Parity Violation in Three-nucleon Systems

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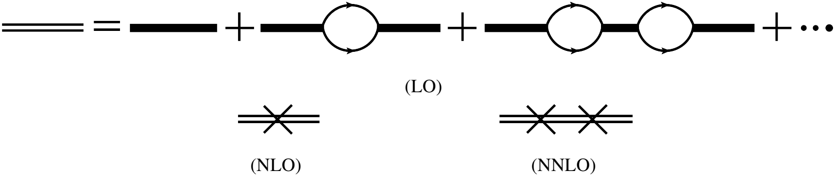
Dibaryon fields make three-body calculations easier

$$\mathcal{L} = \hat{N}^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{2M_N} \right) \hat{N} - \hat{t}_i^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} - \Delta_{(-1)}^{({}^3S_1)} - \Delta_{(0)}^{({}^3S_1)} \right) \hat{t}_i$$
$$- \hat{s}_a^\dagger \left(i\partial_0 + \frac{\vec{\nabla}^2}{4M_N} - \Delta_{(-1)}^{({}^1S_0)} - \Delta_{(0)}^{({}^1S_0)} \right) \hat{s}_a + y_t \left[\hat{t}_i^\dagger \hat{N}^T P_i \hat{N} + H.c. \right]$$
$$+ y_s \left[\hat{s}_a^\dagger \hat{N}^T \bar{P}_a \hat{N} + H.c. \right].$$

- $\hat{N}$ nucleon fields
- $\hat{t}_i$ (dibaryon field) two nucleons in ${}^3S_1$ channel
- $\hat{s}_a$ (dibaryon field) two nucleons in ${}^1S_0$ channel
- Can be matched to theory of only nucleons by integrating out dibaryon fields

The LO dressed deuteron propagator is given by a sum of bubble diagrams

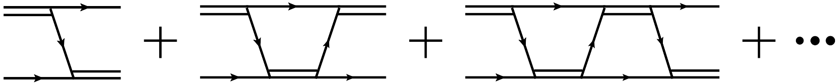


yielding the LO dibaryon propagator

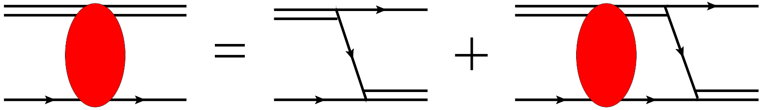
$$iD_{t,s}^{LO}(p_0,\vec{\mathbf{p}}) = \frac{4\pi i}{M_N y_{t,s}^2}\frac{1}{\gamma_{t,s} - \sqrt{\frac{\vec{\mathbf{p}}^2}{4} - M_N p_0 - i\epsilon}}$$

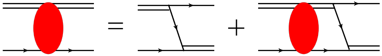
Quartet Channel (*nd* Scattering)

At LO in the quartet channel, *nd* scattering is given by an infinite sum of diagrams.



This infinite sum of diagrams can be represented by an integral equation.





Projecting spin and isospin in the quartet channel and projecting out in angular momentum gives

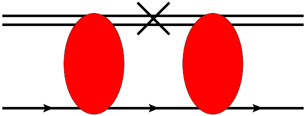
$$t^{\ell}_{0,q}(k,p) = -\frac{y_t^2 M_N}{pk} Q_\ell\left(\frac{p^2+k^2-M_N E - i\epsilon}{pk}\right) - \\ + \frac{2}{\pi}\int_0^\Lambda dq q^2 t^{\ell}_{0,q}(k,q) \frac{1}{\gamma_t - \sqrt{\frac{3\vec{\mathbf{q}}^2}{4} - M_N E - i\epsilon}} \frac{1}{qp} \times \\ Q_\ell\left(\frac{p^2+q^2-M_N E - i\epsilon}{pq}\right),$$

where

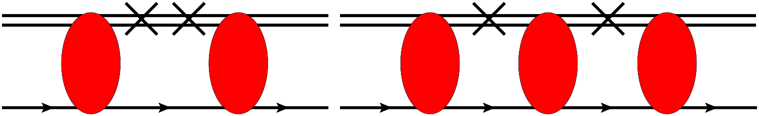
$$Q_\ell(a) = \frac{1}{2}\int_{-1}^{1} dx \frac{P_\ell(x)}{x+a}.$$

