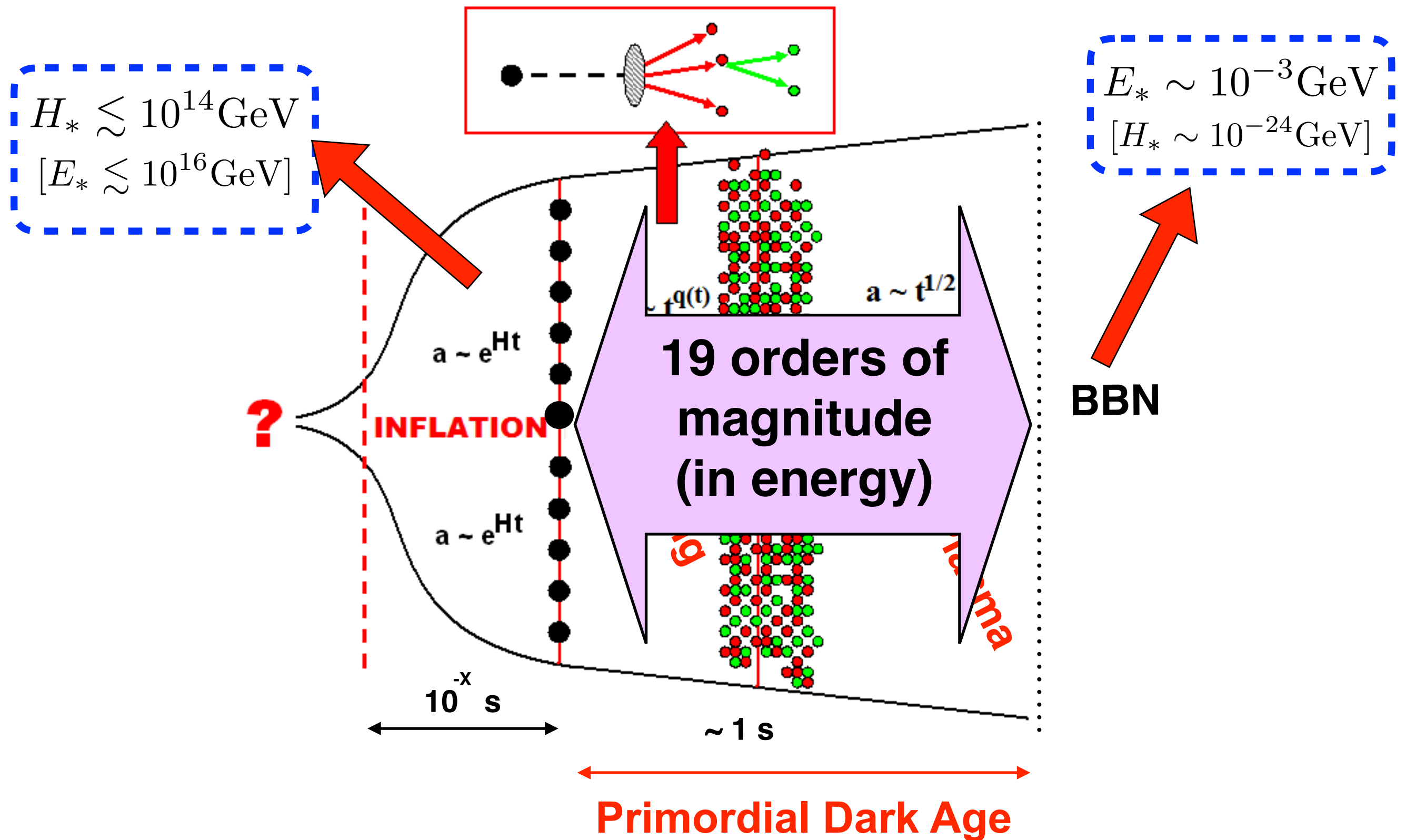
**Ability of LIGO & LISA to probe the
Eq. of state of the early Universe**



The context: The Early Universe



The context: The Early Universe



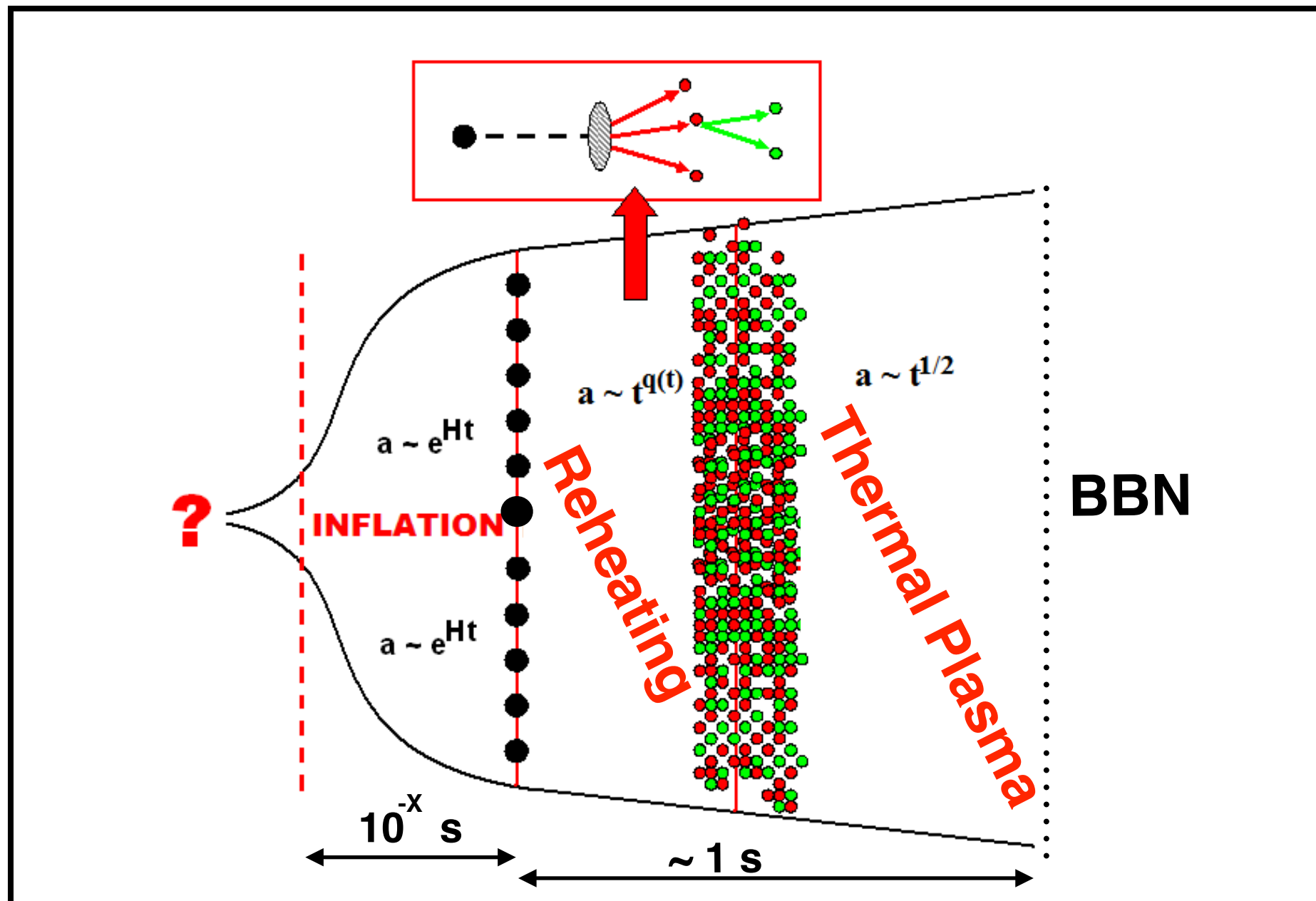
Part 1

**(GRAVITATIONAL)
REHEATING**

Successful Reheating



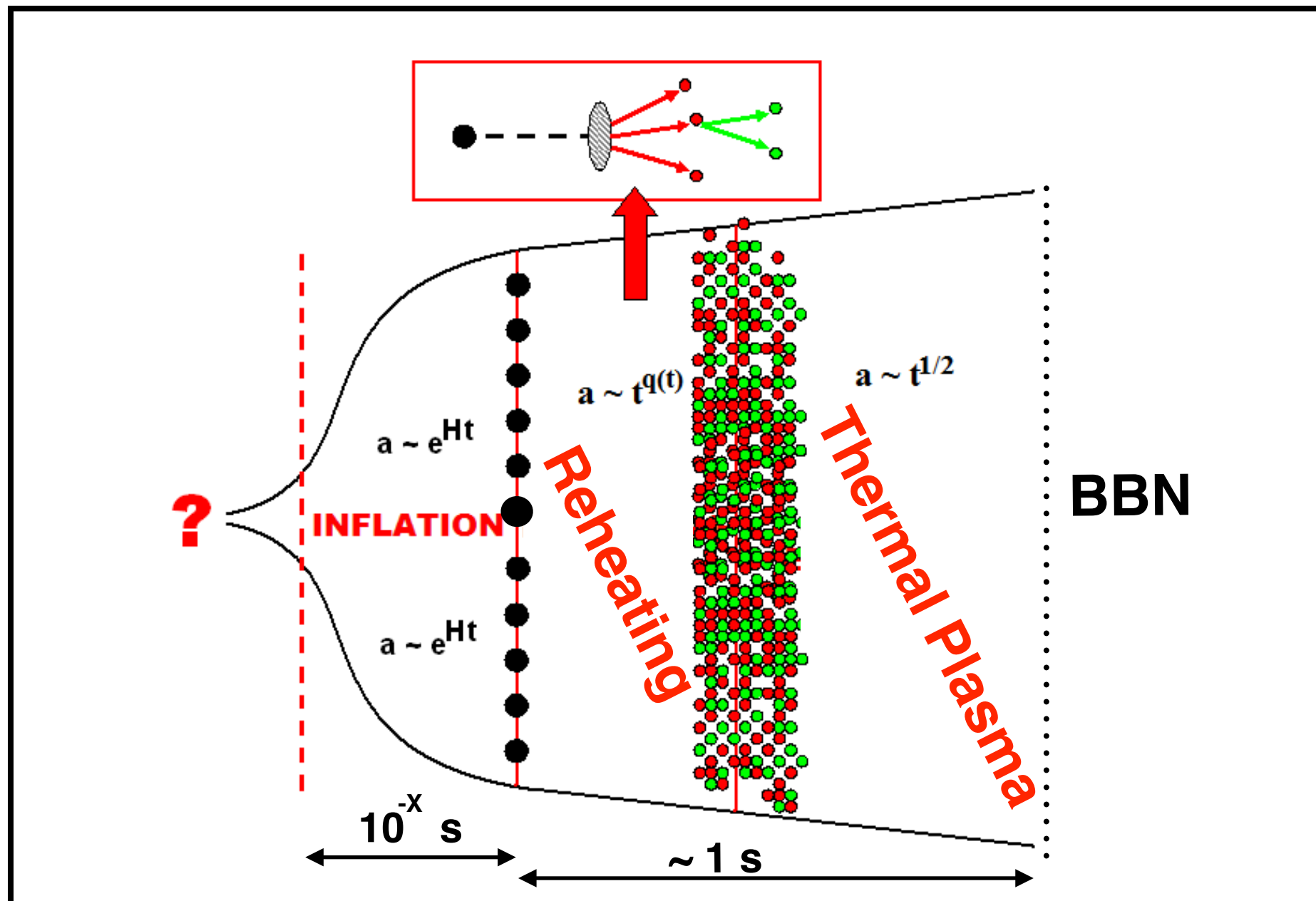
Thermal Ensemble of Relativistic Particle Species



Successful Reheating



Thermal Ensemble of Relativistic Particle Species



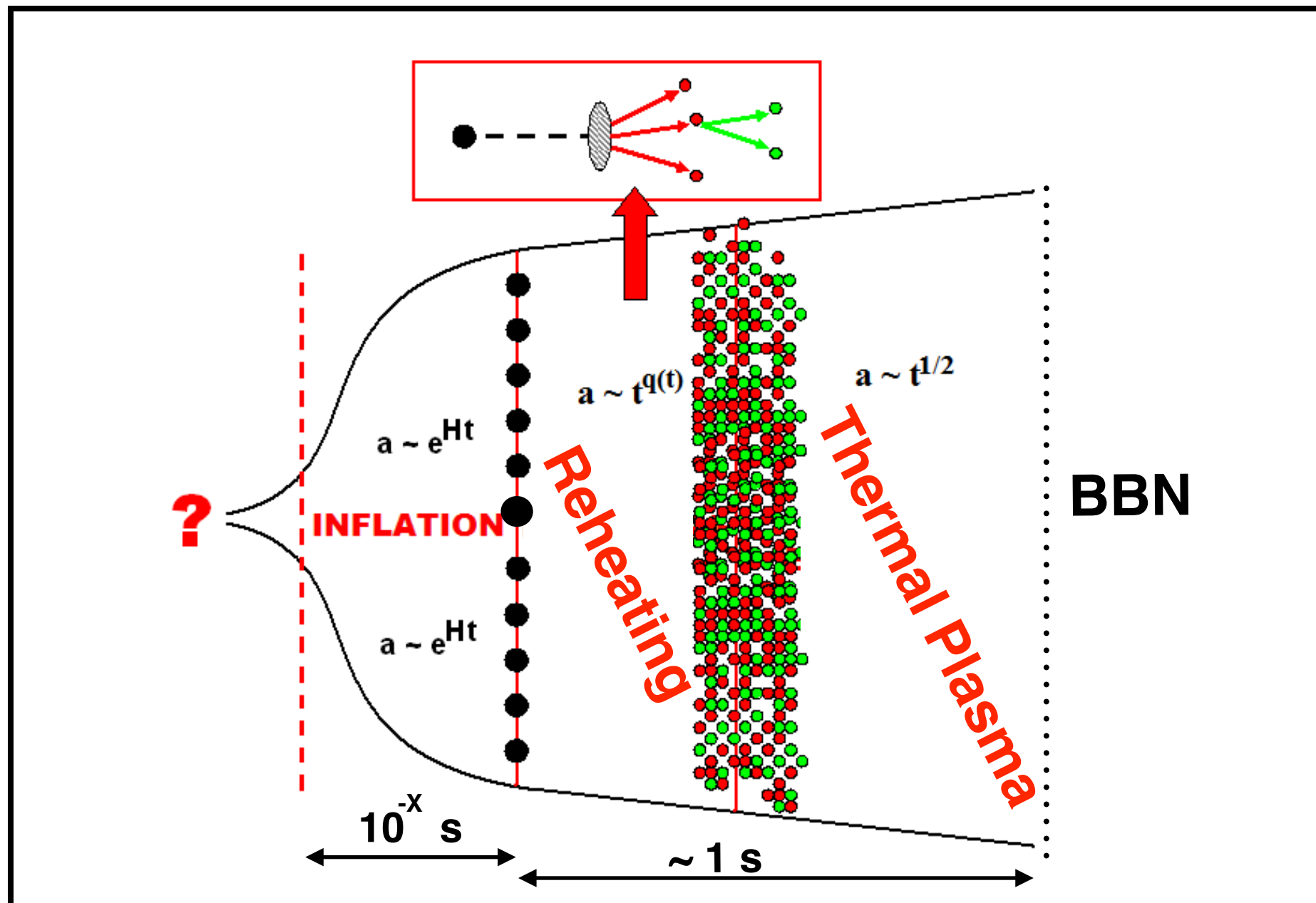
Connection between Particles and Inflationary Sector ?

- * **SM Portals ?**
- * **Mediator fields ?**
- * **No coupling ?**

Successful Reheating



Thermal Ensemble of Relativistic Particle Species



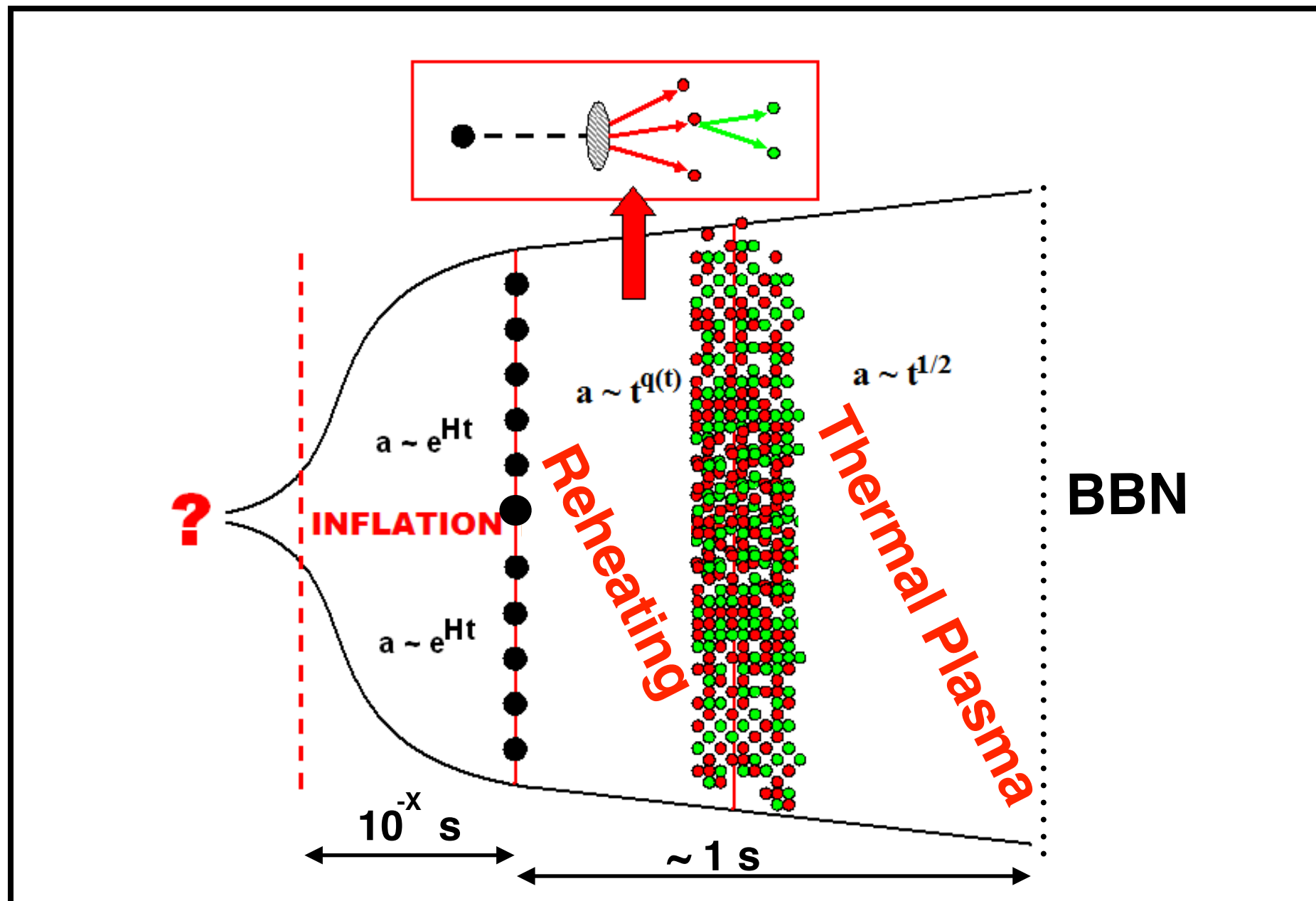
Connection between Particles and Inflationary Sector ?

- * **SM Portals ?**
- * **Mediator fields ?**
- * **No coupling ?**

Successful Reheating



Thermal Ensemble of Relativistic Particle Species



Connection between Particles and Inflationary Sector ?

