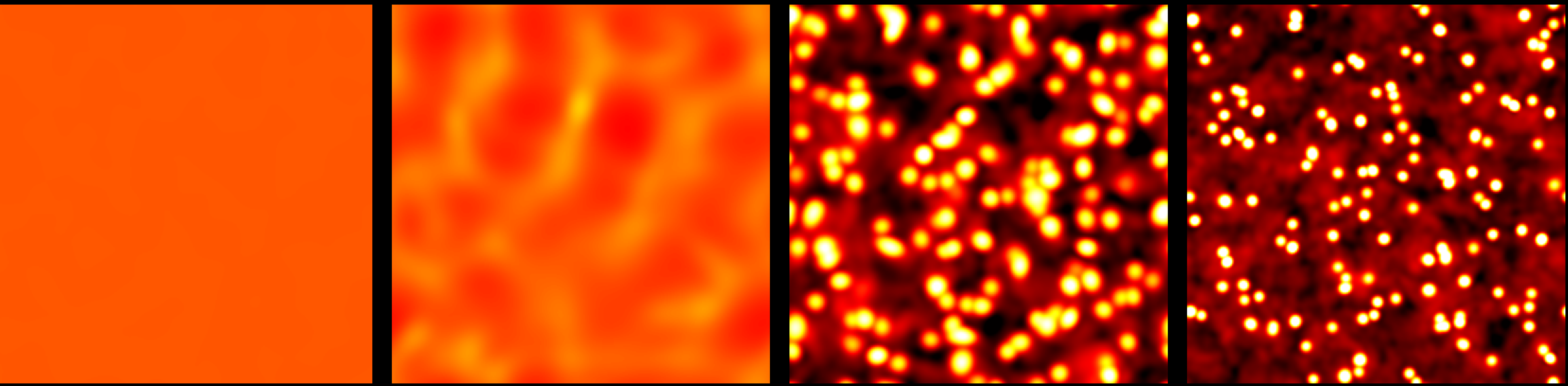
Nonlinear Dynamics After Inflation

* including gravitational effects



UC SANTA BARBARA
Kavli Institute for
Theoretical Physics

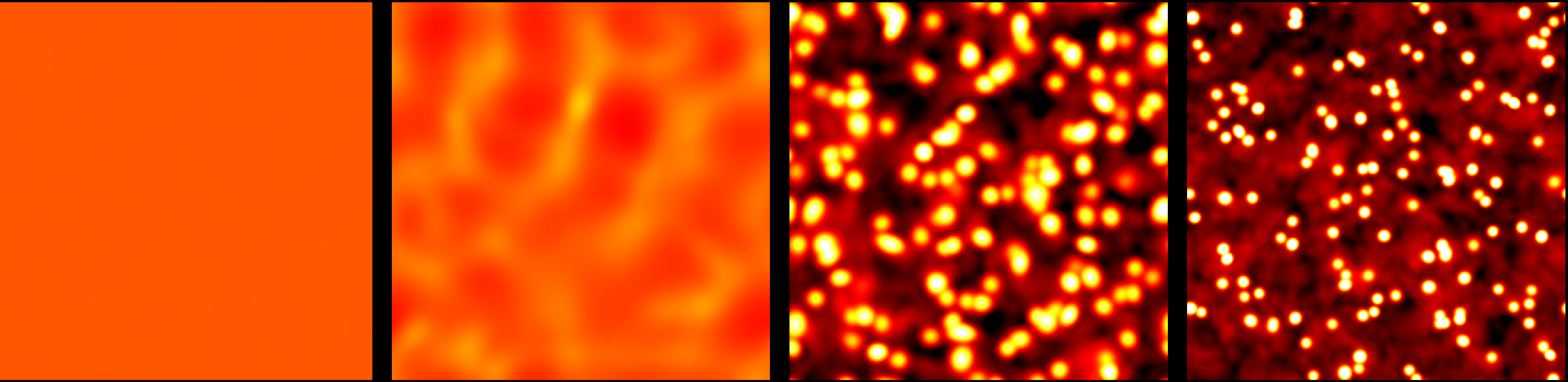
Mustafa A. Amin

Inflationary Reheating
Meets
Particle Physics Frontier

RICE

Nonlinear Dynamics After Inflation

* also applies to moduli or aspects of axion dynamics in the late universe



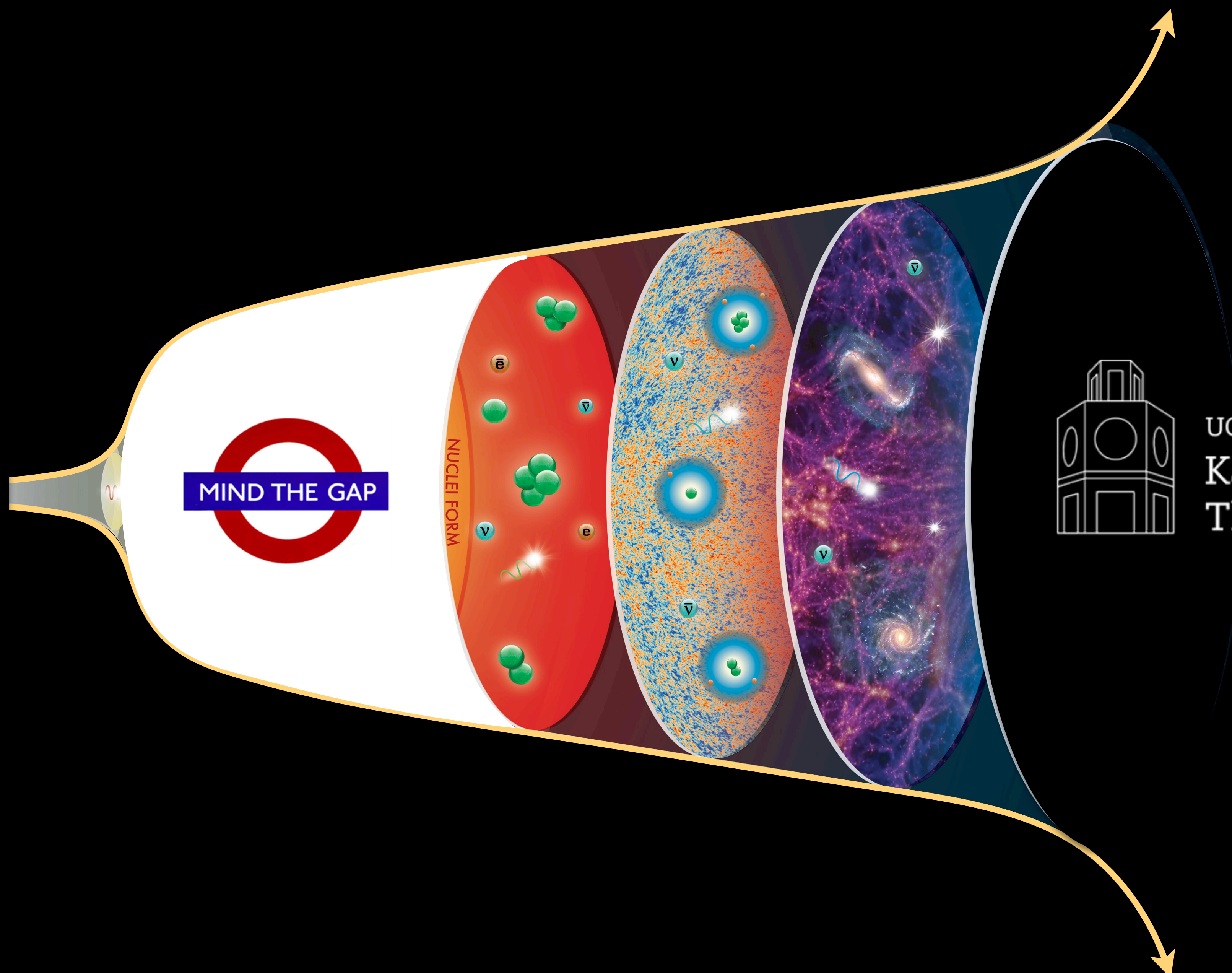
UC SANTA BARBARA
Kavli Institute for
Theoretical Physics

Inflationary Reheating
Meets
Particle Physics Frontier

Mustafa A. Amin

RICE

Non-perturbative Dynamics After Inflation

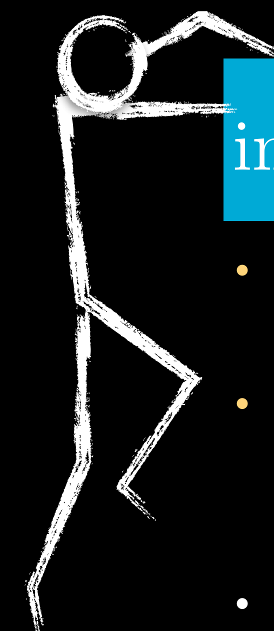


UC SANTA BARBARA
Kavli Institute for
Theoretical Physics

Mustafa A. Amin

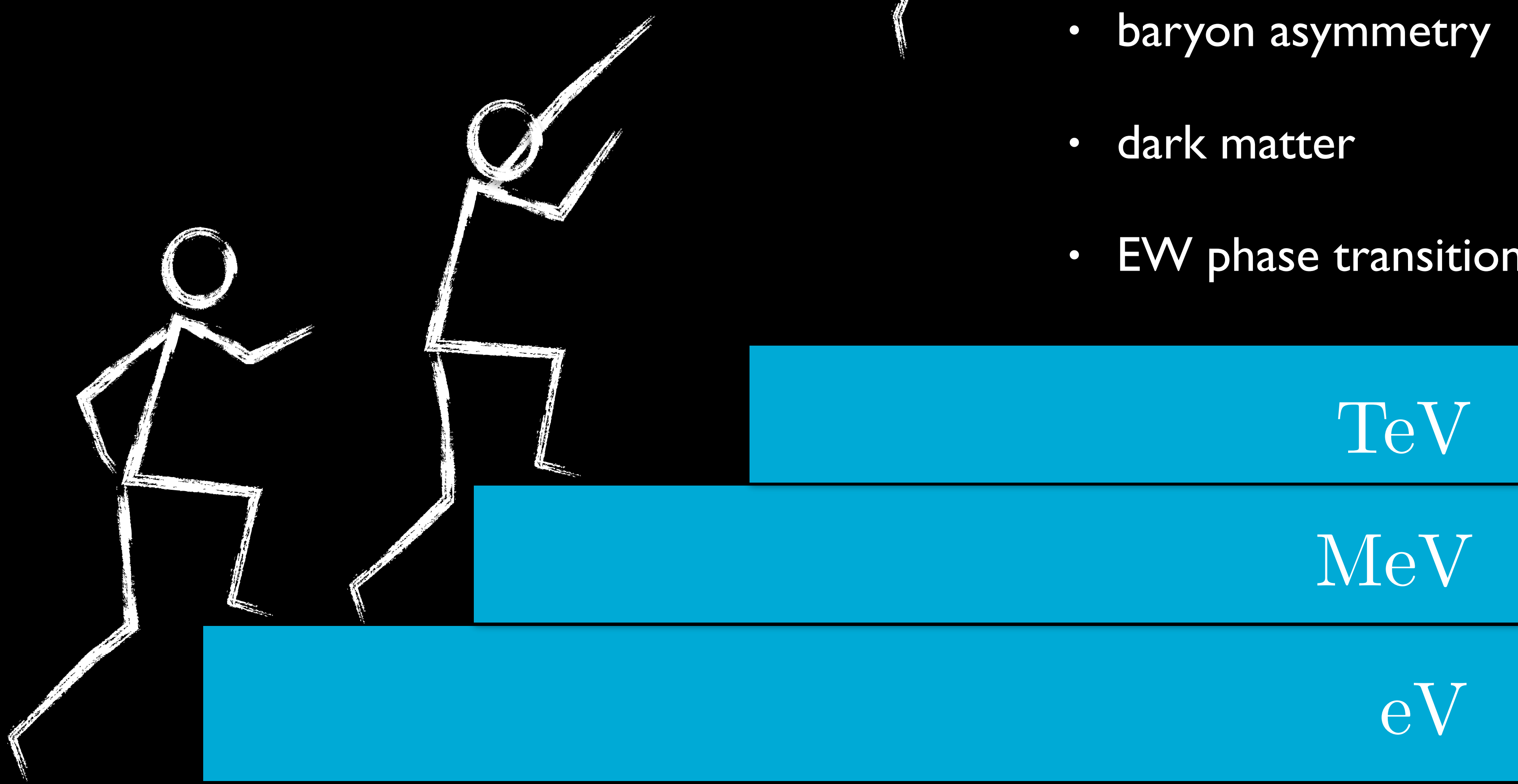
RICE

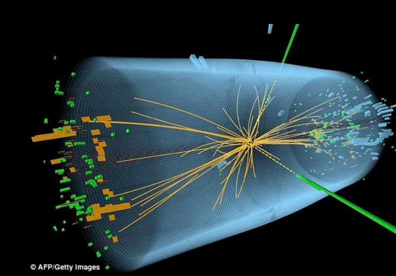
Inflationary Reheating
Meets
Particle Physics Frontier



inflation 10^{16} GeV

- inflation ends
- populate the universe (Standard Model)
- baryon asymmetry
- dark matter
- EW phase transition

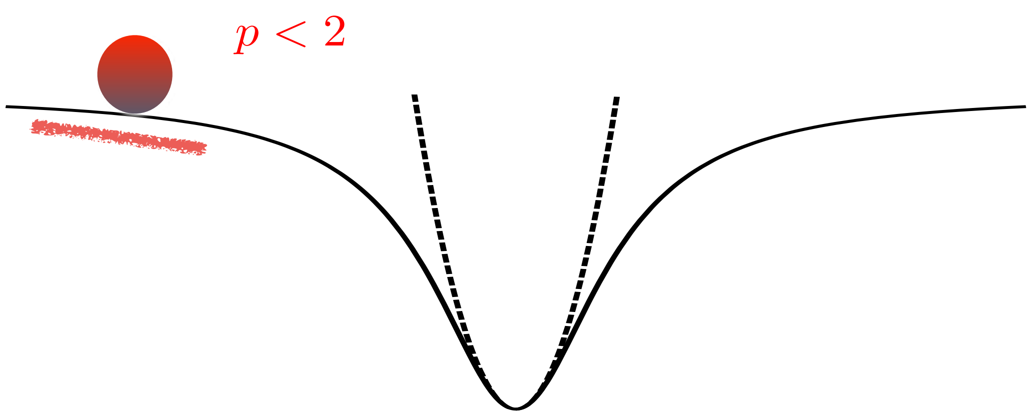


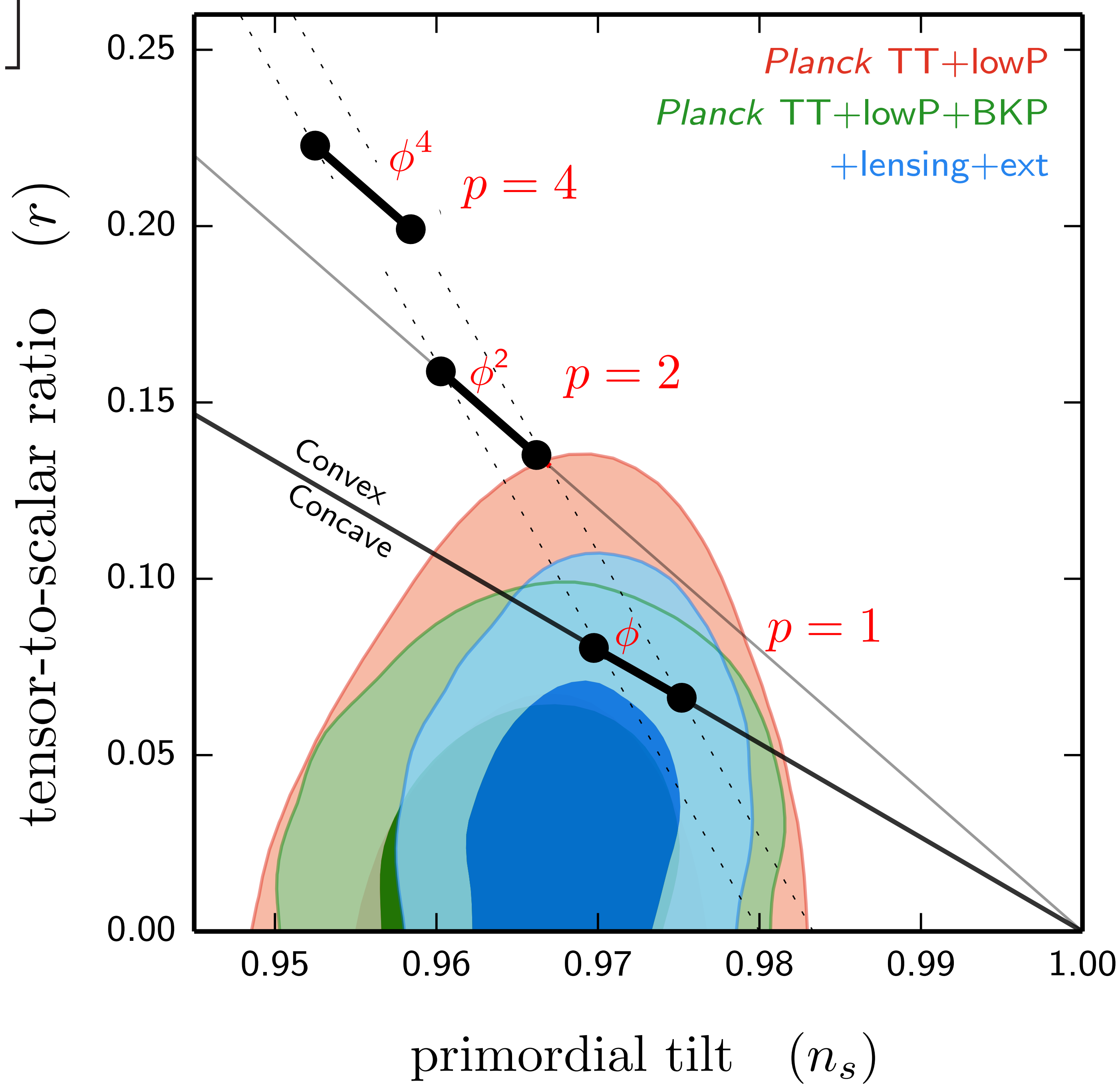


what we “know” about inflation
(simplest case - scalar field driven inflation)

$$S = \int d^4x\sqrt{-g}\left[\frac{m_{\rm pl}^2}{2}R - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$

$V(\phi) \propto \phi^p$

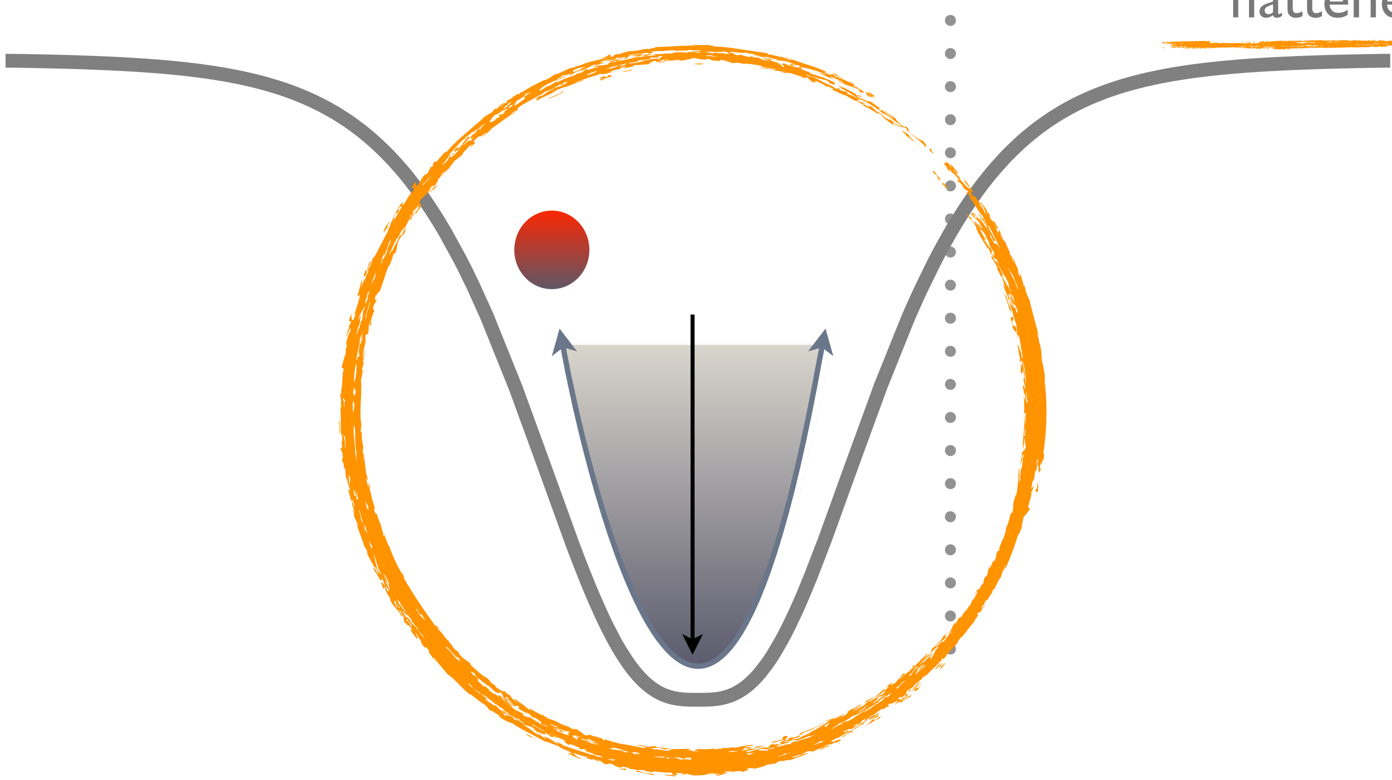




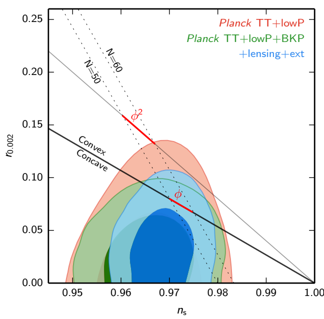
for example:

Starobinsky Inflation (1979/80)
Silverstein & Westhpal (2008)
Kallosh & Linde (2013)

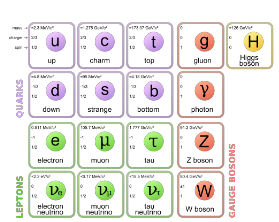
end of inflation



flattened potential $\phi^{p<2}$

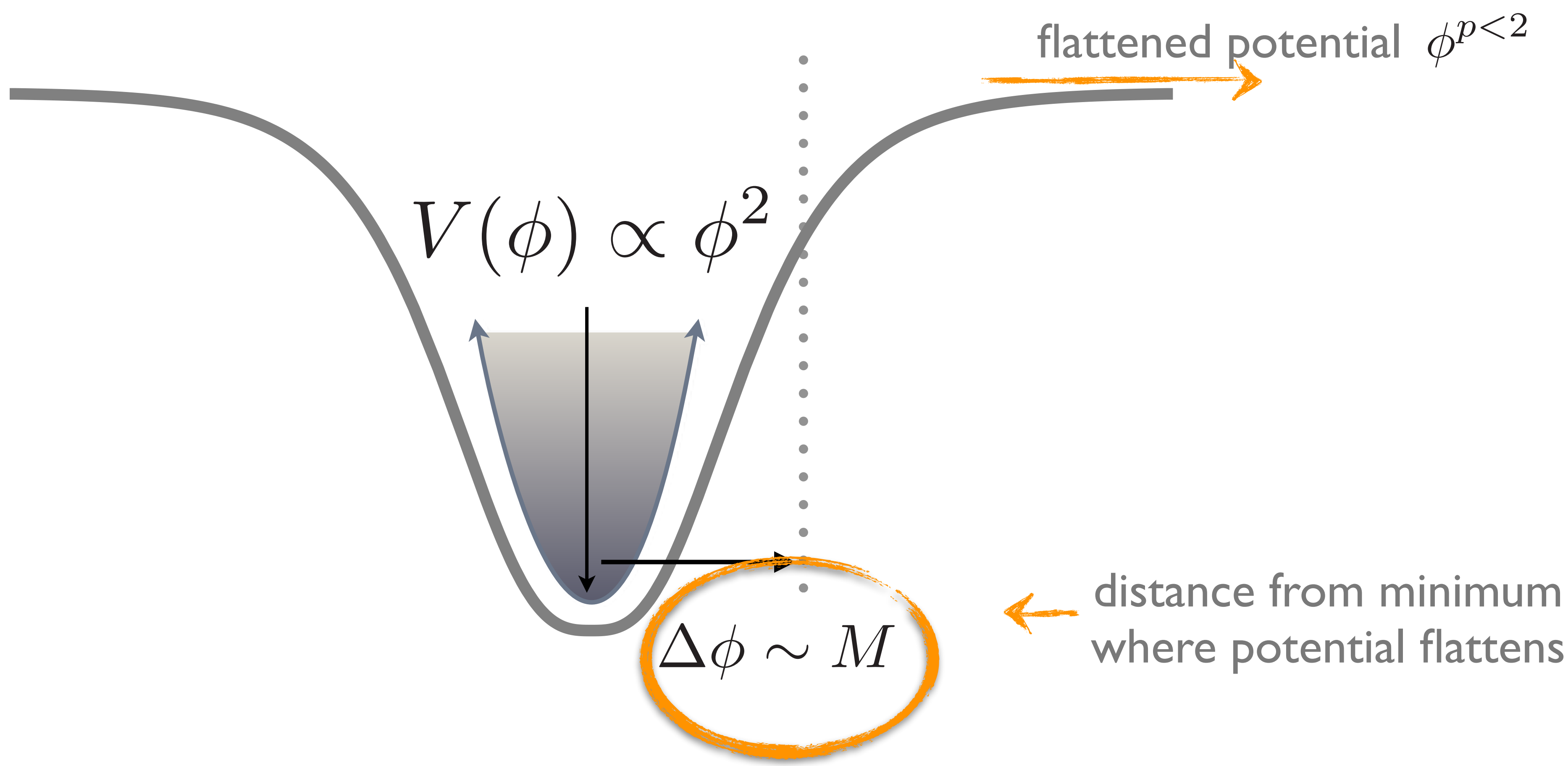


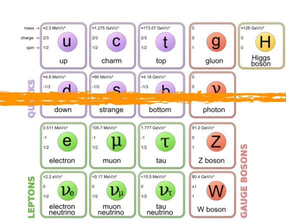
- shape of the potential (self couplings)
- couplings to other fields



$$\chi \;,\psi \; A_\mu$$

end of inflation in “simple” models



- shape of the potential (self couplings)
- ~~couplings to other fields~~  χ , ψ A_μ

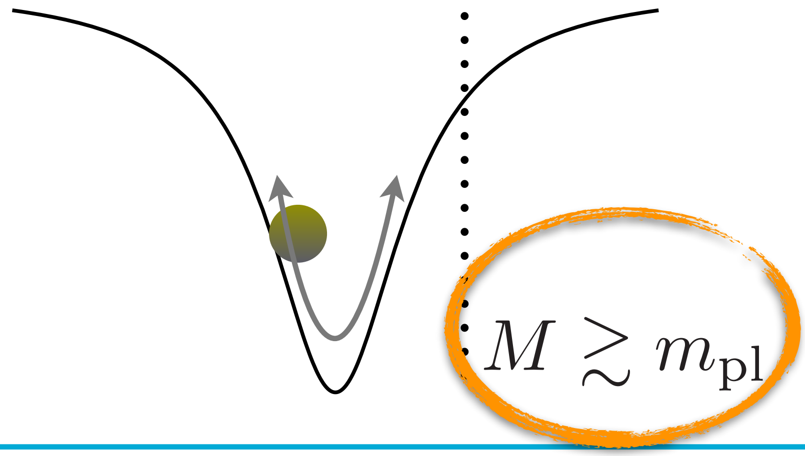
oscillating "free" scalar field - "slow" gravitational instability

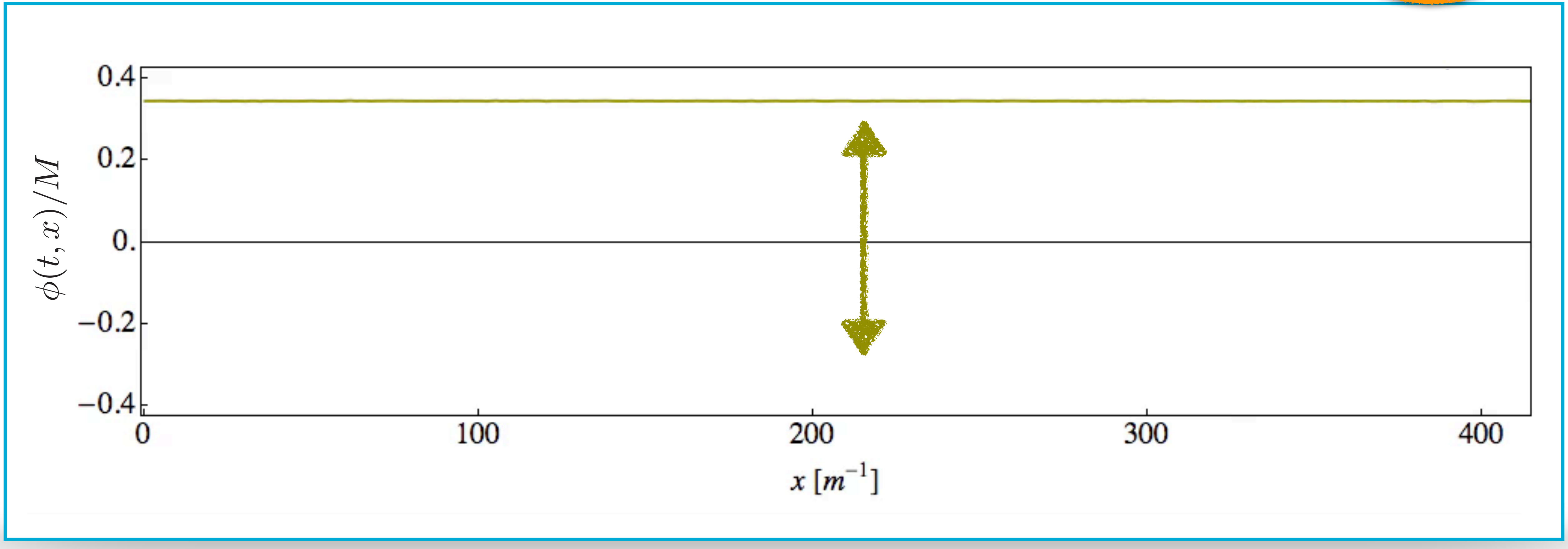
expansion ✓

self-interactions ✗

gravitational int. ✓

$$\Box\phi \approx m^2\phi$$





*similar to a matter dominated universe, see for example: Gilmore, Flauger & Easther (2012), also see Richard's talk

