Cosmological Radiative Transfer

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Outline

- Reionization inside a computer
- SF & RT: two stumbling blocks
- Is OTVET the answer?
- Adaptive Mesh Refinement vs Ray Tracing
- A call for help.

The *very* Brief History of Time

The universe began **HOT** - ended **COLD**

- T > 4000K: hot and photo-*ionized*
- T ~ 4000K: recombination
- T < 4000K: neutral and *cooling down*…
- Today: hot (~8000K) and *ionized* !

Something must have happened in between!

RE-IONIZATION

Imagine If You Were a Hydrogen Atom…

Reionization Inside a Computer

Easy physics: *fast, accurate*
- dark matter dynamics
- gas dynamics
- ionization/recombination/heating

Hard physics:
- star formation ← *phenomenology*
- radiative transfer ← *hard numerics*

Extra physics:
- magnetic fields
- dust
- cosmic rays

Perhaps, not needed ?

Star Formation

Schmidt Law:

$$\frac{d\rho_*}{dt} = \epsilon_{\rm SF} \frac{\rho_{\rm gas}}{\tau_{\rm dyn}}$$

- Must be scale-dependent

Although, observed in large and small galaxies

Radiative Transfer

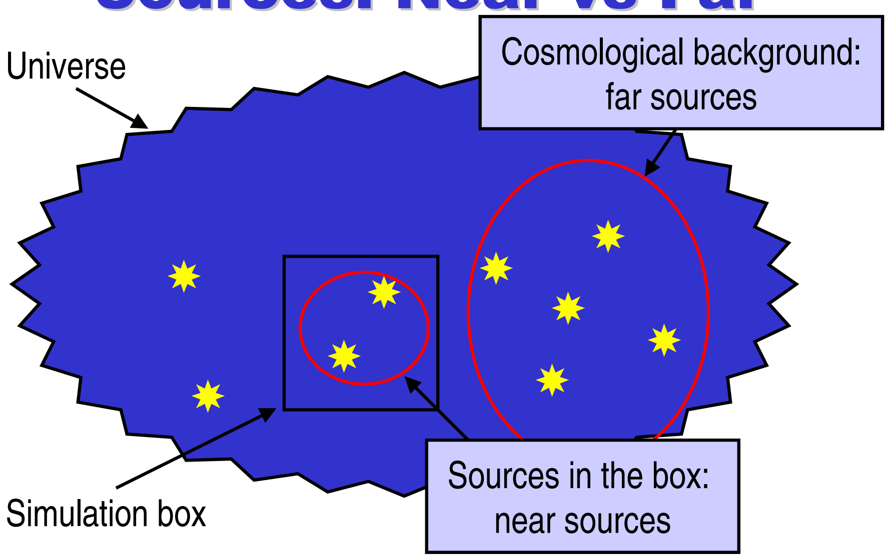
$$\frac{\partial J_\nu}{\partial t} + c\vec{n}\frac{\partial J_\nu}{\partial \vec{x}} = -k_\nu J_\nu + S_\nu$$

Just another Boltzmann equation, but…

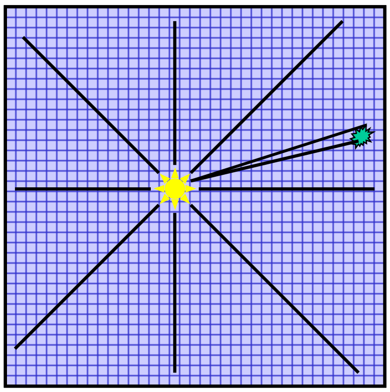
$$\frac{\partial f}{\partial t} + \vec{v}\frac{\partial f}{\partial \vec{x}} - \nabla\phi\frac{\partial f}{\partial \vec{v}} = 0$$

Yet another Boltzmann equation.

Sources: Near vs Far



Ray Tracing



If we want to preserve
the resolution of gas solver:
N(rays) ~ N(surface cells)

↓

Far sources: ~ $N^{5/3} \times N_\nu$

Near sources: ~ $N \times N_S \times N_\nu$

No way!

Moments of Boltzmann Equation

Massive particles:
- Density
- Momentum
- Pressure tensor
- …

Photons:
- Energy density
- Flux
- Radiation Pressure tensor
- …

The hierarchy needs to be closed.

But: photons move with the speed of light!

Eddington Tensor

$$P_R^{ij} = \frac{4\pi}{c}\langle J_\nu n^i n^j \rangle = E_\nu h^{ij}$$

Eddington tensor

$$\mathrm{Trace}(h^{ij}) = 1$$

5 independent components, not 6!

OTVET Approximation

Simple idea:
- use the Eddington tensor from the optically thin regime.

Optically thin regime:
- collect $1/r^2$ contribution from all sources – just like gravity!

Result:
- **O**ptically **T**hin **V**ariable **E**ddington **T**ensor approximation

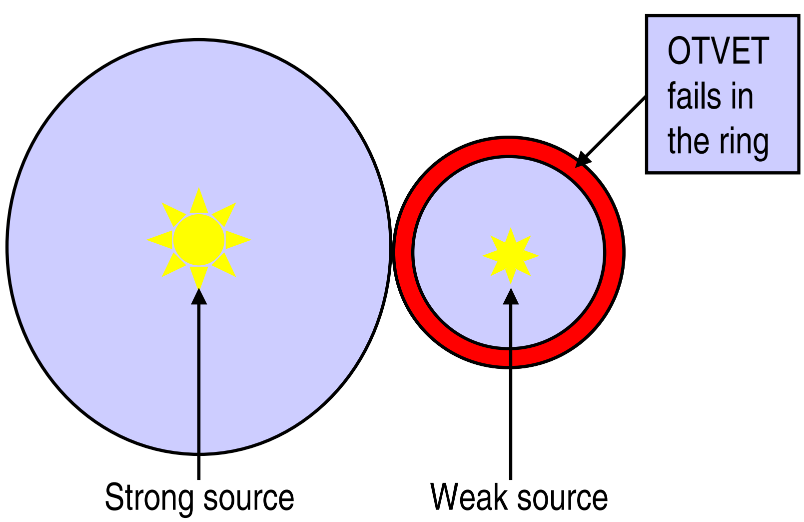
Scaling

- Moments equations (like hydro) ~ $N \times N_\nu$
- Eddington tensor (like gravity) ~ $N \times \log(N)$