Higher Orders

NLO correction is



NNLO corrections are



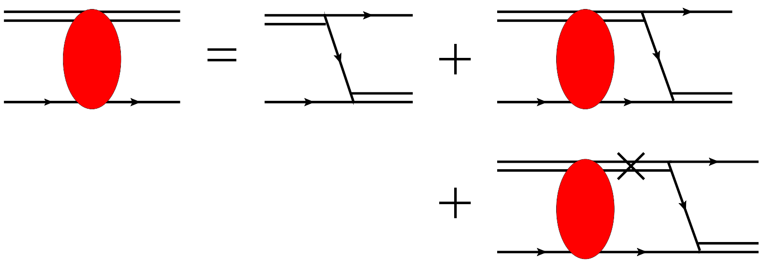
Note the second diagram contains full off-shell scattering amplitude.

Partial Resummation Technique

Denoting $t^{\ell}_{NLO} = t^{\ell}_{0,q} + t^{\ell}_{1,q}$, for the partial resummation technique one finds (Bedaque, Rupak, Grießhammer, and Hammer (2003))

$$t^{\ell}_{NLO}(k,p) = B^{\ell}_{0}(k,p) + B^{\ell}_{1}(k,p) + (K^{\ell}_{0}(q,p,E) + K^{\ell}_{1}(q,p,E)) \otimes t^{\ell}_{NLO}(k,q),$$

with the diagrammatic representation

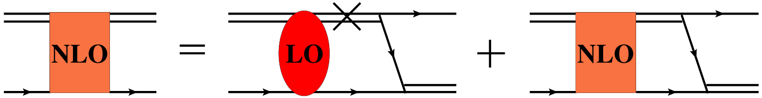


New Full Perturbative technique

Picking out only NLO pieces gives (Vanasse (2013))

$$t_{1,q}^{\ell}(k,p)=B_{1}^{\ell}(k,p)+K_{1}^{\ell}(q,p,E)\otimes t_{0,q}^{\ell}(k,q)+K_{0}^{\ell}(q,p,E)\otimes t_{1,q}^{\ell}(k,q).$$

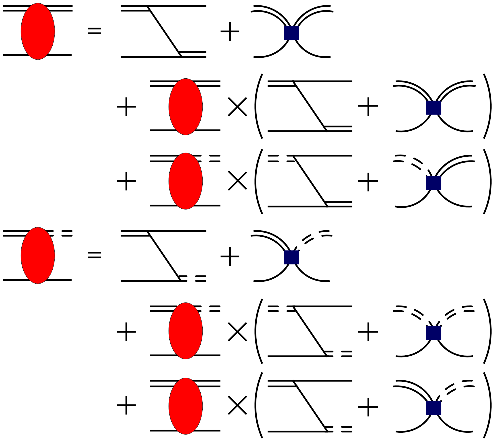
Terms are reshuffled to inhomogeneous term. Kernel at each order is the same. Diagrammatically NLO correction is now given by



Note all corrections are half off-shell.

Doublet Channel *nd* scattering

At LO in the doublet channel, *nd* scattering is given by a coupled set of integral equations