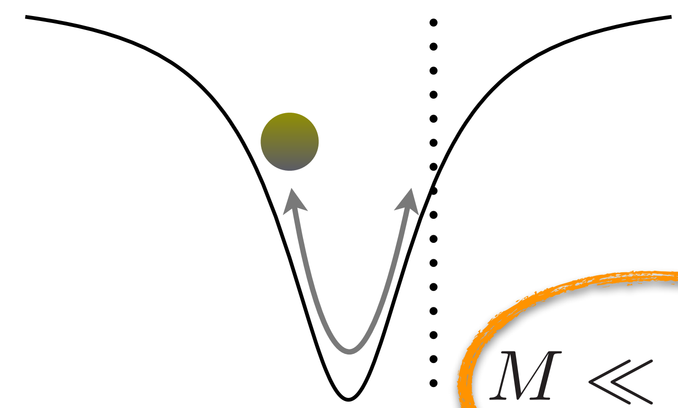
self-interaction instability — "oscillon" formation

expansion ✓

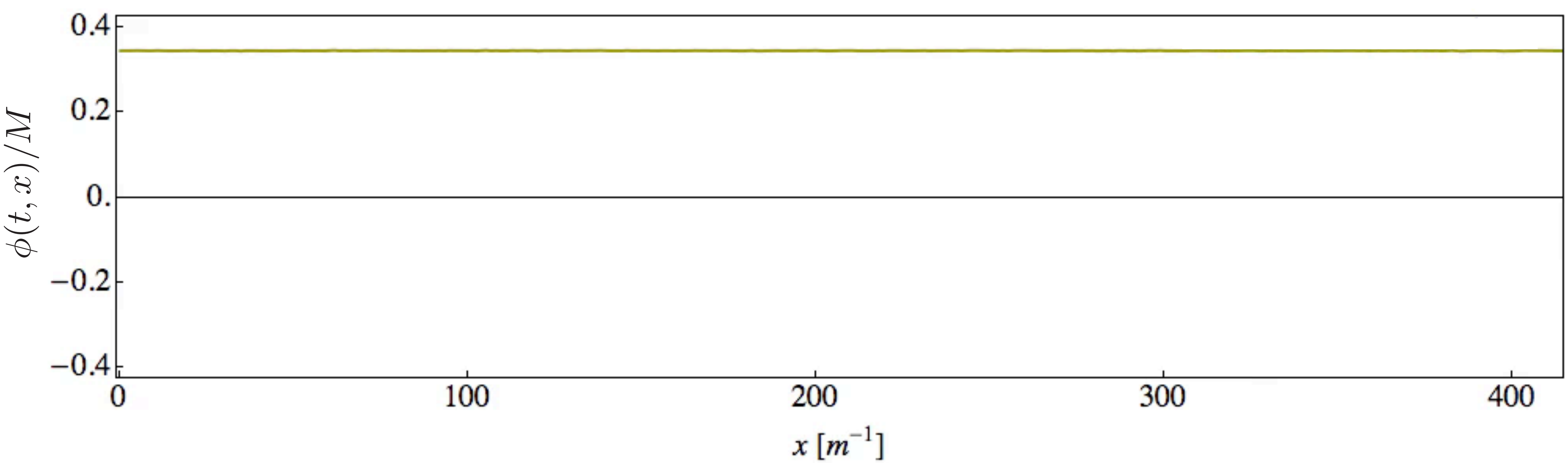
self-interactions ✓

gravitational int. ✗

$$\Box\phi = V'(\phi)$$



$M \ll m_{\rm pl}$



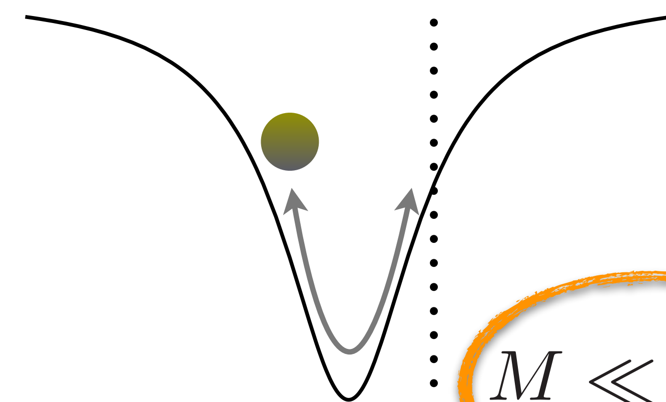
MA (2010) 1006.3075

*without oscillons, but relevant for instabilities, see related (much) earlier work: Khlopov, Malomed & Zeldovich (1985)

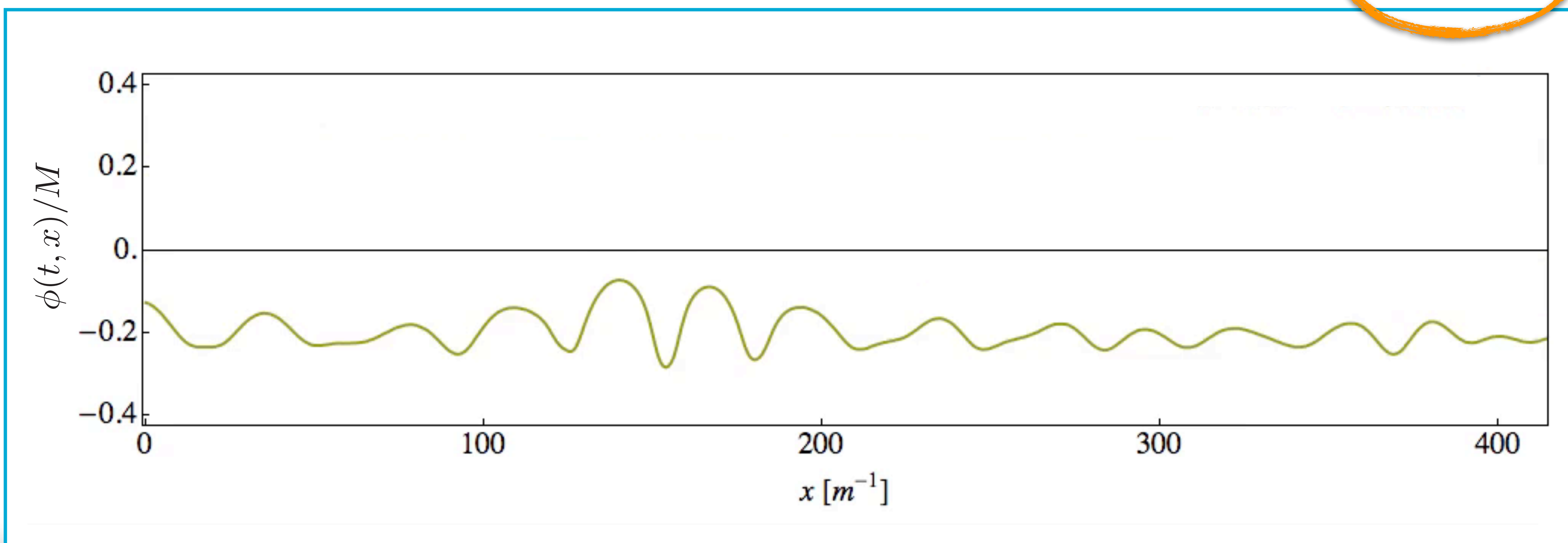
self-interaction instability — “oscillon” formation

- expansion ✓
- self-interactions ✓
- gravitational int. ✗

$$\Box\phi = V'(\phi)$$



$M \ll m_{\rm pl}$



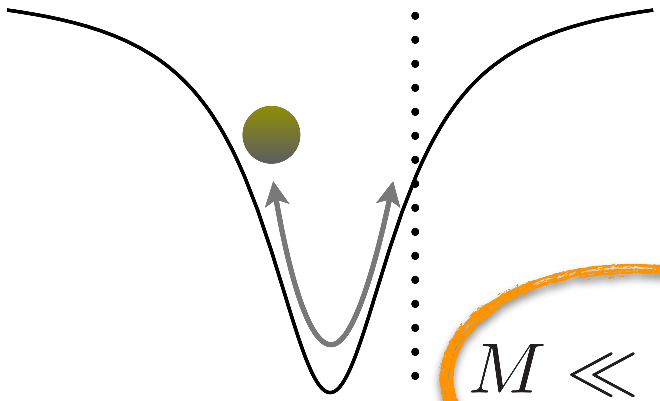
MA (2010) 1006.3075

*without oscillons, but relevant for instabilities, see related (much) earlier work: Khlopov, Malomed & Zeldovich (1985)

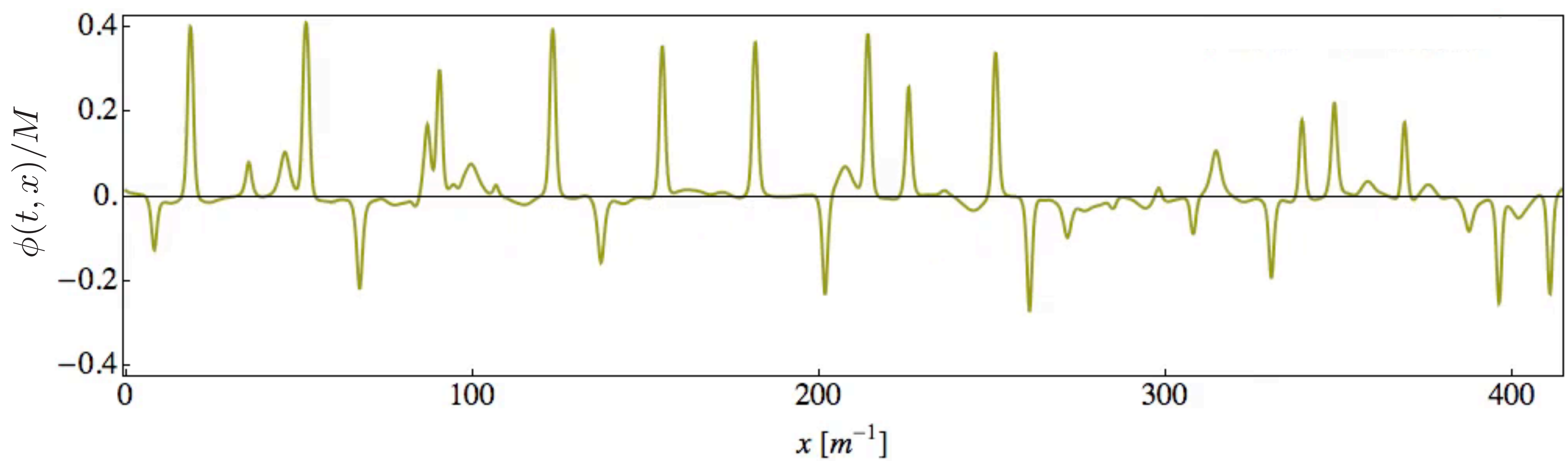
self-interaction instability — "oscillon" formation

- expansion ✓
- self-interactions ✓
- gravitational int. ✗

$$\Box\phi = V'(\phi)$$



$M \ll m_{\rm pl}$



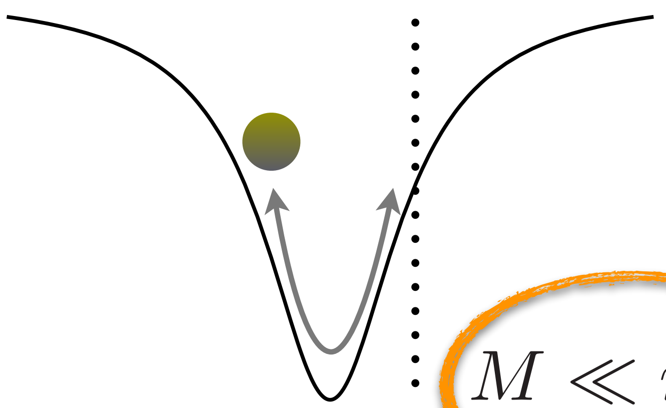
MA (2010) 1006.3075

*without oscillons, but relevant for instabilities, see related (much) earlier work: Khlopov, Malomed & Zeldovich (1985)

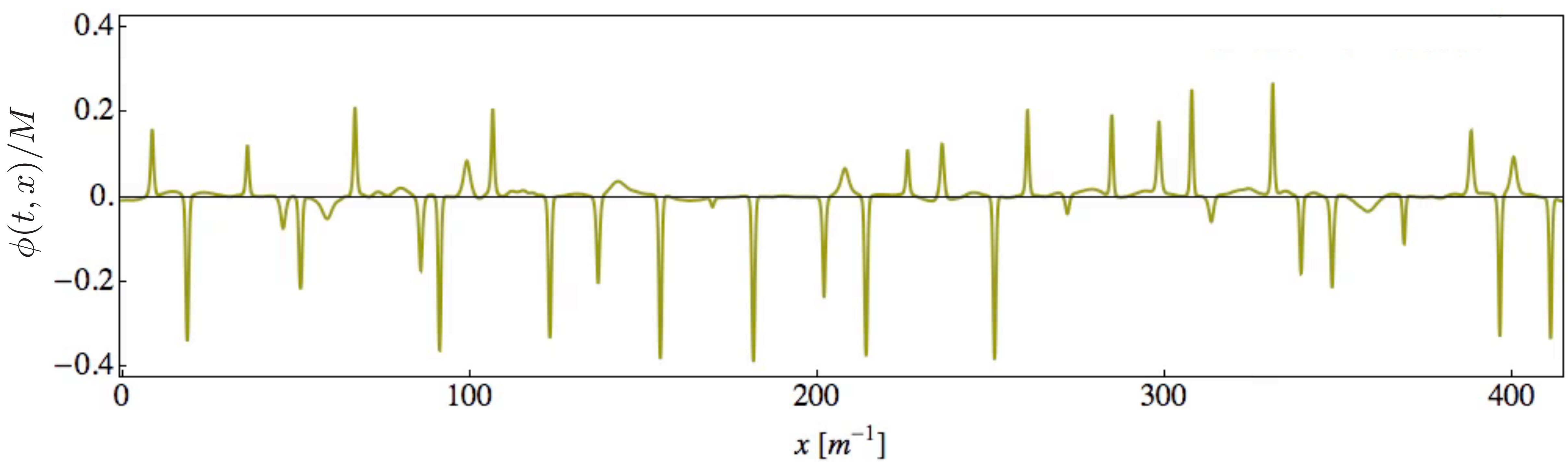
self-interaction instability — "oscillon" formation

- expansion ✓
- self-interactions ✓
- gravitational int. ✗

$$\Box\phi = V'(\phi)$$



$M \ll m_{\rm pl}$



MA (2010) 1006.3075

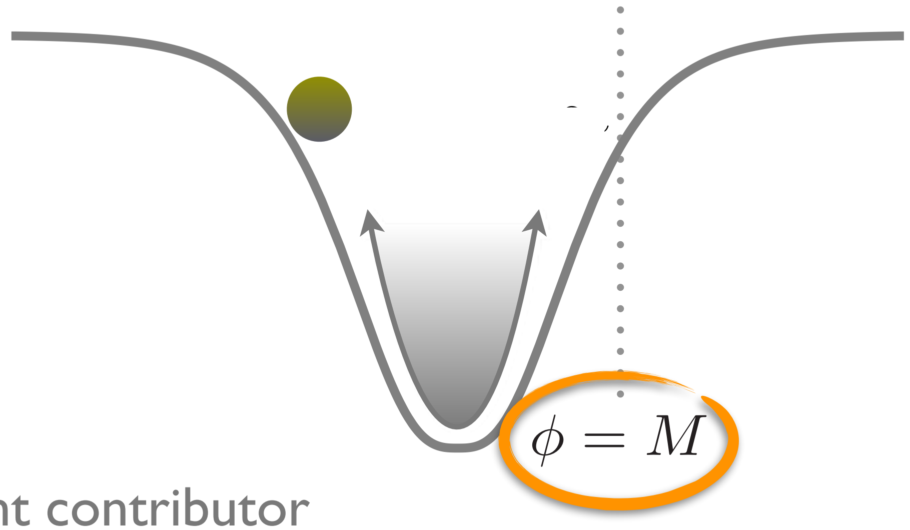
*without oscillons, but relevant for instabilities, see related (much) earlier work: Khlopov, Malomed & Zeldovich (1985)

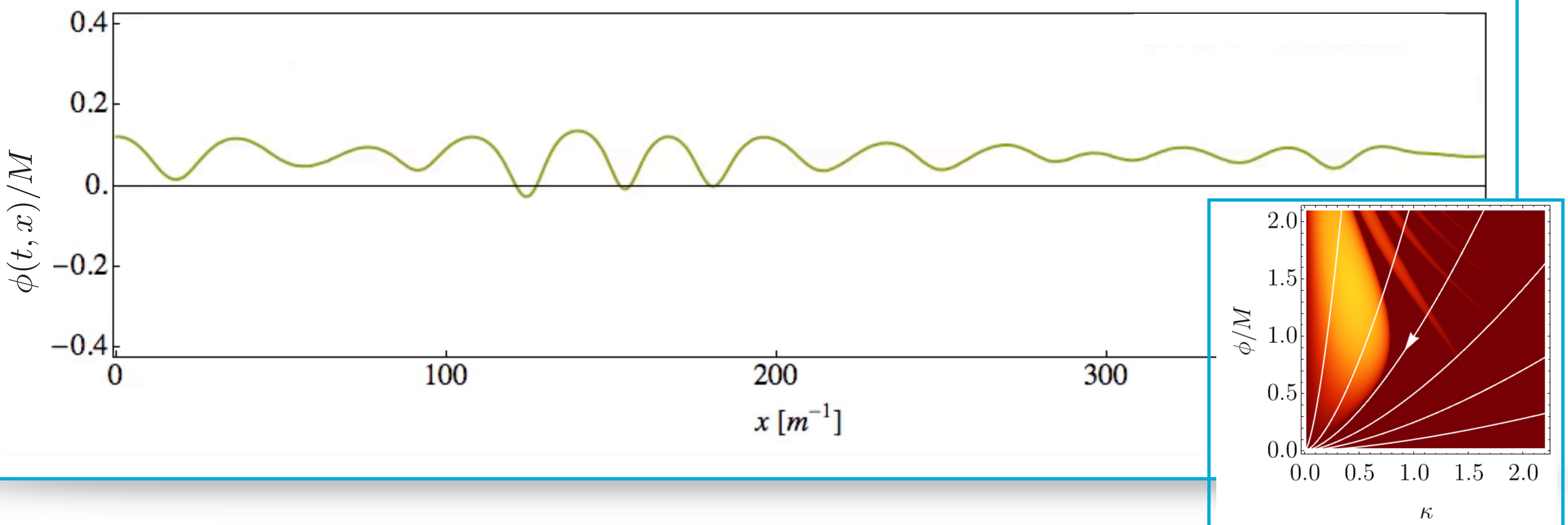
instabilities in an expanding universe

$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} = \frac{\Re[\mu_k]}{H} \sim \frac{m_{\rm pl}}{M} \gg 1$$

general

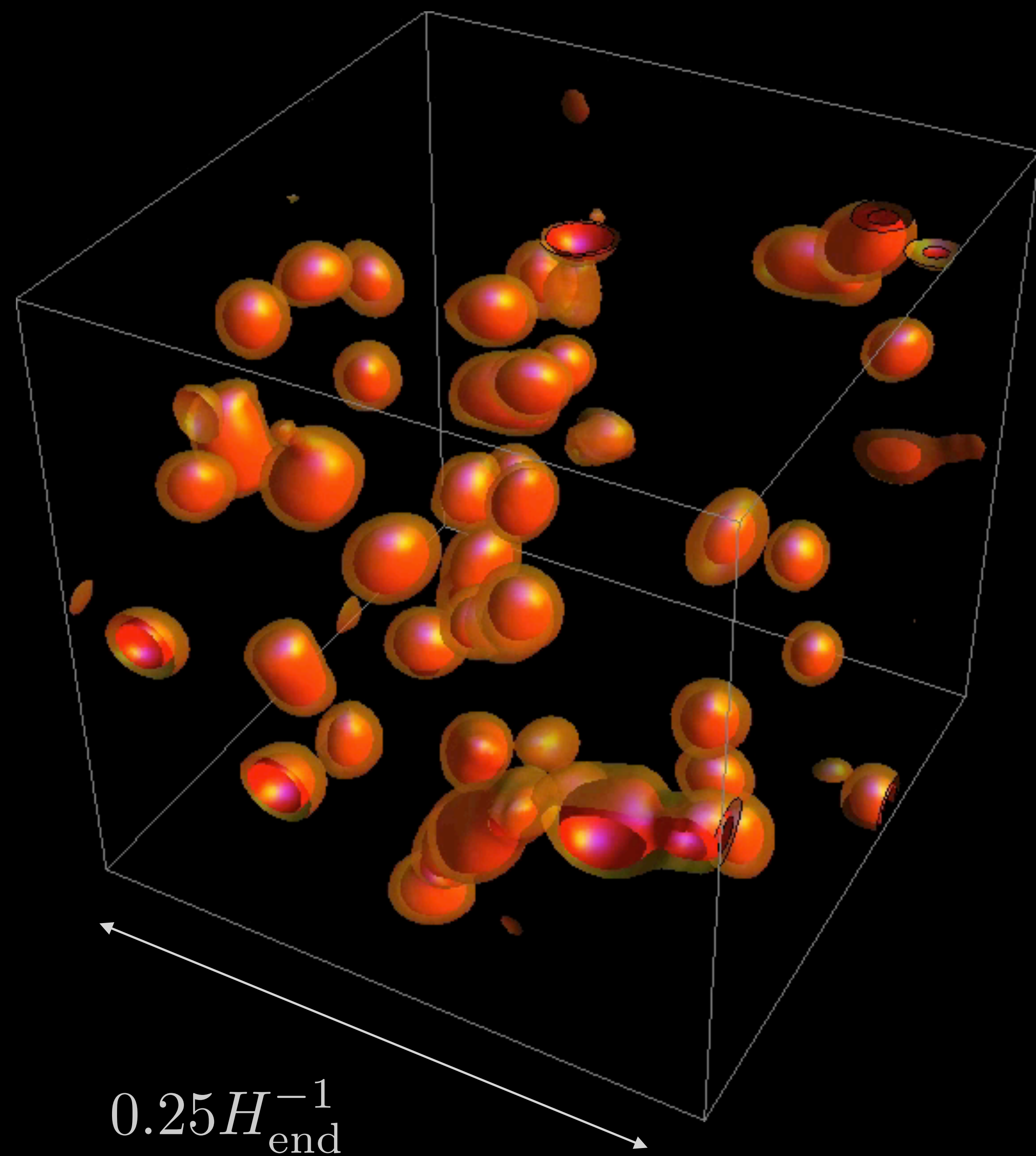
if field is dominant contributor to energy density





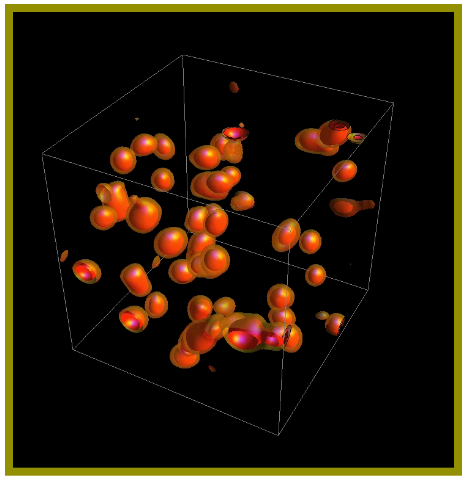
oscillon formation at the end of inflation

- expansion ✓
- self-interactions ✓
- gravitational int. ✗



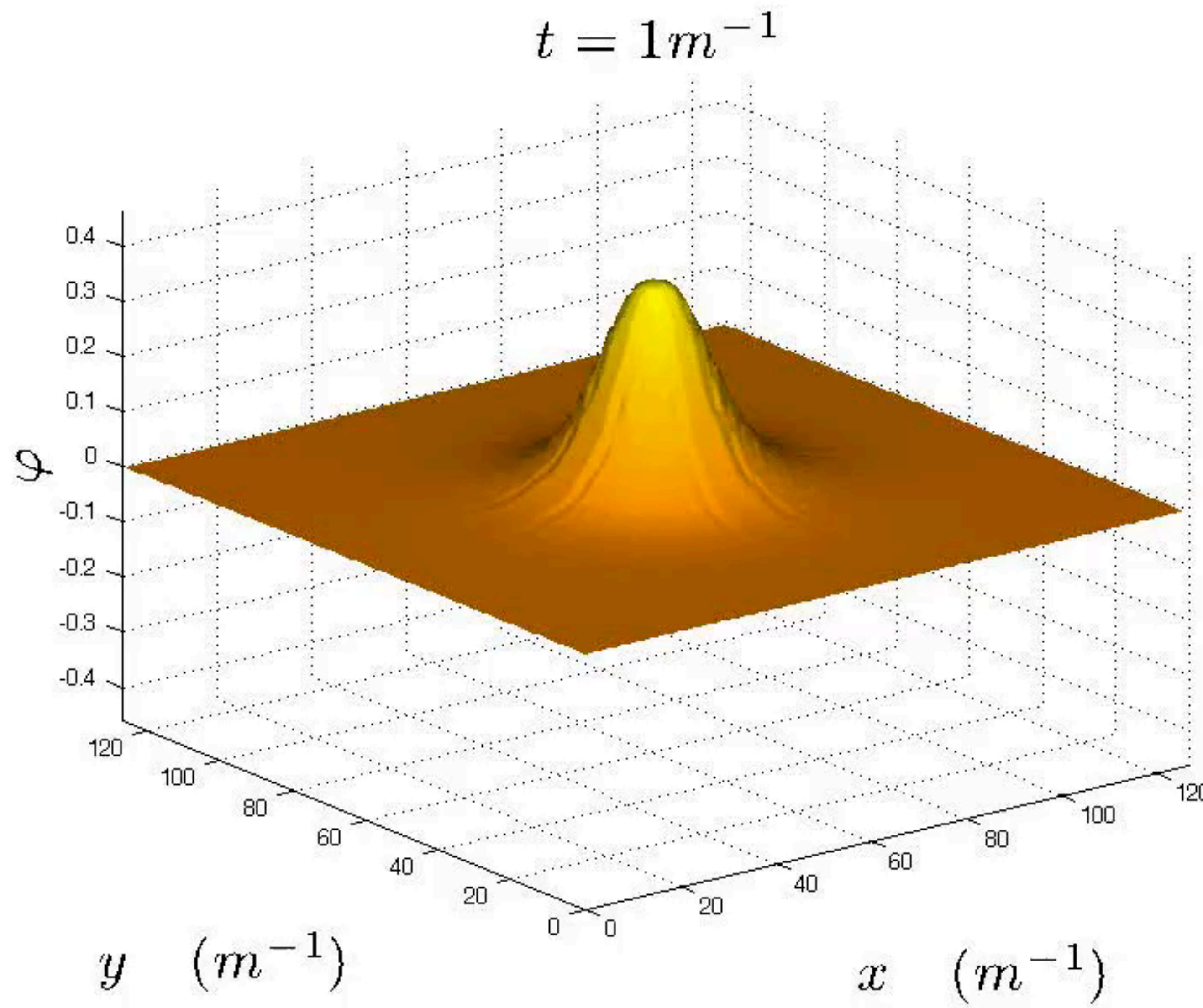
MA, Easther, Finkel, Flauger & Hertzberg (2011)
1106.3335

oscillons?



very inefficient antennas!