- * **SM Portals ?**
- * **Mediator fields ?**
- * **Weak coupling !**

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\text{inf}}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \cancel{g^2\chi^2\phi^2} \right\}$$

Need to excite matter
(to reheat the Universe)

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\text{inf}}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \cancel{g^2\chi^2\phi^2} \right\}$$

Need to excite matter
(to reheat the Universe)



$$\rho_{\text{rad}} = \delta \times 10^{-2} H_*^4$$

**Inflation does
the job !**

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\text{inf}}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2} m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \cancel{g^2 \chi^2 \phi^2} \right\}$$

**Need to excite matter
(to reheat the Universe)**



$$\rho_{\text{rad}} = \delta \times 10^{-2} H_*^4$$

$\delta \lesssim 1$

**Inflation does
the job !**

- | | | | |
|---|-----------------------------|--|---------------------------------------|
| { | $\mathcal{O}(m^2/H_*^2)$ | , quantum – fluct. (<i>light dof</i>) | Linde '83 |
| | $\mathcal{O}(1)$ | , quantum – fluct. (<i>self – interact.</i>) | Starobinsky & Yokoyama '94 |
| | $\mathcal{O}(1)/\xi$ | , non – min grav, $\xi \gtrsim 1$ | Rajantie et al '15 |
| | $\mathcal{O}(1 - 6\xi ^2)$ | , non – min grav, $ 1 - 6\xi \ll 1$ | Ford '87 |

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\text{inf}}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2}m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \cancel{g^2\chi^2\phi^2} \right\}$$

**Need to excite matter
(to reheat the Universe)**



$$\rho_{\text{rad}} = \delta \times 10^{-2} H_*^4$$

$$\delta \lesssim 1,$$

**Inflation does
the job !**

$$\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2 \ll 1$$

Fraction energies radiation-to-total

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\mathcal{L} = \frac{1}{\sqrt{g}} \left\{ \underbrace{(\partial\phi)^2 - V_{\text{inf}}(\phi)}_{\text{inflaton}} + \underbrace{\frac{1}{2} m_{pl}^2 R}_{\text{gravity (GR)}} + \underbrace{(\partial\chi)^2 - V(\chi) - \xi\chi^2 R}_{\text{matter/rad}} - \cancel{g^2 \chi^2 \phi^2} \right\}$$

**Need to excite matter
(to reheat the Universe)**



$$\rho_{\text{rad}} = \delta \times 10^{-2} H_*^4$$

$$\delta \lesssim 1,$$

**Inflation does
the job !**

$$\Delta_* \sim \delta \cdot 10^{-12} \times \left(\frac{H_*}{H_{\text{max}}} \right)^2 \ll 1$$

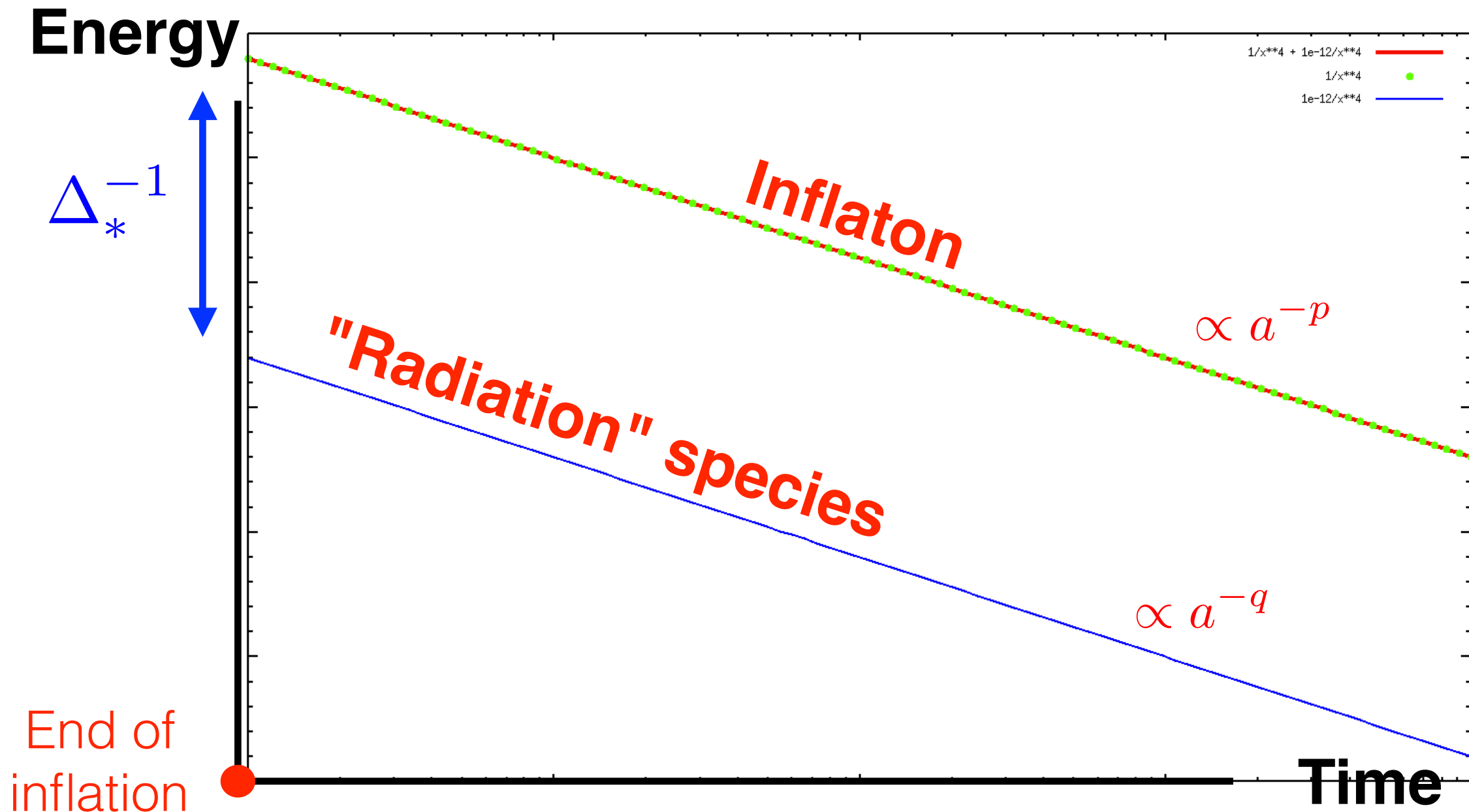
$$\delta \lesssim 1,$$

Fraction energies radiation-to-total

INFLATIONARY SECTOR COUPLED ONLY (minimally) TO GRAVITY

$$\rho_{\text{rad}}^* \ll H_*^2 m_{\text{pl}}^2$$

Radiation excited but subdominant



Rad. Excited

$$\rho_{\text{rad}}^* \ll H_*^2 m_{pl}^2$$

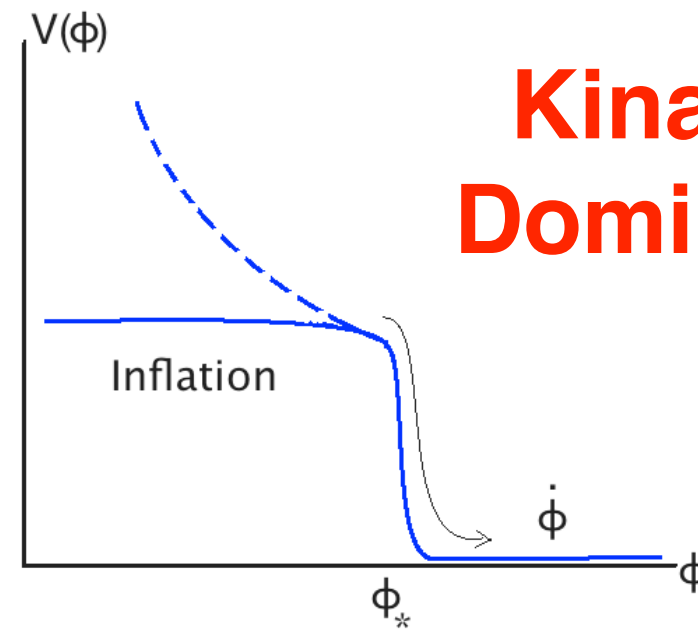
**Rad. produced,
but subdominant**



Rad. Excited

$$\rho_{\text{rad}}^* \ll H_*^2 m_{pl}^2$$

Rad. produced,
and dominant !



Kination-Domination

Energy

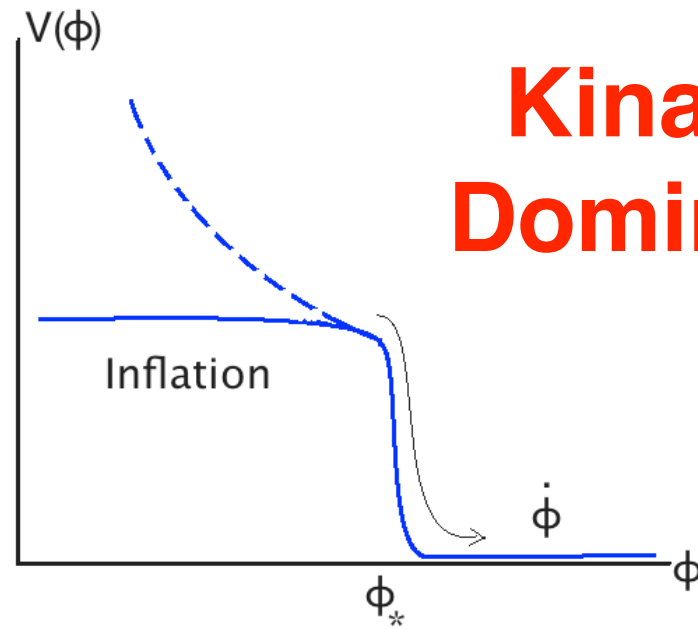
$$\propto \frac{1}{a^6}$$

[Ford '86,
Spokoiny '93]

Rad. Excited

$$\rho_{\text{rad}}^* \ll H_*^2 m_{pl}^2$$

Rad. produced,
and dominant !



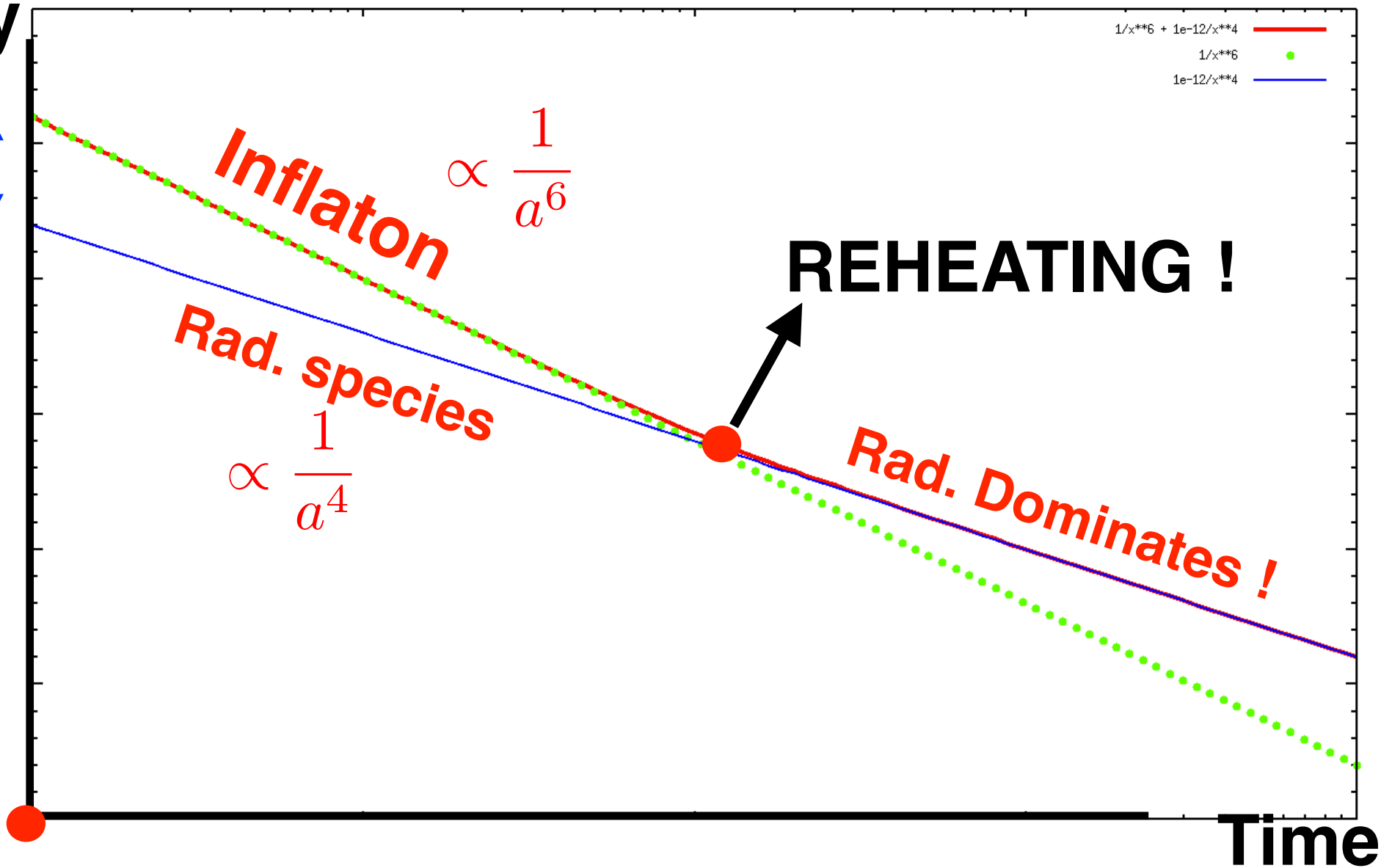
Kination-Domination

$$\text{Energy} \propto \frac{1}{a^6}$$

[Ford '86,
Spokoiny '93]

Energy

$$\Delta_*^{-1}$$

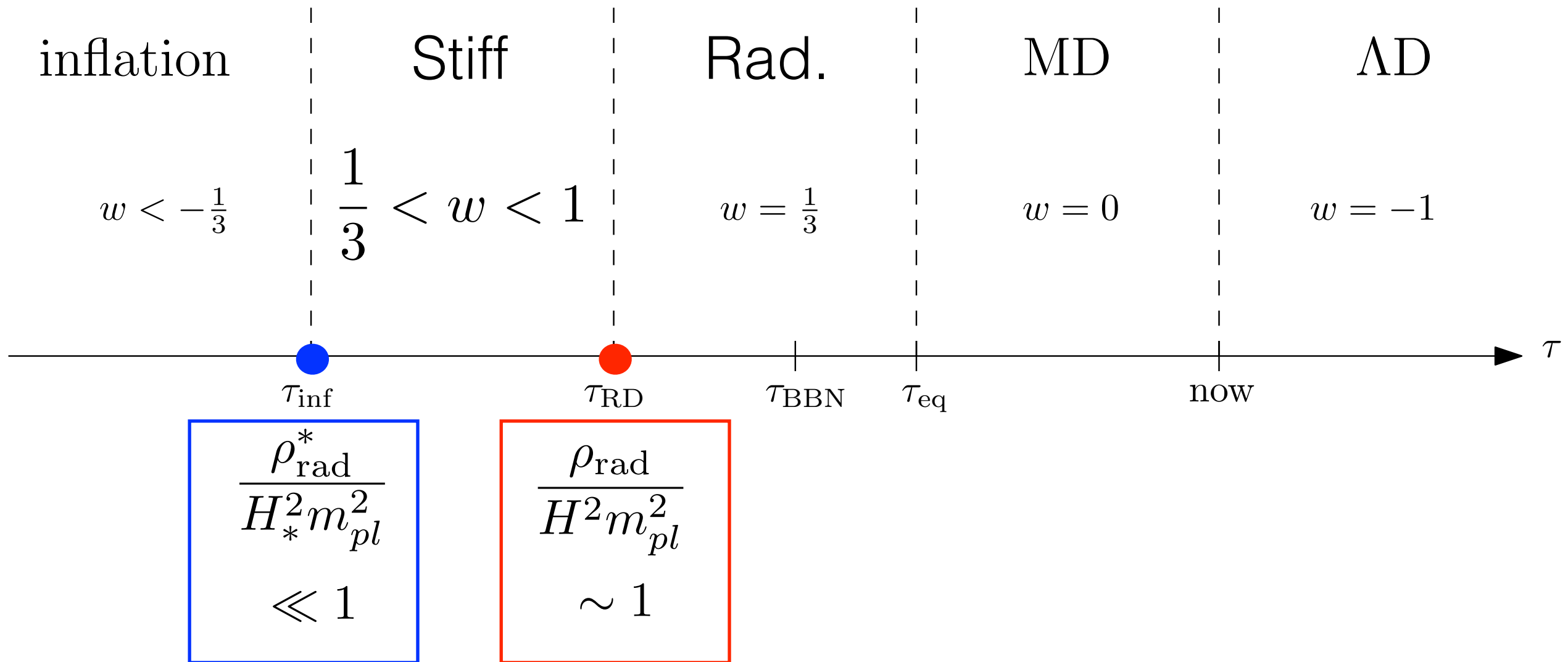


End of
inflation

Time

GRAVITATIONAL REHEATING

Ford '86, Spokoiny '93, Joyce '97,
Giovannini '98/99, Vilenkin & Damour '95,
Peebles & Vilenkin '98, [...], DGF & Tanin '18/19

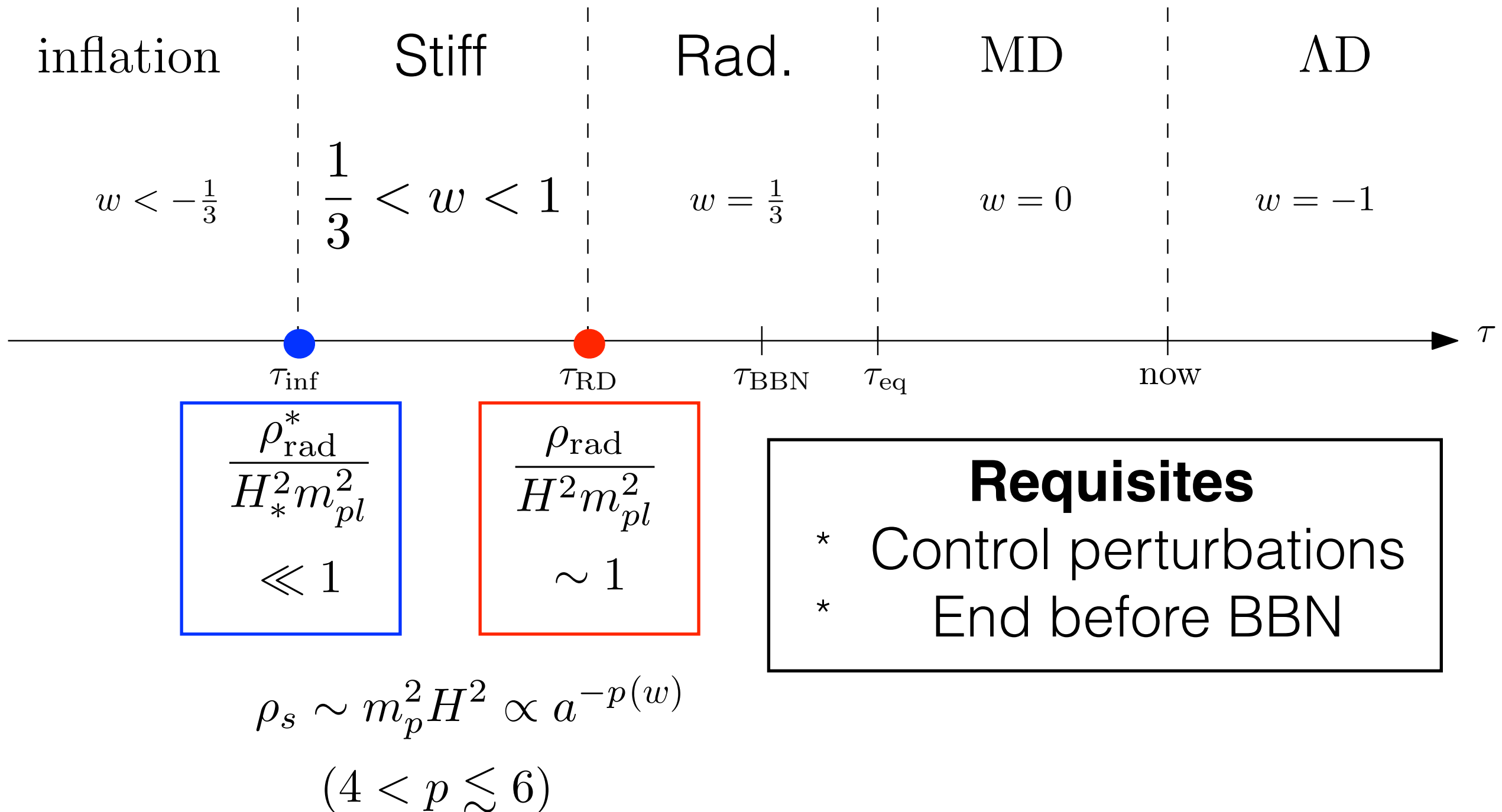


$$\rho_s \sim m_p^2 H^2 \propto a^{-p(w)}$$

$$(4 < p \lesssim 6)$$

GRAVITATIONAL REHEATING

Ford '86, Spokoiny '93, Joyce '97,
Giovannini '98/99, Vilenkin & Damour '95,
Peebles & Vilenkin '98, [...], DGF & Tanin '18/19



GRAVITATIONAL REHEATING

Ford '86, Spokoiny '93, Joyce '97,
Giovannini '98/99, Vilenkin & Damour '95,
Peebles & Vilenkin '98, [...], DGF & Tanin '18/19

$$1/3 < w_s \lesssim 1$$

Stiff Eq. of State

Requisites

- * Control perturbations
- * End before BBN



All good... but we are not done !