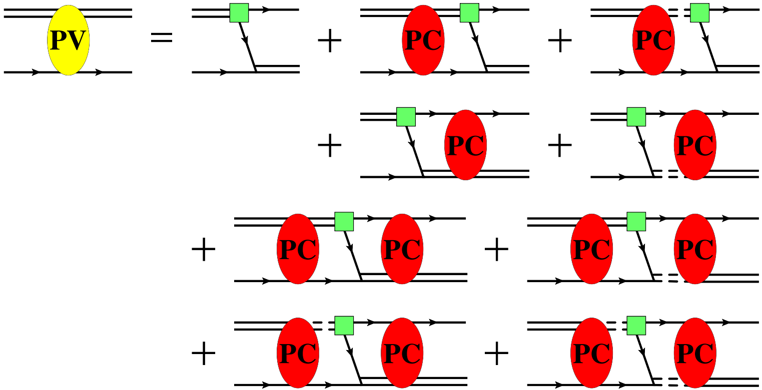
Two-Body Parity Violation

The LO PV Lagrangian in $\mathrm{EFT}_{\not\pi}$ has five LEC's

$$\mathcal{L}_{PV} = -\Big[g^{(^3S_1-^1P_1)} t_i^\dagger \left(N^t \sigma_2\tau_2 i \overset{\leftrightarrow}{\nabla}_i N\right)$$
$$+ g^{(^1S_0-^3P_0)}_{(\Delta I=0)} s_a^\dagger \left(N^T \sigma_2 \vec{\boldsymbol{\sigma}} \cdot \tau_2 \tau_a i \overset{\leftrightarrow}{\nabla} N\right)$$
$$+ g^{(^1S_0-^3P_0)}_{(\Delta I=1)} \epsilon^{3ab} (s^a)^\dagger \left(N^T \sigma_2 \vec{\boldsymbol{\sigma}} \cdot \tau_2 \tau^b \overset{\leftrightarrow}{\nabla} N\right)$$
$$+ g^{(^1S_0-^3P_0)}_{(\Delta I=2)} \mathcal{I}^{ab} (s^a)^\dagger \left(N^T \sigma_2 \vec{\boldsymbol{\sigma}} \cdot \tau_2 \tau^b i \overset{\leftrightarrow}{\nabla} N\right)$$
$$+ g^{(^3S_1-^3P_1)} \epsilon^{ijk} (t_i)^\dagger \left(N^T \sigma_2 \sigma^k \tau_2 \tau_3 \overset{\leftrightarrow}{\nabla}{}^j N\right)\Big] + h.c.,$$

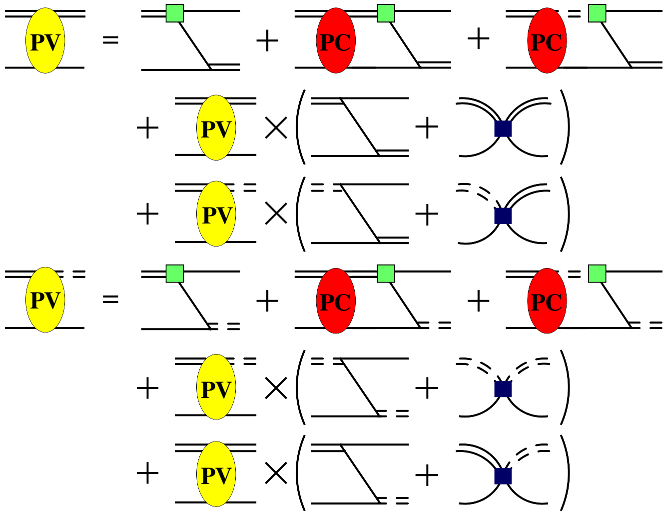
where $\mathcal{I}^{ab} = diag(1,1,-2)$ and $a \overset{\leftrightarrow}{\nabla} b = a(\overset{\rightarrow}{\nabla} b) - (\overset{\rightarrow}{\nabla} a) b$. Contains all possible $S \to P$ transition operators and isospin structures

LO PV given by sum of diagrams

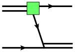


Diagrams with lower two-body PV vertex not shown

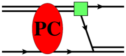
Sum of diagrams can also be represented via integral equation



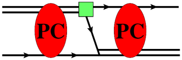
The amplitude can be projected in partial waves of $\vec{\mathbf{J}} = \vec{\mathbf{L}} + \vec{\mathbf{S}}$



$$t_{PV}{}^{JM}_{L'S',LS}(k,p) \sim \mathcal{K}(k,p)^{JM}_{L'S',LS}$$

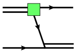


$$t_{PV}{}^{JM}_{L'S',LS}(k,p) \sim \int_0^\infty dq q^2 \mathcal{K}(q,p)^{JM}_{L'S',LS} \mathbf{D}\left(E - \frac{q^2}{2M_N}, \vec{\ell}\right)\left(t_{PC}{}^{JM}_{LS,LS}(k,q)\right)$$



$$t_{PV}{}^{JM}_{L'S',LS}(k,p) \sim \int_0^\infty dq q^2 \int_0^\infty d\ell \ell^2 \left(t_{PC}{}^{JM}_{L'S',L'S'}(p,\ell)\right)^T \mathbf{D}\left(E - \frac{\ell^2}{2M_N}, \vec{\ell}\right)$$
$$\mathcal{K}(q,\ell)^{JM}_{L'S',LS} \mathbf{D}\left(E - \frac{q^2}{2M_N}, \vec{\mathbf{q}}\right)\left(t_{PC}{}^{JM}_{LS,LS}(k,q)\right)$$