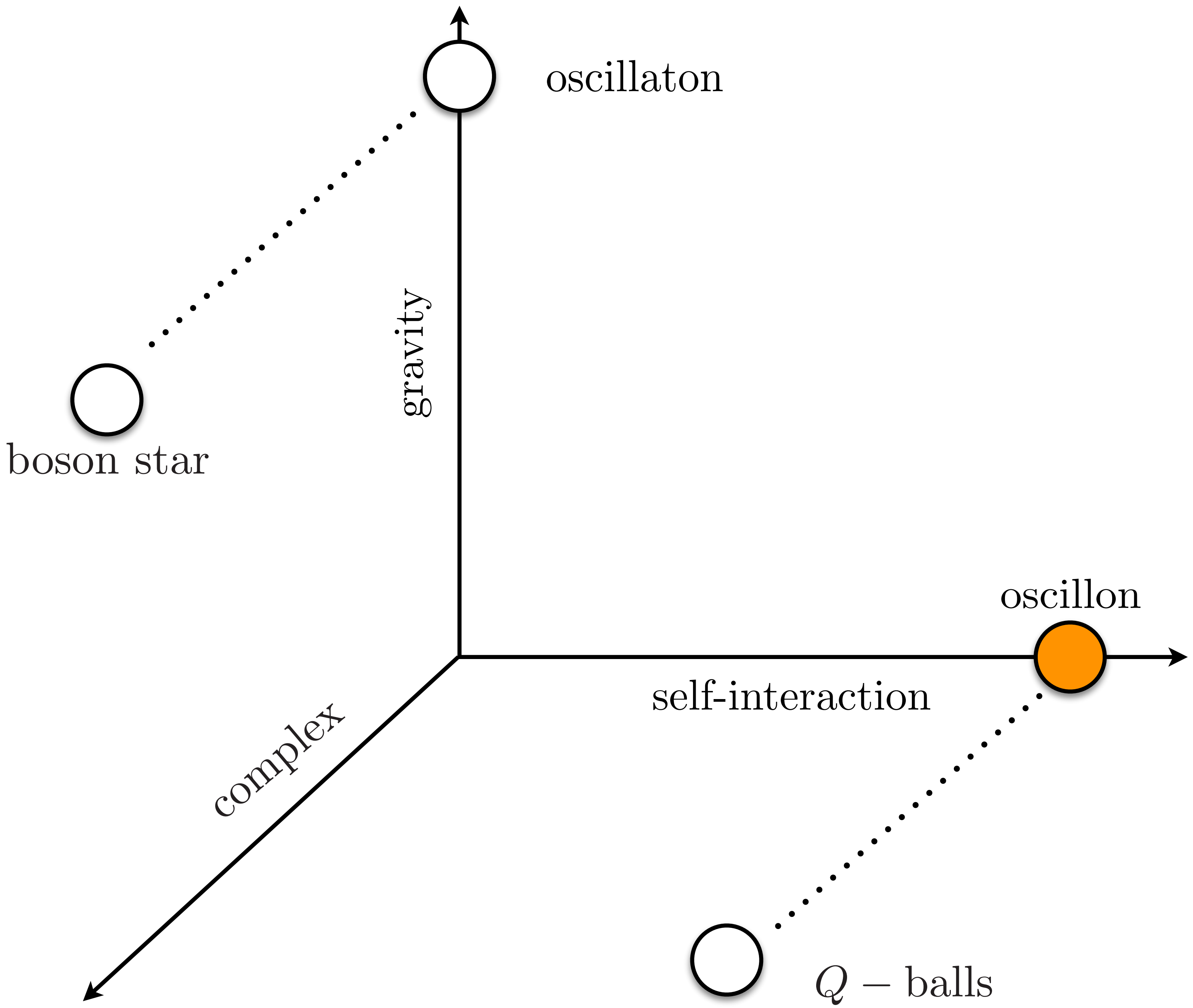
(1) oscillatory (2) spatially localized (3) **very long lived**



For example:
Segur & Kruskal (1987)
MA & Shirokoff (2010) [flat-tops]
Sfakianakis (2012)
Hertzberg (2011)
MA (2013) [non-canonical]
Mukaida et. al (2016)
Salmi & Hindmarsh (2014)
Sakstein & Trodden (2018) [non-canonical]
Antusch, Cefala & Torrenti (2019)
Fodor (2019)

Bogolubsky & Makhankov (1976), Gleiser (1994), Copeland et al. (1995) …

family of scalar field solitons

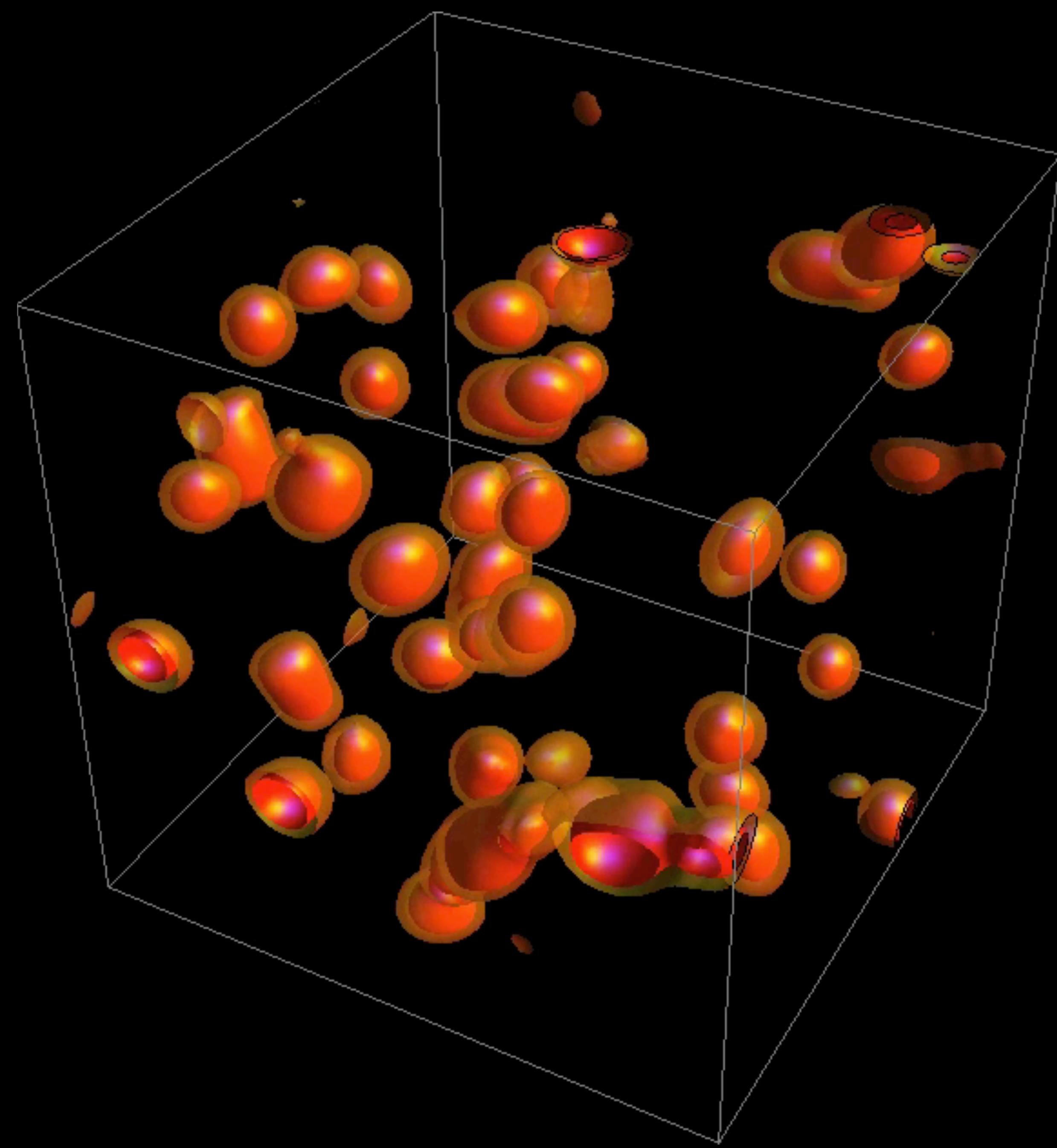


long term dynamics ?

expansion ✓

self-interactions ✓

gravitational int. ?



assuming coupling to other fields is sufficiently weak expansion ✓

self-interactions ✓

gravitational interactions

include gravity ?

- gravitational clustering takes time …
- long time makes it difficult to resolve very fast oscillatory time scale

“non-relativistic” limit

$$\Box\phi + V'(\phi) = 0$$

$$G_{\mu\nu} = \frac{1}{m_{\rm pl}^2} T_{\mu\nu}$$

$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re[e^{-imt}\psi(t,\mathbf{x})]$$

$$\frac{|\nabla|}{m}, \frac{\partial_t}{m} \ll 1$$

$$ds^2 = -(1+2\Phi)dt^2 + a^2(t)(1-2\Phi)d\mathbf{x}^2$$

$$|\Phi| \ll 1$$

↓ non-linear Schrodinger eq.

↓ Poisson eq. + Friedmann eq.

$$V(\phi) = \frac{1}{2}m^2\phi^2 + V_{\rm nl}(\phi)$$

* this is a bit non-rigorous

non-relativistic case

$$\left[i\left(\partial_t + \frac{3}{2}H\right) + \frac{1}{2a^2}\nabla^2 - U'_{\rm nl}(|\psi|^2) - \Phi\right]\psi = 0\,,$$

nonlinear Schrodinger eq.

$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re\left[e^{-imt}\psi(t,\mathbf{x})\right]$$

non-relativistic case

$$\left[i\left(\partial_t + \frac{3}{2}H\right) + \frac{1}{2a^2}\nabla^2 - U'_{\rm nl}(|\psi|^2) - \Phi\right]\psi = 0\,,$$

nonlinear Schrodinger eq.

$$\frac{\nabla^2}{a^2}\Phi = \frac{\beta^2}{2}\left[|\psi|^2 + \frac{1}{2a^2}|\nabla\psi|^2 + U_{\rm nl}(|\psi|^2)\right] - \frac{3}{2}H^2\,,$$

Poisson eq.

$$H^2 = \frac{\beta^2}{3}\overline{\left[|\psi|^2 + \frac{1}{2a^2}|\nabla\psi|^2 + U_{\rm nl}(|\psi|^2)\right]}\,,$$

Friedmann eq.

$$mx^\mu \rightarrow x^\mu$$

length/time units

$$\frac{\psi}{mM} \rightarrow \psi$$

non-linearity

$$\beta \equiv \frac{M}{m_{\rm pl}}$$

Schrodinger Poisson code: Mocz et. al. (2017)

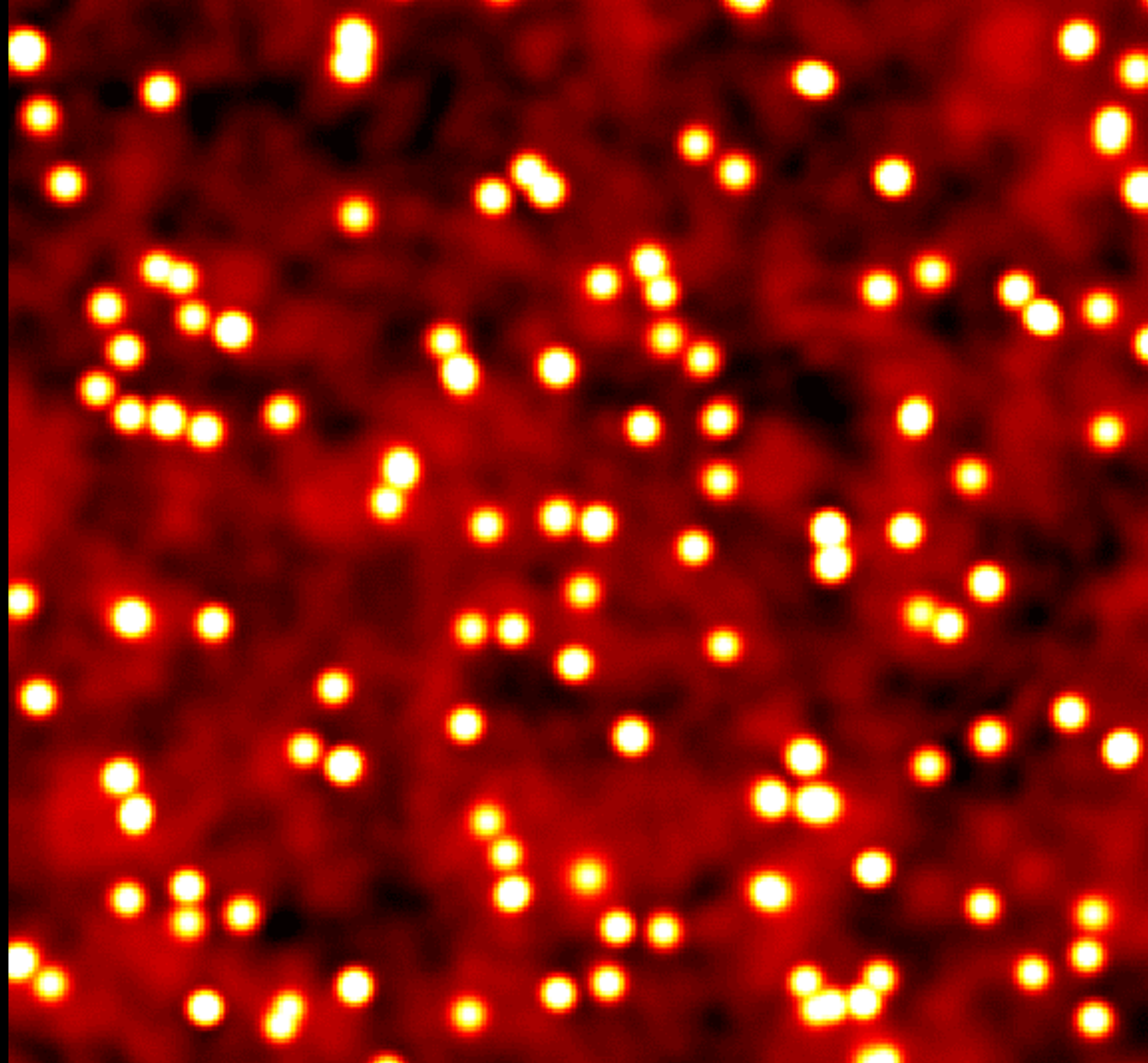
$$\phi(t,\mathbf{x}) = \frac{\sqrt{2}}{m}\Re\left[e^{-imt}\psi(t,\mathbf{x})\right]$$

self-interactions
+ gravity*
(Schrodinger-Poisson)

MA & Mocz (2019) 1902.07261

- expansion ✓
- self-interactions ✓
- gravitational int. ✓
- relativistic? ✗

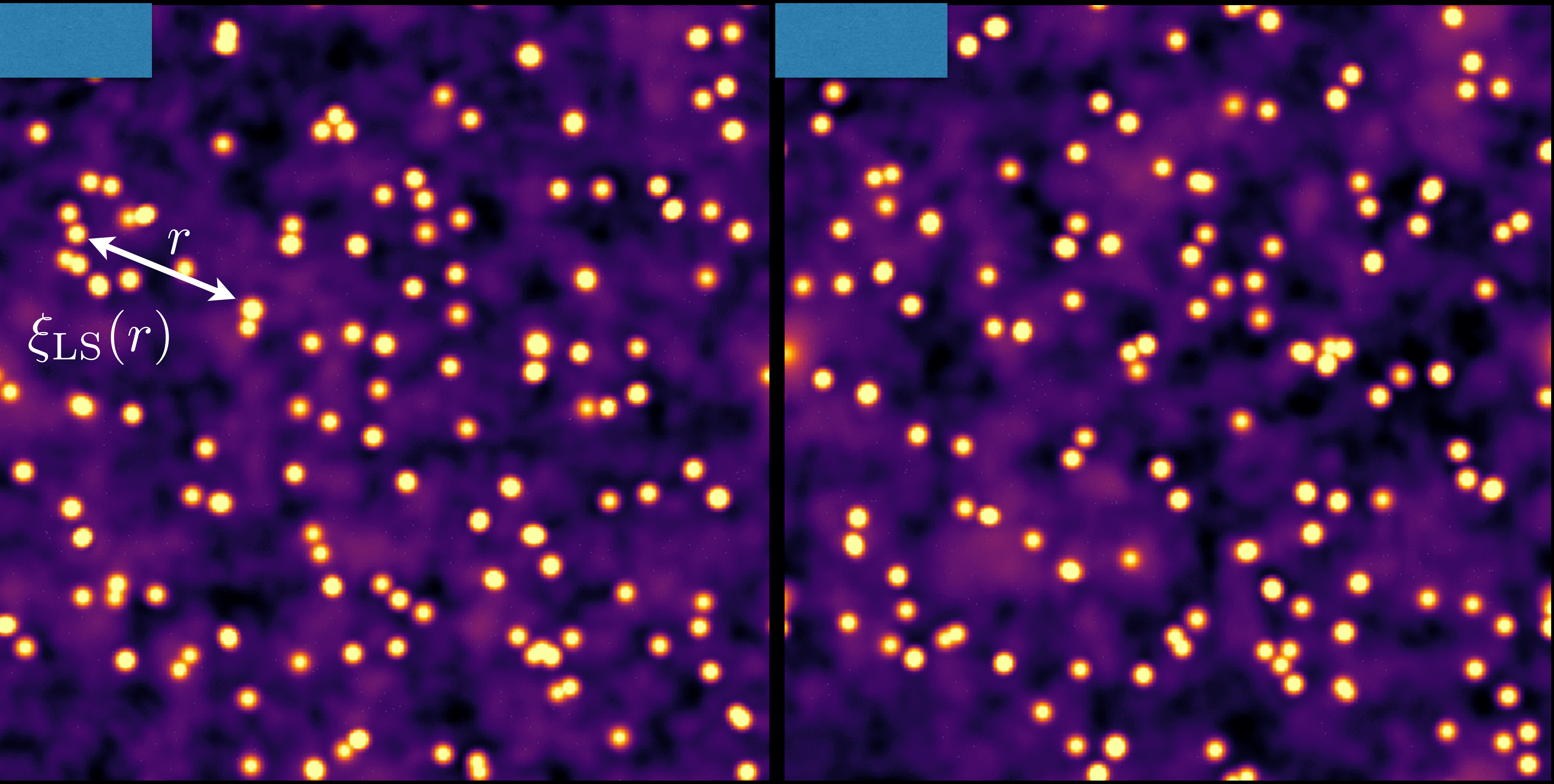
for "passive gravity" case, see Lozanov & MA (2019) 1902.06736



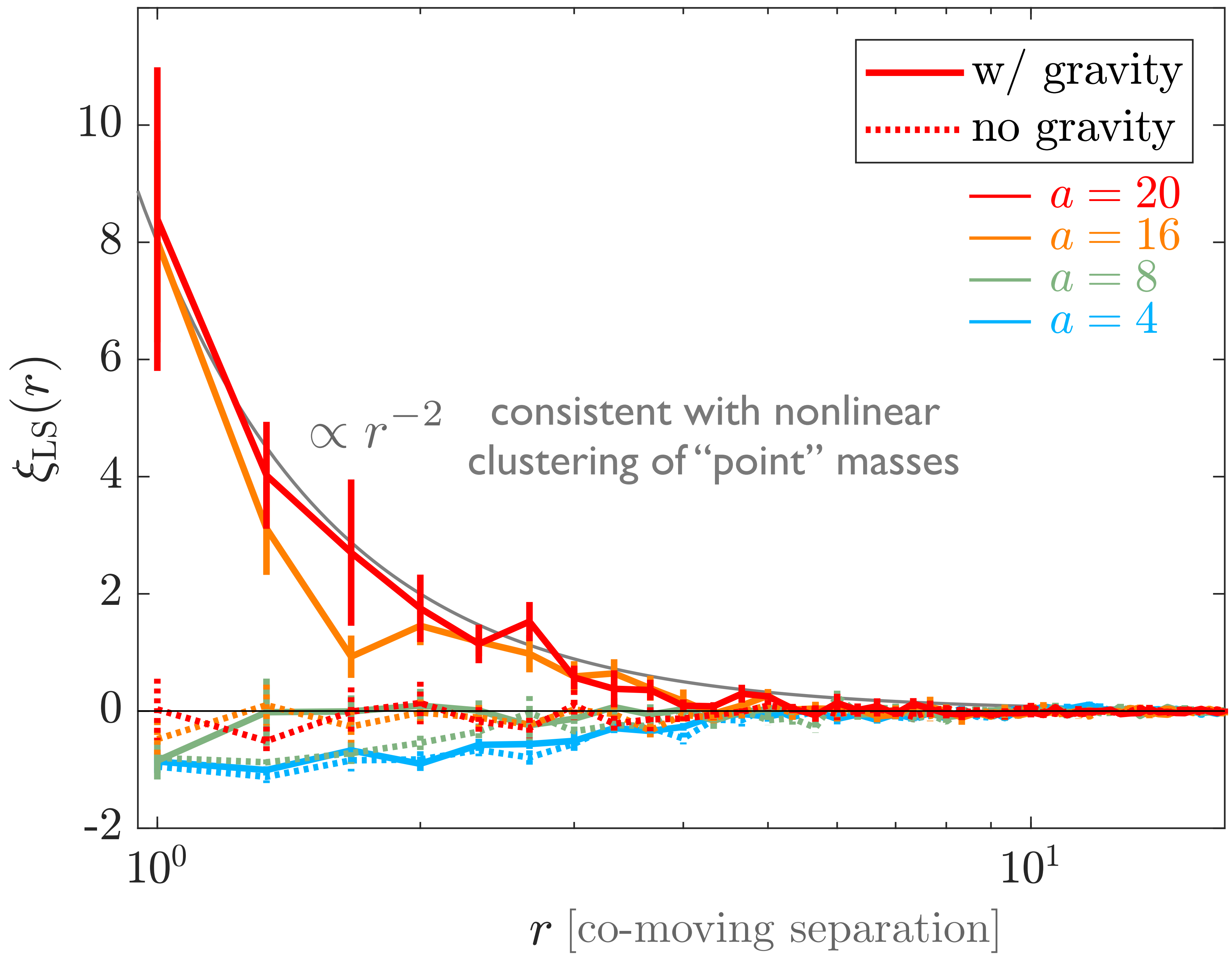
*self interaction more important than gravity initially

**there are a number of caveats in the non-relativistic approximations

gravitational clustering of solitons



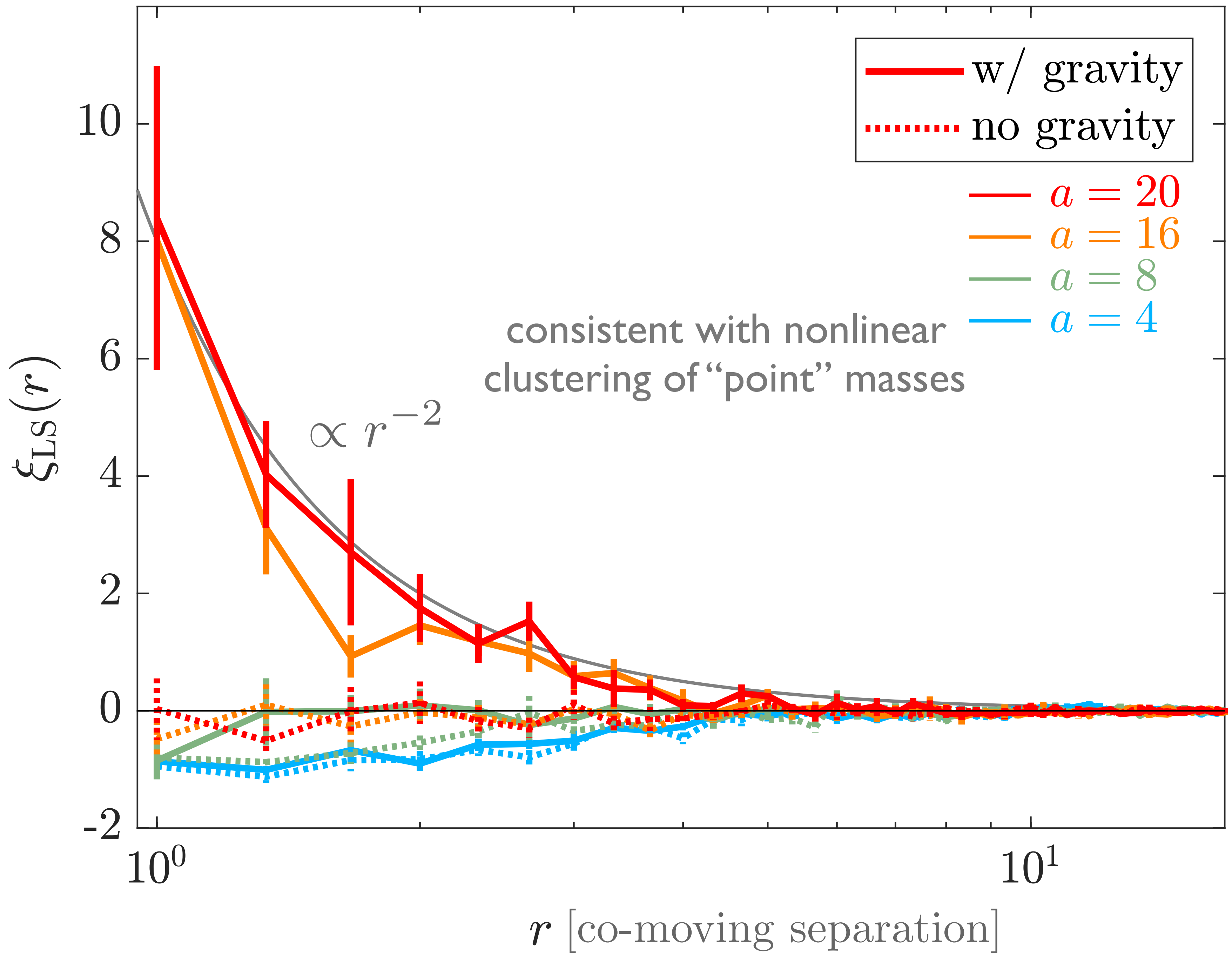
gravitational clustering of solitons



MA & Mocz (2019) 1902.07261

gravitational clustering of solitons

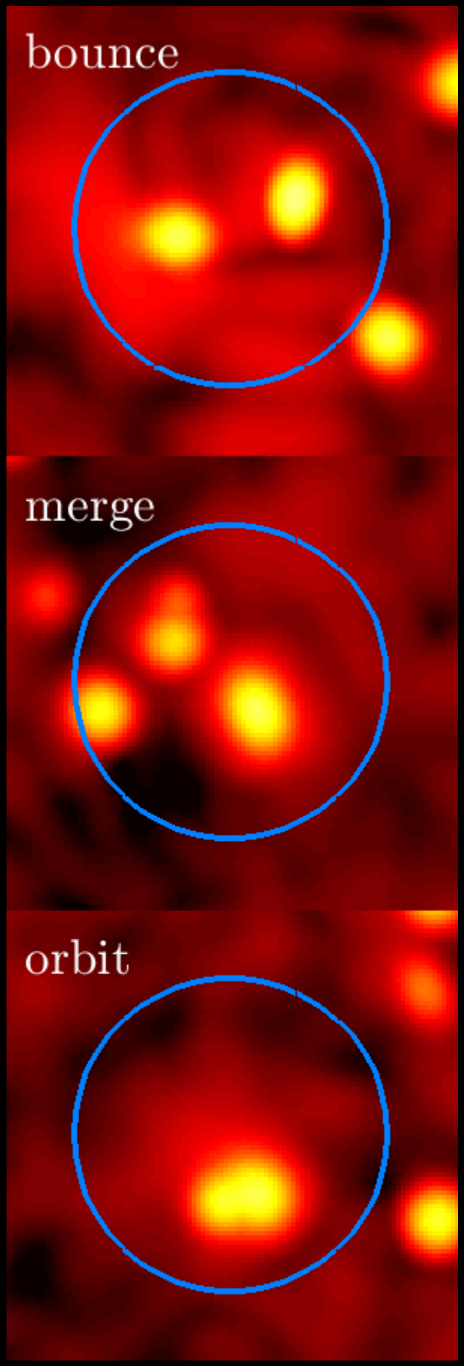
Caution: we don't fully understand (some) velocities of solitons (work in progress)

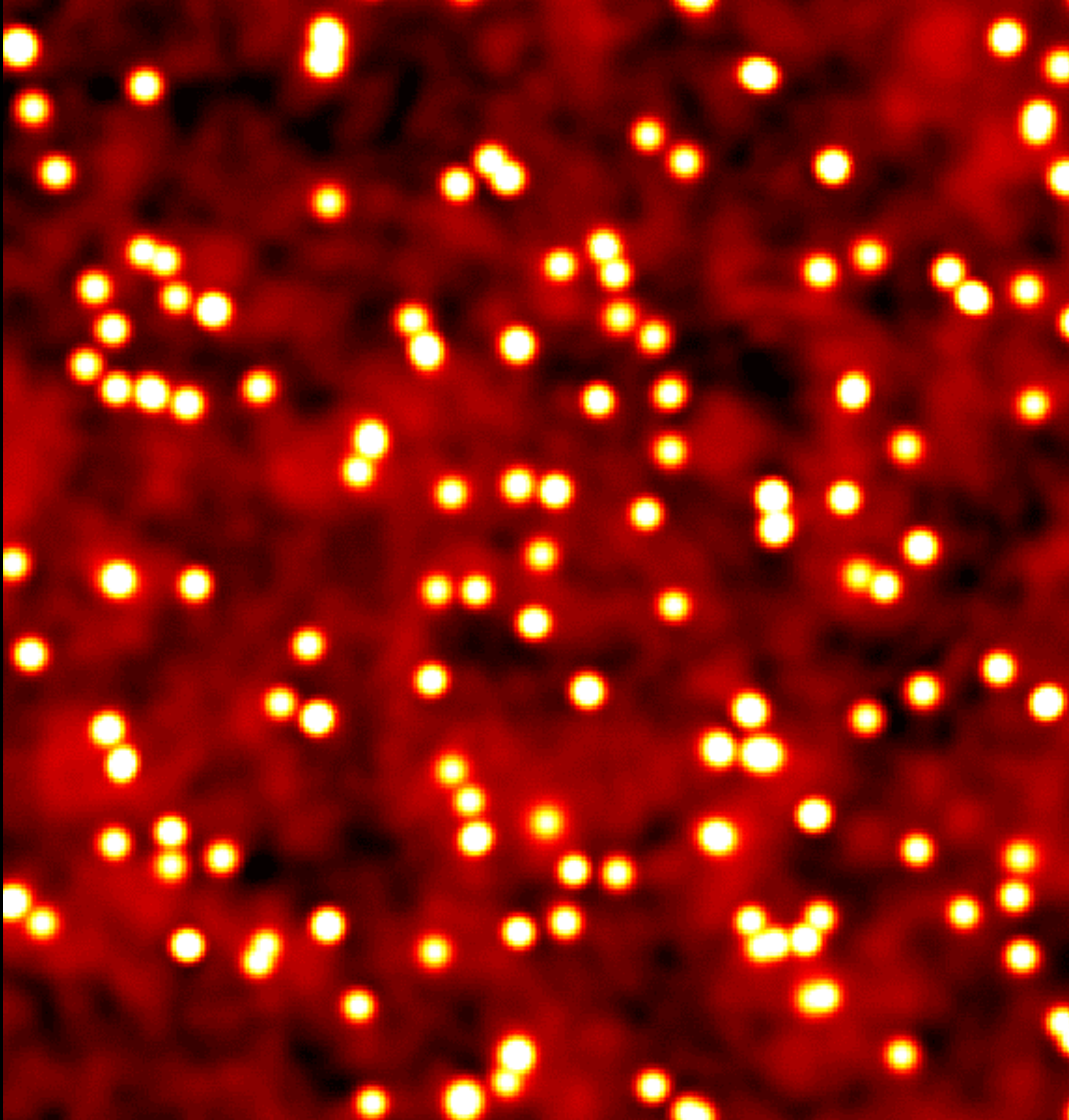


self-interactions

+ gravity*

(Schrodinger-Poisson)