GRAVITATIONAL REHEATING

Ford '86, Spokoiny '93, Joyce '97,
Giovannini '98/99, Vilenkin & Damour '95,
Peebles & Vilenkin '98, [...], DGF & Tanin '18/19

$$1/3 < w_s \lesssim 1$$

Stiff Eq. of State

Requisites

- * Control perturbations
- * End before BBN



All good... but we are not done !

Enhancement of inflationary Gravitational Waves (GW) !

[Giovannini '98/99, ..., Boyle & Buonanno '07, ..., DGF & Tanin '19]

Part 2

GRAVITATIONAL WAVES **(independently of grav. reheating)**

Inflationary GW background

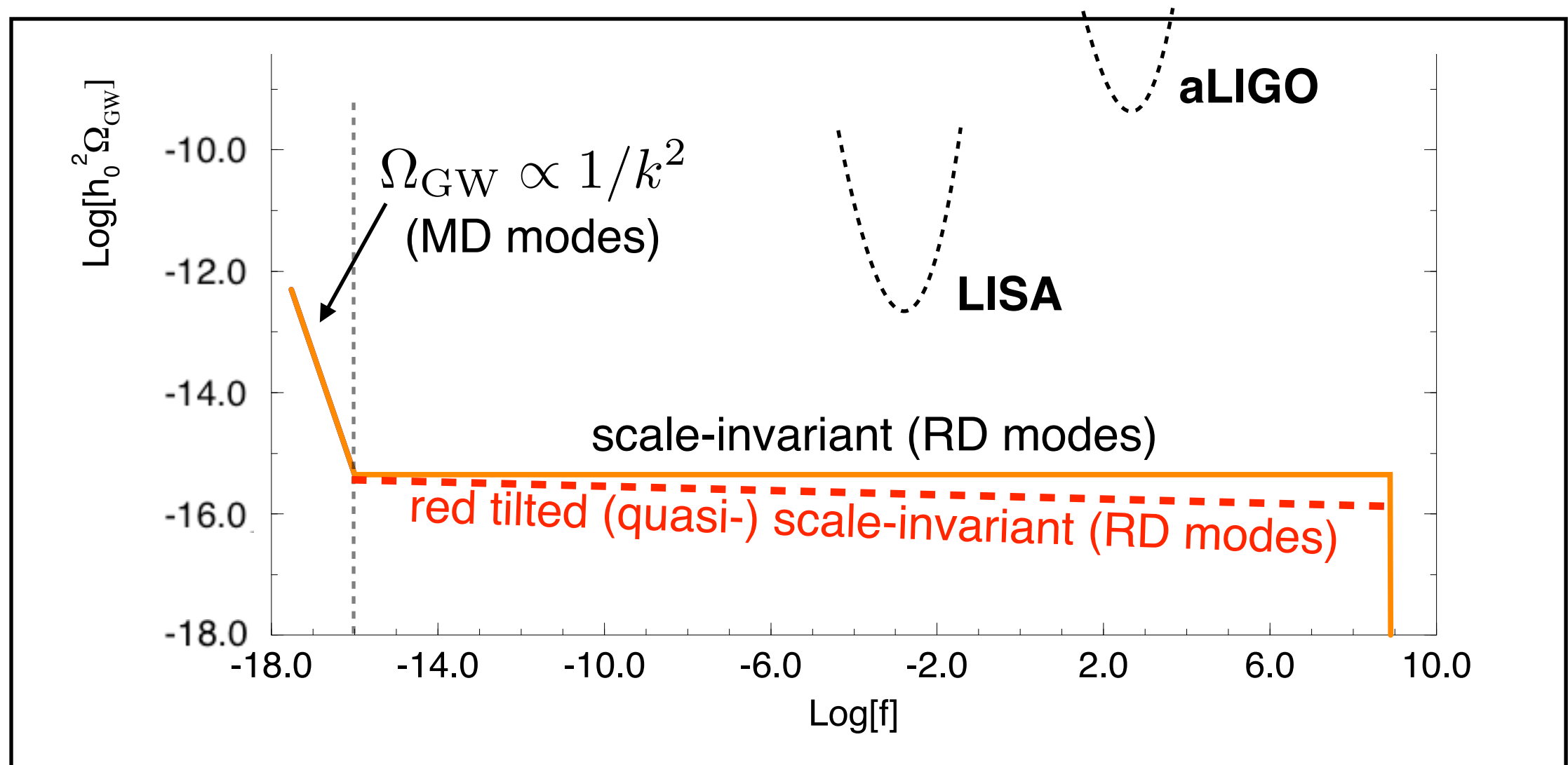
$$\Omega_{\text{GW}}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}} \left(\frac{d\rho_{\text{GW}}}{d \log k} \right)_o = \underbrace{\frac{\Omega_{\text{Rad}}^{(o)}}{24}}_{\text{Transfer Funct.: } T(k) \propto k^0 \text{ (RD)}} \Delta_{h_*}^2(k)$$

$$\Delta_h^2(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p} \right)^2 \left(\frac{k}{aH} \right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

energy scale

Transfer Funct.: $T(k) \propto k^0$ (RD)



Inflationary GW background

$$\Omega_{\text{GW}}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}} \left(\frac{d\rho_{\text{GW}}}{d \log k} \right)_o = \underbrace{\frac{\Omega_{\text{Rad}}^{(o)}}{24}}_{\text{RD modes}} \Delta_{h_*}^2(k)$$

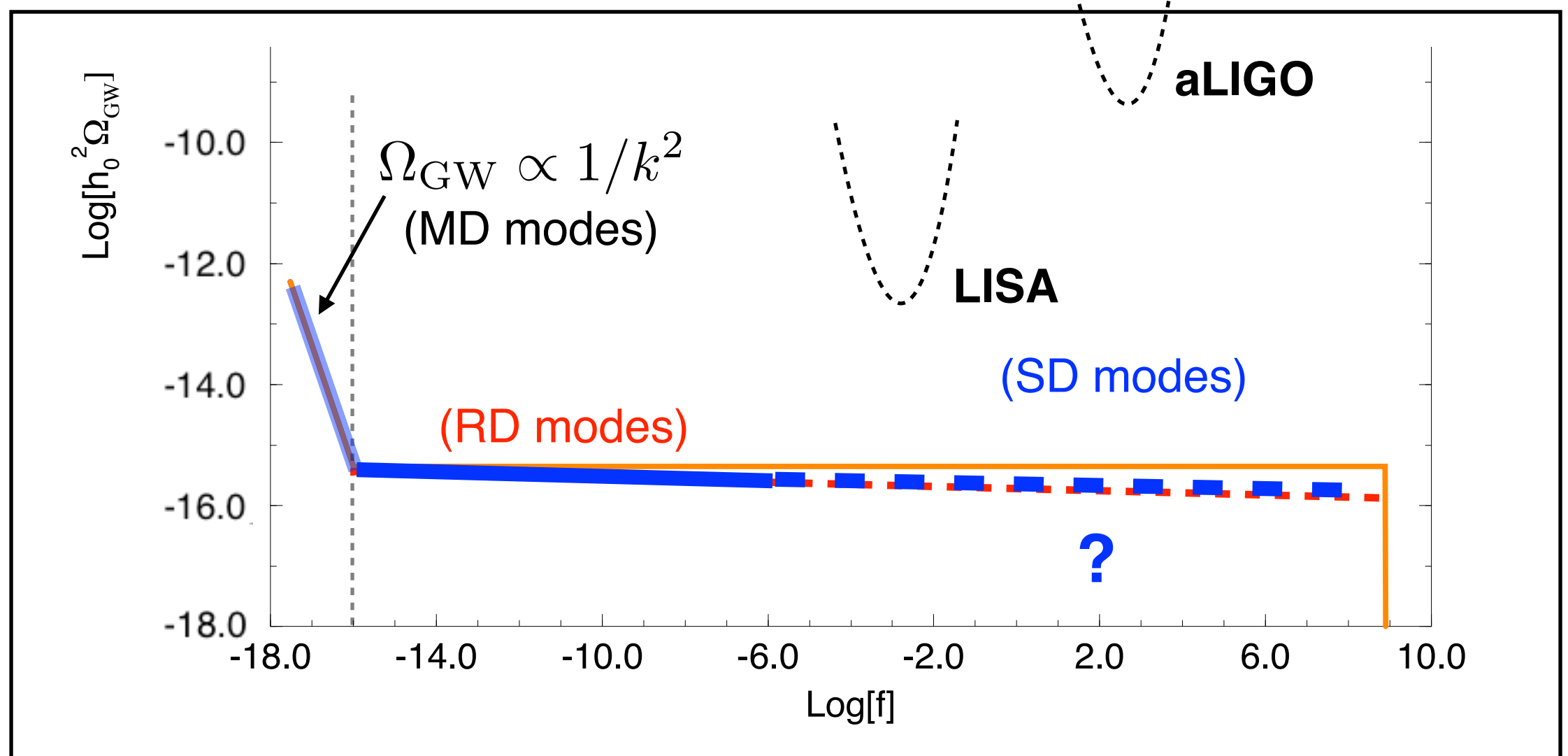
$$\Delta_h^2(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p} \right)^2 \left(\frac{k}{aH} \right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

energy scale

Transfer Funct.: $T(k) \propto k^0$ (RD)

Stiff Period: $T(k) \propto k^2 \frac{(w_s - 1/3)}{(w_s + 1/3)}$



Inflationary GW background

$$\Omega_{\text{GW}}^{(o)}(f) \equiv \frac{1}{\rho_c^{(o)}} \left(\frac{d\rho_{\text{GW}}}{d \log k} \right)_o = \underbrace{\frac{\Omega_{\text{Rad}}^{(o)}}{24}}_{\text{RD modes}} \Delta_{h_*}^2(k)$$

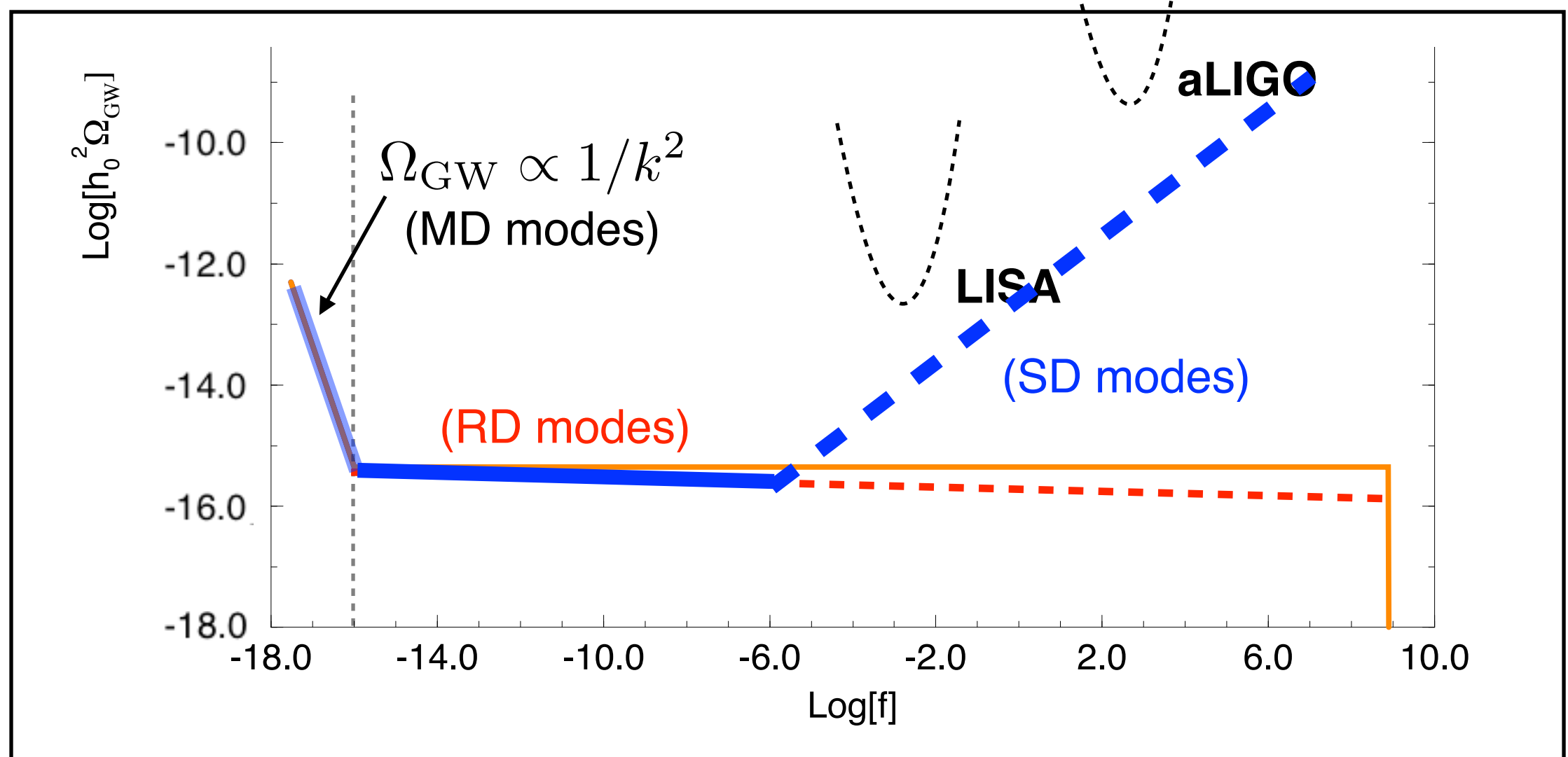
$$\Delta_h^2(k) = \frac{2}{\pi^2} \left(\frac{H}{m_p} \right)^2 \left(\frac{k}{aH} \right)^{n_t}$$

$$n_t \equiv -2\epsilon$$

energy scale

Transfer Funct.: $T(k) \propto k^0$ (RD)

Stiff Period: $T(k) \propto k^2 \frac{(w_s - 1/3)}{(w_s + 1/3)}$



**A couple few pages
computation of the
Transfer function
@ Stiff Domination**

Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

**Rad.
Plateau**

Transfer Funct. Stiff Period
Window \times power-law

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

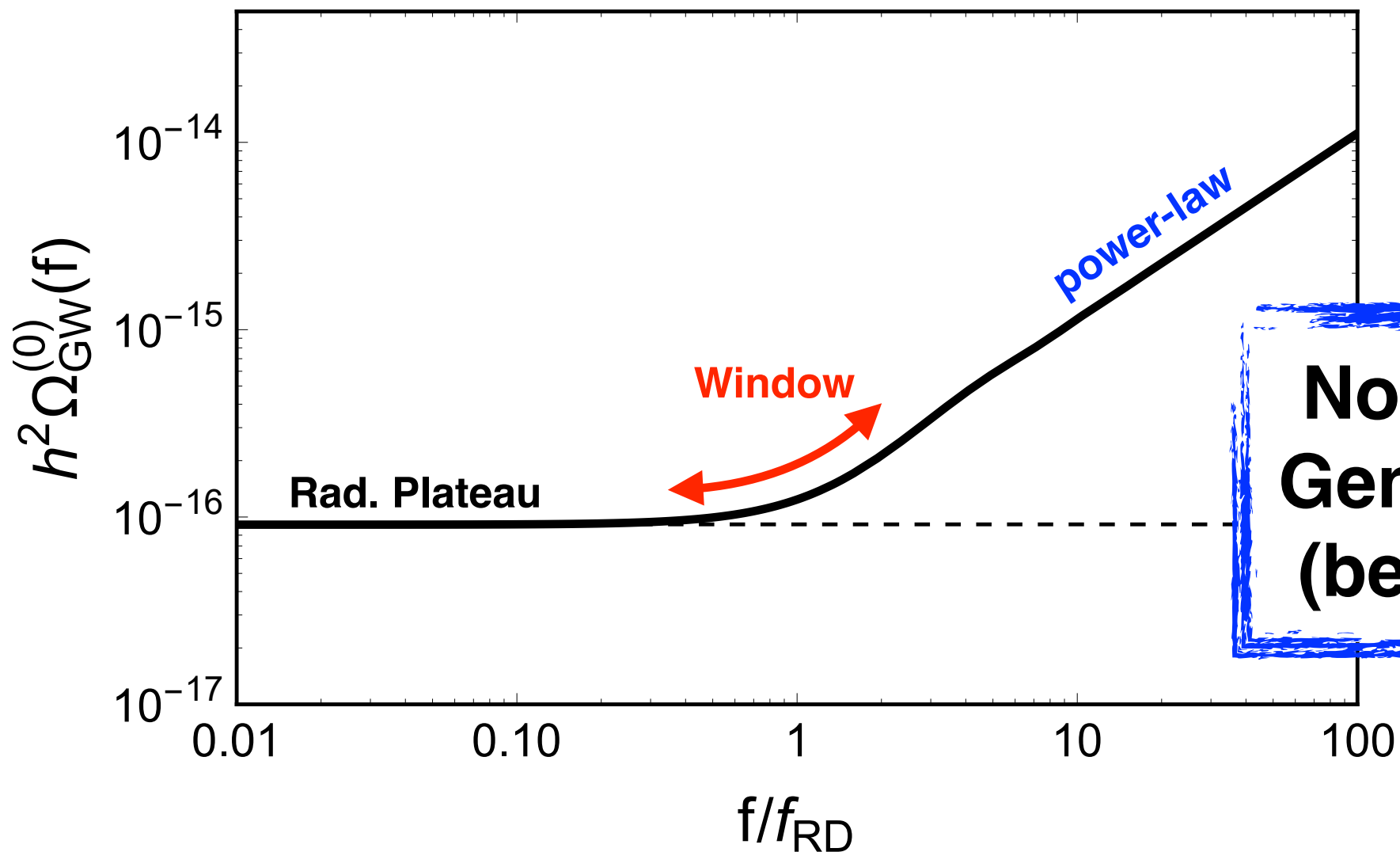
Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