One term of projected $\mathcal{K}(k,p)^{JM}_{L'S',LS}$



is given by

$$\left[\mathcal{K}(k,p)^{J}_{L'S',LS}\right]_{22} = -y_t\left(3g^{^1S_0-^3P_0}_{(\Delta I=0)} - 2g^{^1S_0-^3P_0}_{(\Delta I=1)}\right)4\pi\sqrt{6}(-1)^{1/2-L-J}\delta_{S1/2}\delta_{S'1/2}\sqrt{\bar{L'}}$$
$$\times\, C^{0,0,0}_{L',1,L}\left\{\begin{array}{ccc} L' & 1 & L \\ S & J & S' \end{array}\right\}\frac{1}{kp}(kQ_{L'}(a)+pQ_L(a))$$

where

$$\bar{x} = 2x+1$$

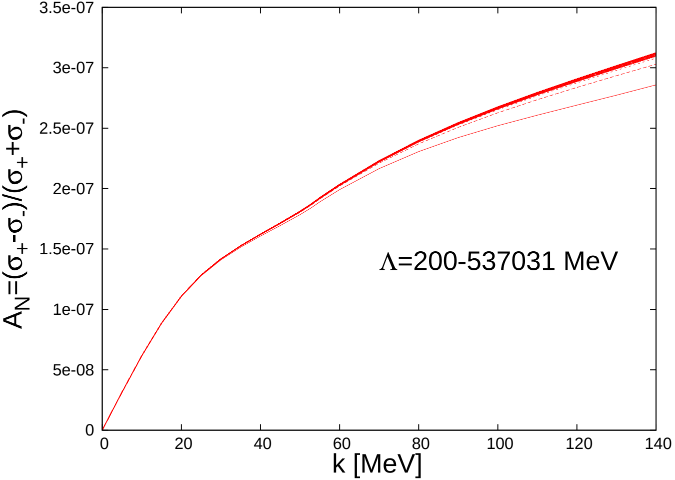
and

$$a = \frac{k^2+p^2-M_NE-i\epsilon}{kp}$$

All Projections given in (Vanasse (2012)). Agree with S-wave to P-wave projections in (Grießhammer, Schindler, and Springer (2012)).

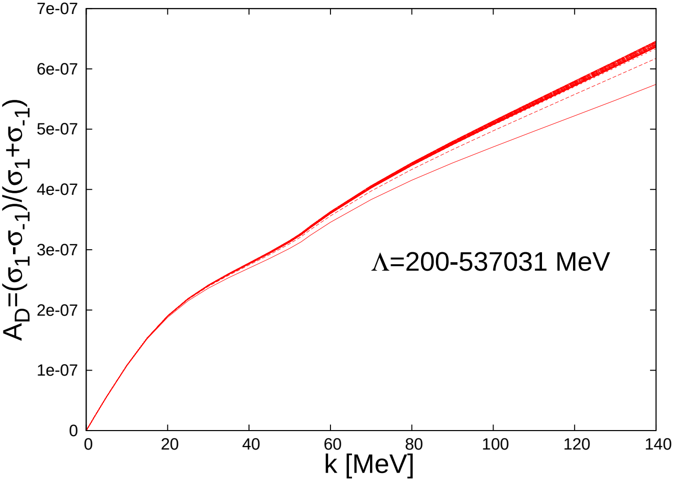
Longitudinal neutron analyzing power

$$A_N = \frac{\sigma_\uparrow - \sigma_\downarrow}{\sigma_\uparrow + \sigma_\downarrow}$$



Longitudinal deuteron analyzing power

$$A_D = \frac{\sigma_1 - \sigma_{-1}}{\sigma_1 + \sigma_{-1}}$$



nd spin rotation

$$\frac{d\phi}{dz} = -\frac{4M_N N}{27k}\mathrm{Re}\left[M^{1/2}_{1^1/2,0^1/2} + 2\sqrt{2}M^{1/2}_{1^3/2,0^1/2} - 4M^{3/2}_{1^1/2,0^3/2} - 2\sqrt{5}M^{3/2}_{1^3/2,0^3/2}\right]$$

Spin rotation prediction in LO $\mathrm{EFT}_{\not\pi}$ is 1.8×10^{-8} rad cm^{-1}, cutoff variation minimal

$$\frac{1}{N}\frac{d\phi}{dz} = \sum_n c_n g_n$$

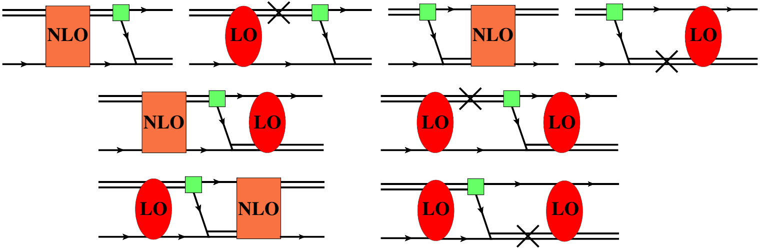
Table: Comparison of EFT calculations for spin rotation $\frac{1}{\rho}\frac{d\phi}{dz}$.