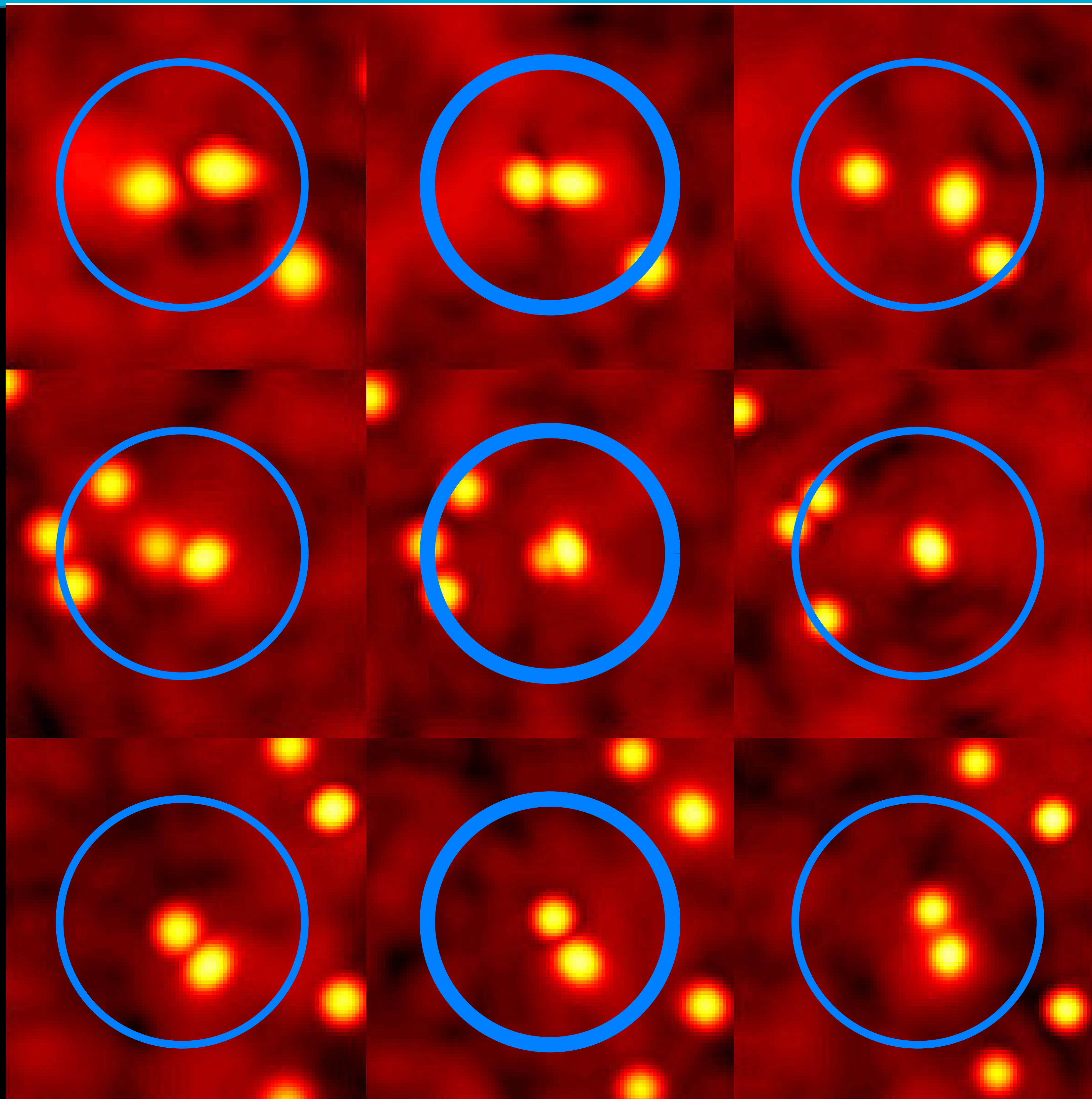


phase dependent interactions

bounce

merger

“binary”



$\phi \propto \Re[\psi]$

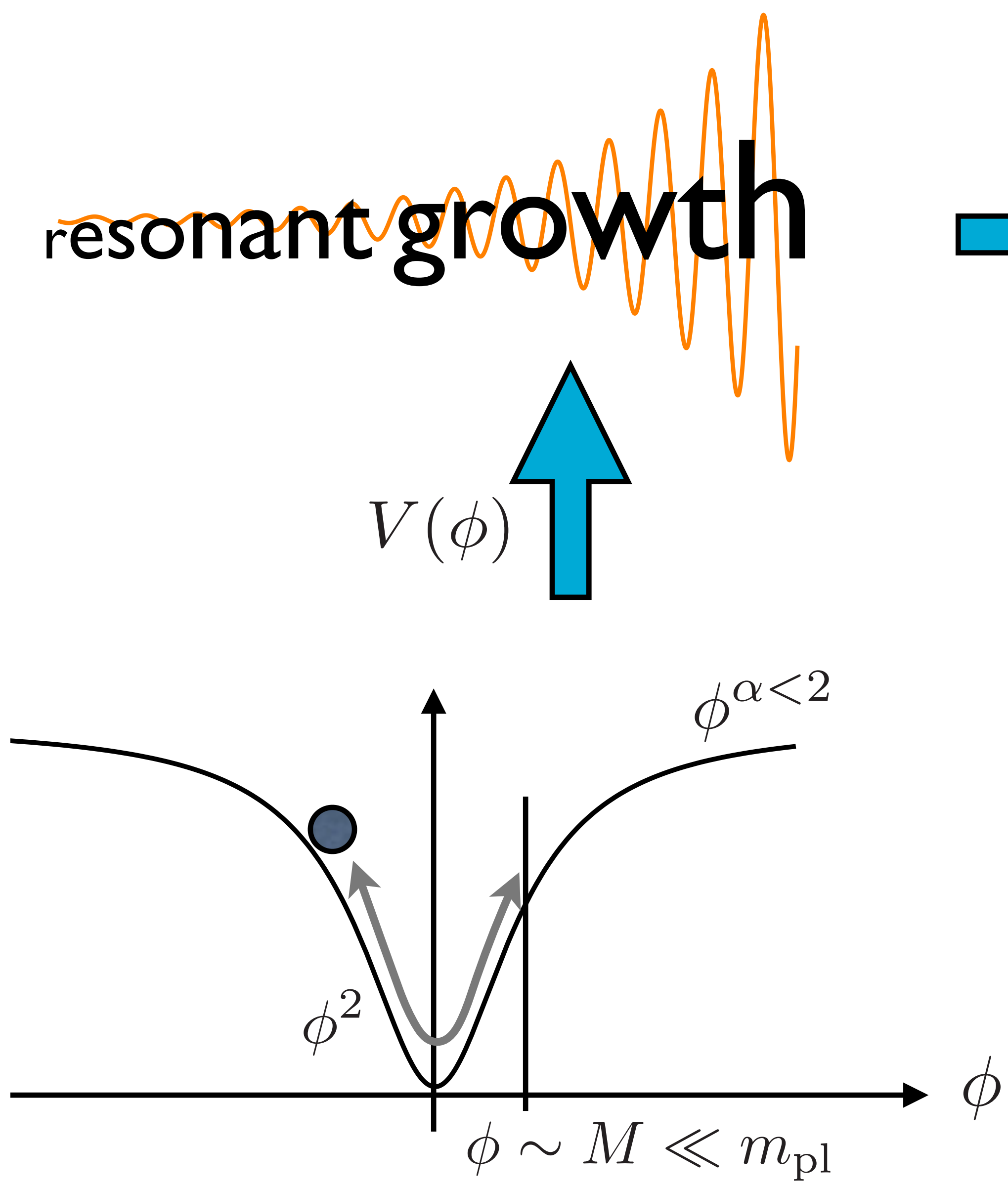
$\psi_a(t, \mathbf{x}) = \Psi_a(\mathbf{x}) e^{-i\nu_a t + \theta_a}$

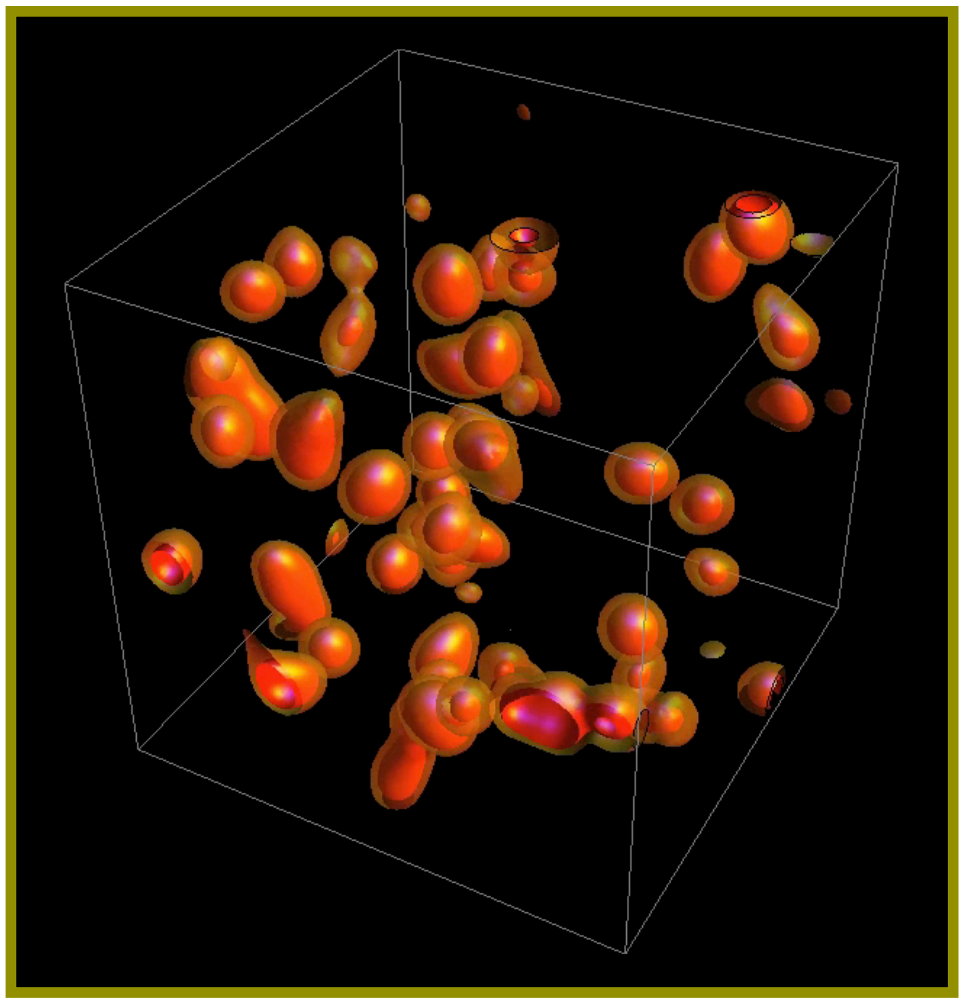
$|\theta_1 - \theta_2| \simeq \pi$

$|\theta_1 - \theta_2| \simeq 0$

EFT of non-topological solitons
— MA & Iqbal (in progress)

summary I



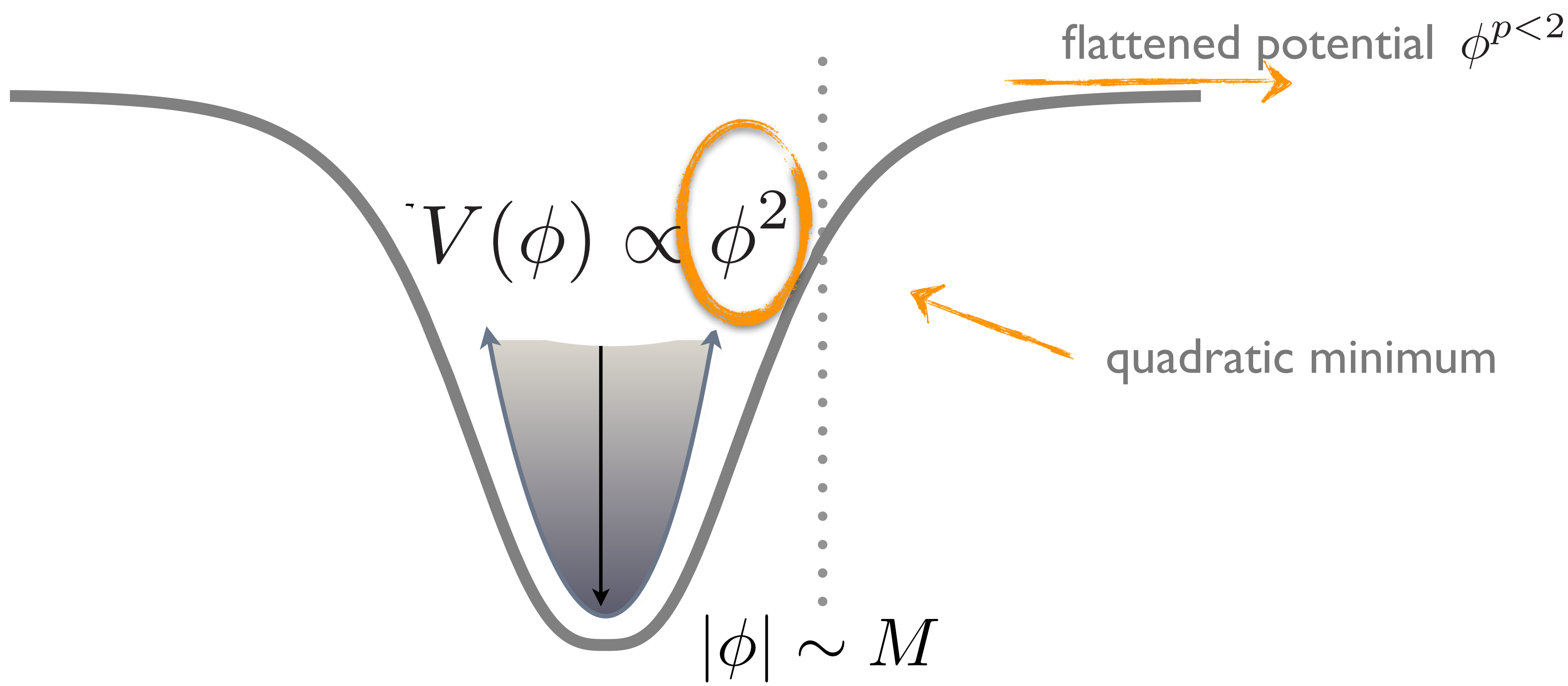


1. oscillons dominate the energy density

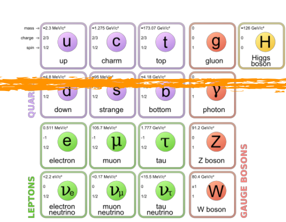
2. they cluster gravitationally

3. can undergo complex scattering

so far, quadratic minima with wings …

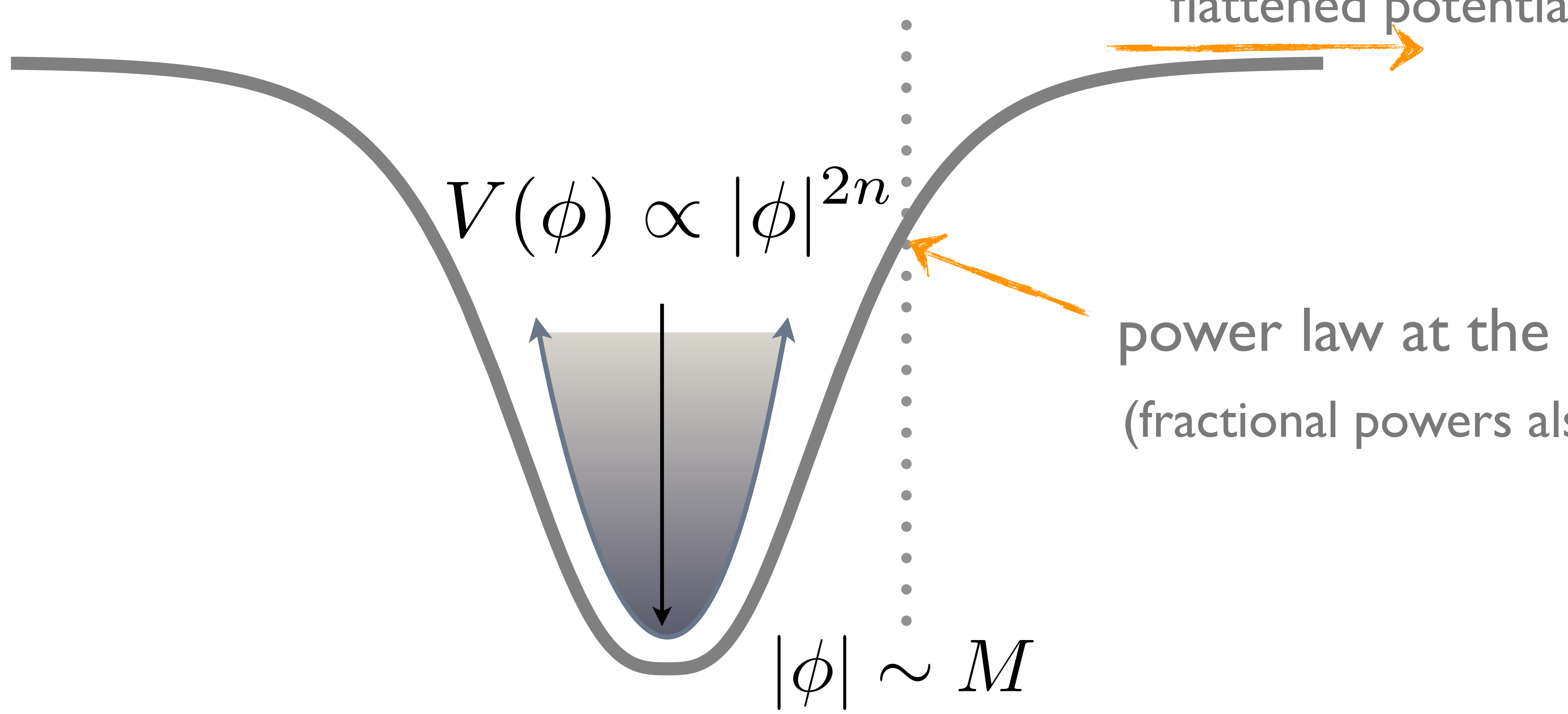


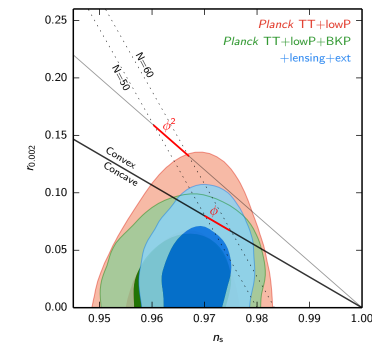
- shape of the potential (self couplings)
- ~~couplings to other fields~~



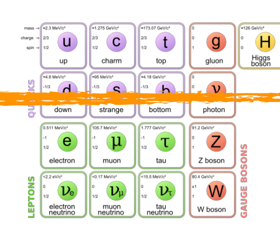
$\chi\ ,\psi$

non-quadratic, power-law minima ?





- shape of the potential (self couplings)
- ~~couplings to other fields~~



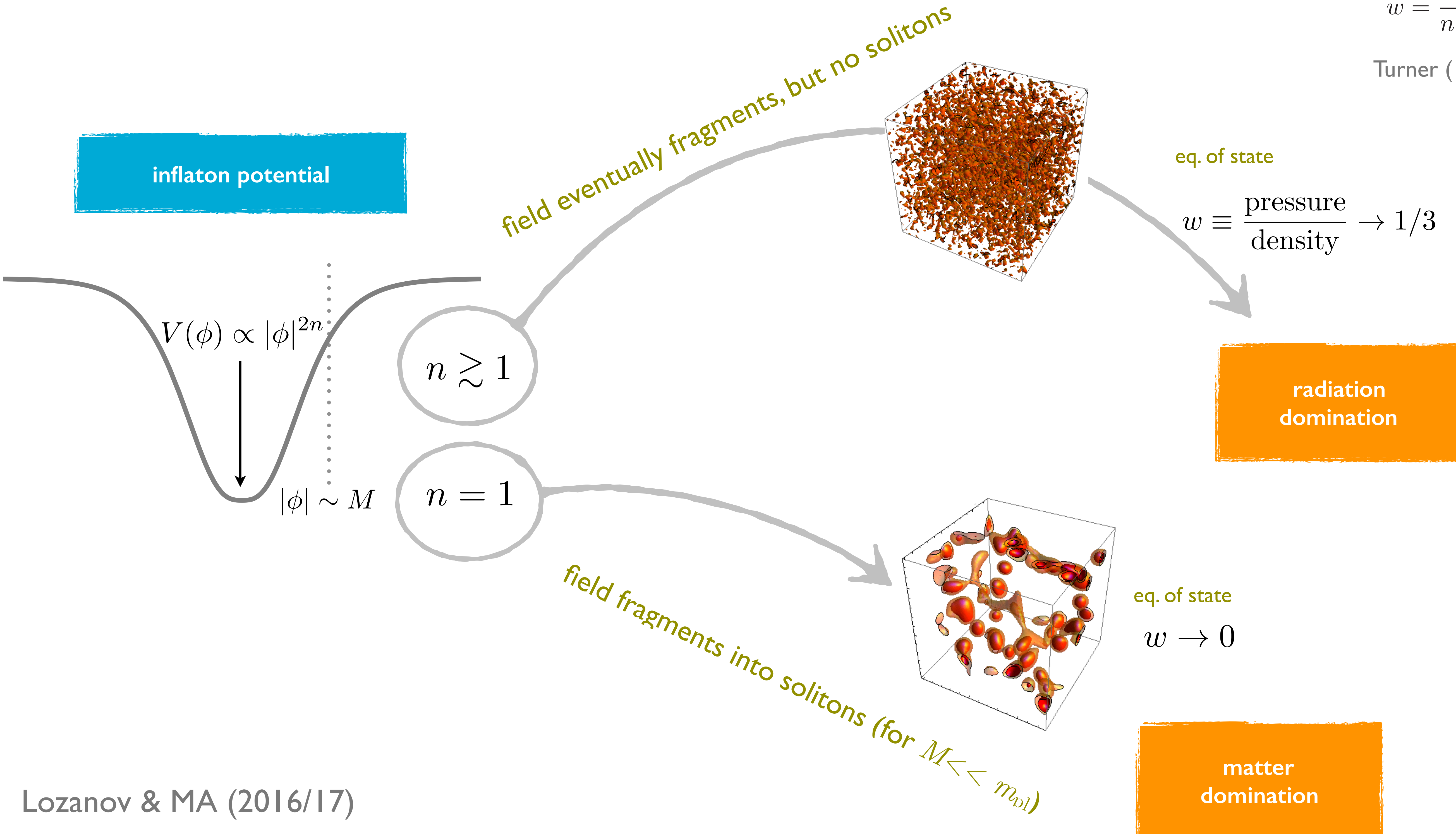
~~$\chi\ ,\psi$~~

dynamics in different power law minima

Homogeneous oscillations

$$w = \frac{n-1}{n+1}$$

Turner (1983)



Lozanov & MA (2016/17)

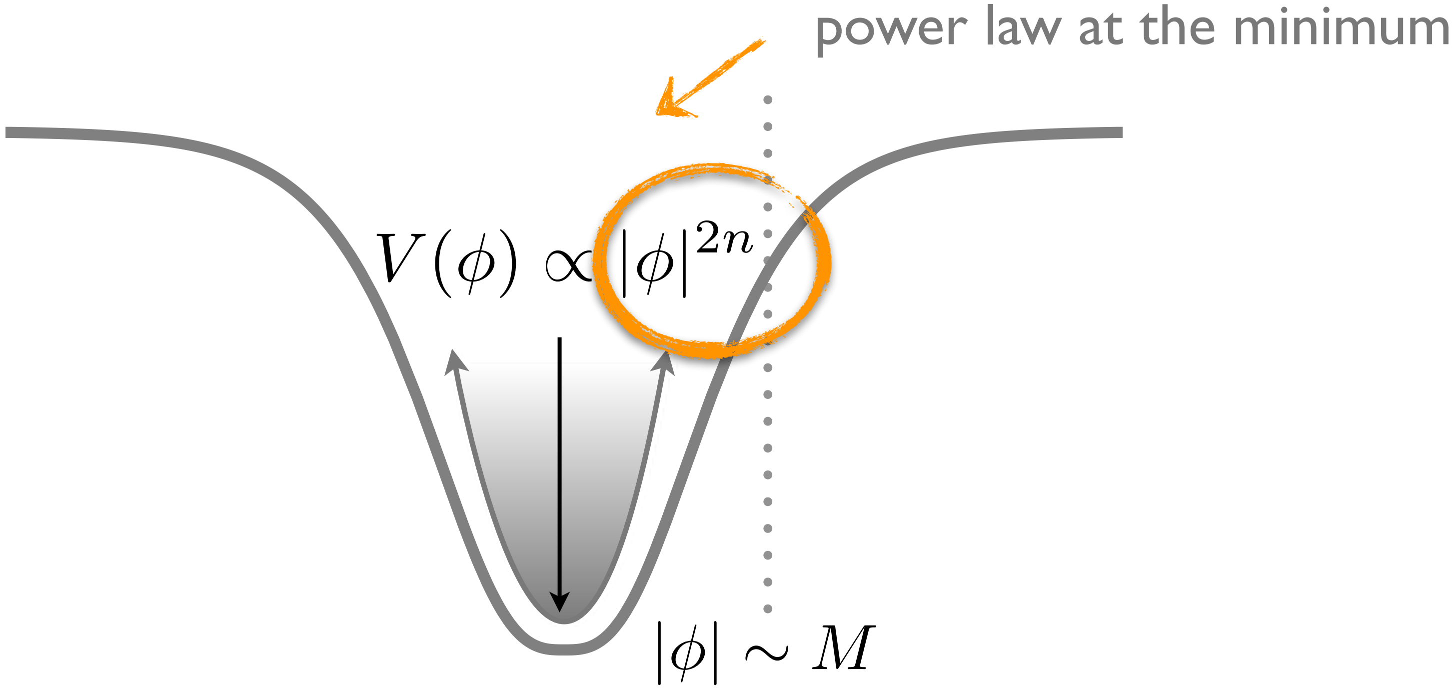
1608.01213, 1710.06851

"equation-of-state" from oscillating fields

the *spatially averaged* equation-of-state of fields

- $(n = 1)$ quadratic minima $w = 0$
- $(n > 1)$ non-quadratic minima $w = 1/3$ (after sufficient time)

why? $\mu_k/H \propto \phi^{-1}$



Lozanov & MA (2016/17)

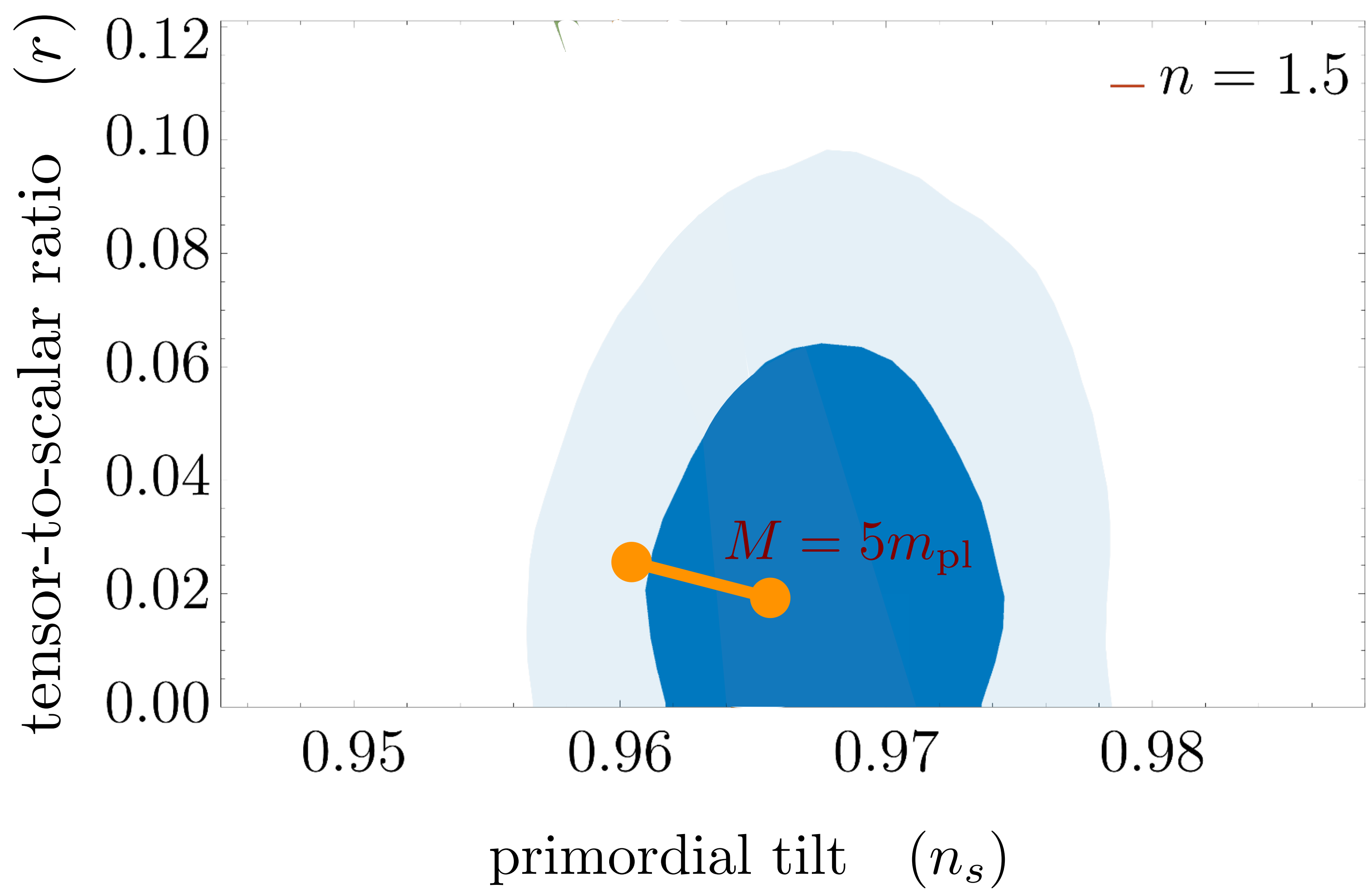
1608.01213, 1710.06851

Implications

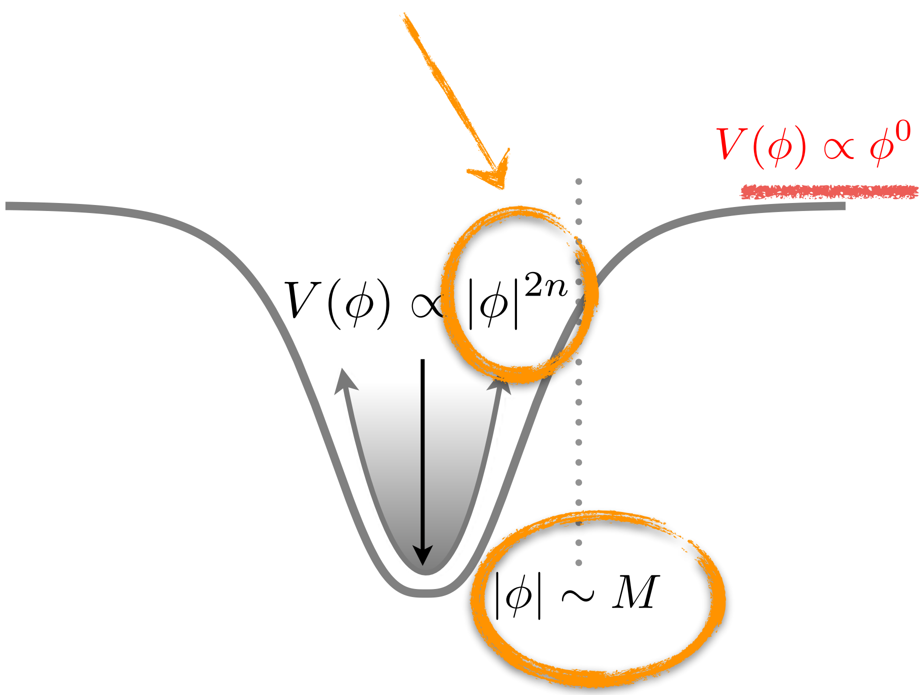
- gravitational
  - homogeneous
  - inhomogeneous