Rad.
Plateau

Transfer Funct. Stiff Period
Window \times power-law



**Not just Grav. RH !
Generic for Stiff Era
(before Rad. Dom.)**

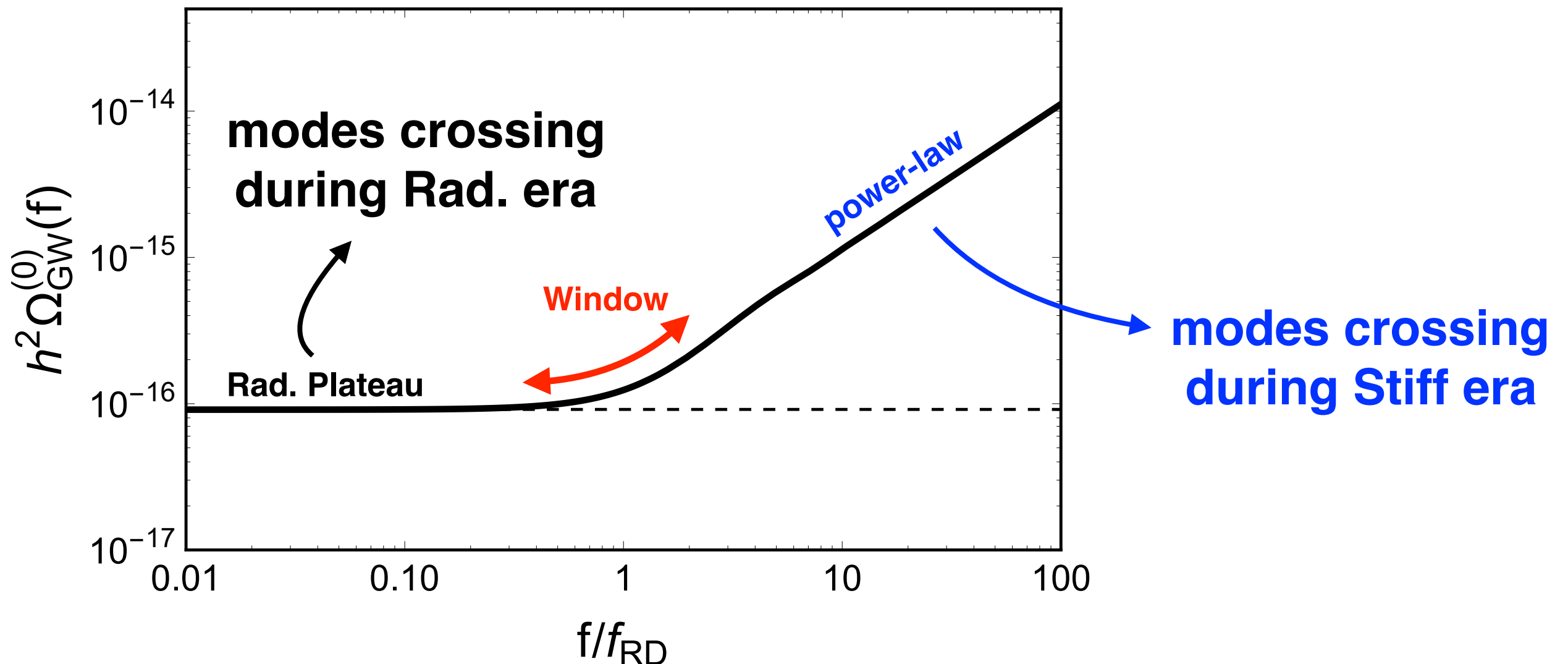
Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

Rad.
Plateau

Transfer Funct. Stiff Period
Window \times power-law



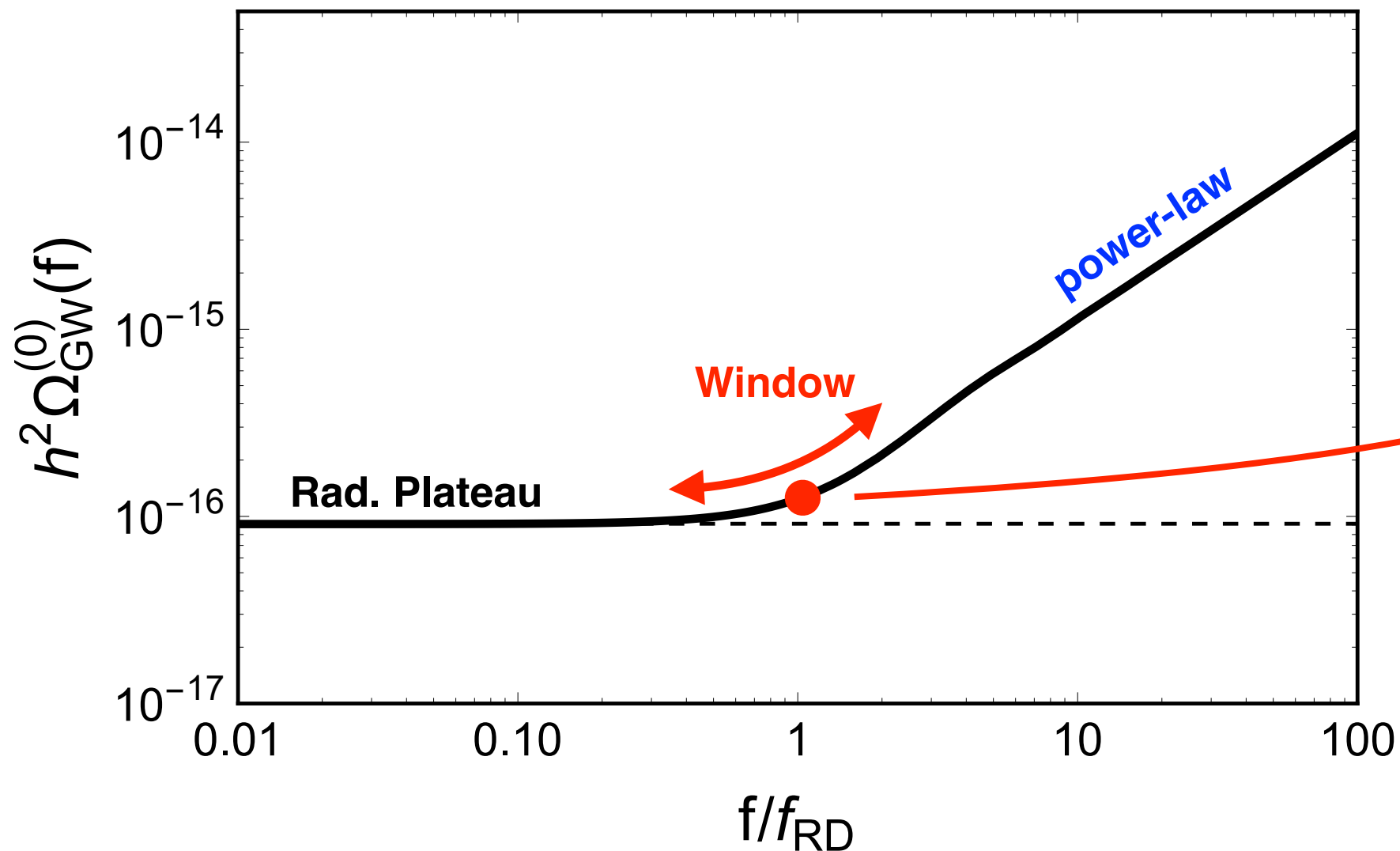
Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

Rad.
Plateau

Transfer Funct. Stiff Period
Window x power-law



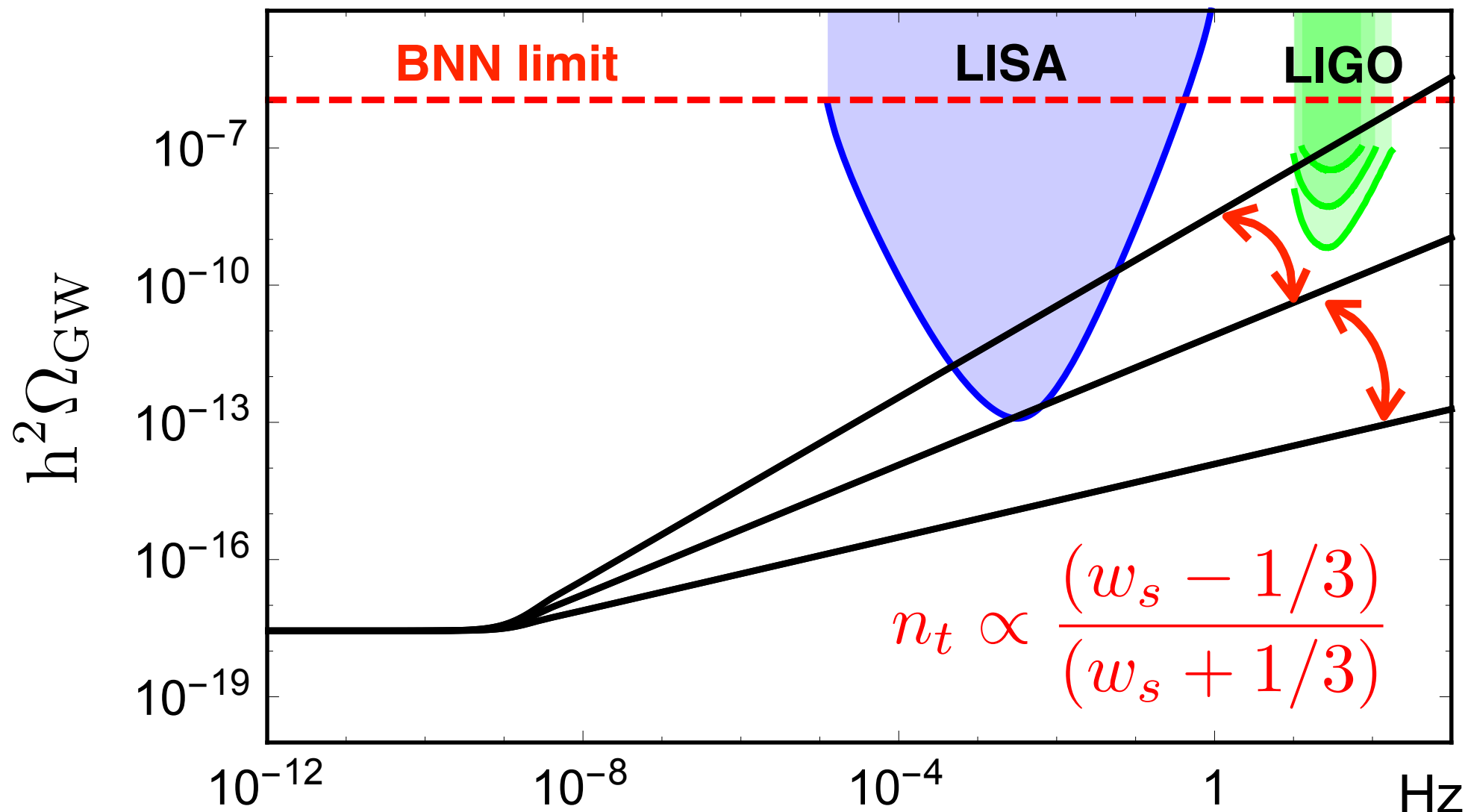
Horizon size @
SD-to-RD transition

Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

Rad. Plateau
Transfer Funct. Stiff Period
Window \times *power-law*

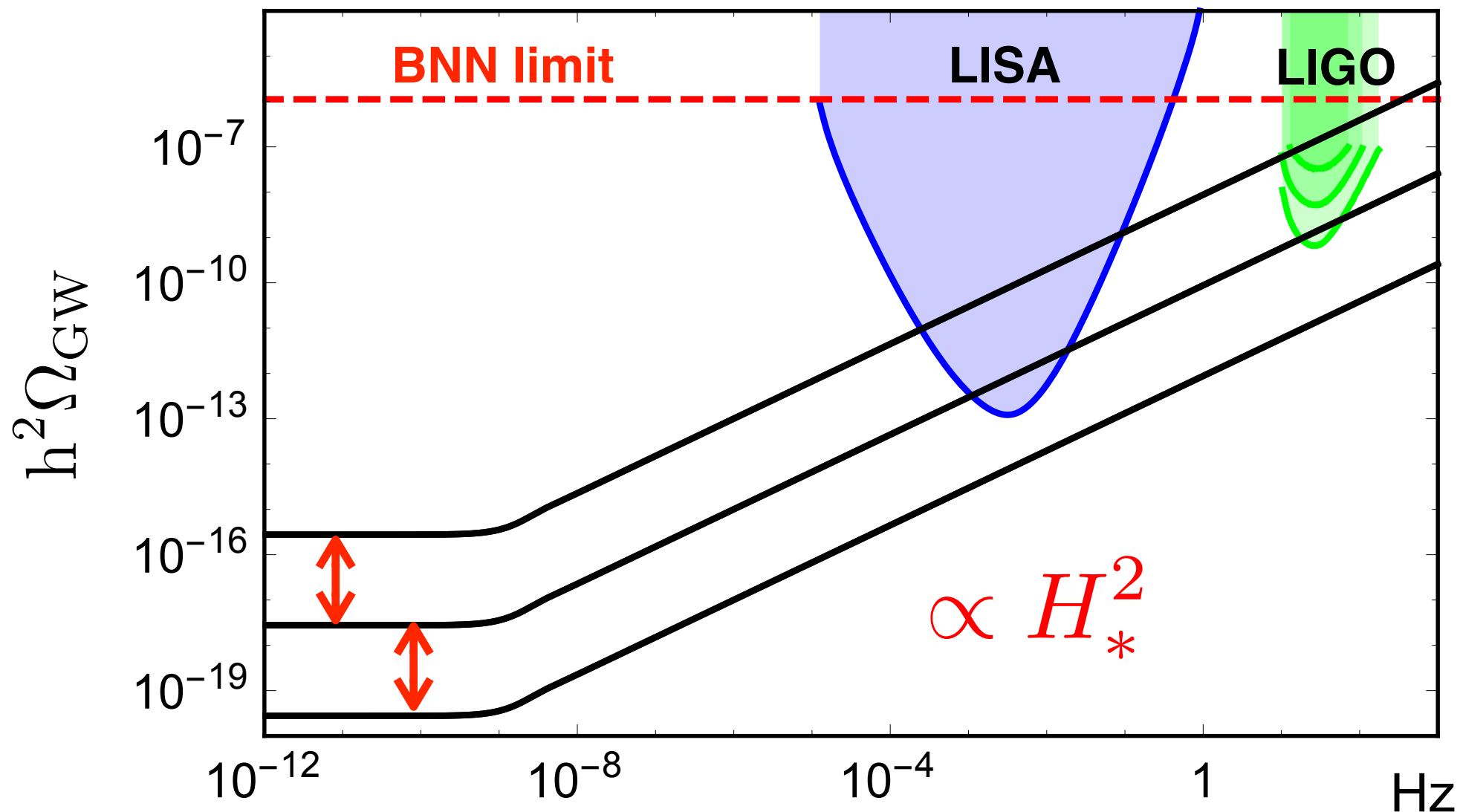


Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

Rad. Plateau
Transfer Funct. Stiff Period
Window \times *power-law*



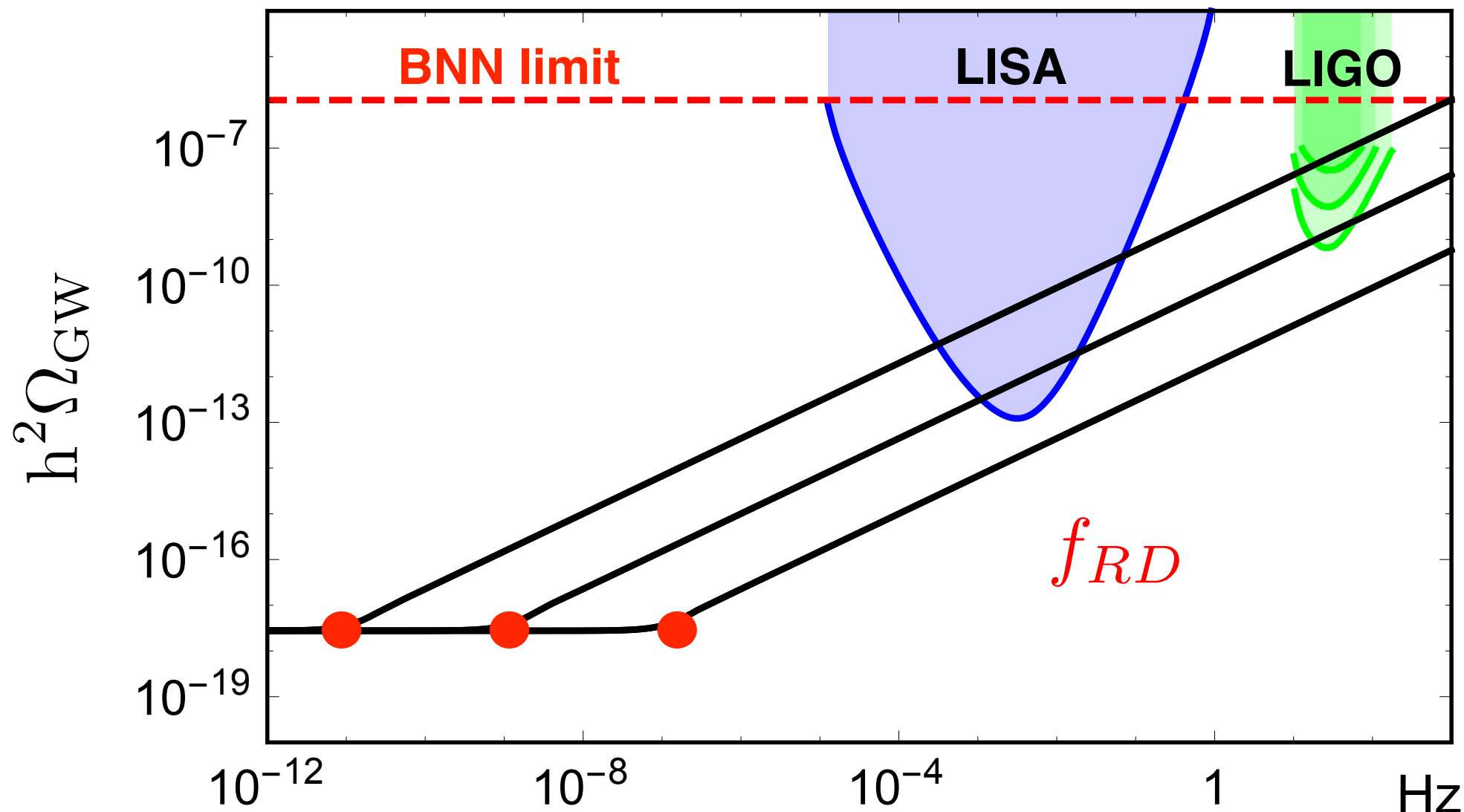
Overall Amplitude
 (Energy Scale Inflation)

Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

Rad. Plateau
Transfer Funct. Stiff Period
Window \times *power-law*



Freq. RD

$$k_{\text{RD}} = a_{\text{RD}} H_{\text{RD}}$$

$$f_{\text{RD}} \equiv k_{\text{RD}} / (2\pi a_0)$$

SD-to-RD transition

Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

**Rad.
Plateau**

Transfer Funct. Stiff Period
Window \times *power-law*

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

$$\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{\text{RD}}})$$

**Energy
Scale
Inflation**

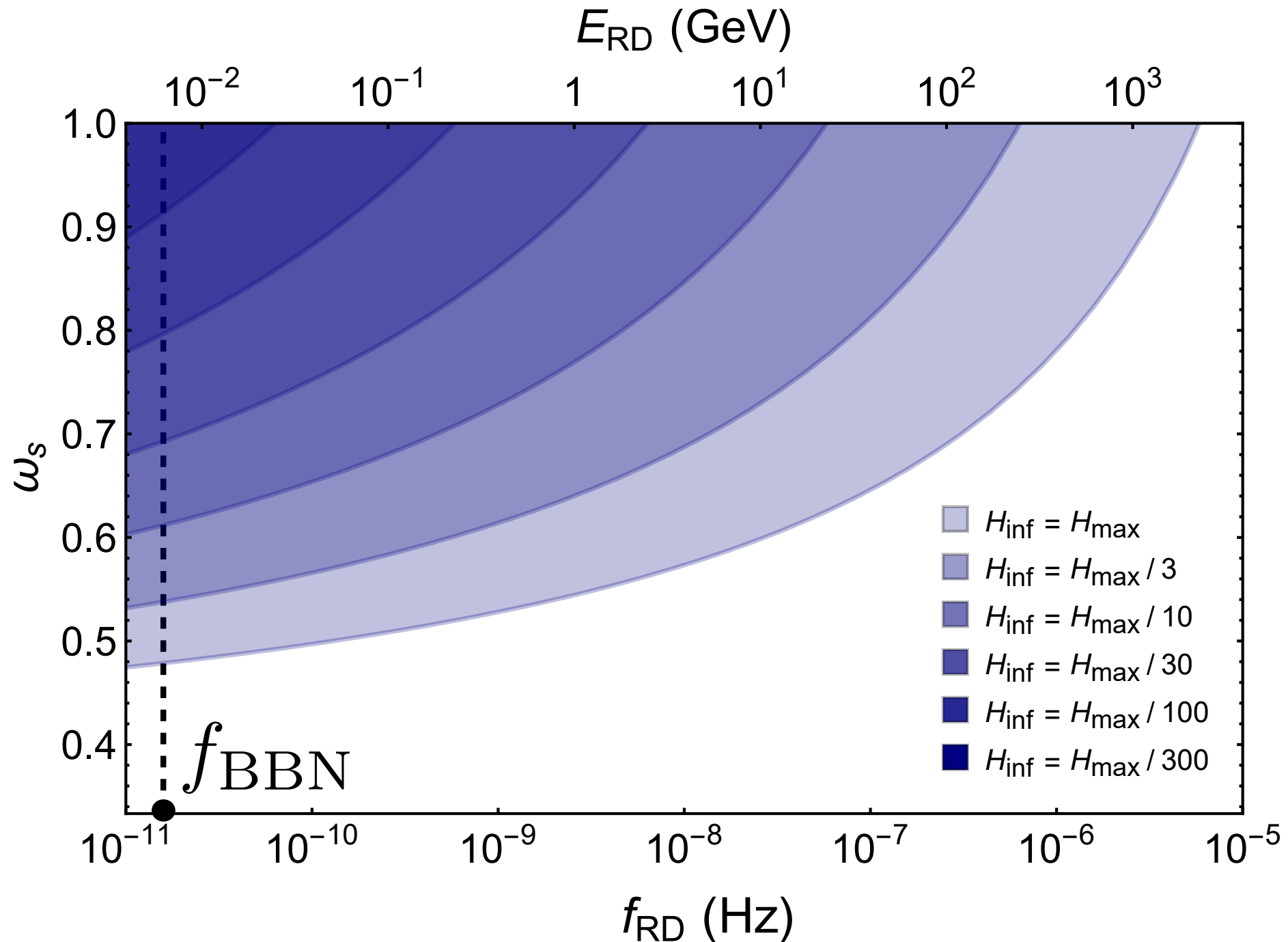
**EoS
Stiff
Period**

**Duration
Stiff
Period**

GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$
Observability @ LISA (~ 2034) Energy Scale EoS Stiff Duration Stiff

GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H}_*, \underline{w}_s, \underline{f}_{RD})$

Observability @ LISA (~ 2034) Energy Scale EoS Stiff Duration Stiff



GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_S}, \underline{f_{RD}})$

Observability @ LISA (~ 2034) Energy Scale EoS Stiff Duration Stiff

$$9.1 \times 10^{10} \text{ GeV} < H_{\text{inf}} < 6.6 \times 10^{13} \text{ GeV}$$

$$0.47 < w_S < 1$$

$$10^{-11} \text{ Hz} \lesssim f_{\text{RD}} < 4.6 \times 10^{-6} \text{ Hz}$$

$$10^{-3} \text{ GeV} \lesssim E_{\text{RD}} < 5.91 \times 10^3 \text{ GeV}$$

Significant fraction of param. space observable !

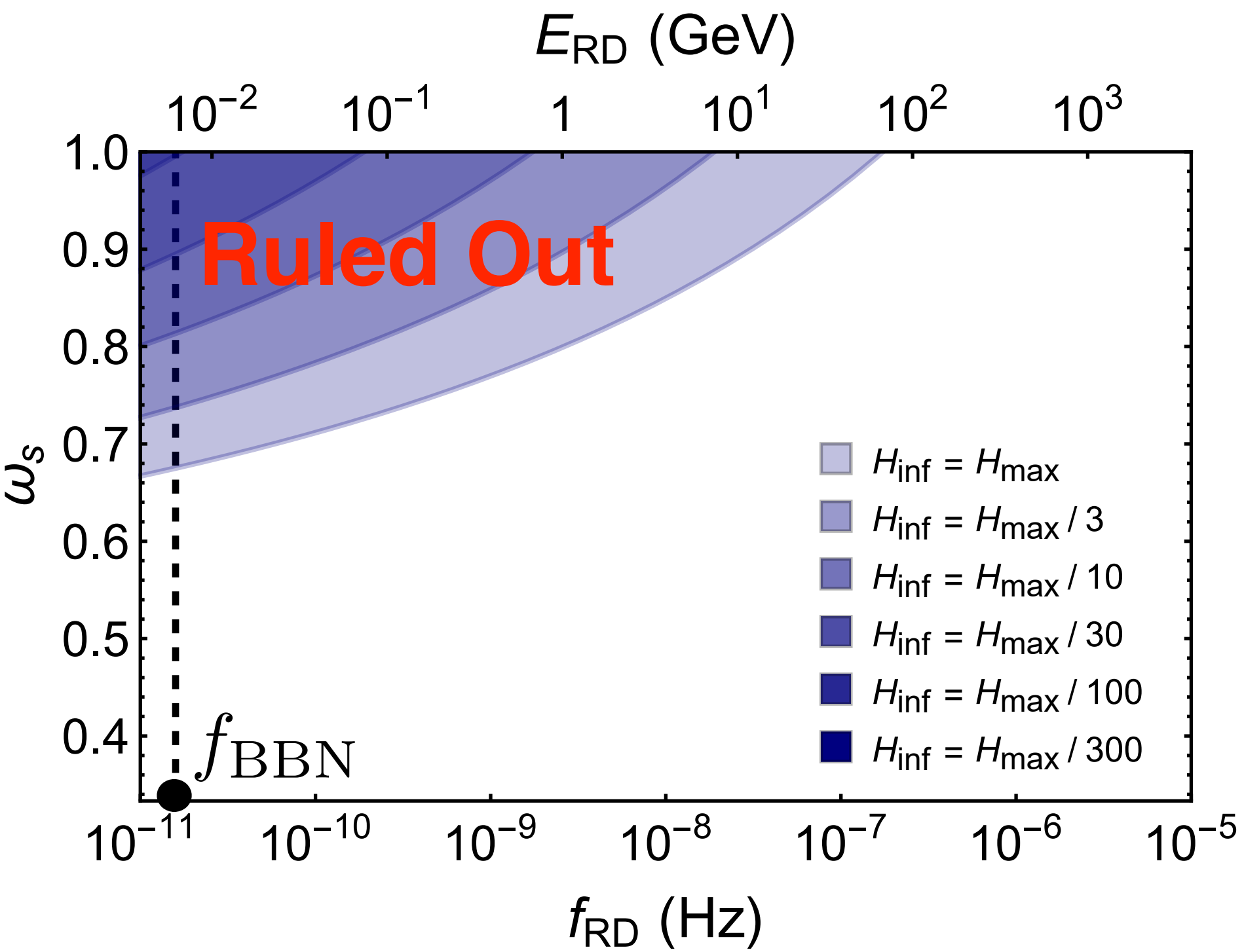
GW background $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$
Observability @ LIGO (today) Energy Scale EoS Stiff Duration Stiff

GW background

$$\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{\omega_s}, \underline{f_{RD}})$$

Observability @ LIGO (today)

Energy Scale EoS Stiff Duration Stiff



**LIGO
reduces
parameter
space
probe-able
by LISA !**

GW background

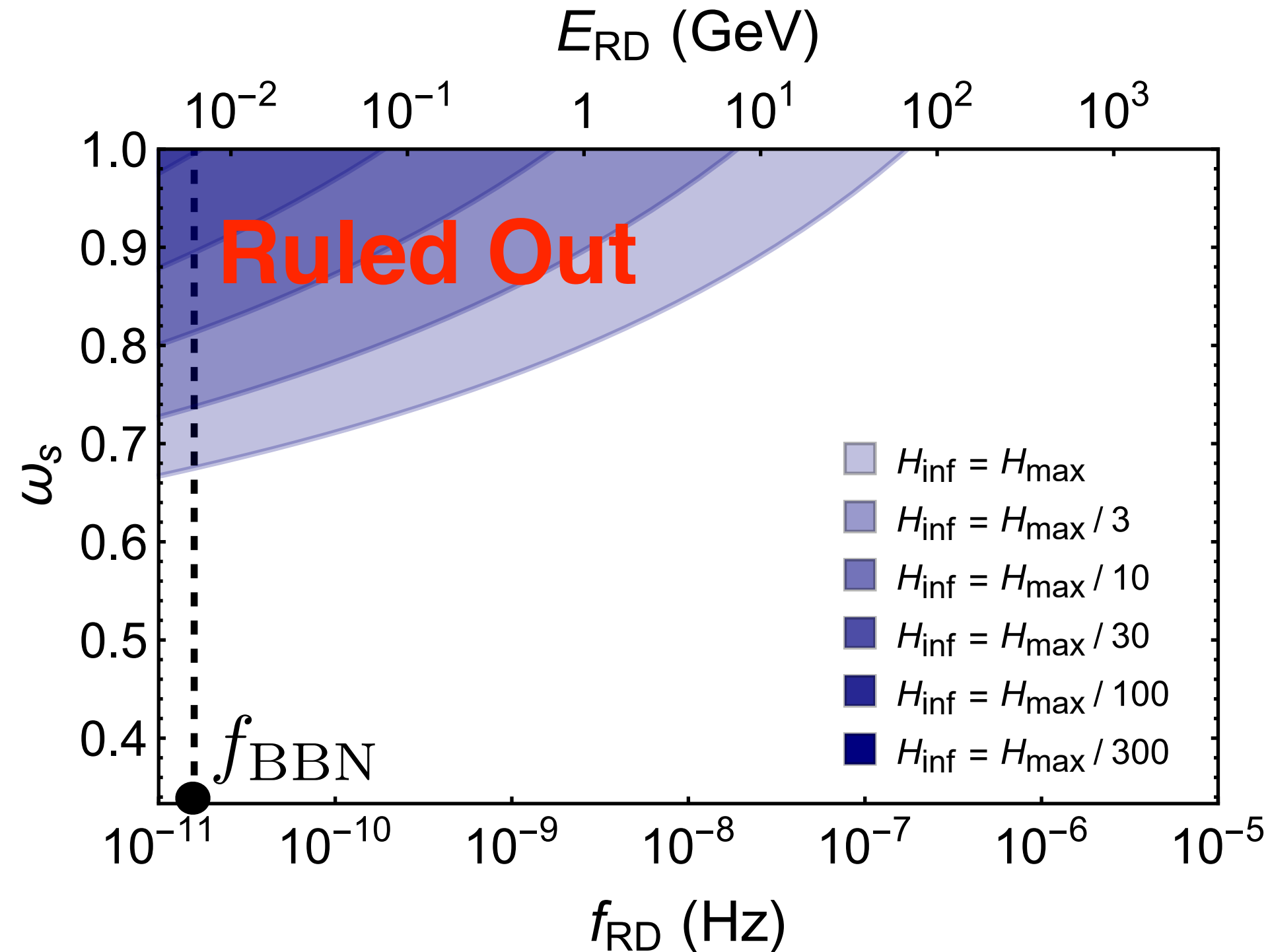
$$\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}})$$

Observability @ LIGO (today)

Energy
Scale

EoS
Stiff

Duration
Stiff

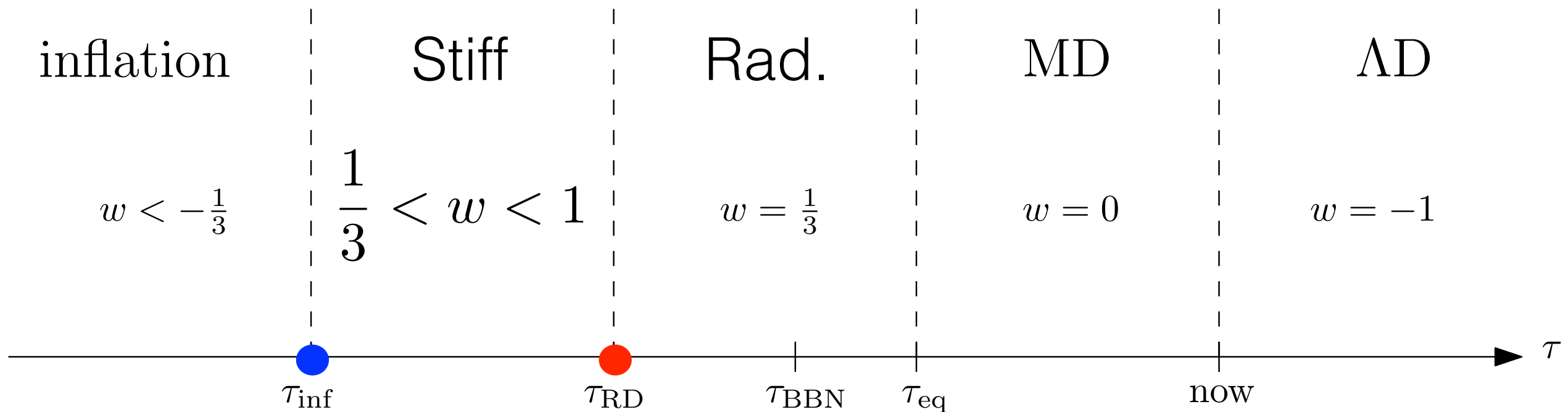


Let's look first
at consistency
of scenarios

Part 3

The trouble with gravitational reheating

BACK to ... GRAVITATIONAL REHEATING



$$\frac{\rho_{\text{rad}}^*}{H_*^2 m_{\text{pl}}^2} \ll 1$$

$$\frac{\rho_{\text{rad}}}{H^2 m_{\text{pl}}^2} \sim 1$$

$$\Omega_{\text{GW}}^{(\text{TOT})} = \int \left(\frac{d\rho_{\text{GW}}}{d \log f} \right) \frac{d \log f}{\rho_c}$$

- Requisites**
- * Control perturbations
 - * End before BBN
 - * **Not too many GWs**

BIG BANG NUCLEOSYNTHESIS

Expansion rate (Rad. Dom): ~ Extra relativistic species

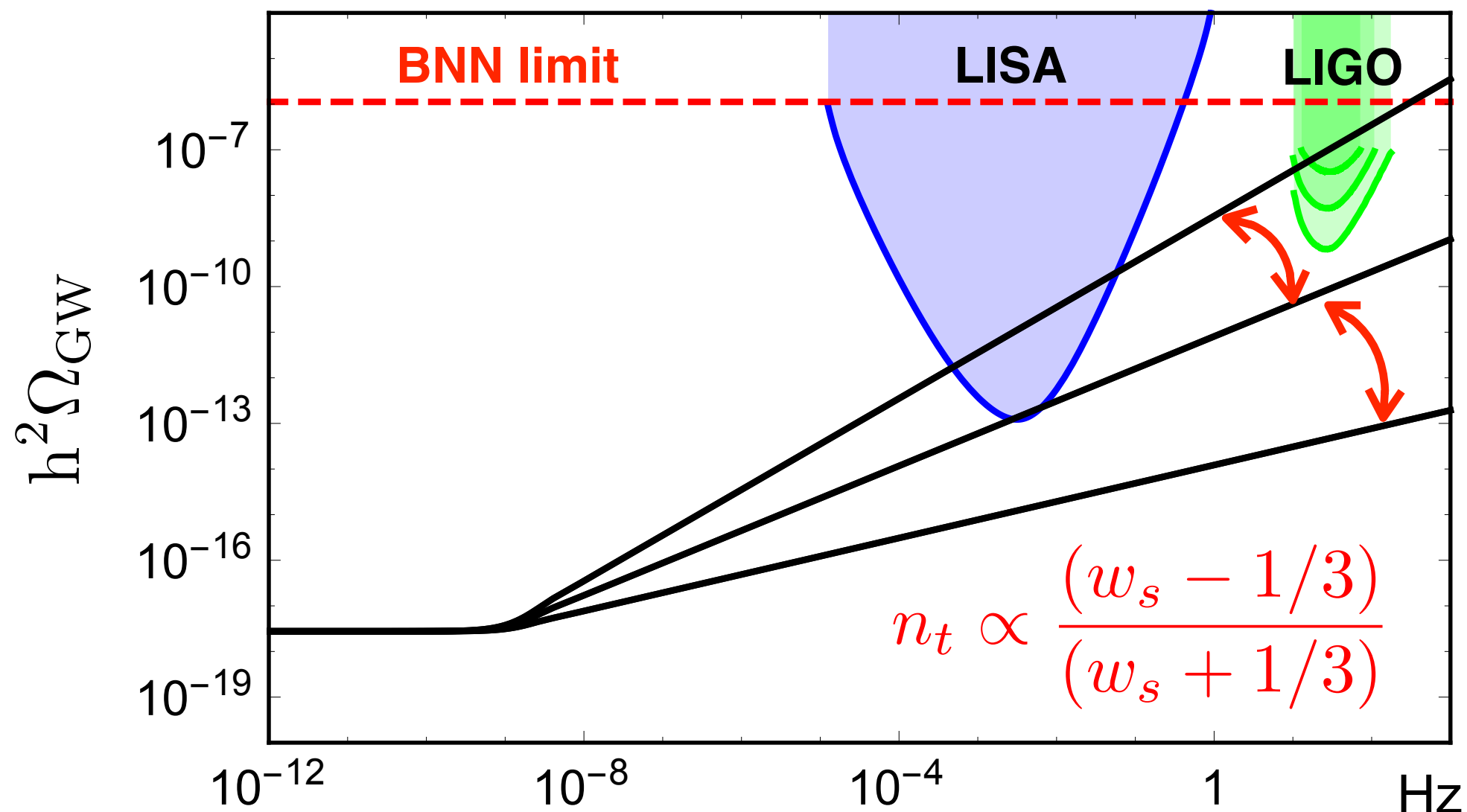
$$\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \leq 1.12 \times 10^{-6}$$