Coefficient	LO [rad MeV$^{-1/2}$]	NLO [rad MeV$^{-1/2}$]
$g^{^3S_1-^1P_1}$	10.4-10.7	7.2-7.8
$g^{^3S_1-^3P_1}$	20.1 - 21.1	15.3-18.7
$3g^{^1S_0-^3P_0}_{(\Delta I=0)} - 2g^{^1S_0-^3P_0}_{(\Delta I=1)}$	1.9-3.1	1.8-2.8

LO EFT calculation (Vanasse (2012)), NLO EFT calculation (Grießhammer, Schindler, and Springer (2012)) using partial resummation technique.

NLO 3-Body PV

NLO PV amplitude is given by type of diagrams below. NLO box is the half off shell NLO amplitude.



(Note not all diagrams given here)

As shown by (Schindler and Grießhammer (2010)) no NLO PV three-body force for *Nd* scattering should exist.

Asymptotic Behavior (Bedaque Numbers)

Going to Wigner basis asymptotic form of *nd* scattering integral equation is

$$t_\lambda^{(\ell)}(p) = \frac{8\lambda}{\sqrt{3}\pi}(-1)^\ell \int_0^\infty \frac{dq}{q} Q_l\left(\frac{p}{q}+\frac{q}{p}\right) t_\lambda^{(\ell)}(q)$$

$(\lambda = 1)$: Wigner-symmetric combination
$(\lambda = -1/2)$: Wigner antisymmetric combination
Equation is scaleless and must have solution of form

$$t_\lambda^{(\ell)}(p) = p^{-s_\ell^\lambda - 1}$$

partial wave ℓ	$s_\ell(\lambda = 1)$	$s_\ell(\lambda = -\frac{1}{2})$
0	$1.00624\ldots i$	$2.16622\ldots$
1	$2.86380\ldots$	$1.77272\ldots$
2	$2.82334\ldots$	$3.10498\ldots$

(Grießhammer 2005)

Using Fierz rearrangements only single derivative PV 3B forces in $^2S_{1/2} - {}^2P_{1/2}$ are ((Grießhammer and Schindler (2010)))

$$i\mathcal{M}\left[{}^2S_{\frac{1}{2}} \to {}^2P_{\frac{1}{2}}, p, q\right]^{\text{Wigner}}_{\text{3NI}} = A_{\text{3NI}}\left(H^{(\Delta I=0)}_{\text{PV}} + \tau^3 H^{(\Delta I=1)}_{\text{PV}}\right)\begin{pmatrix} 2 & 0 \\ 0 & 0 \end{pmatrix}$$

tree-level PV diagrams are given by

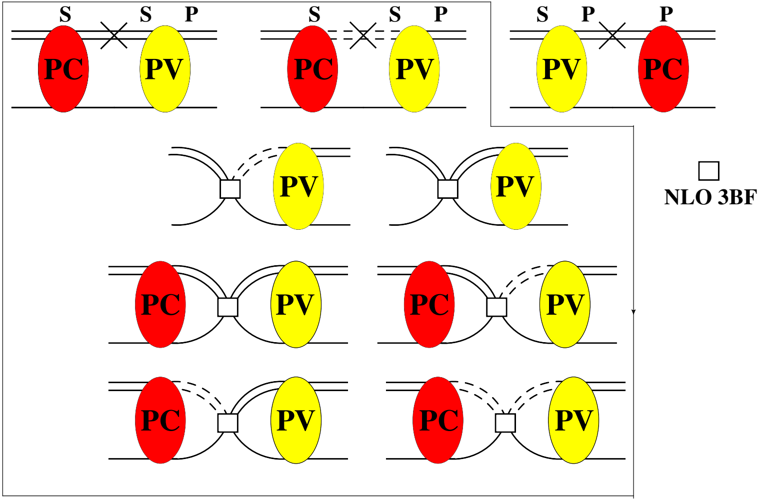
$$i\mathcal{M}\left[{}^2S_{\frac{1}{2}} \to {}^2P_{\frac{1}{2}}, p, q\right]^{\text{Wigner}}_{\text{2NI}} = A^{(a)}_{\text{2NI}}\begin{pmatrix} 0 & 0 \\ \mathcal{S}_1 + \mathcal{T} & \mathcal{S}_1 - \mathcal{T} \end{pmatrix} + A^{(b)}_{\text{2NI}}\begin{pmatrix} 0 & \mathcal{S}_1 + \mathcal{T} \\ 0 & \mathcal{S}_1 - \mathcal{T} \end{pmatrix}$$