- non-gravitational (typically more fields needed, and more model dependent)

eq. of state & CMB observables



power law at the minimum



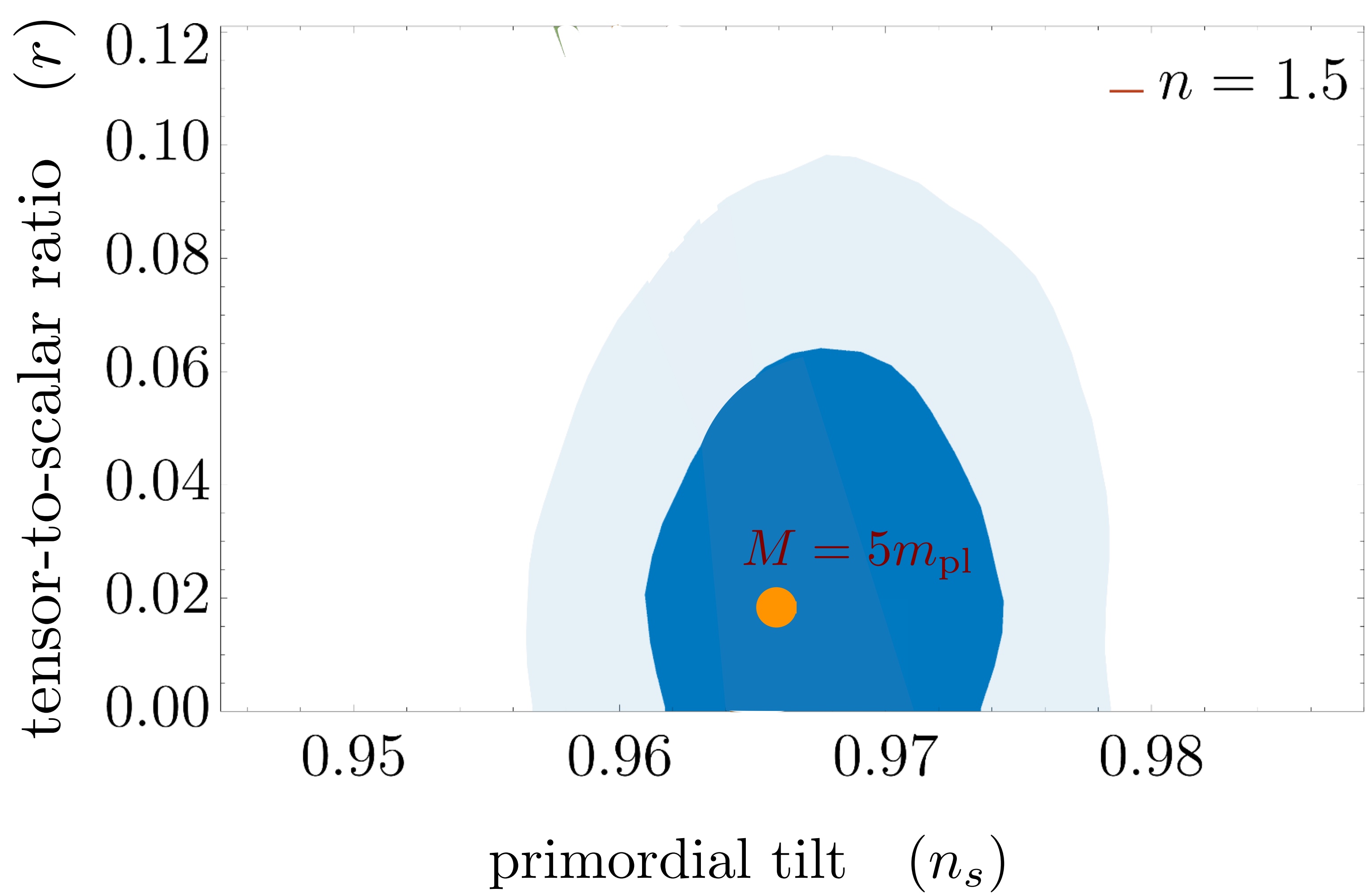
scale fo flattening

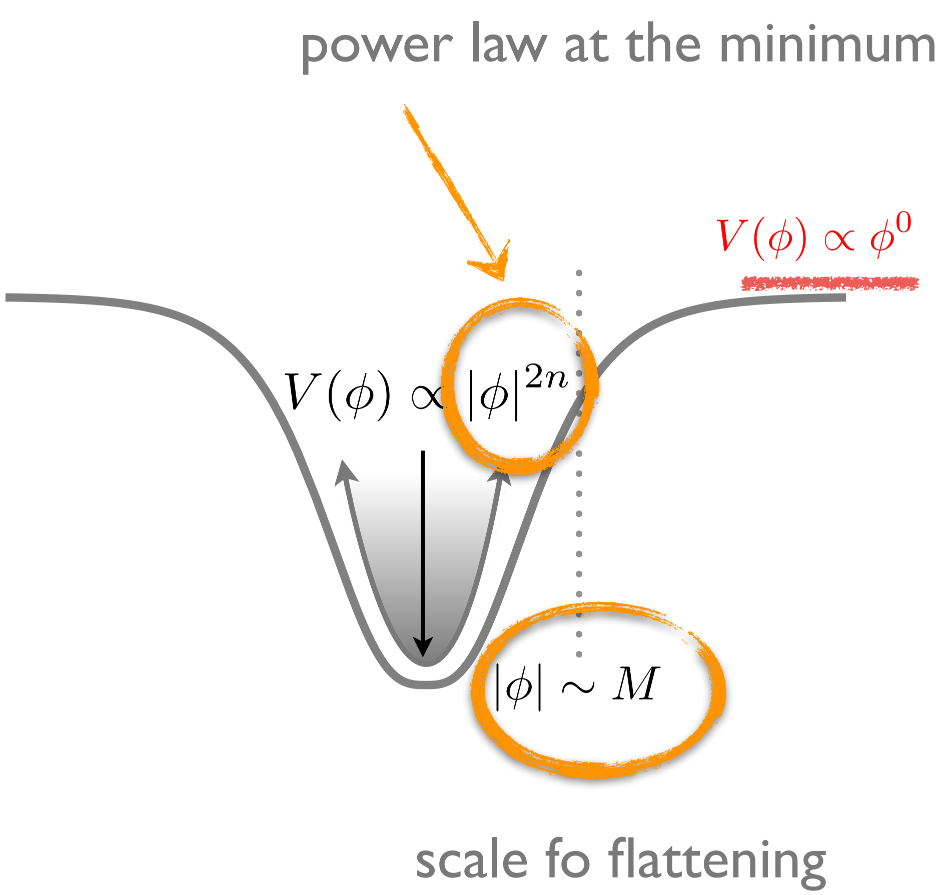
MA & Lozanov 2017 [1608.01213, 1710.06851]

also see: Kamionkowski & Munoz (2014), Cook et. al (2015)

* non-quadratic minimum
  $n \neq 1$
* no oscillons here,

reduction in uncertainty





$$\Delta N_{\rm rad} \sim \begin{cases} 1 & M \lesssim 10^{-2} m_{\rm Pl}\,, \\ \frac{n+1}{3} \ln\left(\frac{\kappa}{\Delta\kappa}\frac{10M}{m_{\rm Pl}}\right) & M \gtrsim 10^{-2} m_{\rm Pl}\,. \end{cases}$$

$n \neq 1$

* non-quadratic minimum

Lozanov & MA (2017) [1608.01213, 1710.06851]

*Caveat: all other fields are assumed to be light and massless.

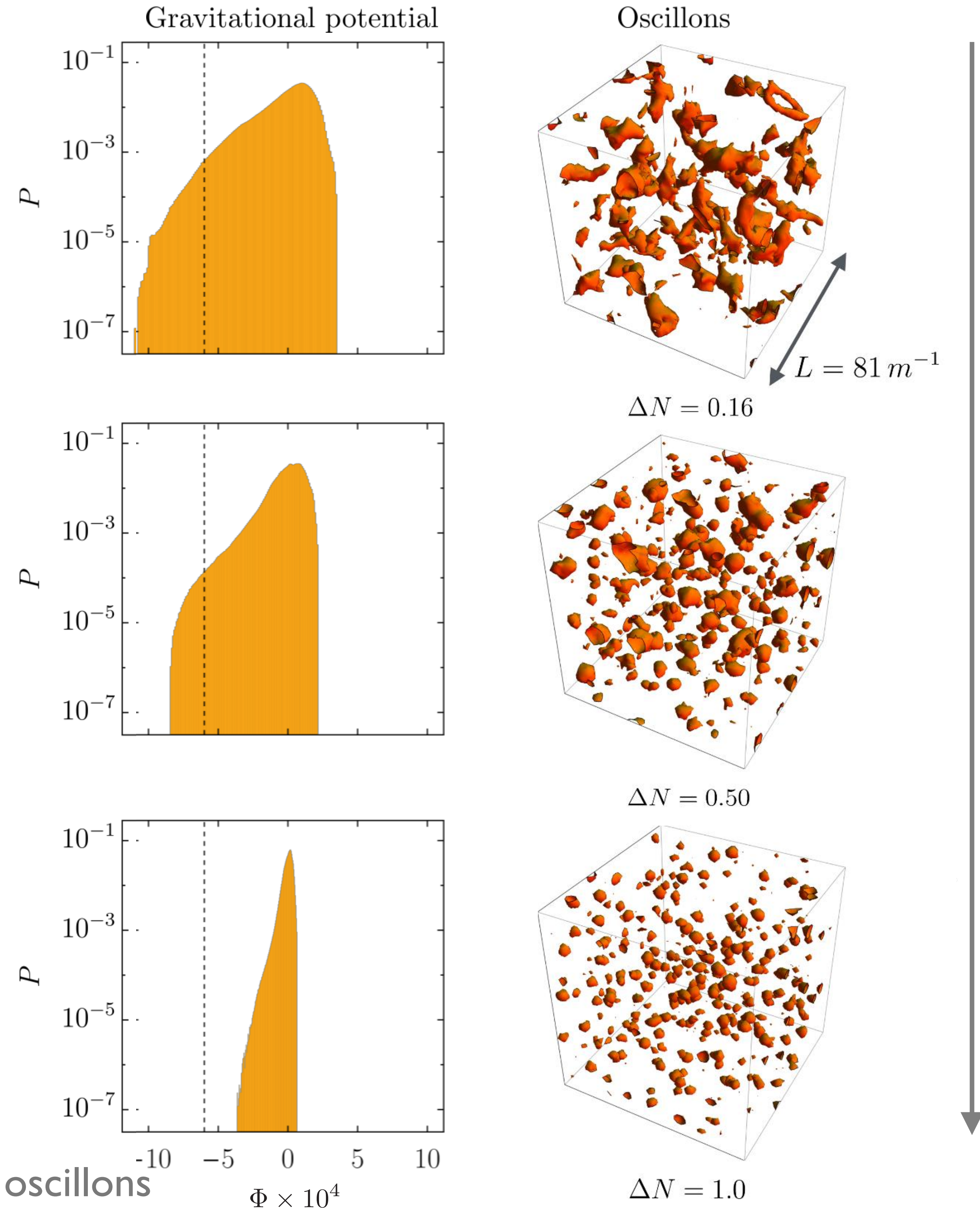
primordial black hole formation from solitons?

1902.06736 Lozanov & MA (2019)

$$\Phi \lesssim \text{few} \times 10^{-3}$$

Not easy to form *PHBs*
from *individual solitons* from self resonance

Can *accidental over-densities* in solitons ***lead to*** PBHs (Cotner et. al 2018/19)



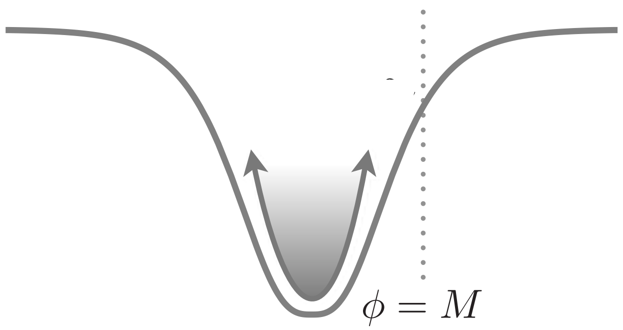
Also see Kou, Tian & Zhou (2019) for recent GR simulations of oscillons
Also see T. Giblin's talk on GR effects in preheating

expansion ✓

self-interactions ✓

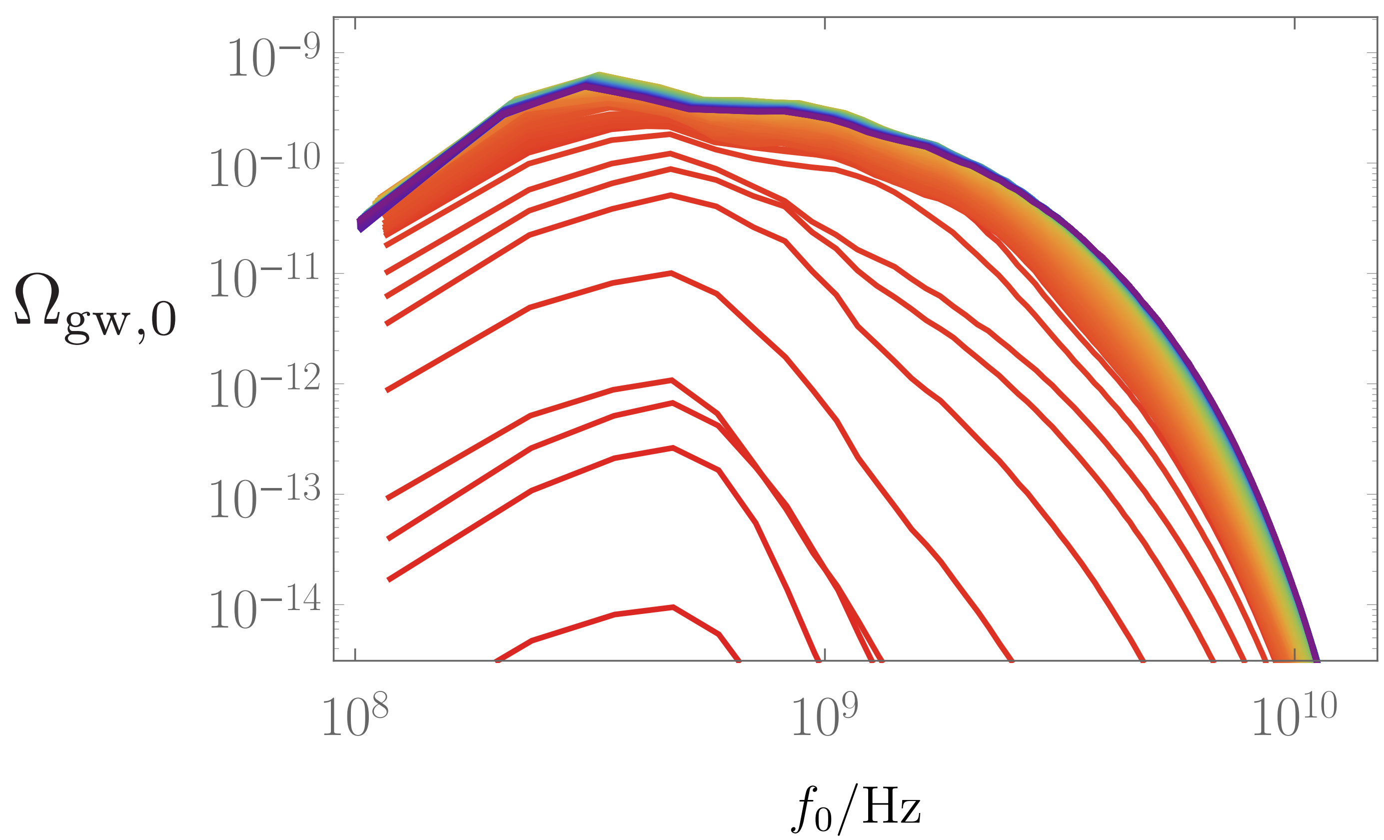
gravitational int. ✗

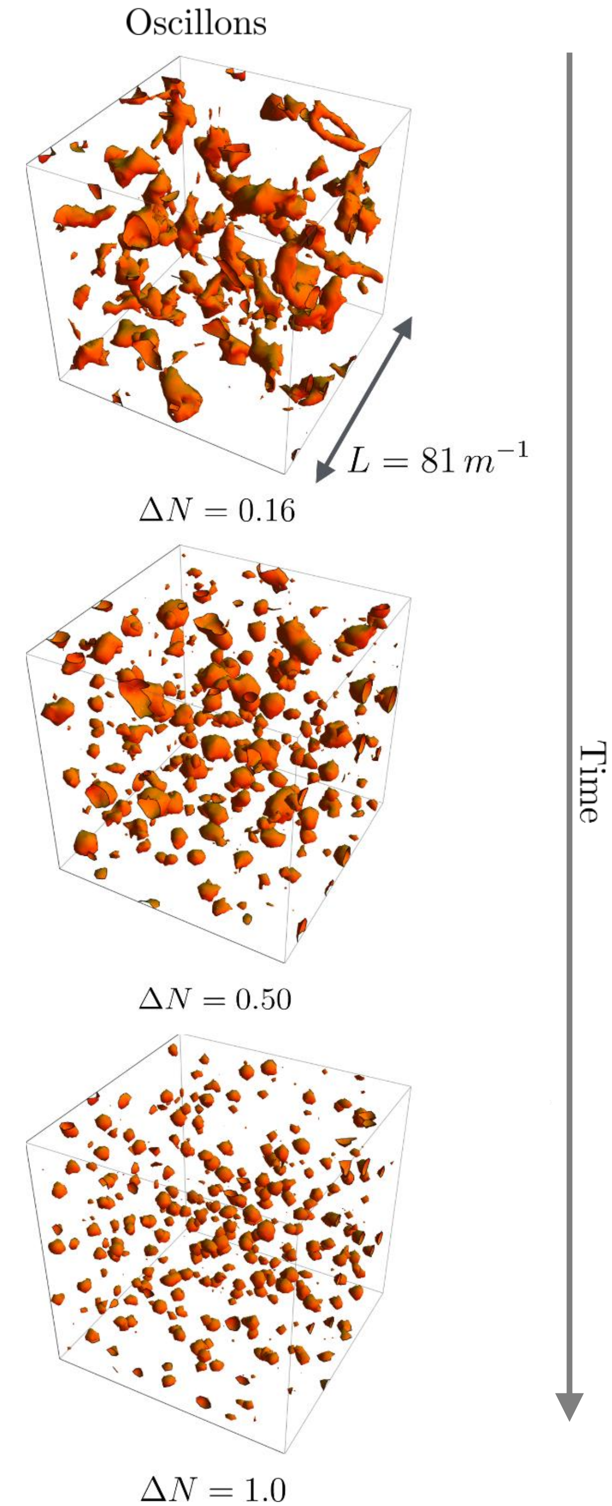
gravitational waves (mainly from formation)



$$\frac{\text{growth-rate of fluctuations}}{\text{expansion rate}} \sim \frac{m_{\rm pl}}{M} \gg 1$$

$$\Omega_{\rm gw,0} \sim 10^{-6}\left(\frac{M}{m_{\rm Pl}}\right)^2 \lesssim \mathcal{O}[10^{-9}]$$

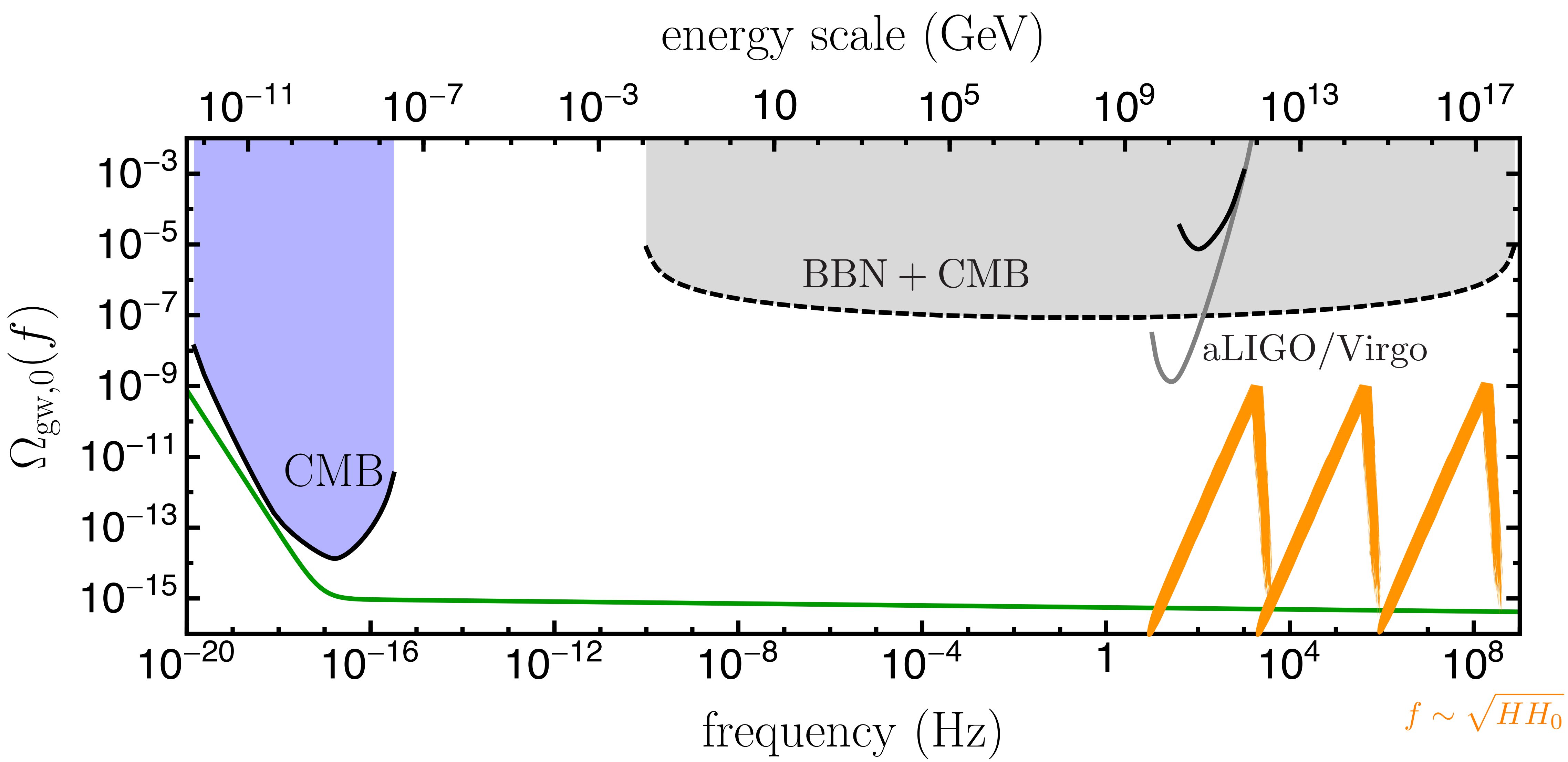




Lozanov & MA (2019) 1902.06736

Earlier work: Zhou et. al (2013). Also see Kitajima, Soda & Urakawa (2018)

gravitational waves



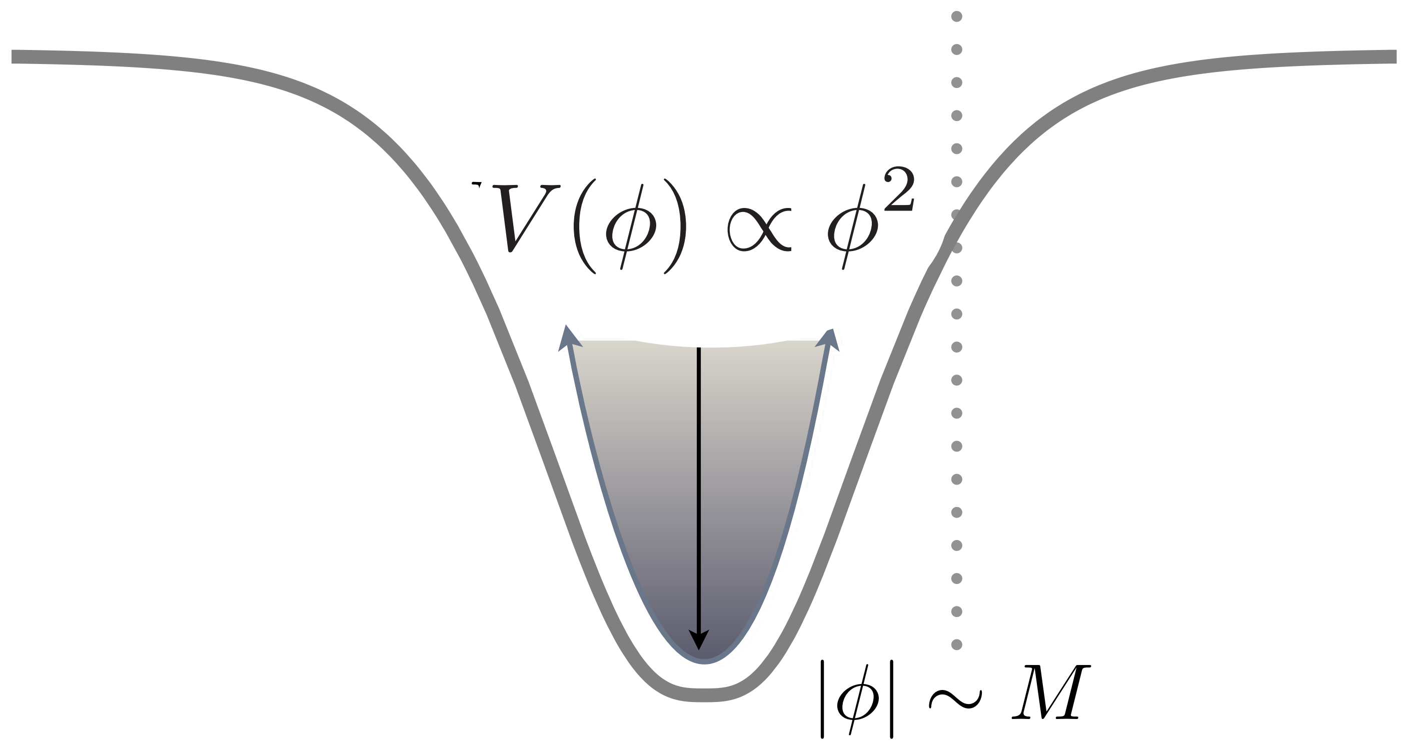
also potential constraints from Neff from CMB S4

limits adapted from Lasky et. al (2015)

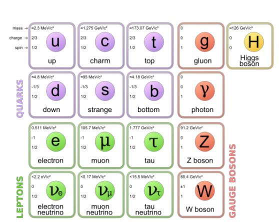
caveat* early universe g-waves amplitude depend on assumptions of expansion history (see Kane, Sinha & Watson (2015)

Earlier work on g-waves from end of inflation: Khlebnikov & Tkachev (1996), Easther, Giblin, Lim (2006/07), Dufaux et. al (2007)

coupling to other fields?
— model dependent answers



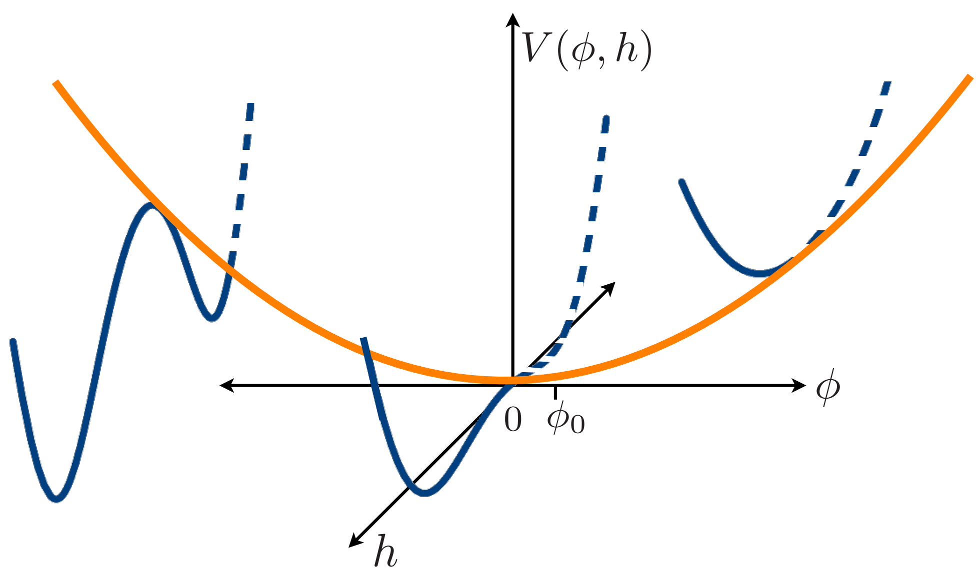
- shape of the potential (self couplings)
- couplings to other fields