$$\Delta N_\nu = 0.2 \text{ (95\% C.L.) [latest CMB]}$$

BIG BANG NUCLEOSYNTHESIS

Expansion rate (Rad. Dom): \sim Extra relativistic species

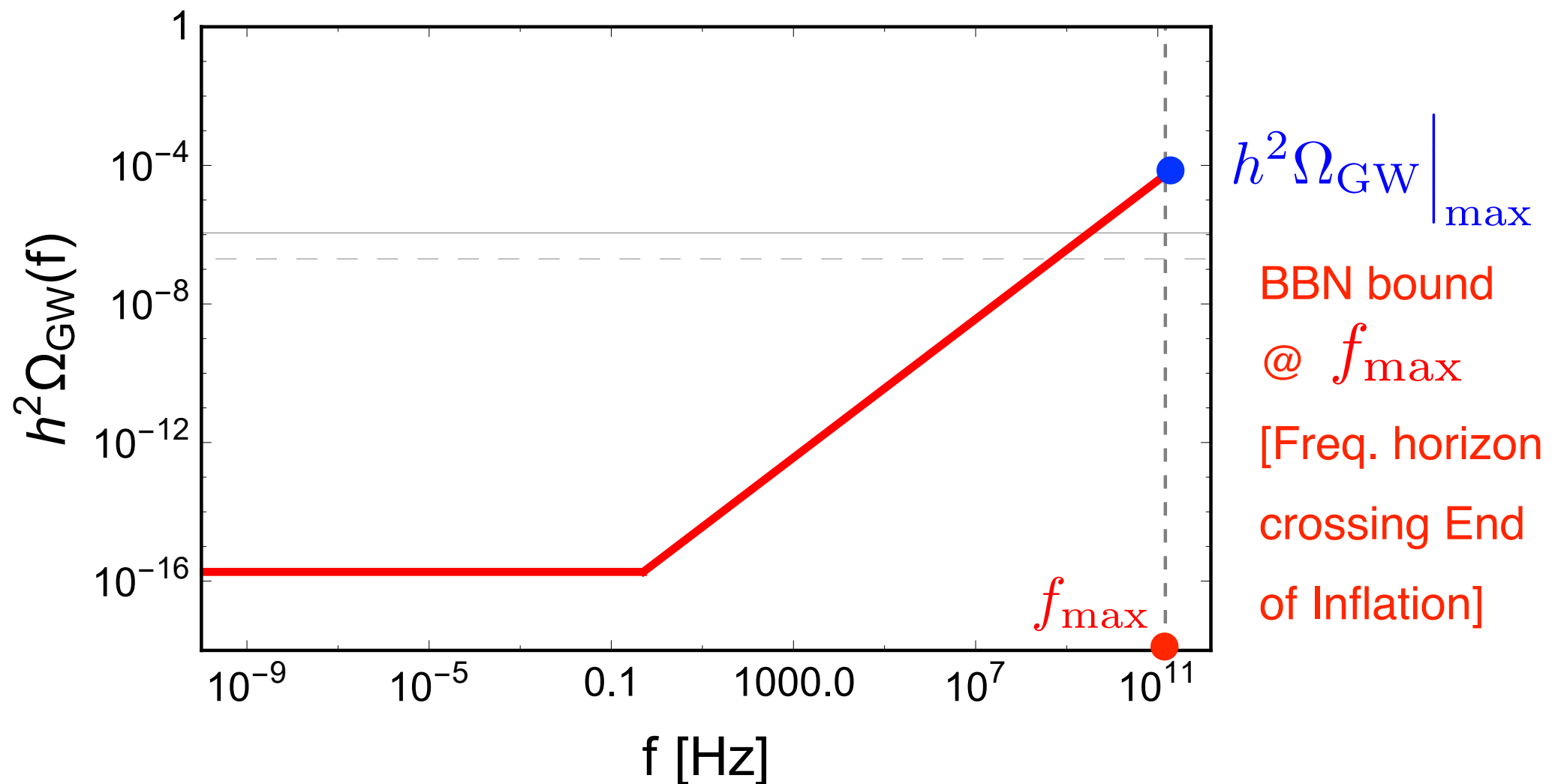
$$\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \leq 1.12 \times 10^{-6}$$



BBN: $\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \leq 1.12 \times 10^{-6}$

Grav. Reheating: $\Omega_{\text{GW}}(f) \propto (f/f_{\text{RD}})^2 \left(\frac{w_s - 1/3}{w_s + 1/3} \right)$

Monotonically growing signal !



BBN: $\int \frac{df}{f} h^2 \Omega_{\text{GW}}(f) \leq 1.12 \times 10^{-6}$

Grav. Reheating: $\Omega_{\text{GW}}(f) \propto (f/f_{\text{RD}})^{2\left(\frac{w_s - 1/3}{w_s + 1/3}\right)}$

Monotonically growing signal !

BBN bound @ f_{max} [Freq. horizon crossing End of Inflation]

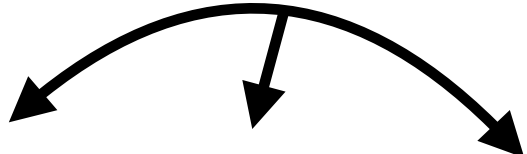
$$h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{\text{RD}}) \lesssim 10^{-6}$$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{\text{RD}}) \lesssim 10^{-6}$

Grav. Reheating: $\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2, \quad \delta \lesssim 1,$

$$f_{\text{RD}} = f_{\text{RD}}(H_*, w_s, \Delta_*) = f_{\text{RD}}(H_*, w_s, \delta)$$



$h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{\text{RD}}) \lesssim 10^{-6}$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, w_s; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

↓

$$2 \leq f(w_s) \leq 2.54$$

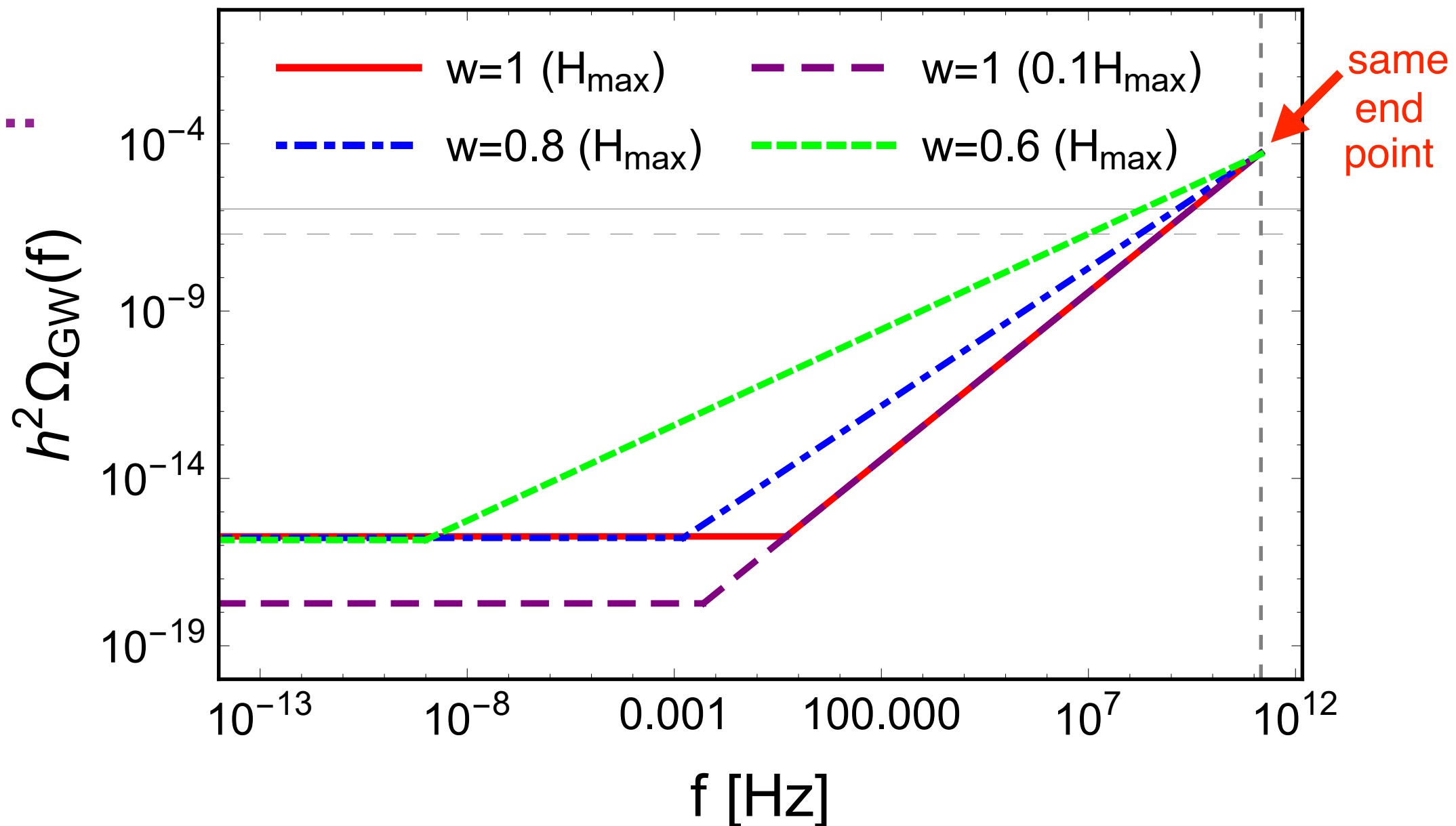
($w_s = 1/3$) ($w_s = 1$)

$$f(w_s) \equiv \frac{2^{\frac{3(1-w_s)}{1+3w_s}} \Gamma^2 \left(\frac{5+3w_s}{2+6w_s} \right)}{\left(\frac{2}{1+3w_s} \right)^{\frac{4}{1+3w_s}} \Gamma^2 \left(\frac{3}{2} \right)}$$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ...



BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\underbrace{H_*, w_s}_{\text{crossed out}}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

Why ?



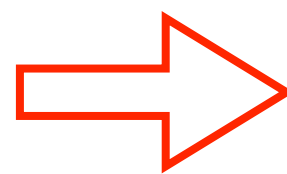
BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\underbrace{H_*, w_s}_{\text{crossed out}}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

Why ?

$$\left. \begin{aligned} \rho_{\text{rad}} &\propto H_*^4 a^{-4} \\ \rho_{\text{GW}} &\propto H_*^4 a^{-4} \end{aligned} \right\}$$



$$\frac{\rho_{\text{GW}}}{\rho_{\text{rad}}} \sim \text{const.}$$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}$, $\delta \lesssim 1$,

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

$$\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2$$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}, \quad \delta \lesssim 1,$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

So ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} \simeq \frac{\text{const.}}{\delta} \lesssim 10^{-6} \quad \Leftrightarrow \quad \delta \gtrsim 50$

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}$, $\delta \lesssim 1$,

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

So ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} \simeq \frac{\text{const.}}{\delta} \lesssim 10^{-6} \iff \delta \gtrsim 50$ (!)

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}$, $\delta \lesssim 1$

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

So ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} \simeq \frac{\text{const.}}{\delta} \lesssim 10^{-6} \iff \delta \gtrsim 50$ (!)

$\frac{\rho_{\text{GW}}}{\rho_{\text{rad}}} \sim \text{const.} \gg 1$ **Universe dominated by GWs!**

BBN: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s, f_{RD}) \lesssim 10^{-6}$

Grav. Reheating: $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (H_*, w_s; \delta) \lesssim 10^{-6}$, $\delta \lesssim 1$,

However ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} (\cancel{H_*}, \cancel{w_s}; \delta) \simeq \underbrace{2.1 \cdot 10^{-5}}_{\text{const.}} \times \underbrace{f(w_s)}_{\text{mild dependence}} \times \underbrace{\frac{1}{\delta}}_{\text{initial fraction}}$

So ... $h^2 \Omega_{\text{GW}}^{(0)} \Big|_{\text{max}} \simeq \frac{\text{const.}}{\delta} \lesssim 10^{-6} \iff \delta \gtrsim 50$ (!)

CMB: $\delta \gtrsim 200$ (!!)

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

- 1) Either we modify Grav. Reheating**
- 2) We use modified gravity in Inflationary Sector**
- 3) We couple the inflaton and reheat via such couplings**

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

2) We use modified gravity in Inflationary Sector

3) We couple the inflaton and reheat via such couplings

Standard (P)reheating

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

2) We use modified gravity in Inflationary Sector

I'm very happy with General Relativity !

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

2) We use modified gravity in Inflationary Sector

But if you are not ...

Y. Watanabe and E. Komatsu, Phys. Rev. **D75**, 061301 (2007), gr-qc/0612120.

Y. Watanabe, Phys. Rev. **D83**, 043511 (2011), 1011.3348.

A. A. Starobinsky, Phys. Lett. **B91**, 99 (1980), [,771(1980)].

A. De Felice and S. Tsujikawa, Living Rev. Rel. **13**, 3 (2010), 1002.4928.

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

$$\Delta_* \equiv \frac{\rho_{\text{rad}}}{3m_p^2 H_*^2} = \frac{\delta}{300} \left(\frac{H_*}{m_p} \right)^2 \longrightarrow \mathcal{N}_f \Delta_*$$

All \mathcal{N}_f fields
same properties !

$$\begin{aligned} \delta &= \delta_1 \times \mathcal{N}_f, \\ \mathcal{N}_f &\gtrsim \mathcal{O}(10^3) \end{aligned}$$

**Ad hoc
tuning !**

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH ?



(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !

$m_\chi^2 < 0$ @ Stiff Period,

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !
 $m_\chi^2 < 0$ @ Stiff Period, but
self-interactions regularize

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !

**Corrected in DGF & Byrnes '16
Phys.Lett. B767 (2017) 272-277
Arxiv: 1604.03905**

(standard) Grav. Reheating incompatible with BBN/CMB !

Therefore...

1) Either we modify Grav. Reheating

Radiation field is the SM Higgs ? We need non-min coupling

$$\mathcal{L}_\chi = (\partial\chi)^2 + \lambda\chi^4 - \xi\chi^2 R$$

Standard Grav. RH wrong !

$$\delta \sim \mathcal{O}(10^3) \frac{\xi^2}{\lambda} \gg 1$$

$$\lambda > 0 \text{ (stability)}, \quad \xi \gtrsim 1$$

Grav.
Reheating
OK !

Corrected in DGF & Byrnes '16
Phys.Lett. B767 (2017) 272-277
Arxiv: 1604.03905

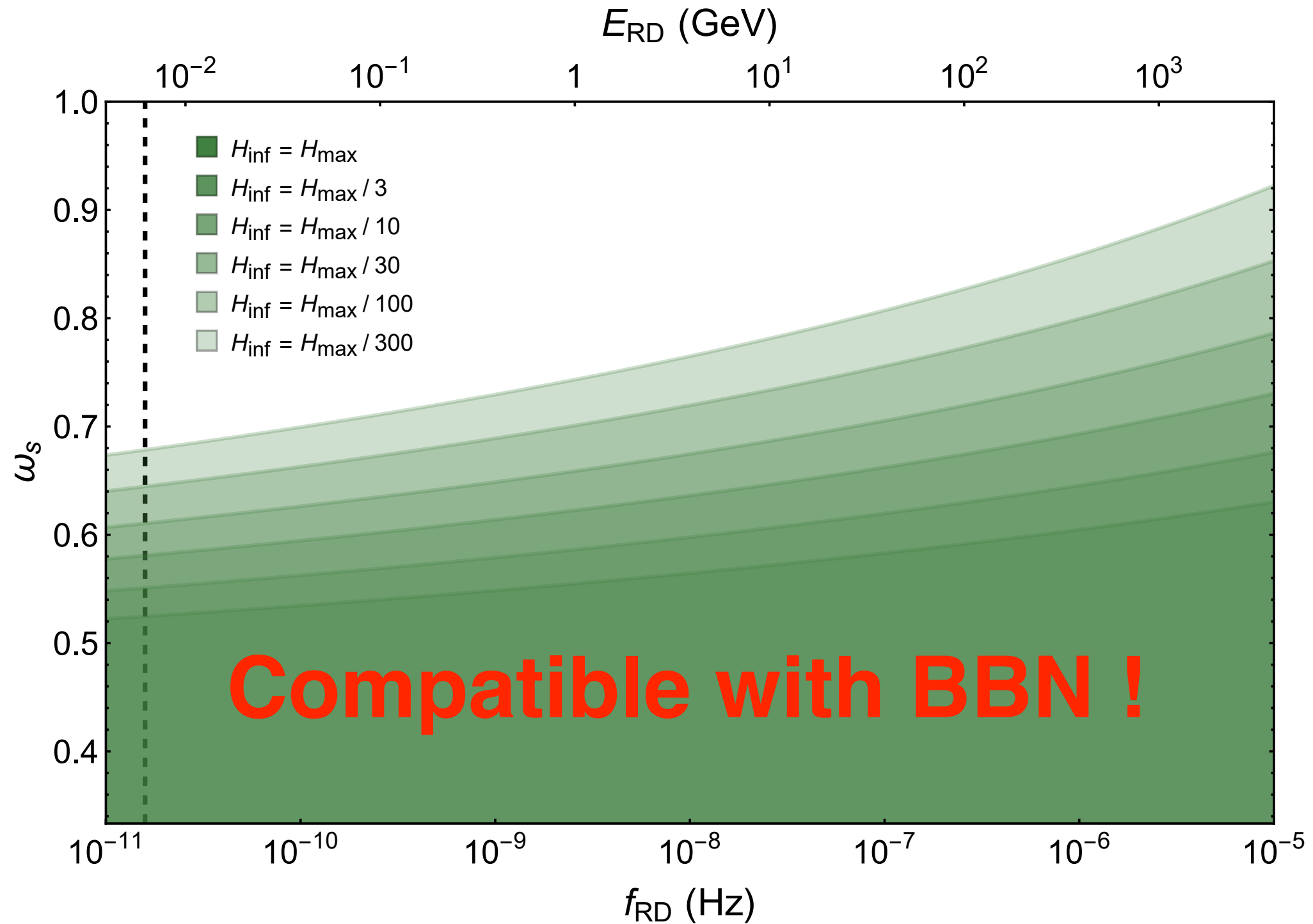
See also [1803.07399](#)
[1905.06823](#) for generic $\lambda\chi^4$