where

$$\mathcal{S}_1 = 3g^{({}^3S_1 - {}^1P_1)} + 2\tau_3 g^{({}^3S_1 - {}^3P_1)}, \mathcal{T} = 3g^{({}^1S_0 - {}^3P_0)}_{(\Delta I=0)} + 2\tau_3 g^{({}^1S_0 - {}^3P_0)}_{(\Delta I=1)}$$

PC scattering amplitudes are diagonal in Wigner basis, therefore NLO PV diagrams do not contain element in upper left of Wigner basis matrix. Hence, **no NLO 3B PV force**

The NLO PV amplitude can be calculated with



Note, three-body force is PC

Asymptotic behavior of NLO PV $^2S_{1/2} - {}^2P_{1/2}$ scattering

$$\frac{1}{16\pi}\frac{\Lambda^{2-s_1}}{\sqrt{s_0^2+(s_1-2)^2}}\left[(\rho_t+\rho_s)\left\{CD^{^2P_{1/2}}\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{s_0}{s_1-2}\right)\right)\right.\right.$$
$$\left.+B^{^2P_{1/2}}|H^{^2P_{1/2}}|\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{s_0}{s_1-2}\right)+\mathrm{Arg}(H^{^2P_{1/2}})\right)\right\}$$
$$\left.+(\rho_t-\rho_s)CE^{^2P_{1/2}}\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{s_0}{s_1-2}\right)\right)\right]$$
$$+\frac{4H_{NLO}(\Lambda)}{3\pi^2\Lambda^2}CD^{^2P_{1/2}}\frac{1}{\sqrt{1+s_0^2}(2-s_1)}\Lambda^{3-s_1}\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)-\arctan(s_0)\right)+b,$$

where $H_{NLO}(\Lambda)$ is NLO PC 3B force

$$H_{NLO}(\Lambda)=-\Lambda\frac{3\pi(1+s_0^2)}{128}(\rho_t+\rho_s)\frac{\left(1-\frac{1}{\sqrt{1+4s_0^2}}\sin\left(2s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{1}{2s_0}\right)\right)\right)}{\sin^2\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)-\arctan(s_0)\right)}+\cdots$$

Asymptotic behavior of NLO PV ${}^2S_{1/2} - {}^4P_{1/2}$ scattering

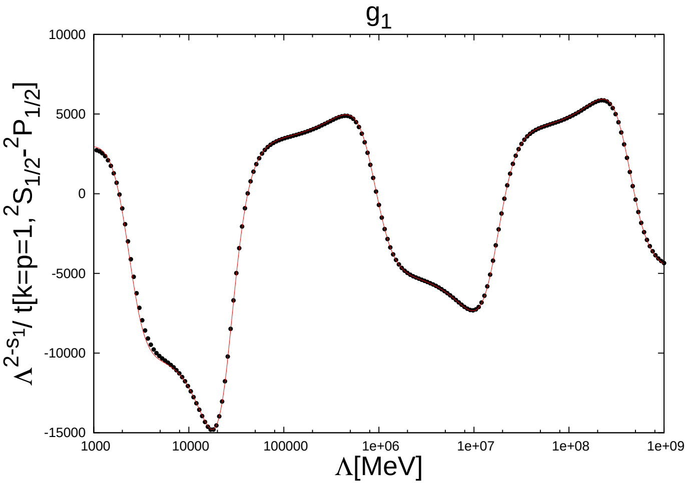
$$
\frac{1}{16\pi}\frac{\Lambda^{2-s_1}}{\sqrt{s_0^2+(s_1-2)^2}}\left[(\rho_t+\rho_s)CD^{{}^4P_{1/2}}\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{s_0}{s_1-2}\right)\right)\right.
$$

$$
+(\rho_t-\rho_s)CE^{{}^4P_{1/2}}\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{s_0}{s_1-2}\right)\right)
$$