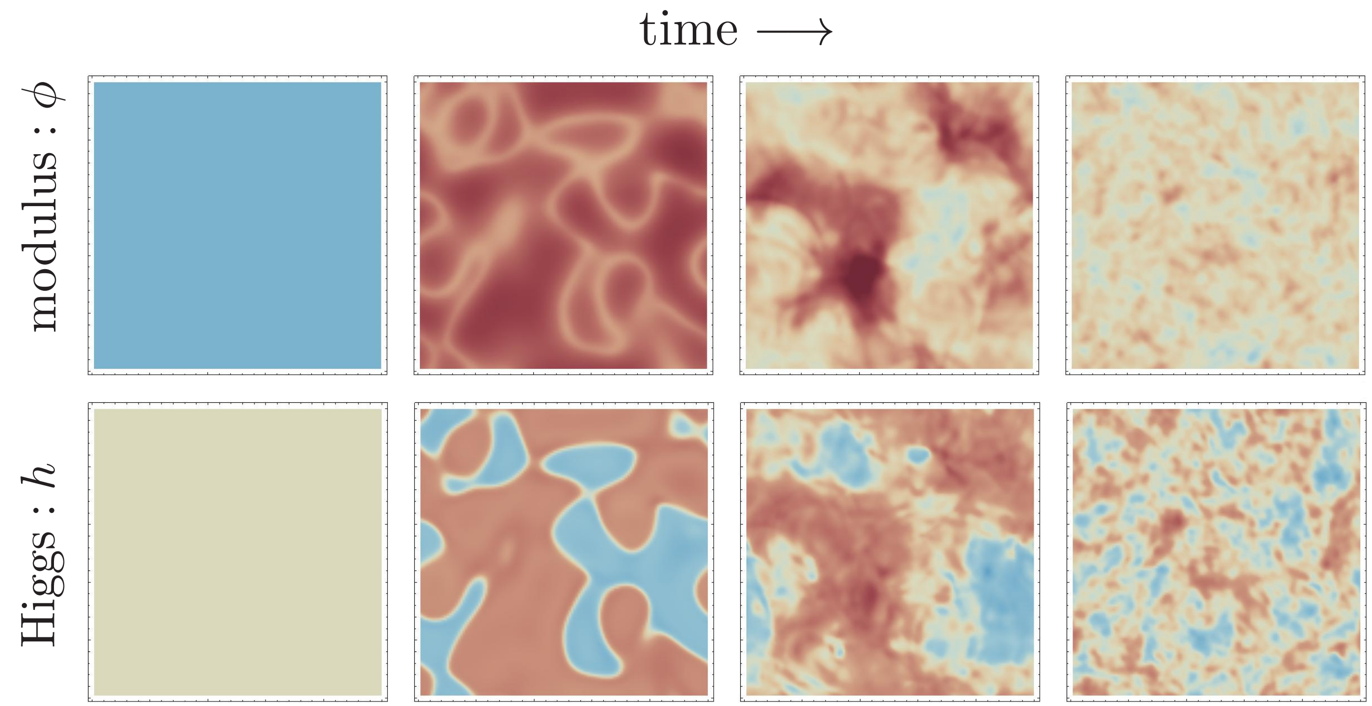


χ , ψ A_μ

example: Higgs - modulus/inflaton system

MA, J. Fan, K. Lozanov & M. Reece (2018) [1802.00444]



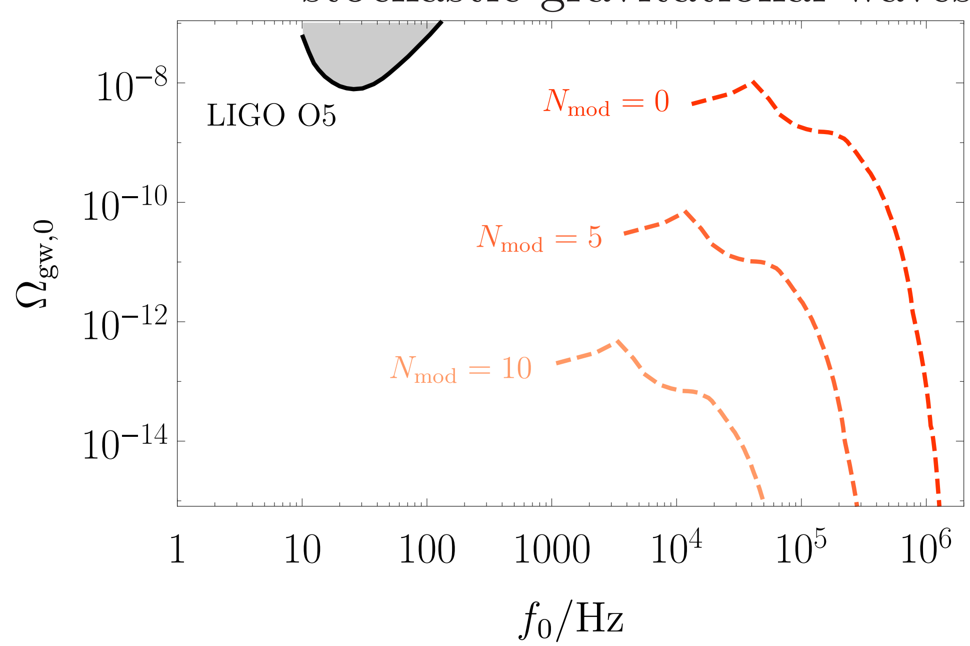


$$\frac{1}{2}m_\phi^2\phi^2 + \frac{M^2}{f}(\phi - \phi_0)\left(h^\dagger h - \frac{v^2}{2}\right) + \lambda(h^\dagger h)^2$$

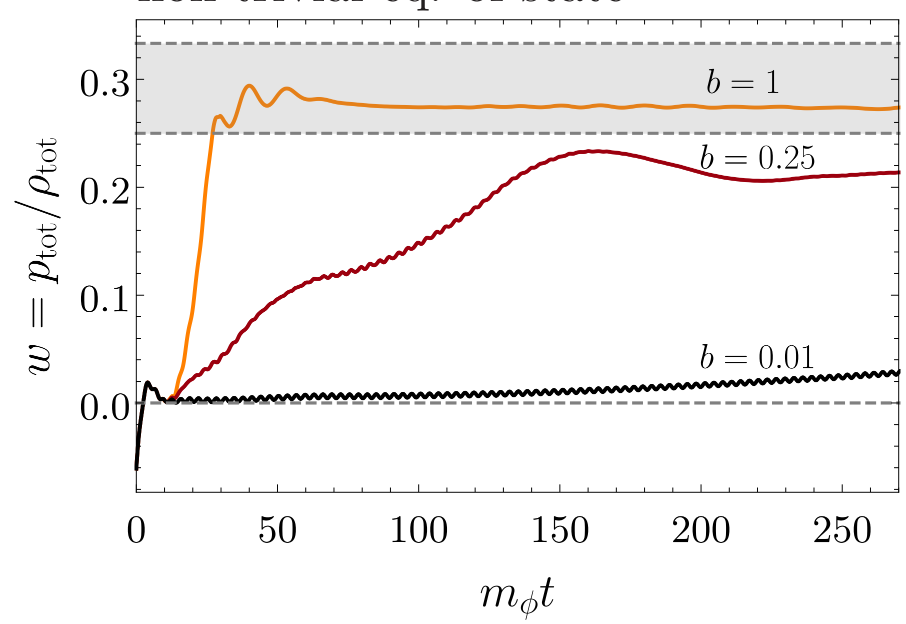
$$\frac{M^4}{2\lambda f^2 m_\phi^2} \to 1 \Leftrightarrow \text{rapid fragmentation}$$

fine tuning $\Leftrightarrow \frac{\phi_0}{f} \ll 1$

stochastic gravitational waves



non-trivial eq. of state



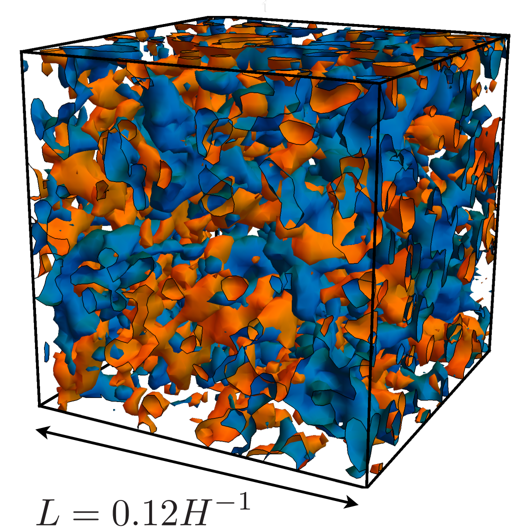
GFiRe: a Gauge Field integrator for Reheating

Kaloian D. Lozanov & MA [arXiv:1911.06827]

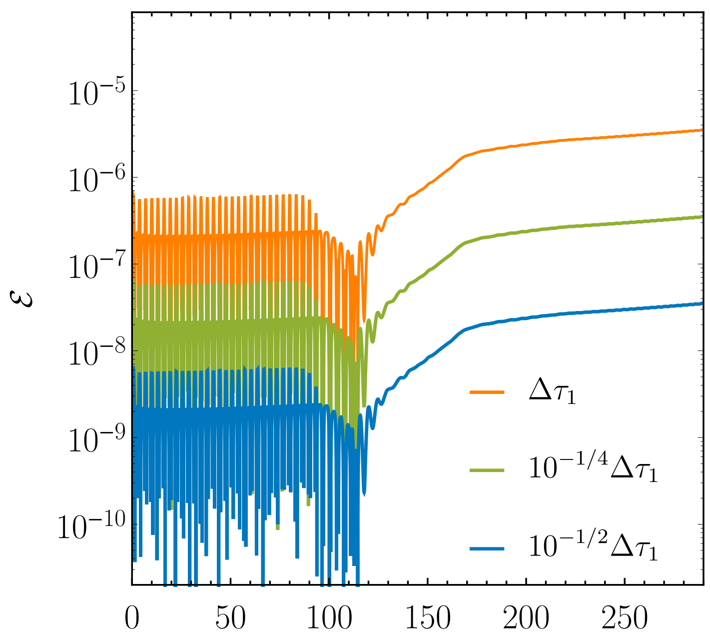
New algorithm and code, **GFiRe**, to simulate nonlinear dynamics of *Charged Scalars Fields* coupled to *Abelian Gauge Fields* in an expanding universe

- algorithm uses link variables and includes self-consistent expansion
- algorithm is symplectic (arbitrary order) and has "exact" preservation of Gauss constraint

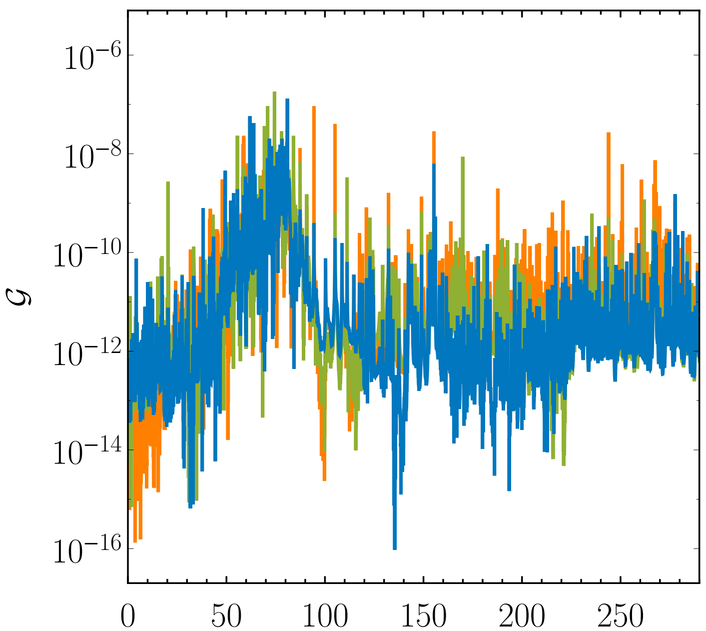
Resonant Production of Electric & Magnetic Fields



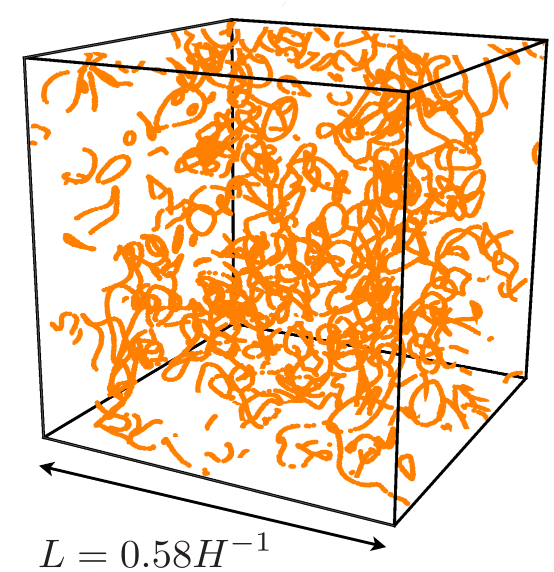
Energy Constrain Violation



Gauss Constrain Violation



Formation & Evolution of "Local" cosmic Strings

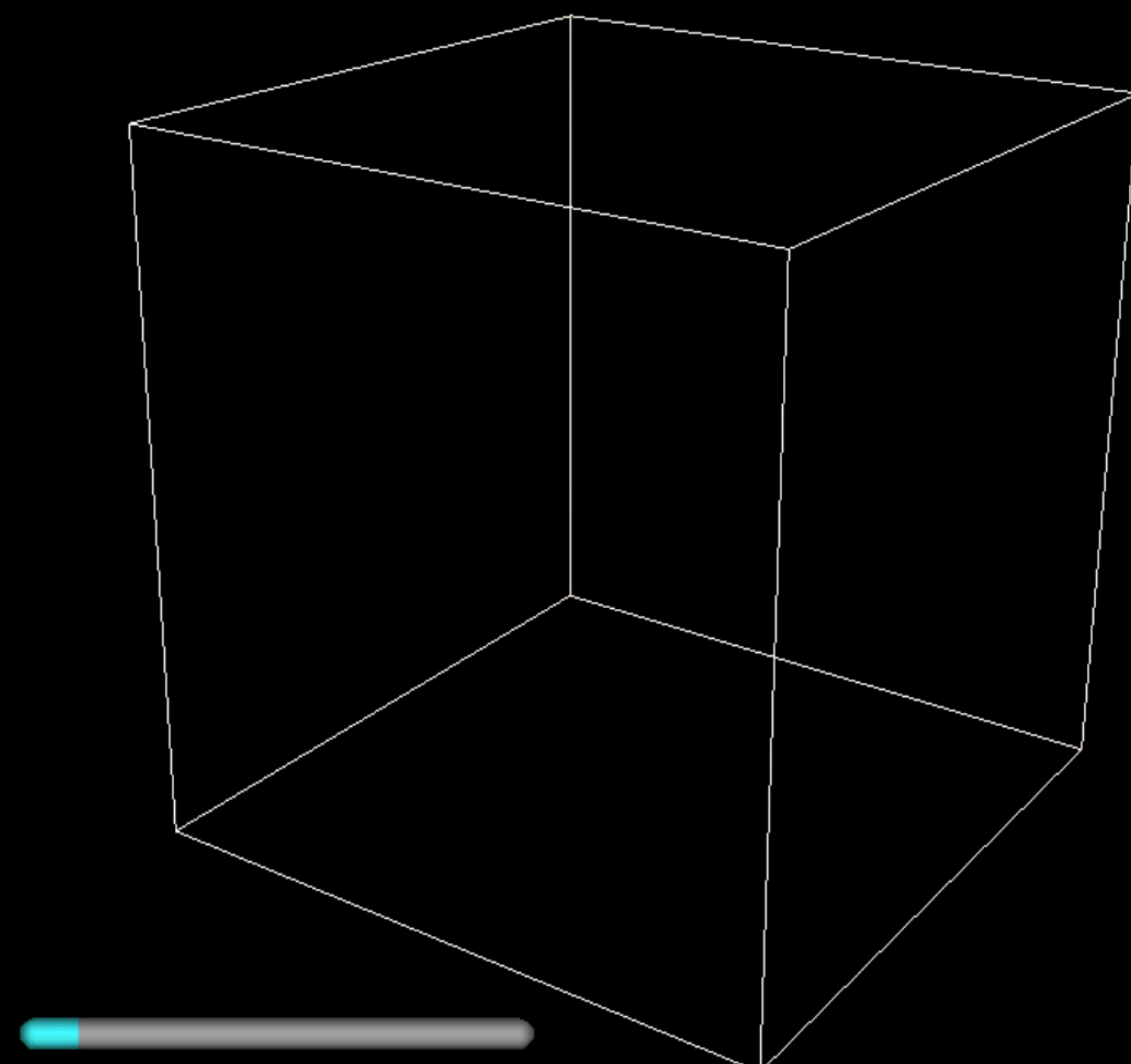


Also see related works by Figueroa et. al (2015), Adshead, Giblin, Scully/Weiner (2015/17)

formation and decay of strings

Kaolian Lozanov & MA [arXiv:1911.06827]

$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\mathcal{R} - (D_\mu\phi)^\dagger D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

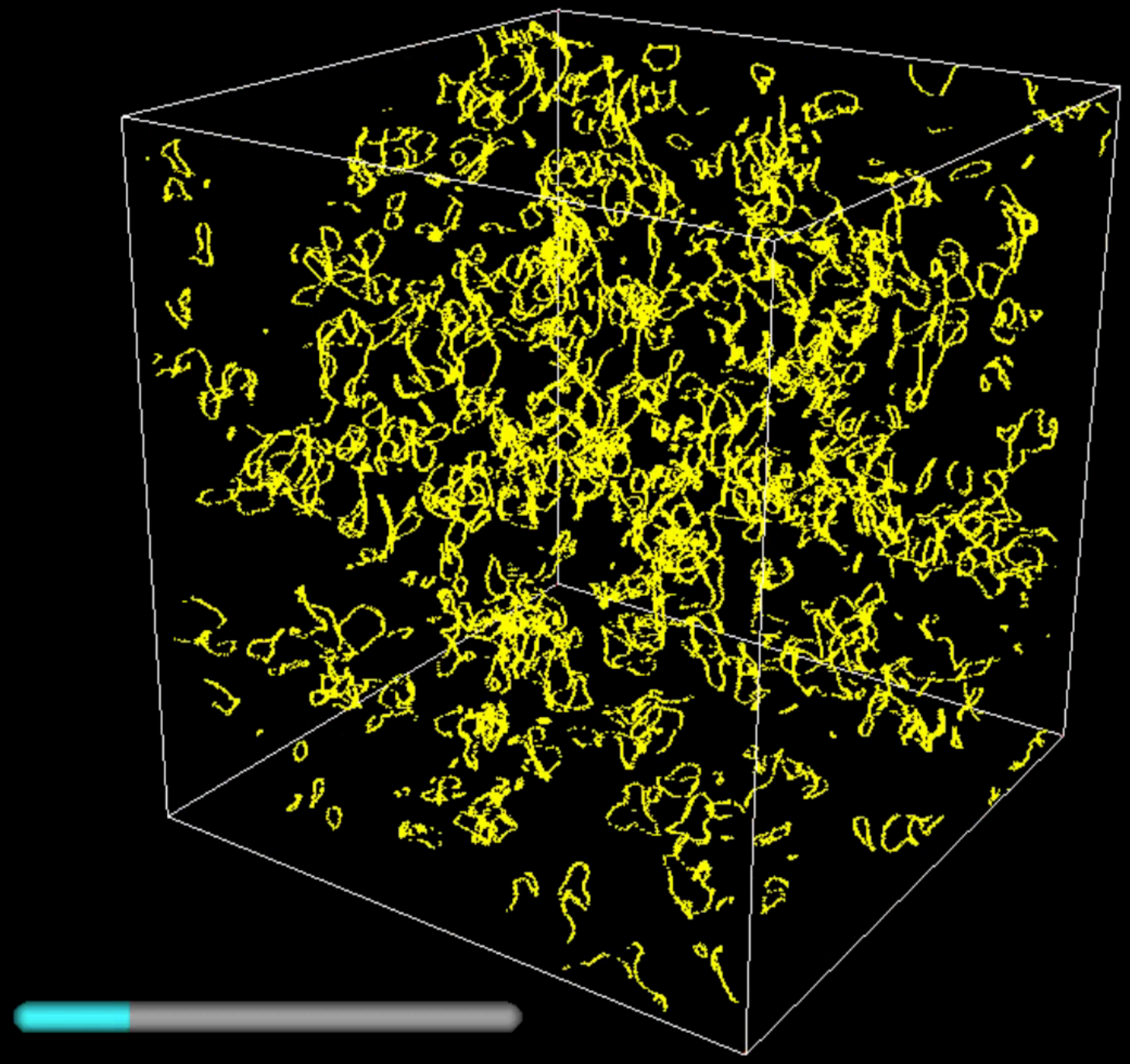


GFiRe: **G**auge **F**ield **i**ntegrator for **Re**heating

formation and decay of strings

Kaolian Lozanov & MA [arXiv:1911.06827]

$$S = \int d^4x \sqrt{-g} \left[\frac{1}{16\pi G} \mathcal{R} - (D_\mu \phi)^\dagger D^\mu \phi - V(|\phi|) - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right]$$

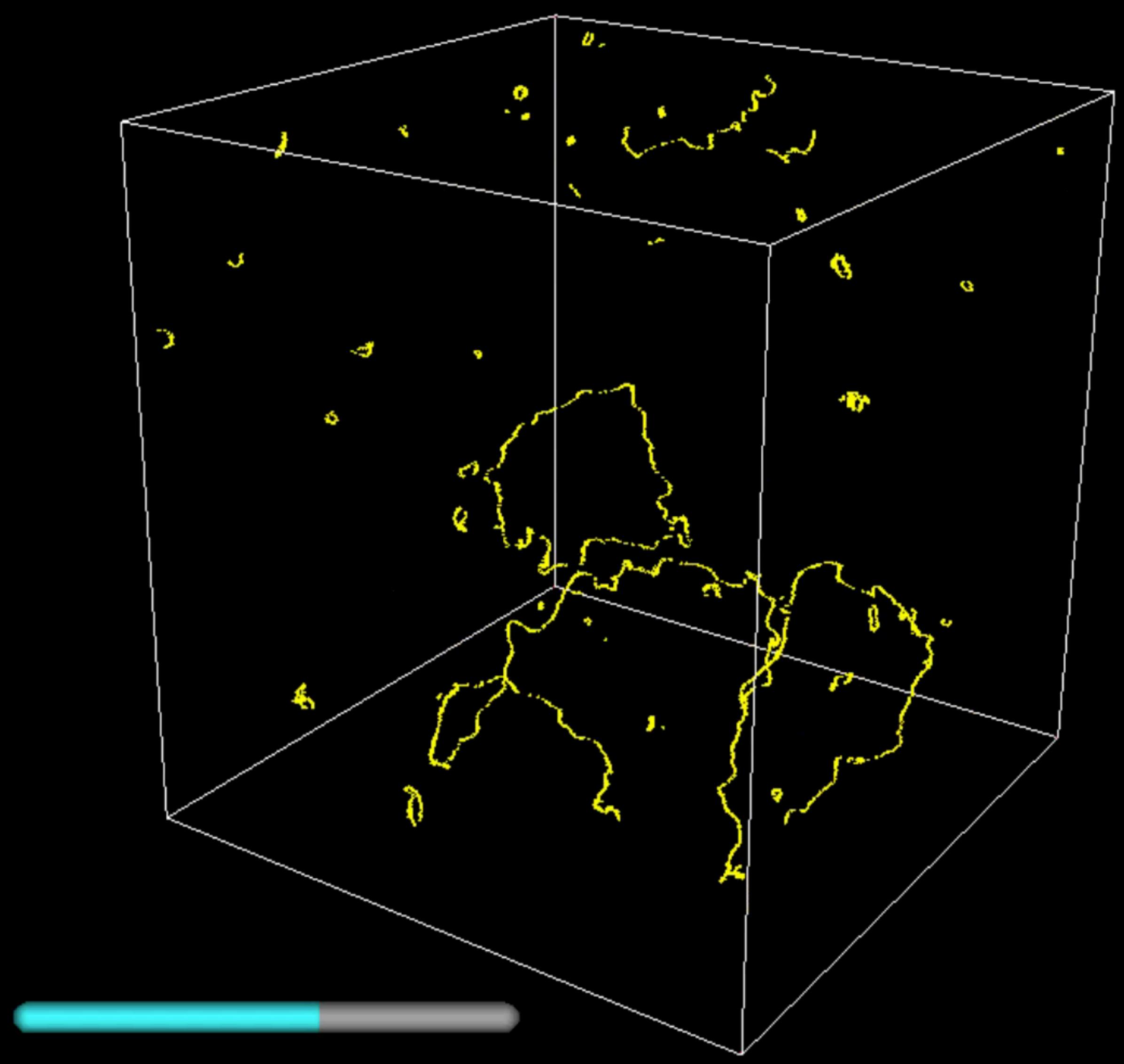


GFiRe: **G**auge **F**ield **i**ntegrator for **Re**heating

formation and decay of strings

Kaolian Lozanov & MA [arXiv:1911.06827]

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{16\pi G}\mathcal{R} - (D_\mu\phi)^\dagger D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

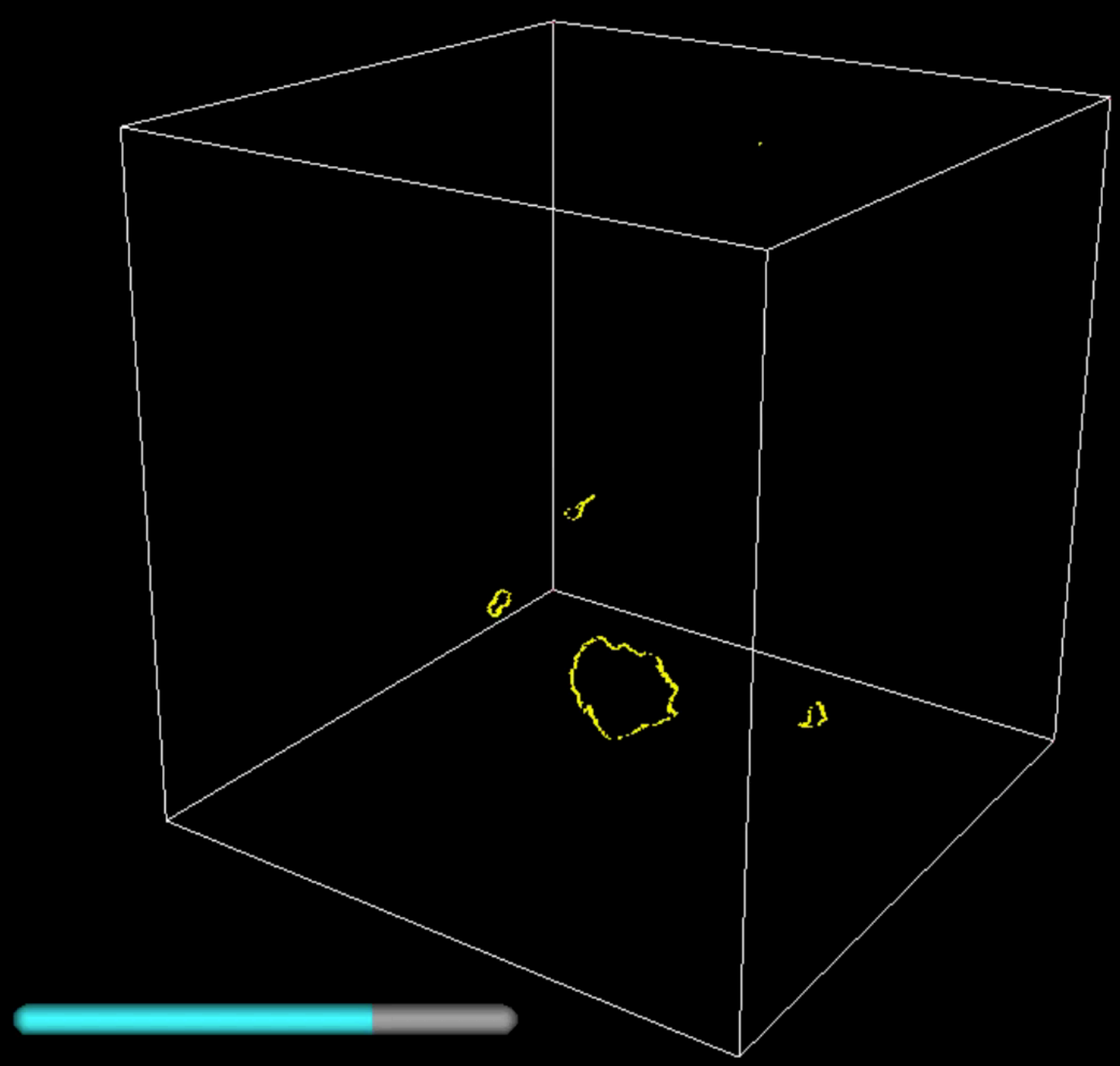


GFiRe: Gauge Field integrator for Reheating

formation and decay of strings

Kaolian Lozanov & MA [arXiv:1911.06827]

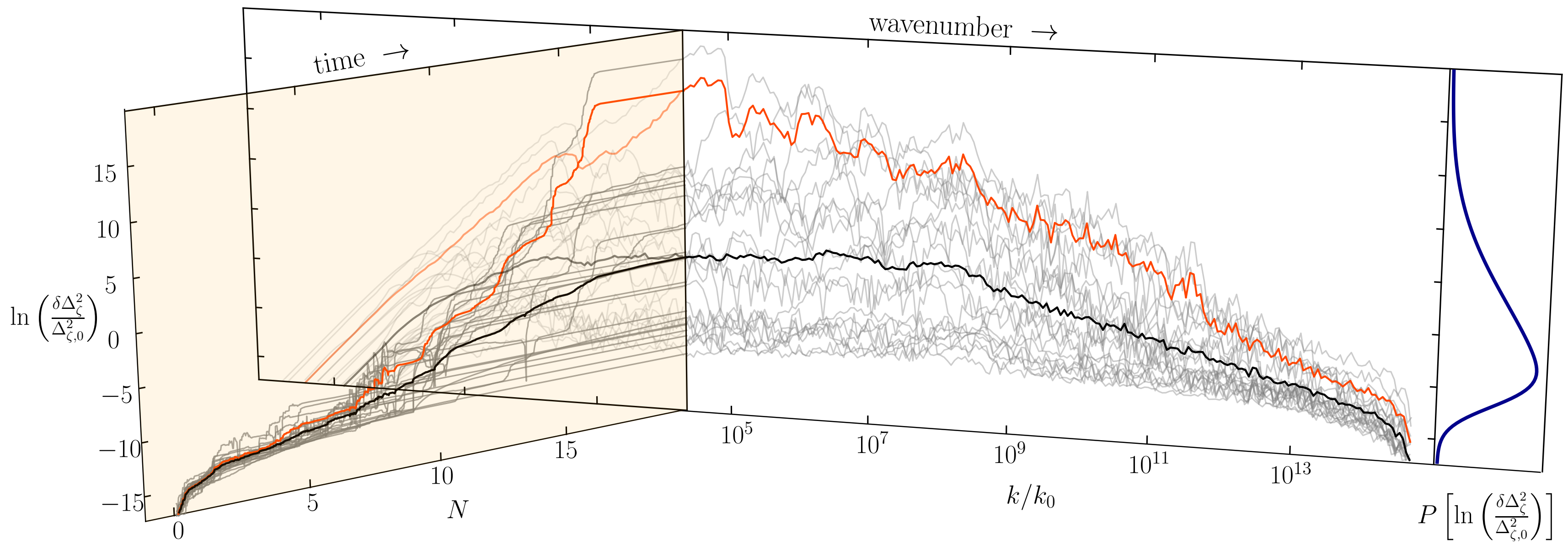
$$S = \int d^4x\sqrt{-g}\left[\frac{1}{16\pi G}\mathcal{R} - (D_\mu\phi)^\dagger D^\mu\phi - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right]$$