Advantages:

- **fast**
- controlled: the error of the approximation can be measured posteriori
- no scaling with number of sources: works for any N_S
- energy density and flux are conserved **exactly**

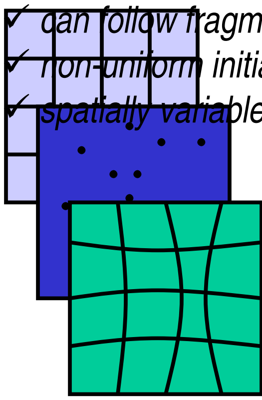
Where It Fails



Do We Have a Choice?

XX Century

XXI Century

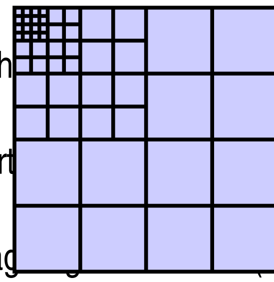


✓ *can follow fragmentation*
✓ *non-uniform initial conditions*
✓ *spatially variable resolution*

• Eulerian sch

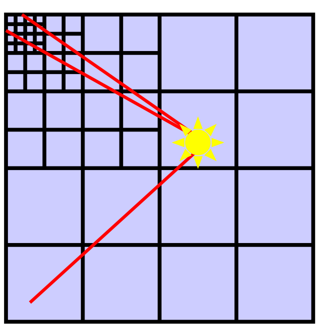
• Smooth Part s” (SPH)

• Arbitrary Lag LE)



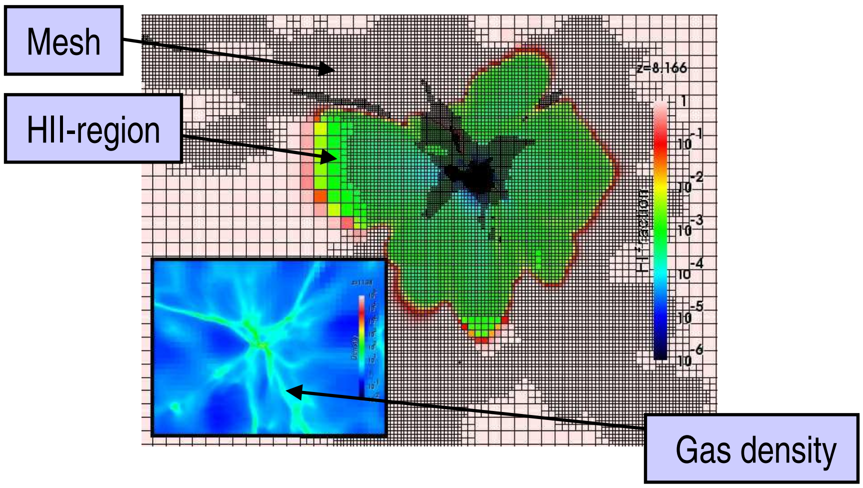
• Adaptive Mesh Refinement (AMR)

AMR And Ray Tracing



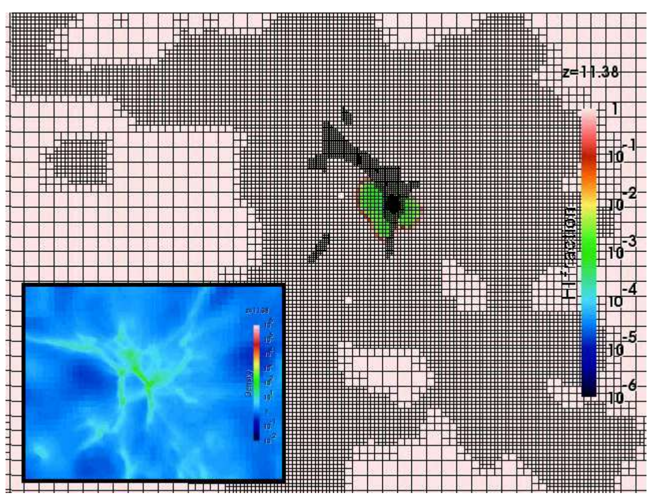
Ray Tracing on the AMR mesh would require unrealistically many rays – as if the whole mesh was refined.

AMR & OTVET



OTVET is mesh-based, just like hydrodynamics!

AMR & OTVET



RT vs N-body

Radiative Transfer		**N-body**
Ray tracing	⇦⇨	Direct summation
OTVET	⇦⇨	Hydrodynamics
?	⇦⇨	PM
???	⇦⇨	P^{3}M, Tree, AMR

Conclusions

- Reionization is a complex process: its complete understanding will crucially depend on our ability to simulate radiative transfer with a fast and accurate solver.
- Large computational boxes will be required, because HII regions are biased and non-trivially shaped.
- As AMR takes over, we are left with few options for doing RT on adaptive meshes.
- Graduate students: ***YOU ARE THE ONLY HOPE!***

The End

Title