Part 4

**BBN/CMB constraints:
further implications**

BBN Bound

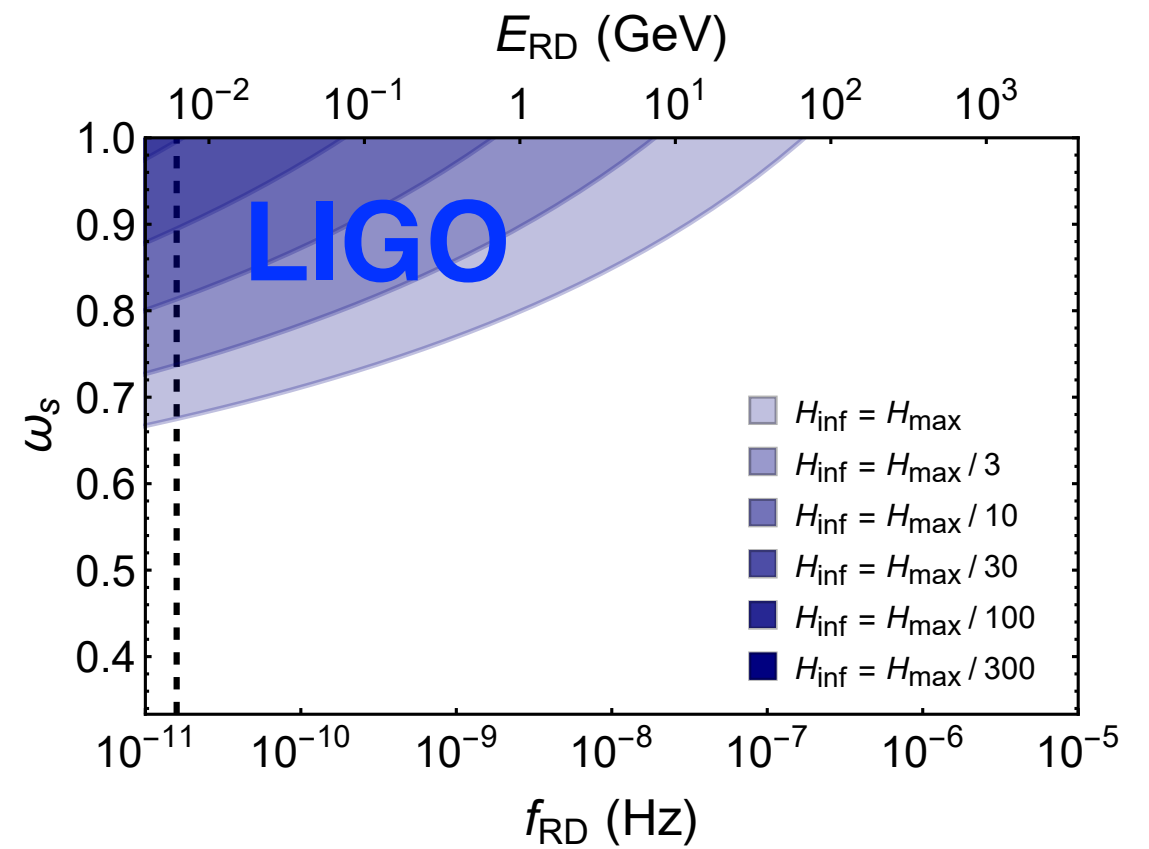
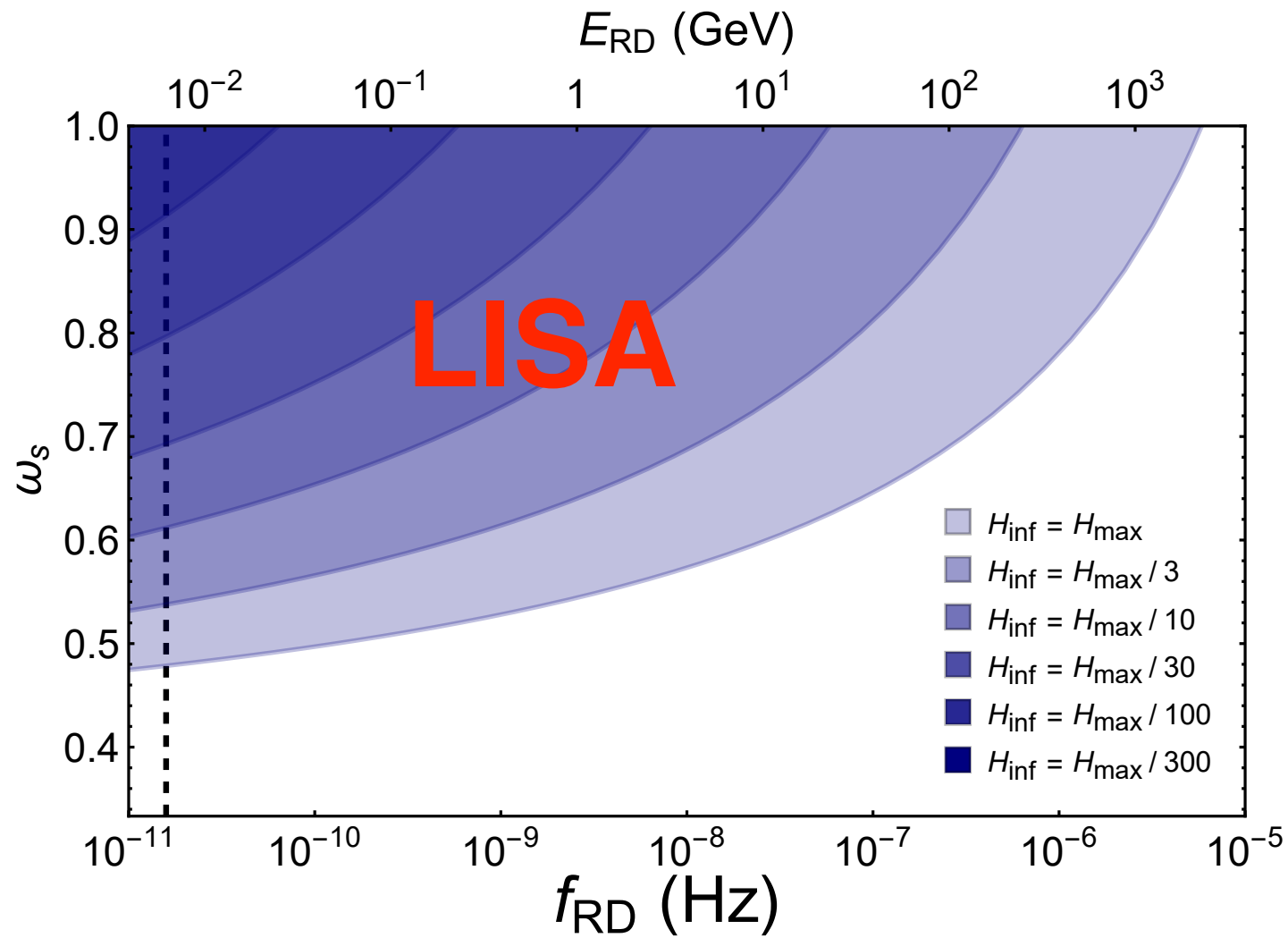
$$\Omega_{\text{GW}}^{(0)}(f; \underbrace{H_*}_{\text{Energy Scale}}, \underbrace{w_s}_{\text{EoS Stiff}}, \underbrace{f_{\text{RD}}}_{\text{Duration Stiff}}) \lesssim 10^{-6}$$



BBN Bound

$$\Omega_{\text{GW}}^{(0)}(f; \underbrace{H_*}_{\text{Energy Scale}}, \underbrace{w_s}_{\text{EoS Stiff}}, \underbrace{f_{\text{RD}}}_{\text{Duration Stiff}}) \lesssim 10^{-6}$$

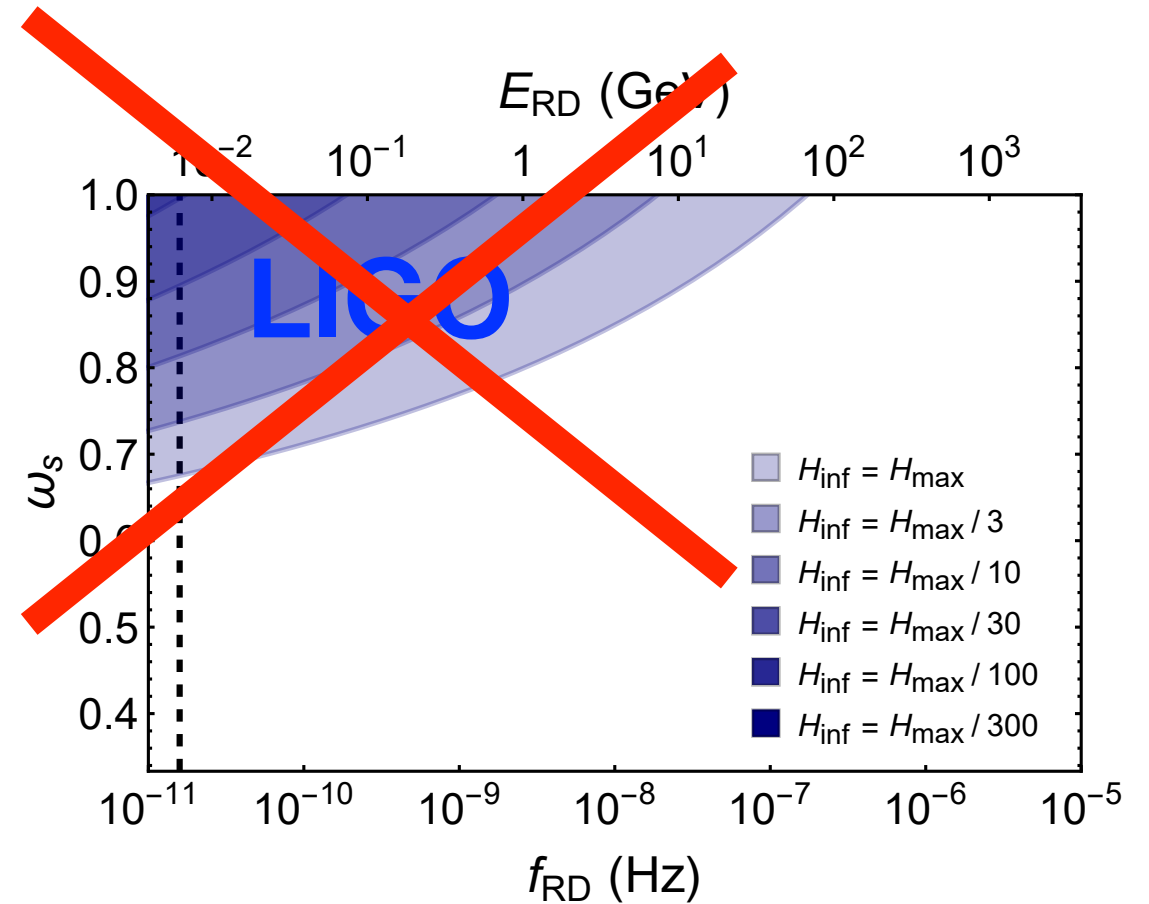
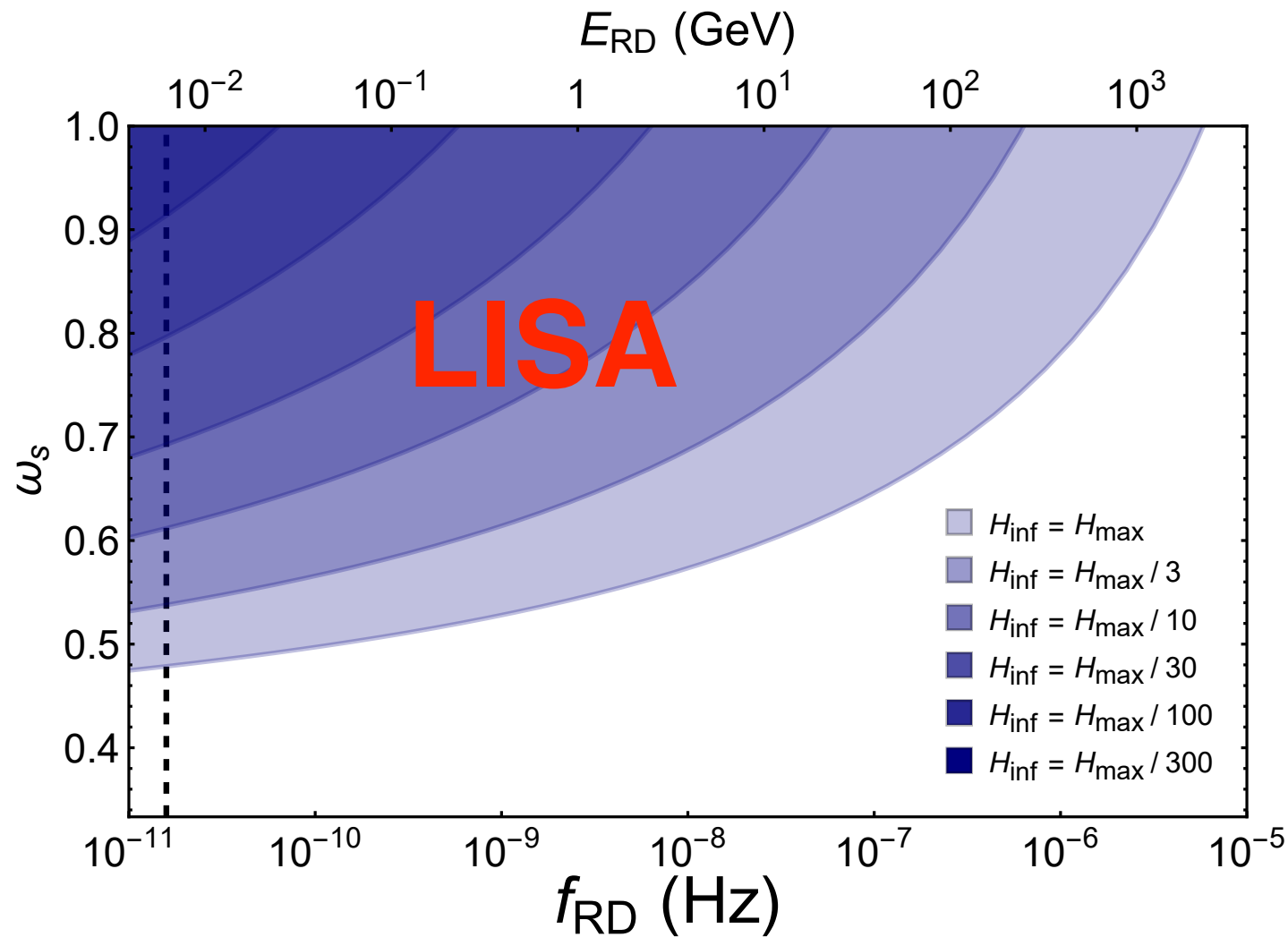
Energy Scale EoS Stiff Duration Stiff



BBN Bound

$$\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$$

Energy Scale EoS Stiff Duration Stiff

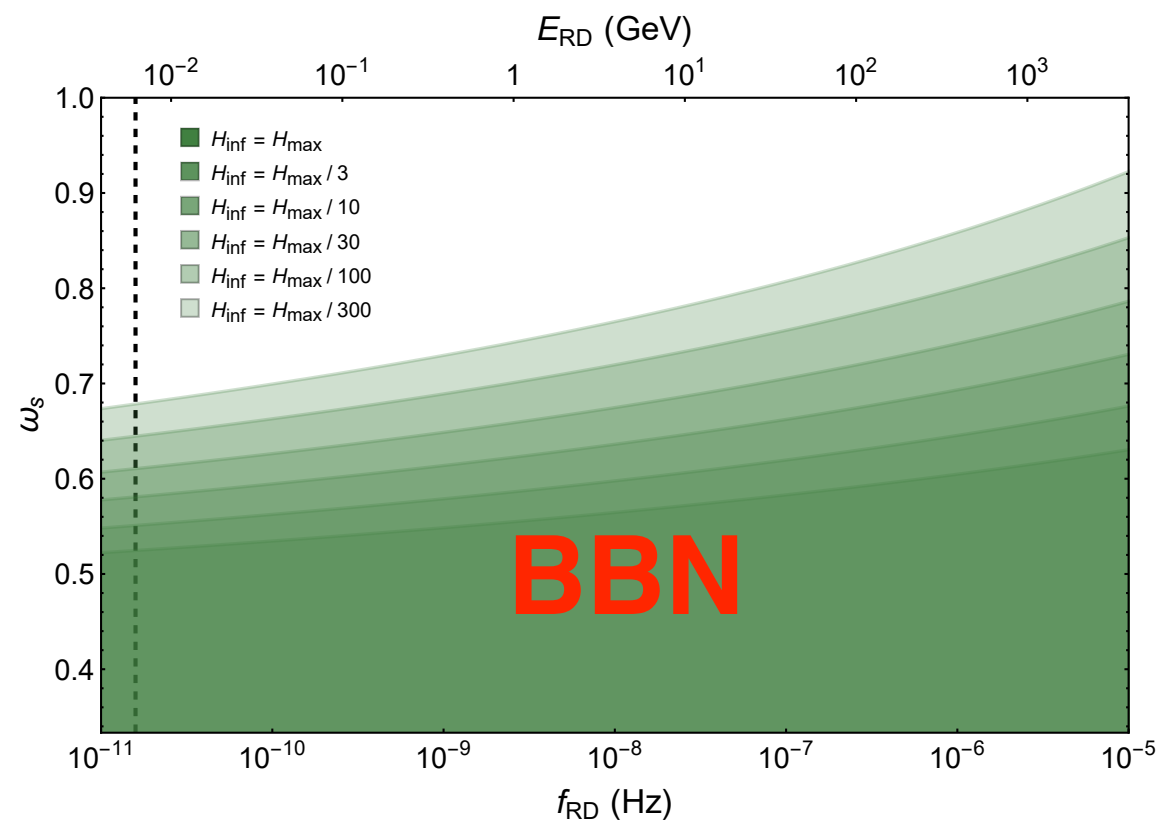
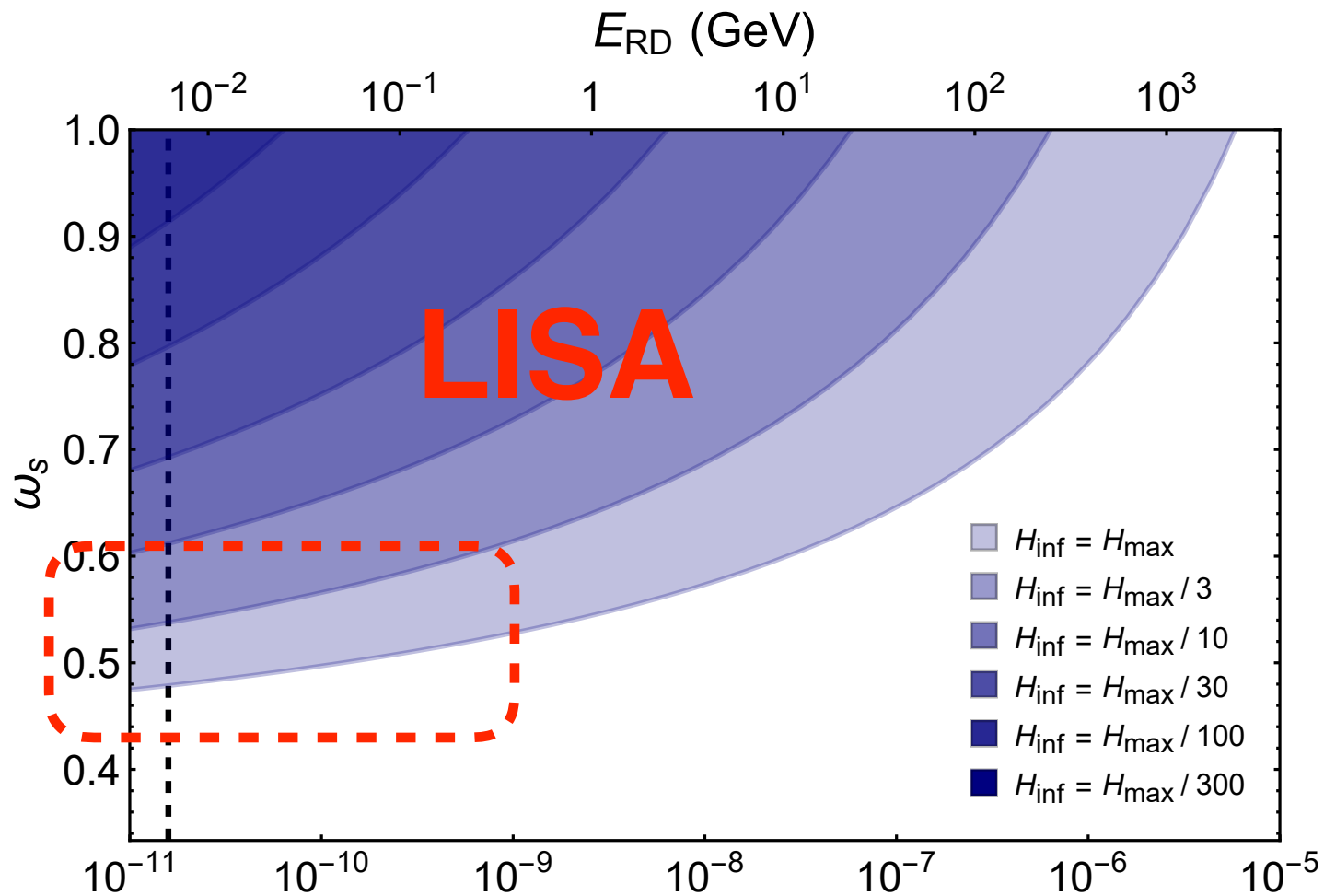


LIGO cannot probe parameter space compatible with BBN !