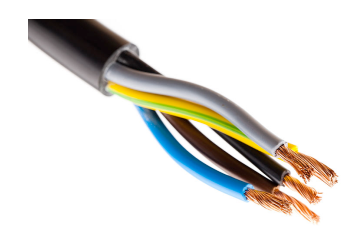


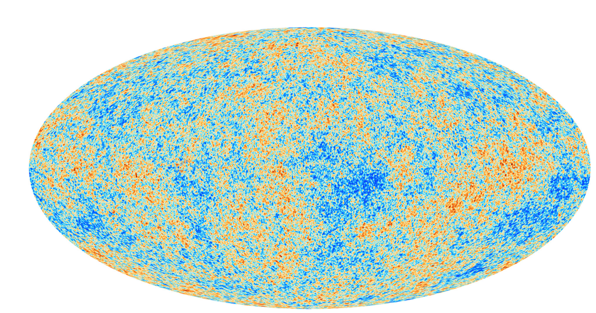
GFiRe: **G**auge **F**ield **i**ntegrator for **Re**heating

sufficiently complex models of inflation and reheating

appropriate for sufficiently complex models of inflation



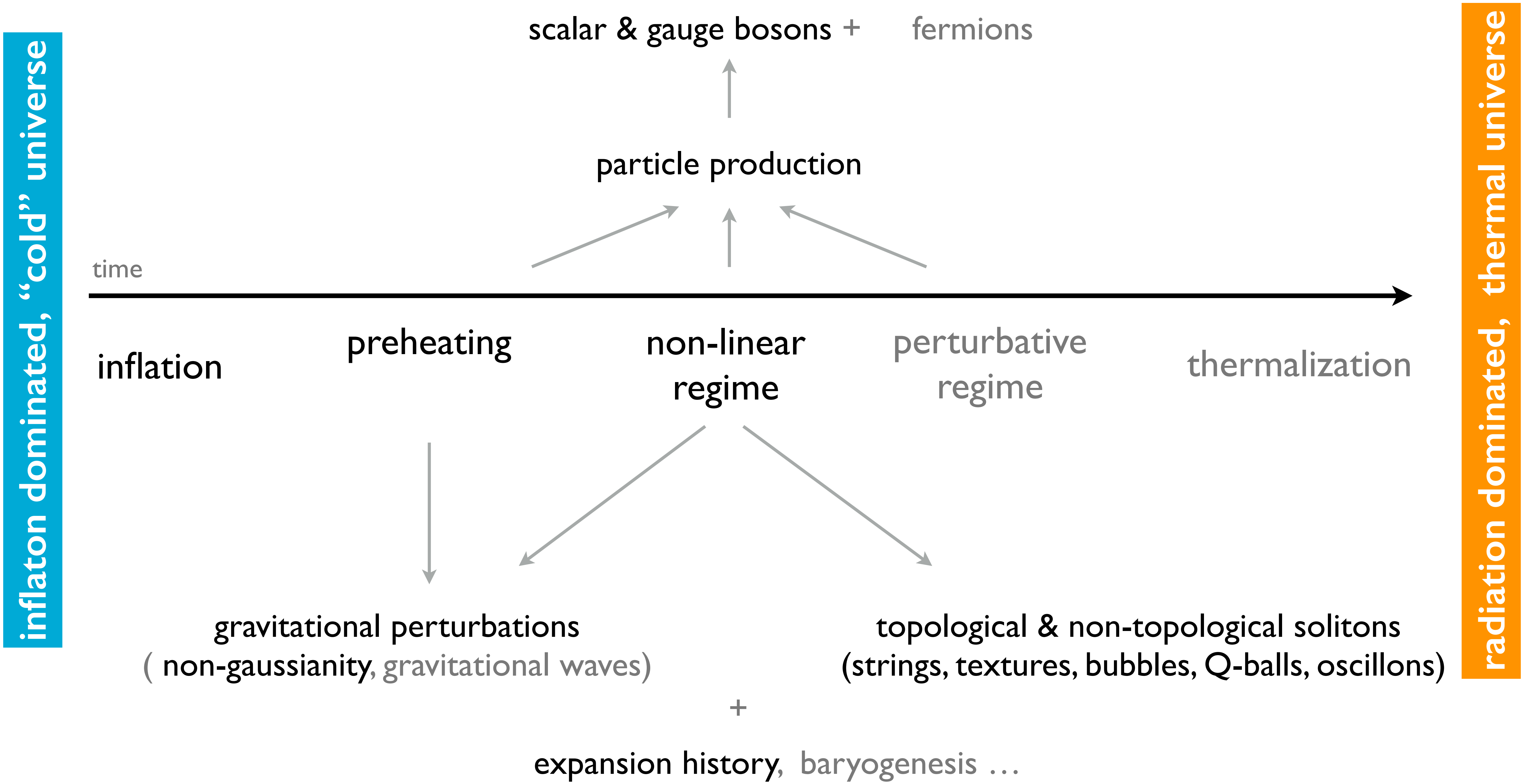




- Wires to Cosmology
  (MA & Baumann 1512.02637)
- Multifield Stochastic Particle Production
  (MA, Garcia, Wen & Xie 1706.02319)
- Stochastic Particle Production in deSitter Space
  (Garcia, MA, Carlsten & Green 1902.06736)
- Curvature Perturbations from Stochastic Particle Production during Inflation
  (Garcia, MA & Green 2001.09158)

Also see recent work by D. Green in 2015. Early work in context of noise in preheating: Zanchin et. al 1997, Bassett 1998

in general, there can be lot more going on

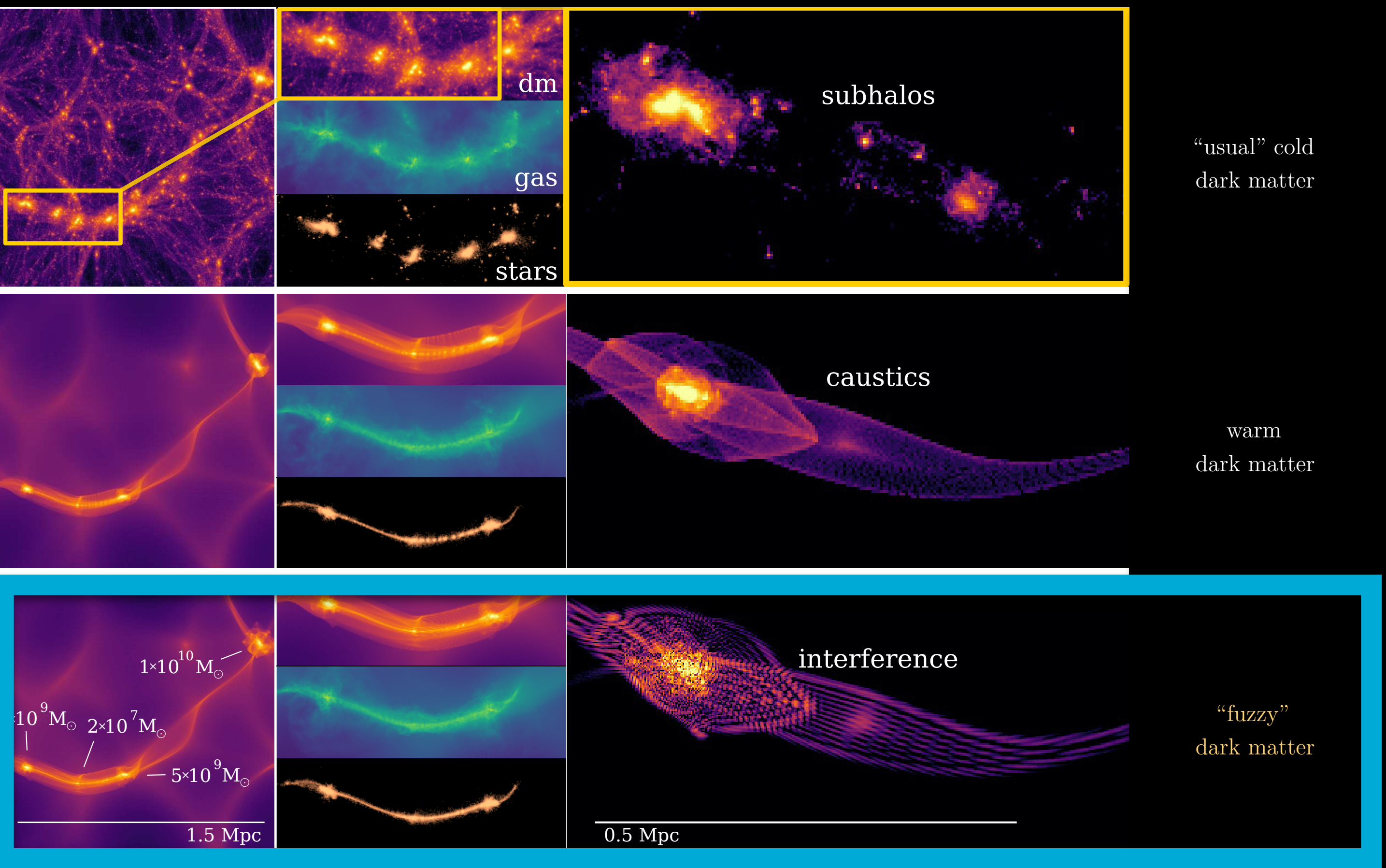


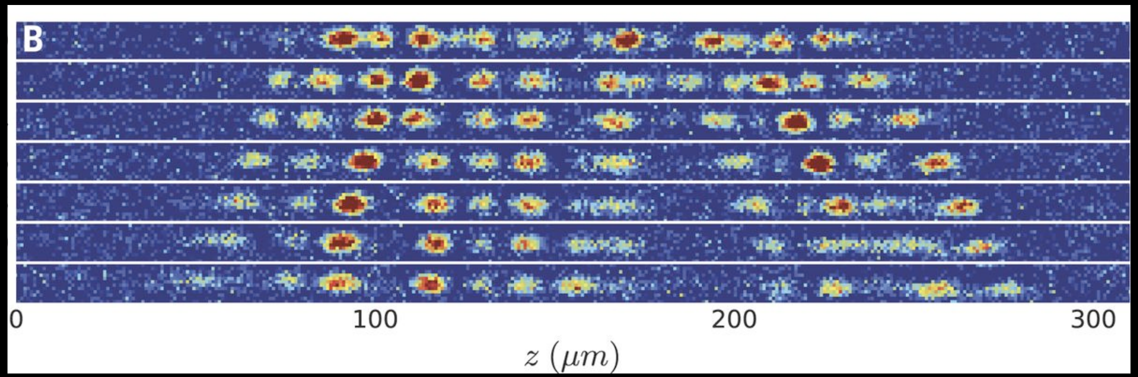
from review: MA, Kaiser, Karouby & Hertzberg (2013)

generality & novel connections

- Axionic dark matter
- **Solitons in cold-atom BECs**

Mocz et. al + MA (2019)

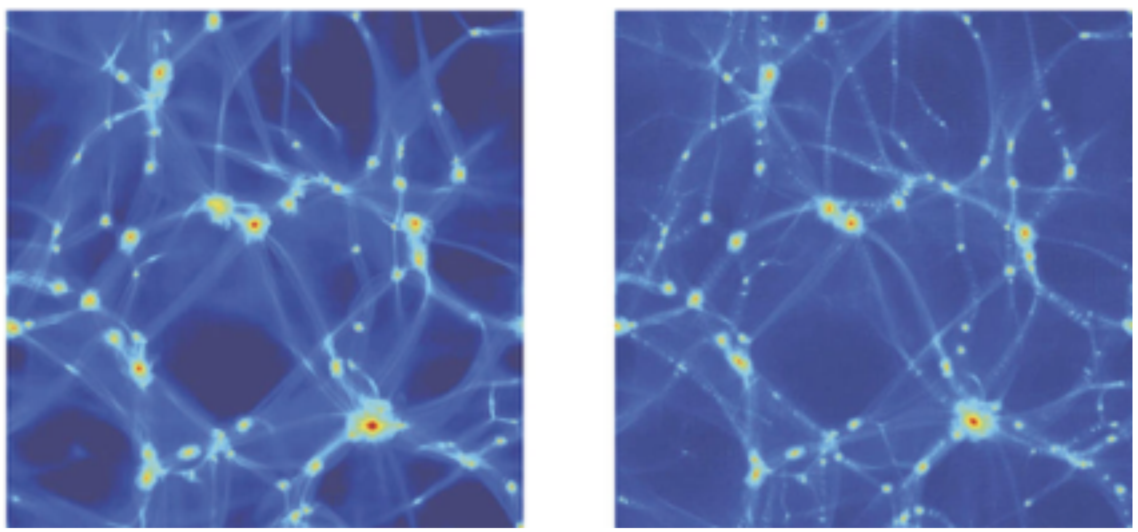


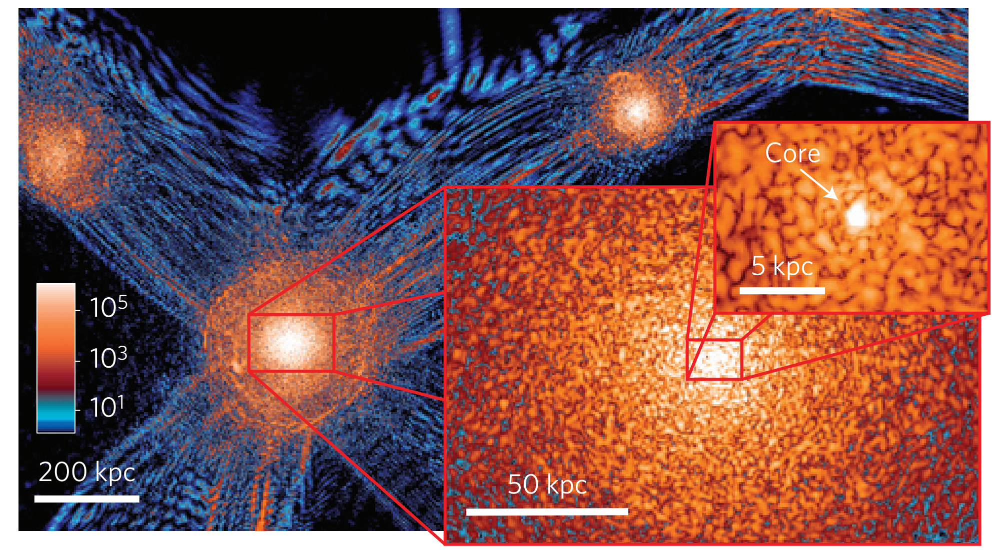


Nguyen, Luo & Hulet (2017)

dark matter:
axion-like fields

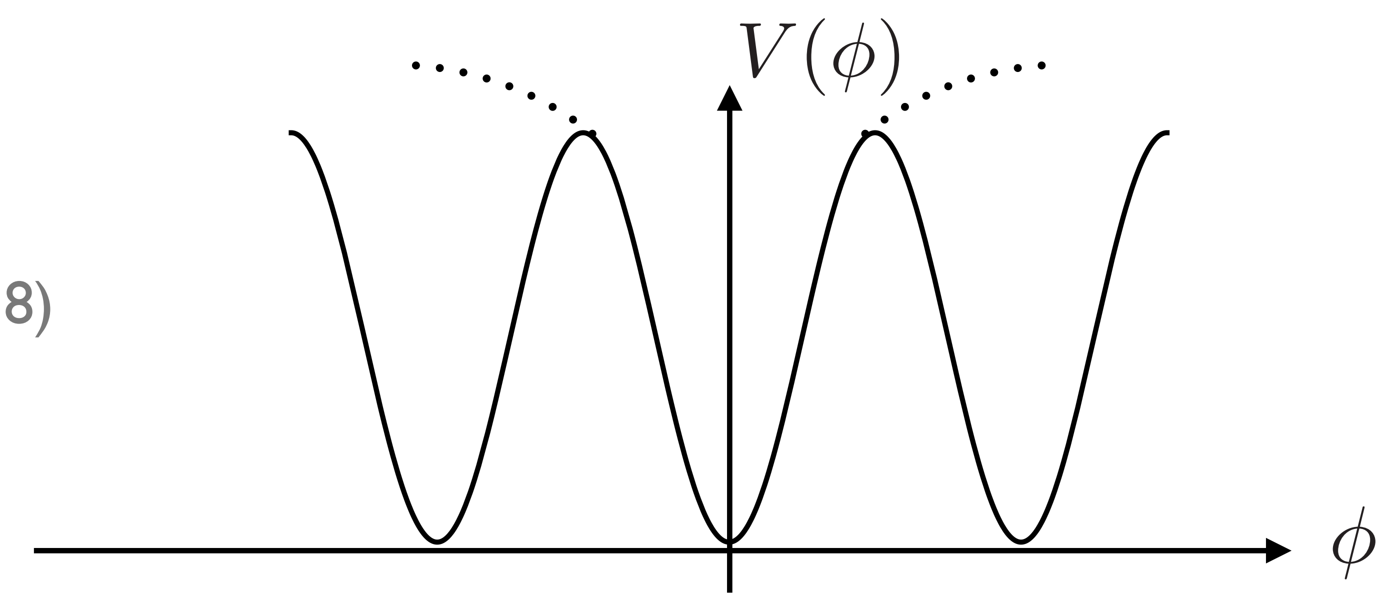
(a) ψDM (b) CDM





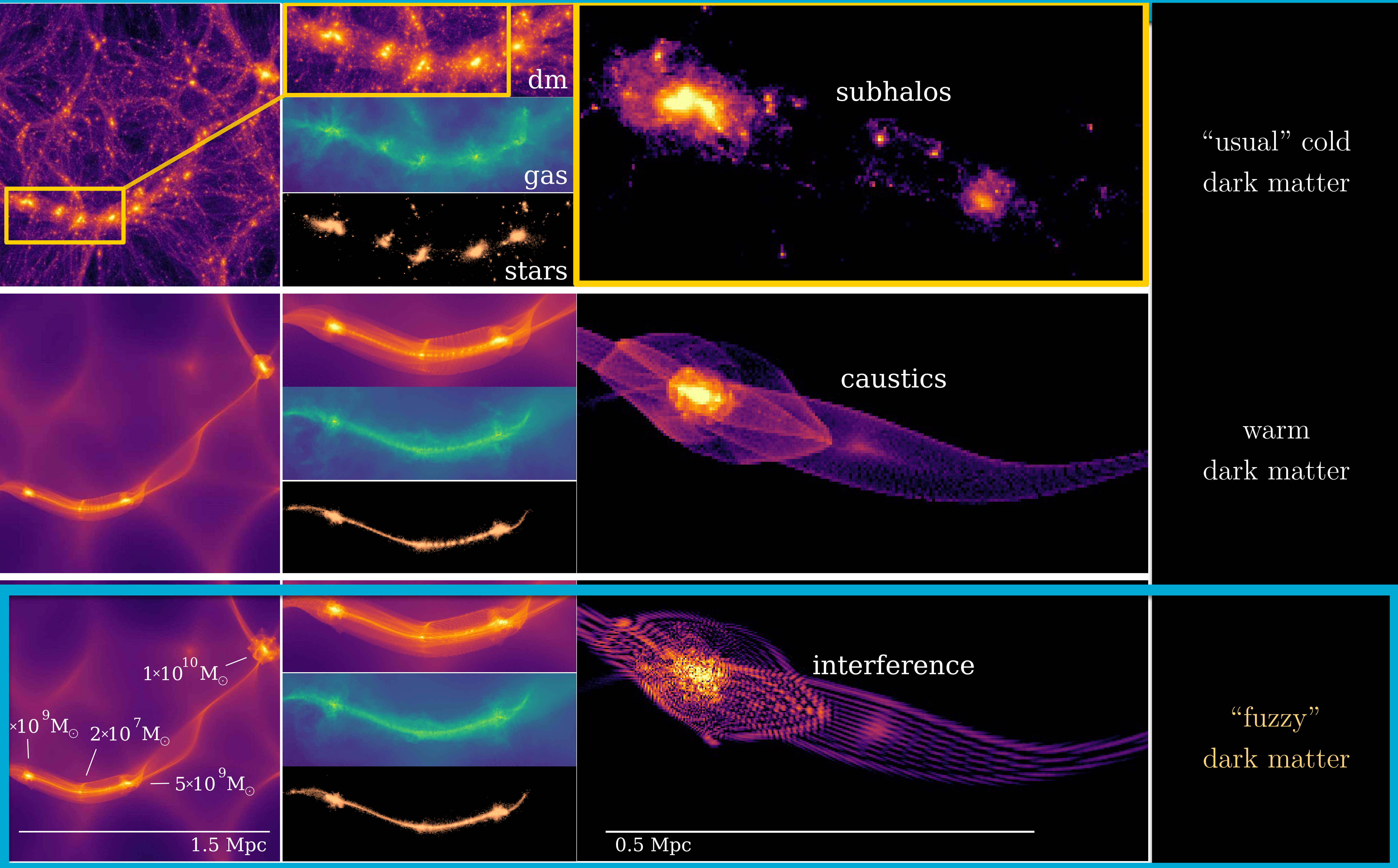
Schive et. al (2014)

for example:
Peccei & Quinn (1977)
Hogan & Reece (1988)
Kolb & Tkachev (1994)
Hu, Barkana & Gruzinov (2000)
Marsh & Silk (2014)
Guth, Hertzberg & Prescod-Weinstein (2018)
Niemeyer & Engels (2016)
Hui et. al (2016)
Arvanitaki et. al (2009/19)
Mocz et. al (2019)

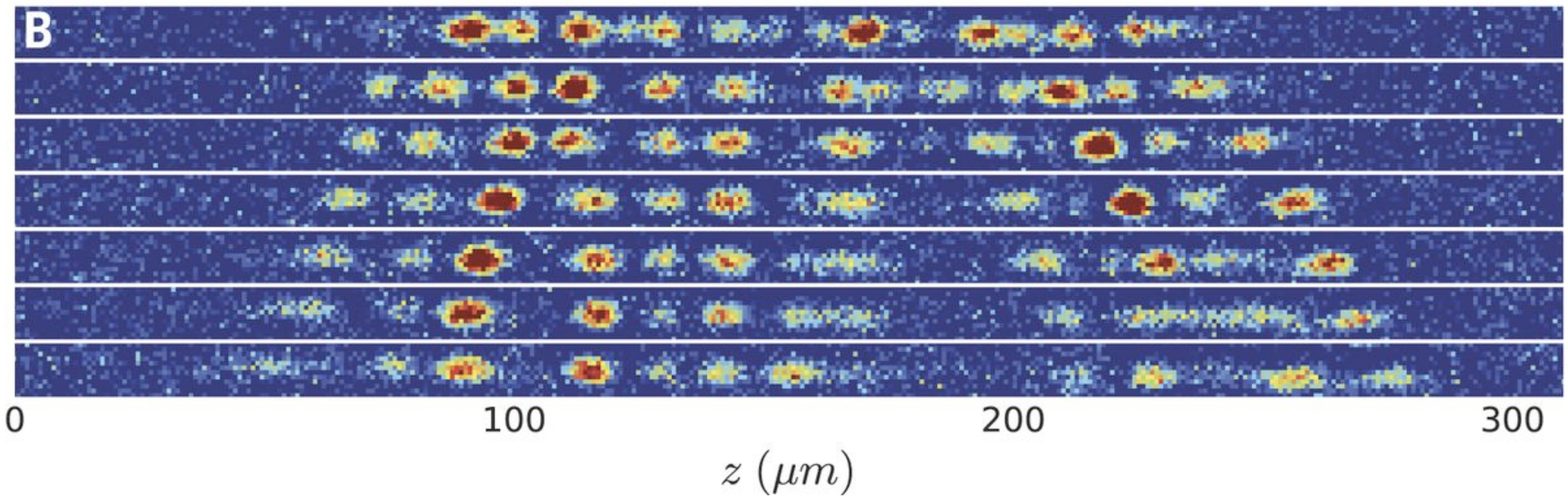


structure formation with light scalar fields

Mocz, +MA, et. al (2019)



related solitons in BECs



Nguyen, Luo & Hulet (2017)

nonlinear Klein Gordon — nonlinear Schrodinger eq.

$$\partial_t^2\phi - c^2\nabla^2\phi + \partial_\phi V(\phi) = 0$$

$$\partial_t^2\varphi - c_\mathrm{s}^2\nabla^2\varphi + \partial_\varphi \mathcal{V}(\varphi) = 0$$

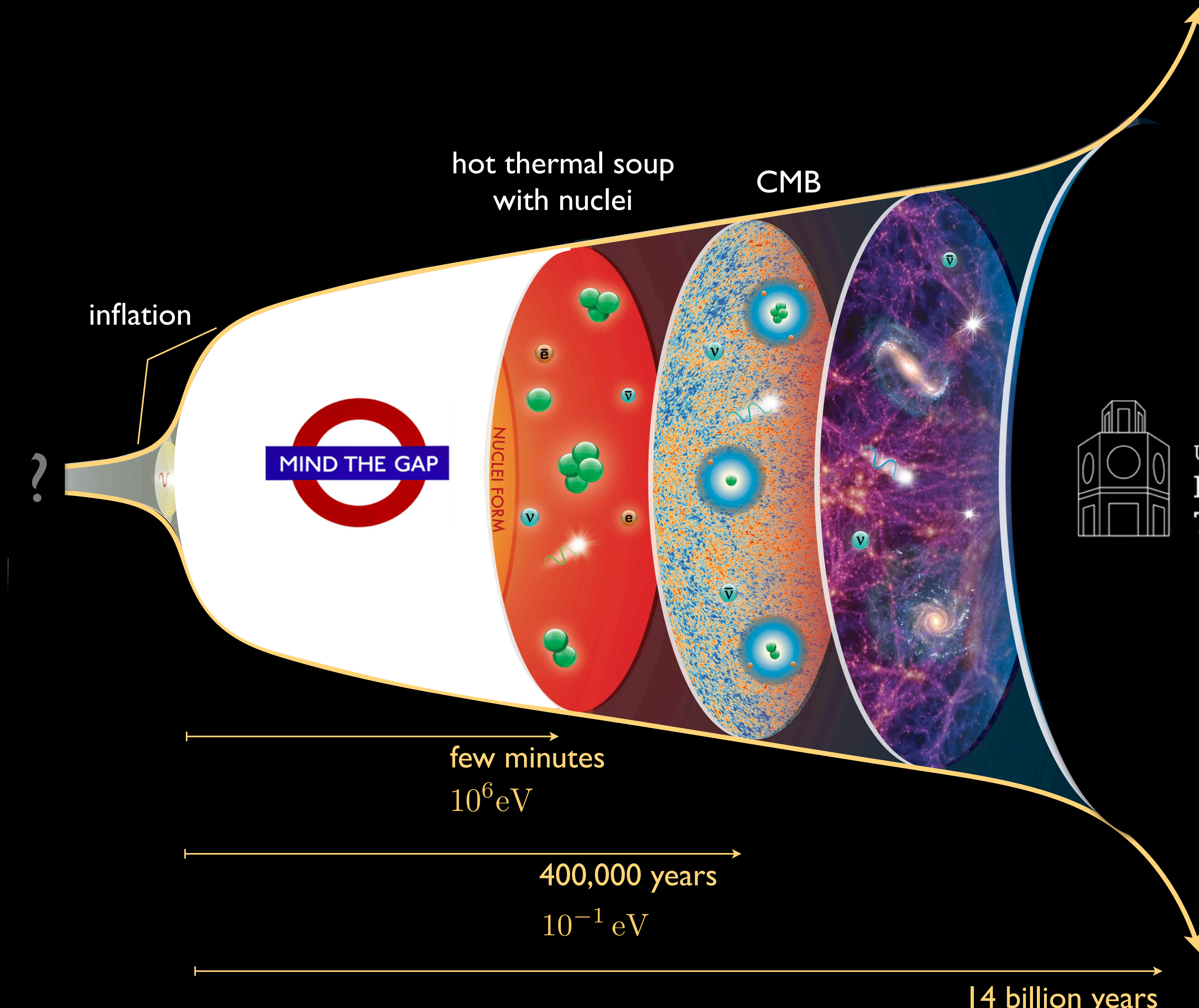
relative phase between different condensates

see Jonathan Braden's work on bubble nucleation

$$i\partial_t\psi = \left[-\frac{1}{2m}\nabla^2 + U'(|\psi|^2)\right]\psi$$

non-relativistic

non-perturbative dynamics at the end of inflation?



UC SANTA BARBARA
Kavli Institute for
Theoretical Physics

Nonlinear Dynamics after Inflation

+ with connections to

axions & condensed matter systems + Higgs & gauge fields + disordered systems

theoretical/numerical results

1. instability in oscillating fields
2. formation of solitons
3. eq. of state

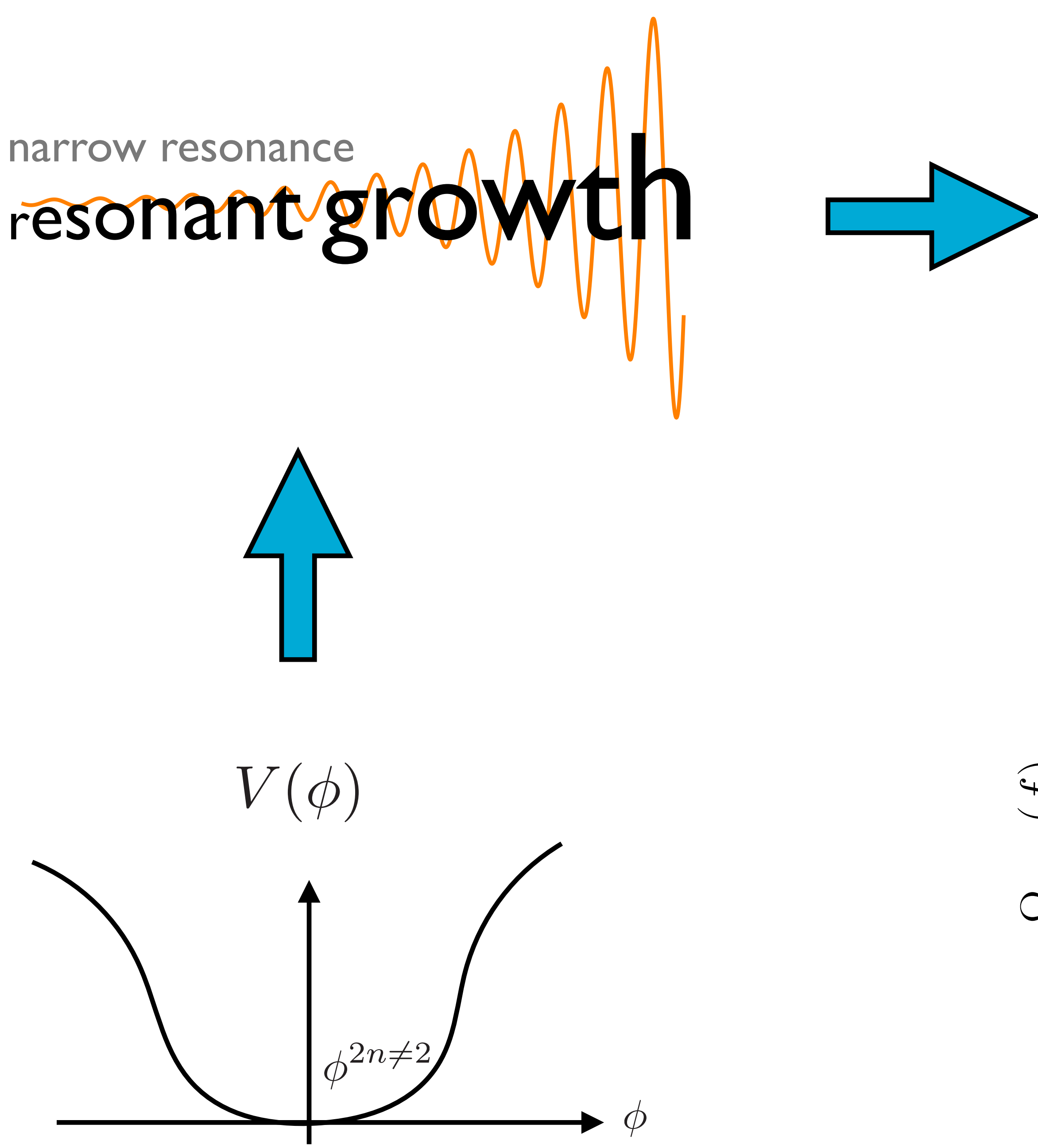
obs. implications

1. expansion history
2. gravitational waves
3. structure formation

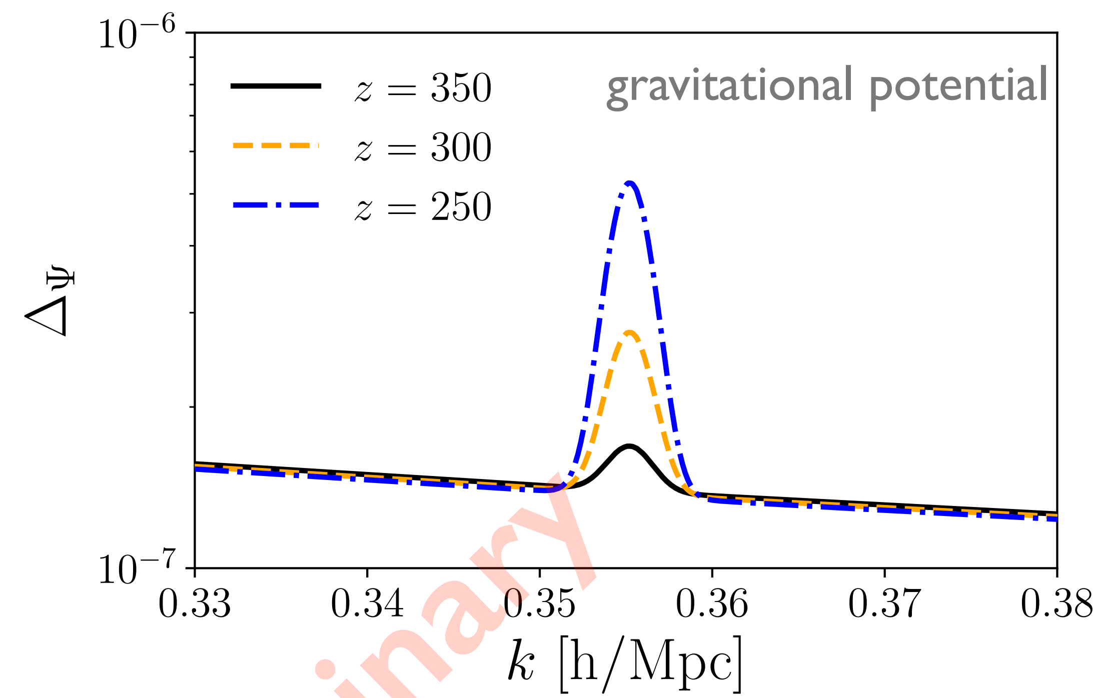
what else can these general results be useful for ?

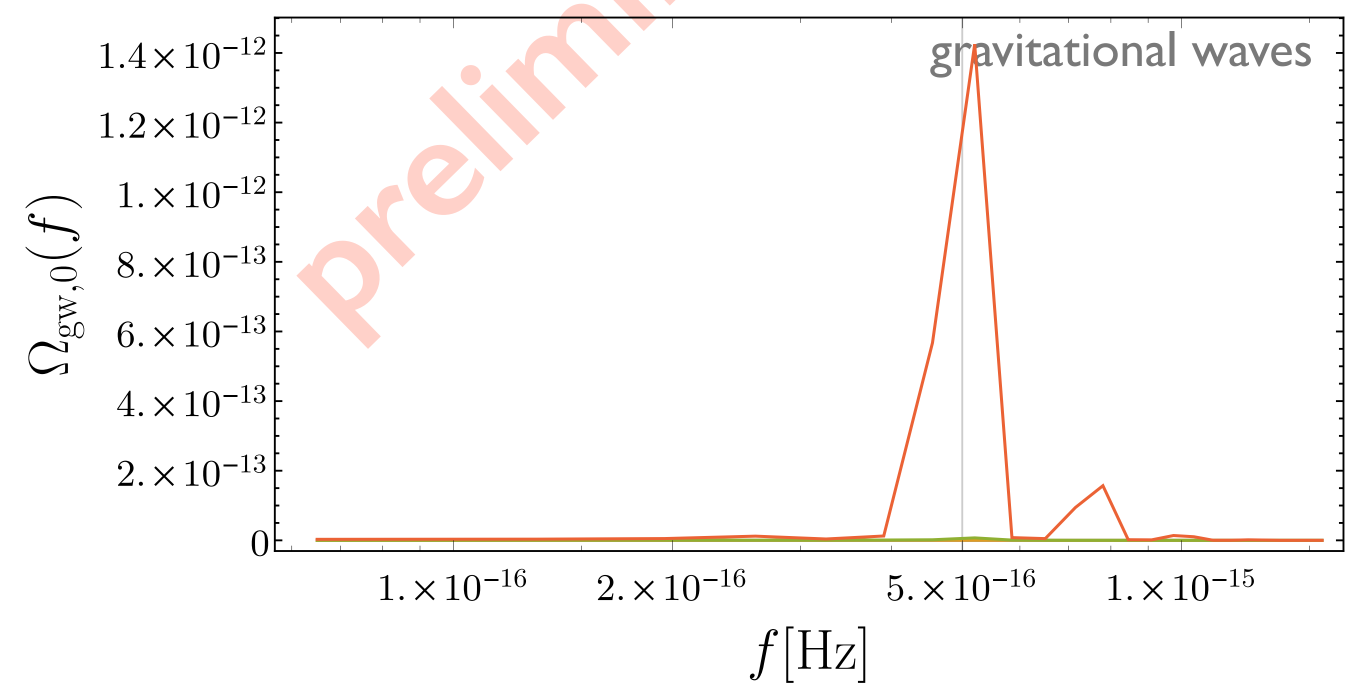
thanks

“Hubble Tension” resolution — some novel implications



*no-solitons





Smith, Poulin & MA (2019)
MA, Lozanov & Smith (in progress)

Also see: Karwal & Kamionkowski (2016), Poulin et. al (2018), Agrawal et. al (2019).

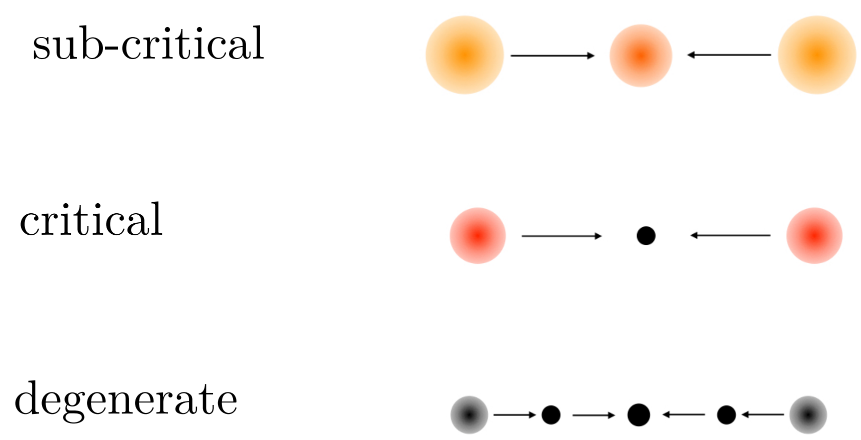
Numerical GR ✓

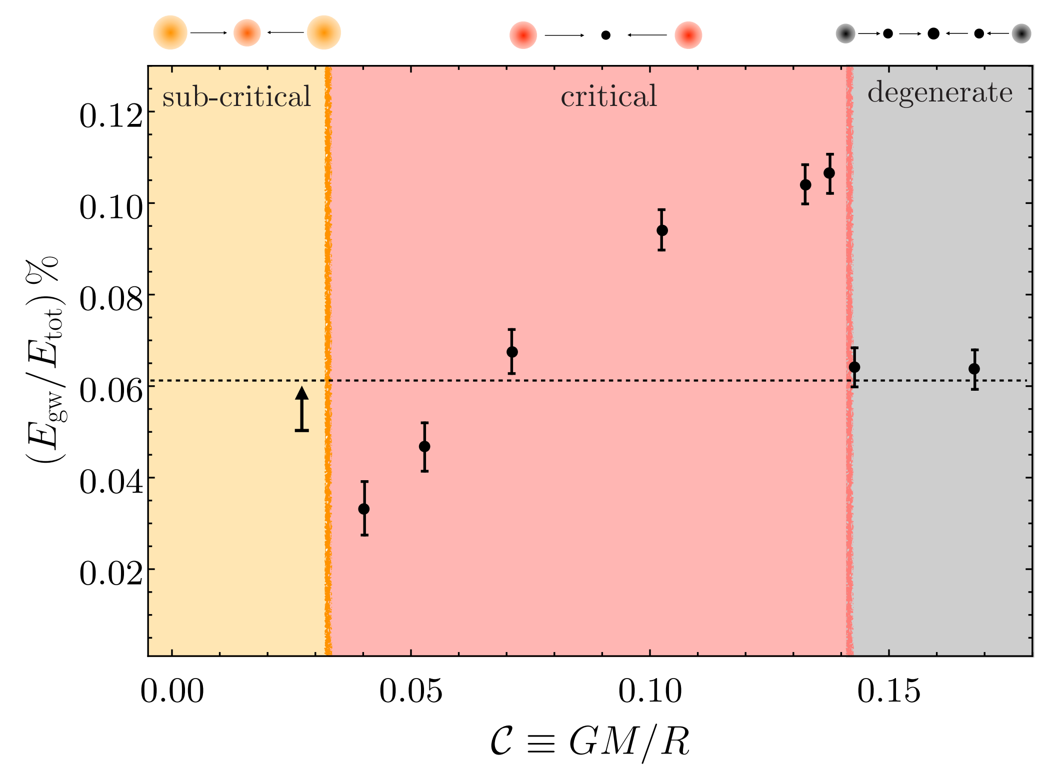
self-interactions ✗

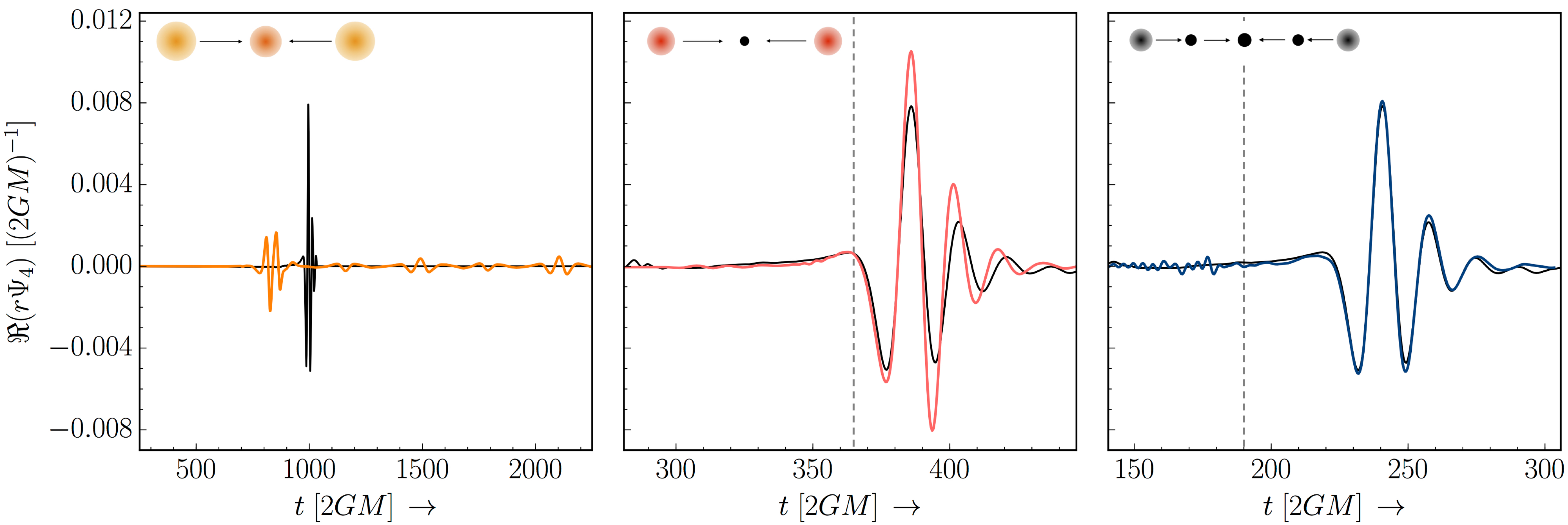
formation ✗

"ultra-compact" soliton collision

Helfer, Lim, Garcia & MA (2018)





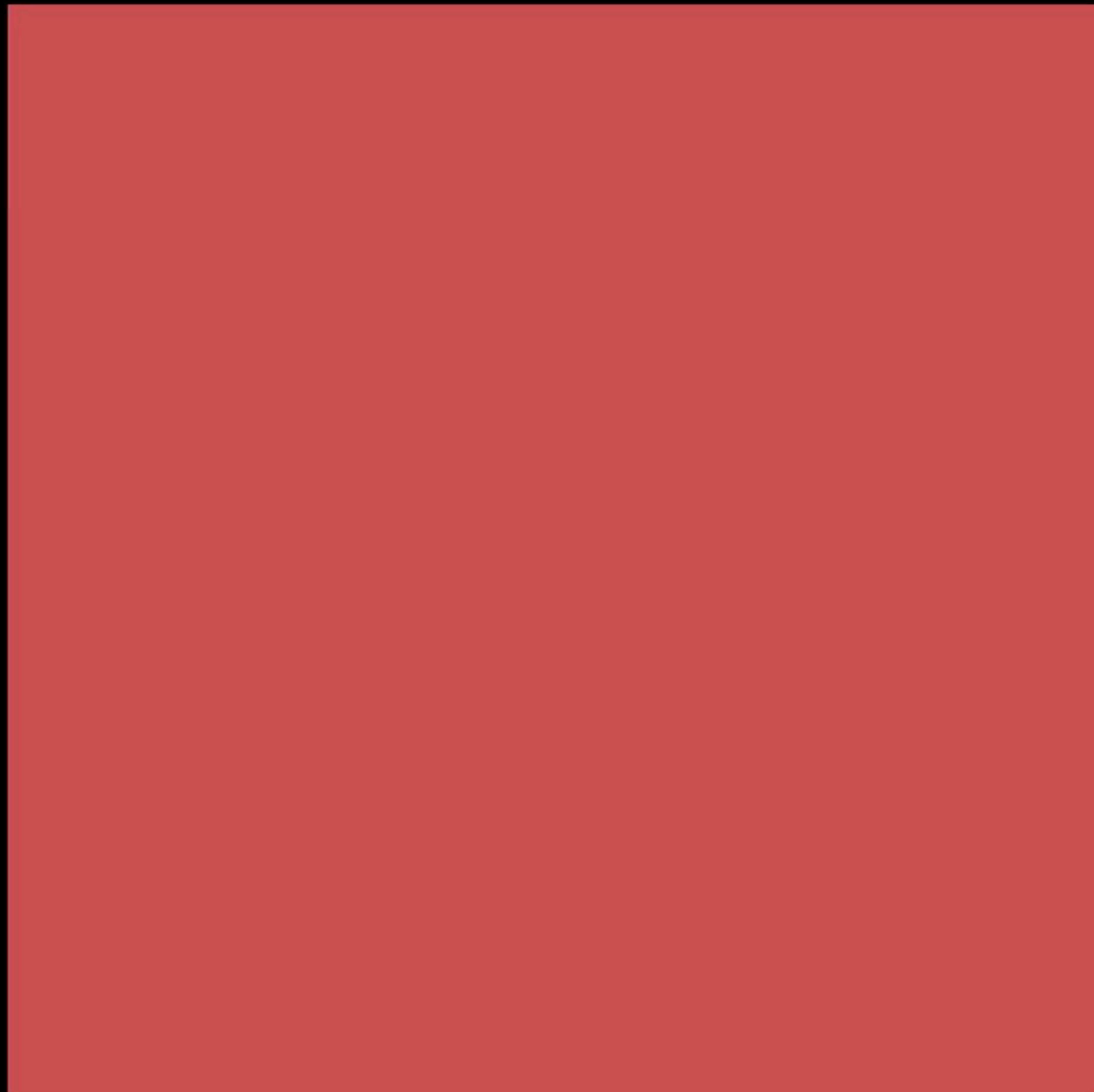


relativistic* ✓

self-interactions ✓

gravitational int. ✗

a=1.21



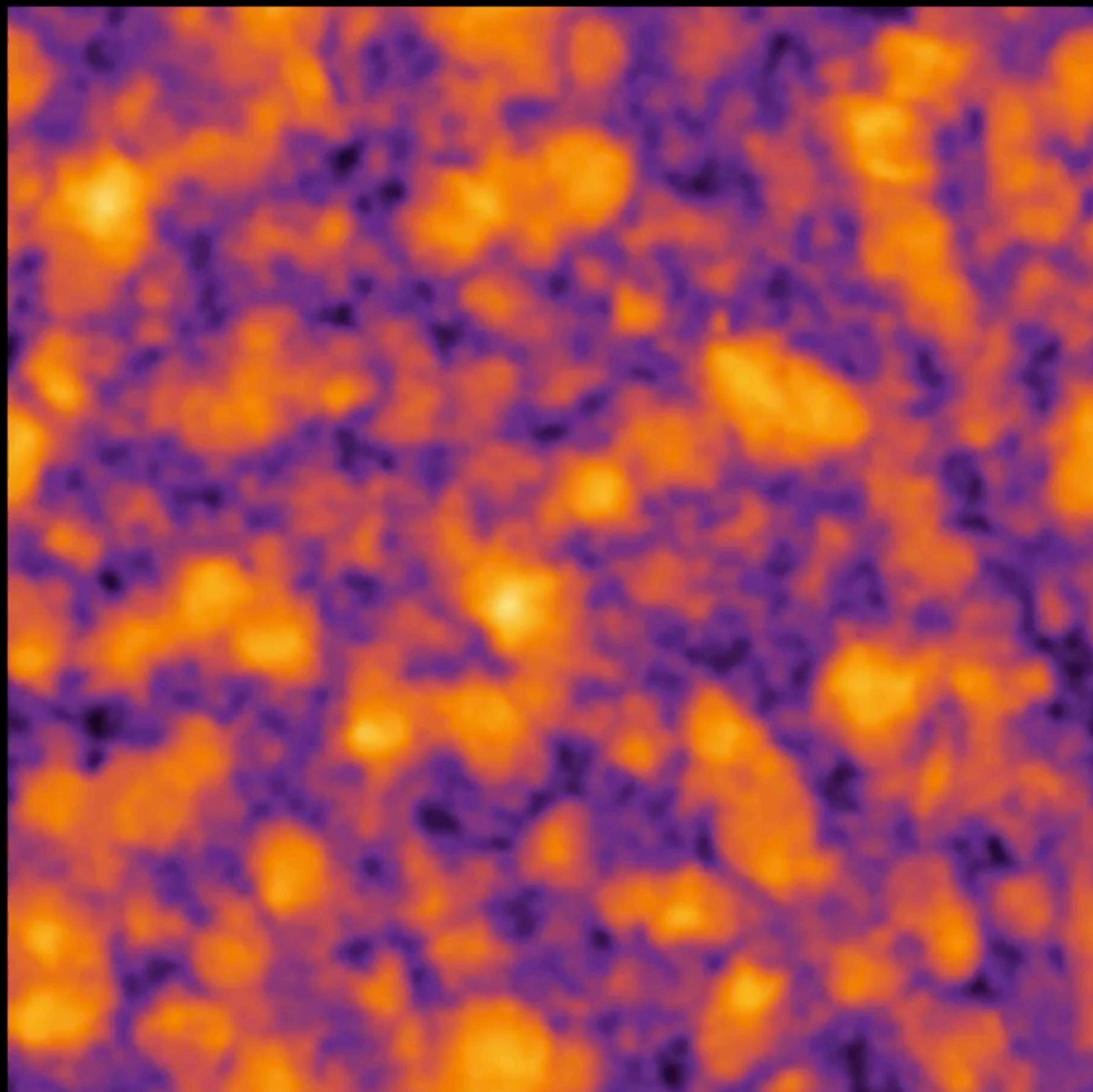
Lozanov & MA (2017)

relativistic* ✓

self-interactions ✓

gravitational int. ✗

a=1.93

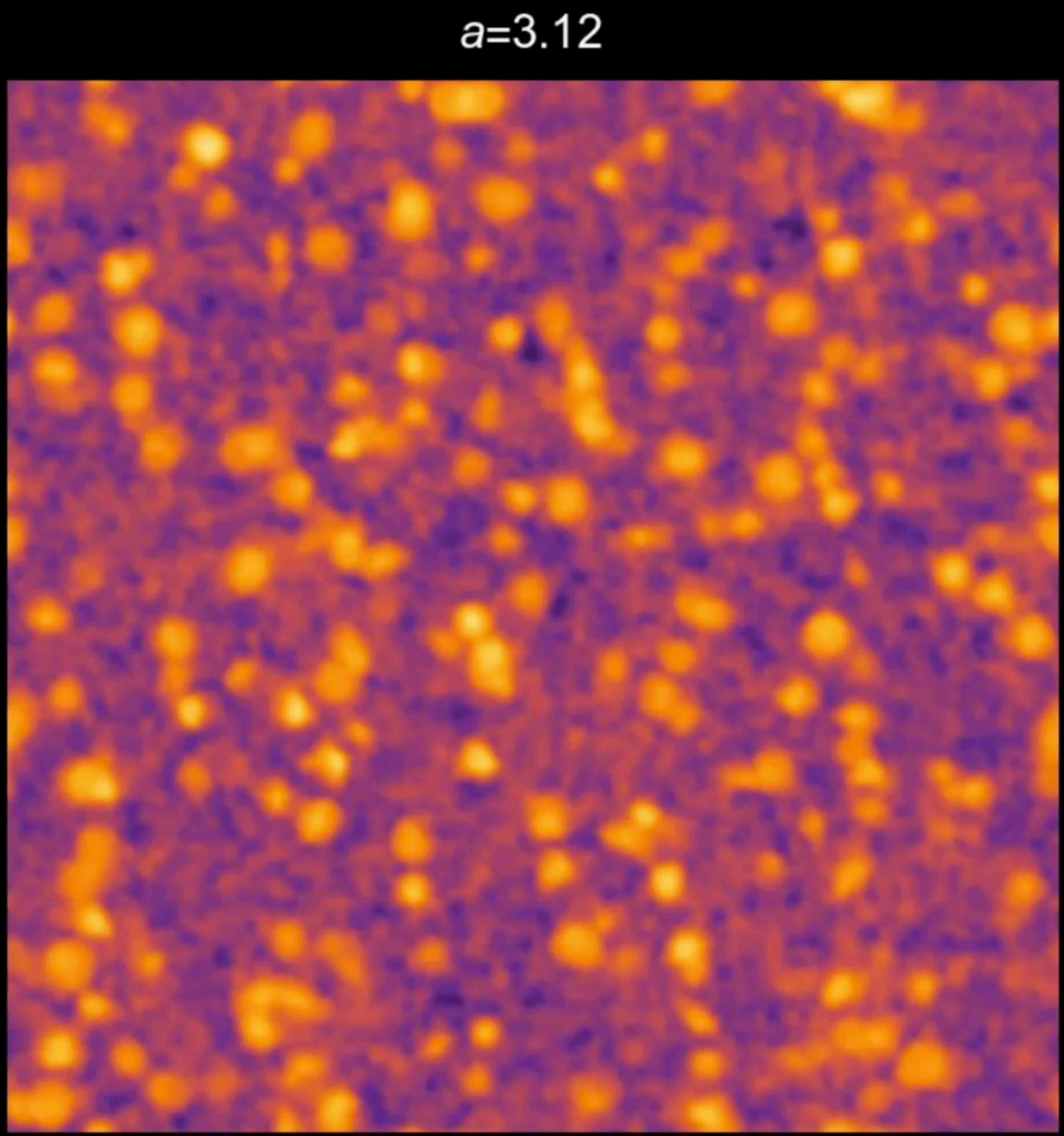


Lozanov & MA (2017)

relativistic* ✓

self-interactions ✓

gravitational int. ✗



Lozanov & MA (2017)

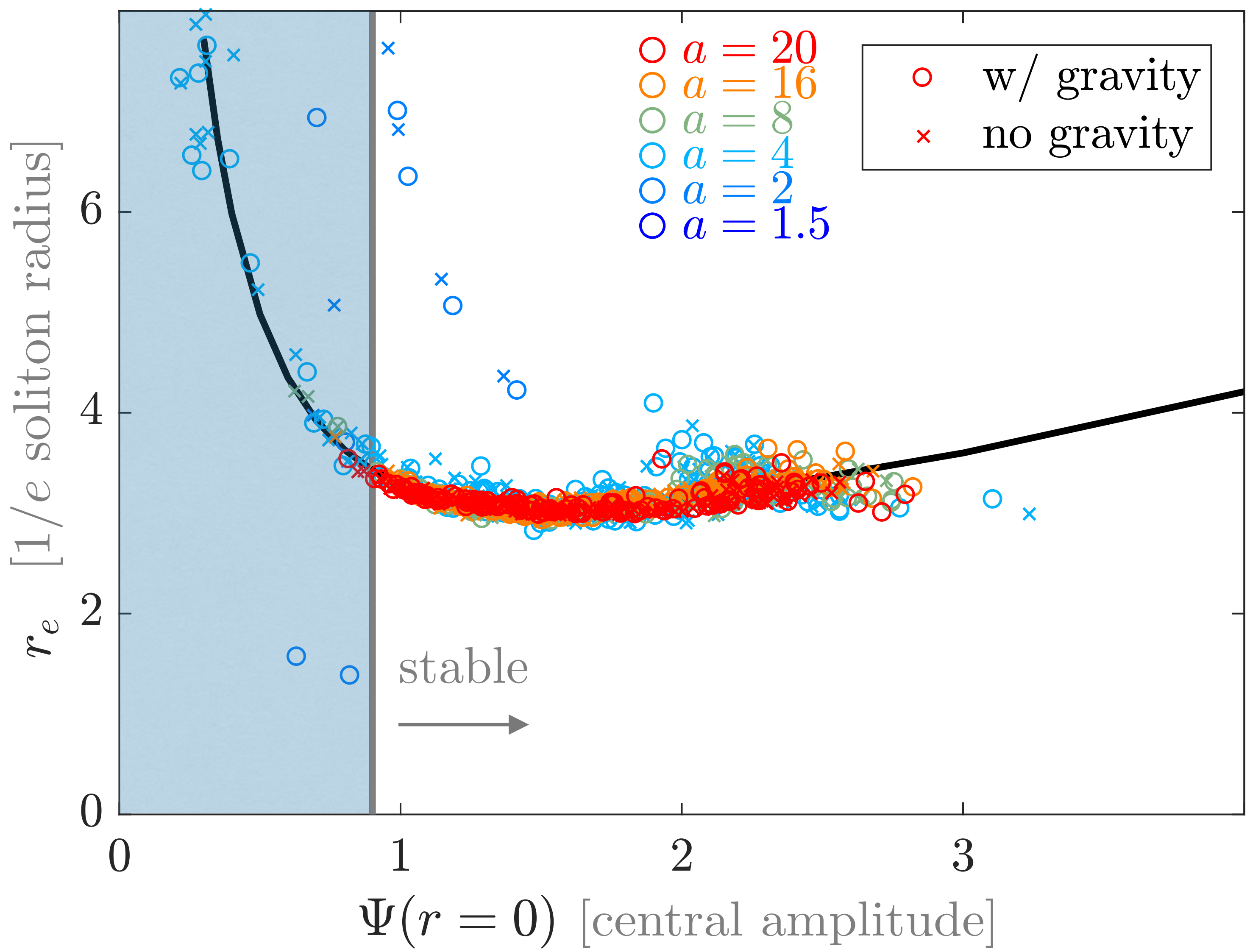
individual solitons stability

$$\psi(t,r) = e^{-i\nu t}\Psi(r)$$

$$\mathcal{N} \equiv \int d^3r\Psi^2(r)$$

stable iff:
Vakhitov Kolokolov (1973)

$$\frac{d\mathcal{N}}{d(-\nu)} > 0$$



— stability with gravitational interactions needs to be investigated