BBN Bound $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$

LISA ?

Energy Scale EoS Stiff Duration Stiff



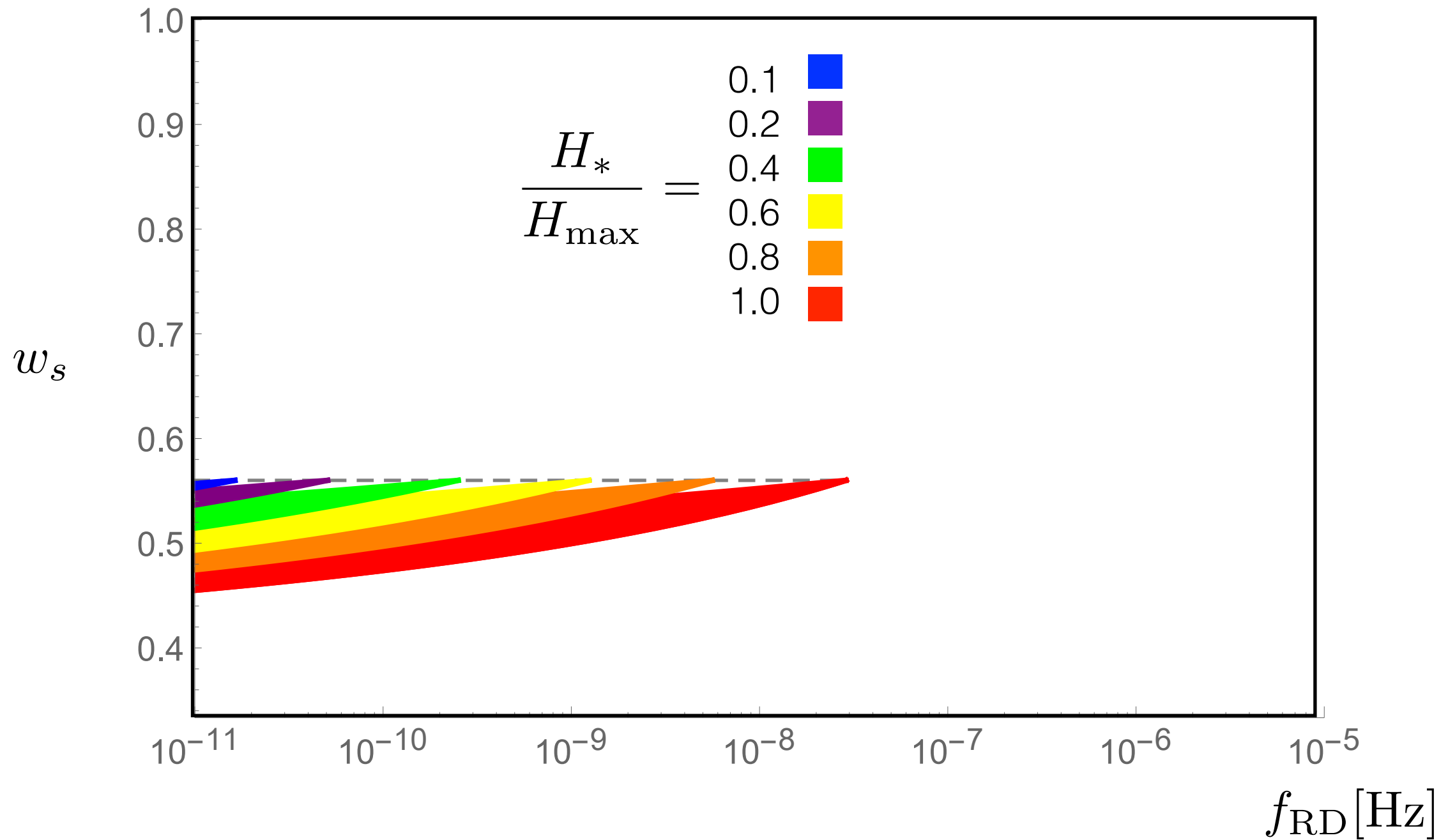
BBN Bound $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$

LISA ✓

Energy
Scale

EoS
Stiff

Duration
Stiff



Part 5

Outlook

(+ take-home message)

OUTLOOK

0) Reheating w/o couplings requires imagination:

Grav. Reheating or Modified Gravity

1) (Standard) Grav. Reheating is inconsistent

Too many GWs (violates BBN/CMB bounds)

2) Inf. sectors only (minimally) coupled

to gravity inconsistent unless:

i) Inflation ~ Modify gravity: I don't want to

ii) $O(1000)$ spectator fields identical: ad hoc tuning

iii) SM Higgs + Non-Min coupling: works (not observable)

3) Stiff Era (in general): not observable @ LIGO, barely @ LISA

TAKE HOME MESSAGE

**If you go stiff ...
check LIGO/BBN/CMB**

Part 6

Muchas gracias
por vuestra atención !



Part 7

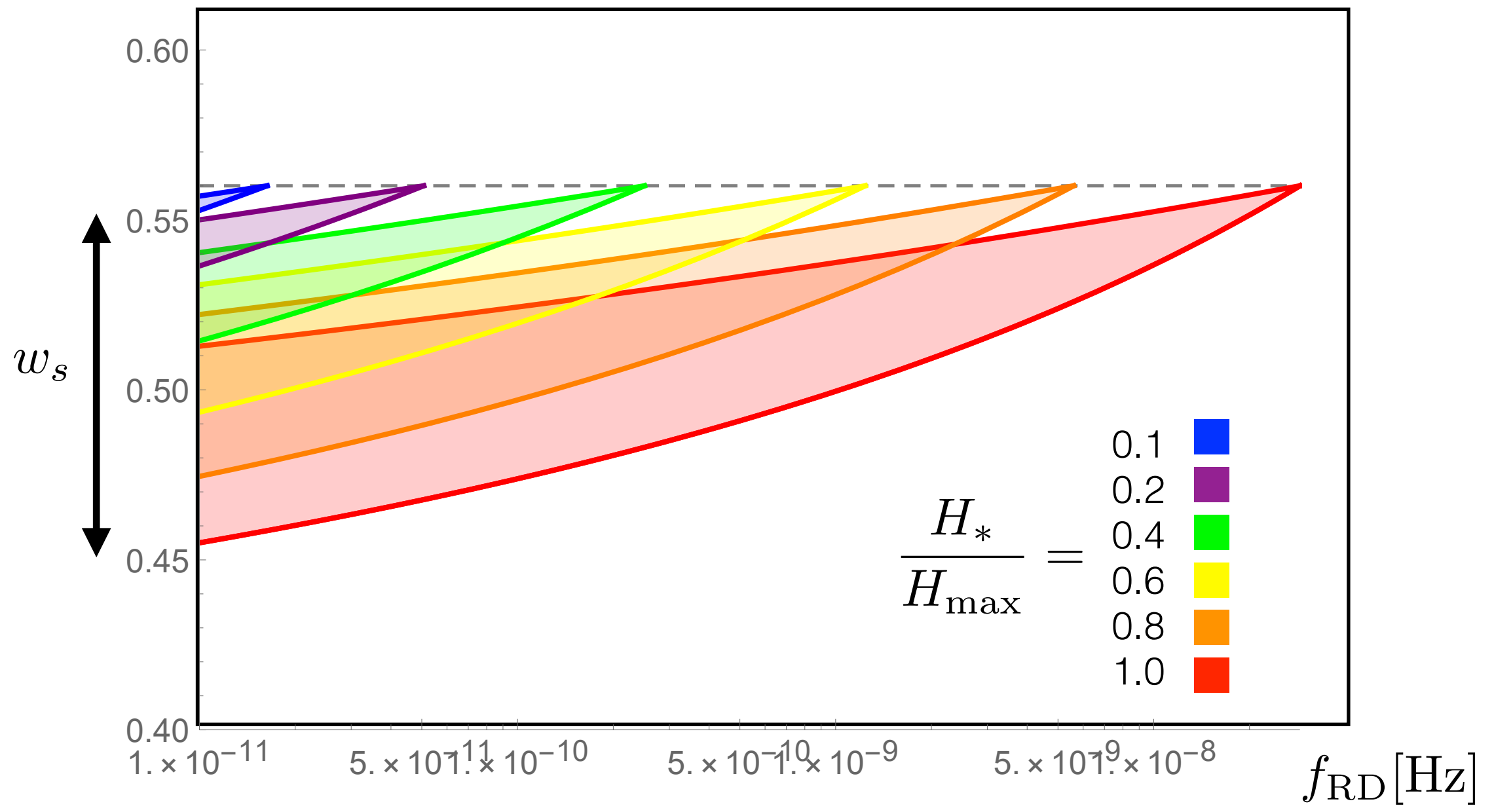
Backslides

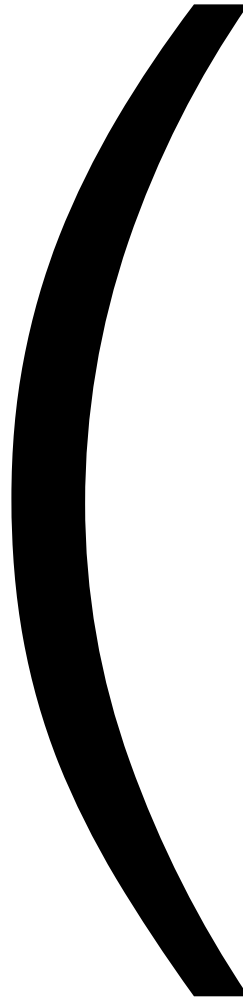
BBN Bound $\Omega_{\text{GW}}^{(0)}(f; \underline{H_*}, \underline{w_s}, \underline{f_{RD}}) \lesssim 10^{-6}$

LISA ✓

Energy Scale EoS Stiff Duration Stiff

ZOOM





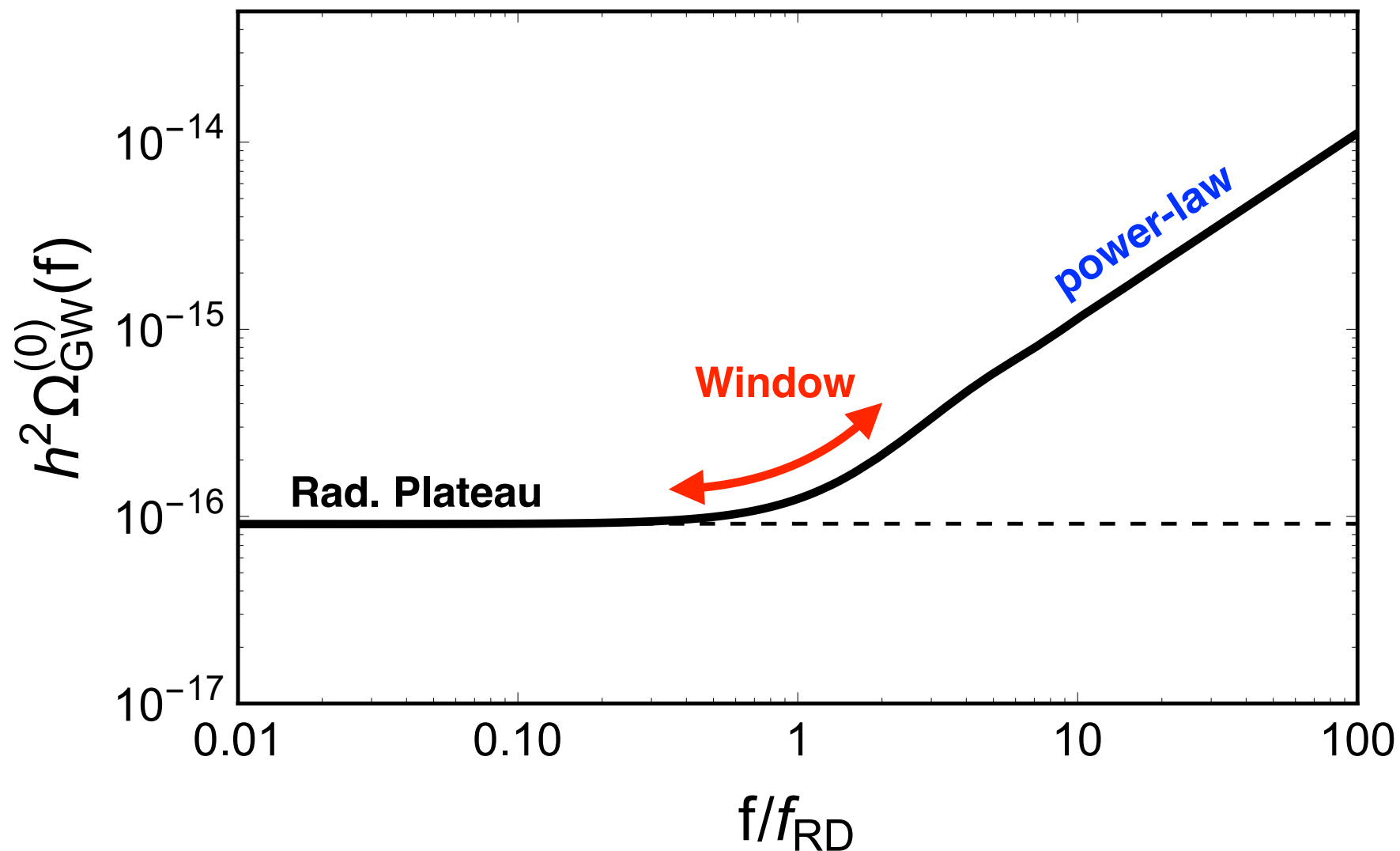
Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$

**Rad.
Plateau**

Transfer Funct. Stiff Period
Window \times *power-law*



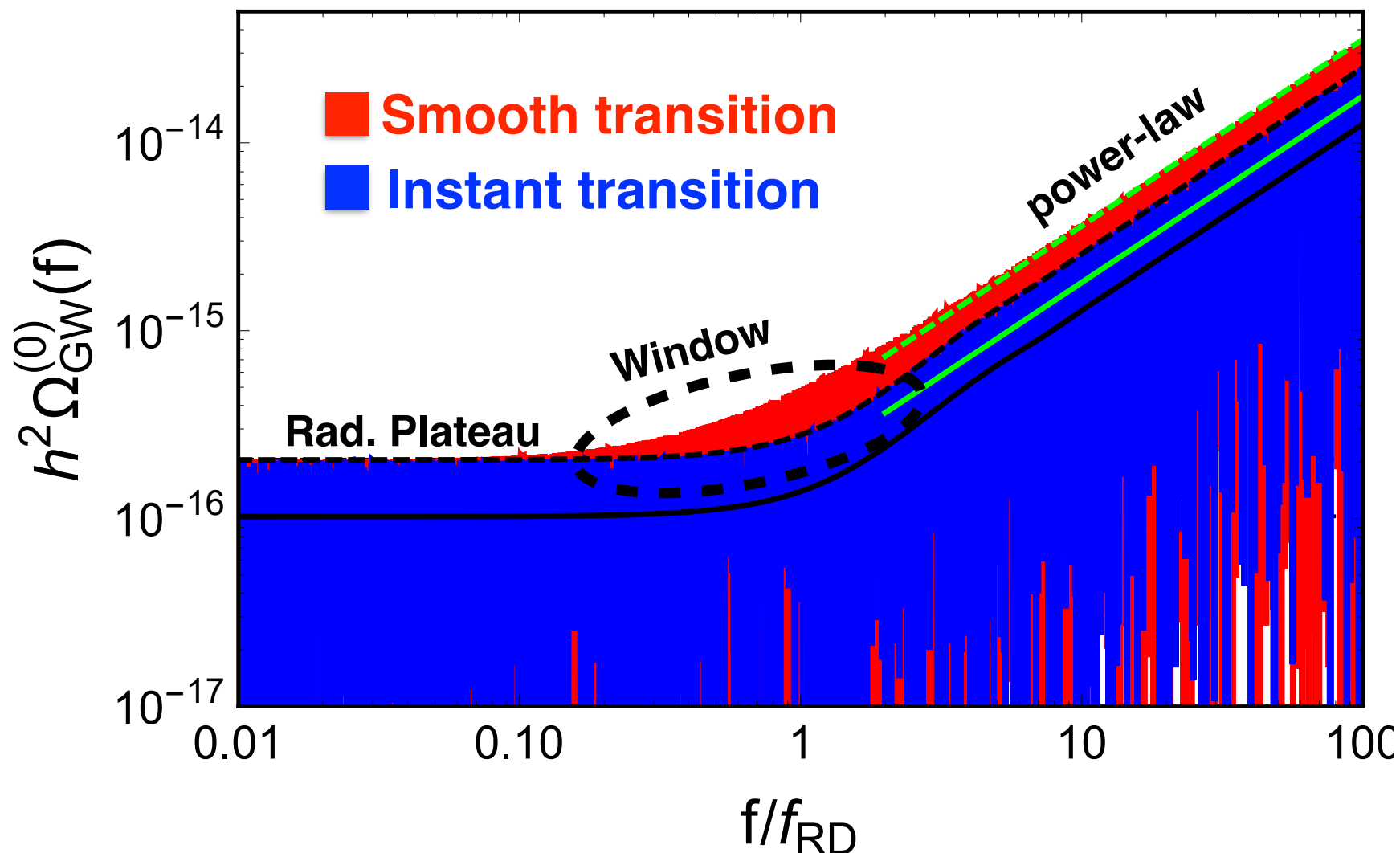
Inflationary GW background

$$\Omega_{\text{GW}}^{(0)}(f) = \underbrace{\Omega_{\text{GW}}^{(0)}|_{\text{plateau}}}_{\text{Rad. Plateau}} \times \underbrace{\mathcal{W}(f/f_{\text{RD}}) \times \mathcal{A}_s \left(\frac{f}{f_{\text{RD}}}\right)^{n_t(w_s)}}_{\text{Transfer Funct. Stiff Period Window} \times \text{power-law}}$$

Rad.
Plateau

Transfer Funct. Stiff Period
Window \times power-law

$$\Omega_{\text{GW}}^{(0)}|_{\text{plateau}} \simeq 2 \cdot 10^{-16} \left(\frac{H_*}{H_{\text{max}}}\right)^2$$



**Real signal:
highly oscillatory**

**Stochastic Signal:
average measurement**

$$\langle \dot{h}_{ij}(f) \dot{h}_{ij}(f) \rangle = \mathcal{P}_h(f)$$