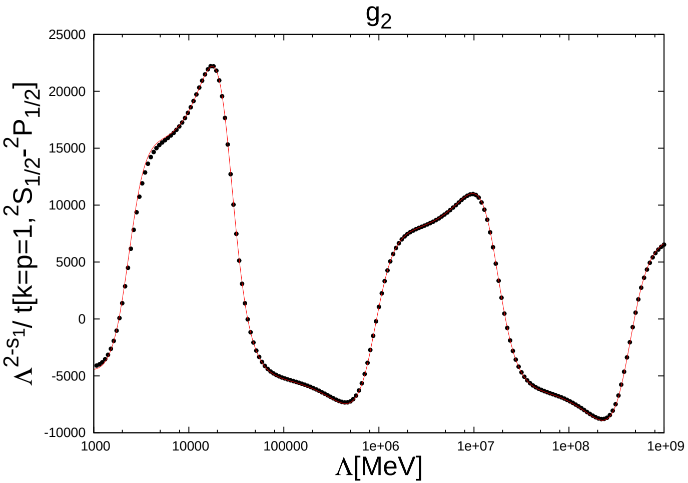
$$
\left.+4\rho_t B^{{}^4P_{1/2}}|H^{{}^4P_{1/2}}|\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)+\arctan\left(\frac{s_0}{s_1-2}\right)+\mathrm{Arg}(H^{{}^4P_{1/2}})\right)\right]
$$

$$
+\frac{4H_{NLO}(\Lambda)}{3\pi^2\Lambda^2}CD^{{}^4P_{1/2}}\frac{1}{\sqrt{1+s_0^2}(2-s_1)}\Lambda^{3-s_1}\sin\left(s_0\ln\left(\frac{\Lambda}{\Lambda^*}\right)-\arctan(s_0)\right)+b
$$

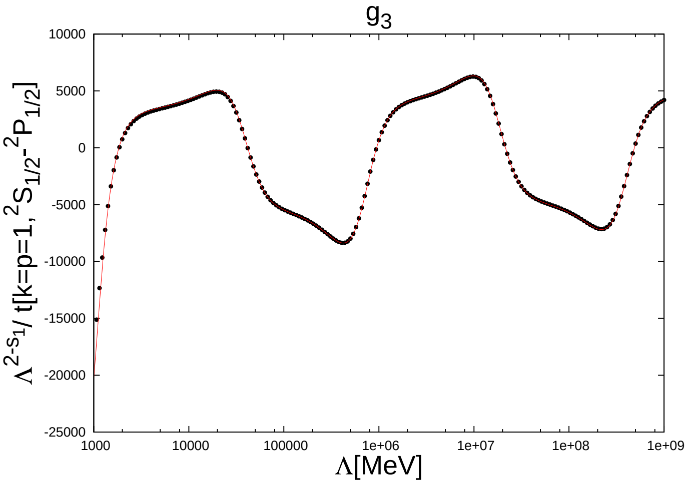
$g_1 \rightarrow g^{{}^3S_1-{}^1P_1}$



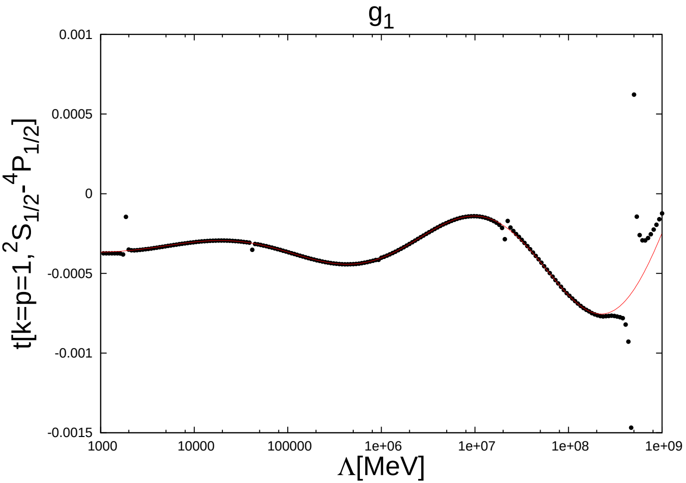
$g_2 \to g^{^3S_1-^3P_1}$



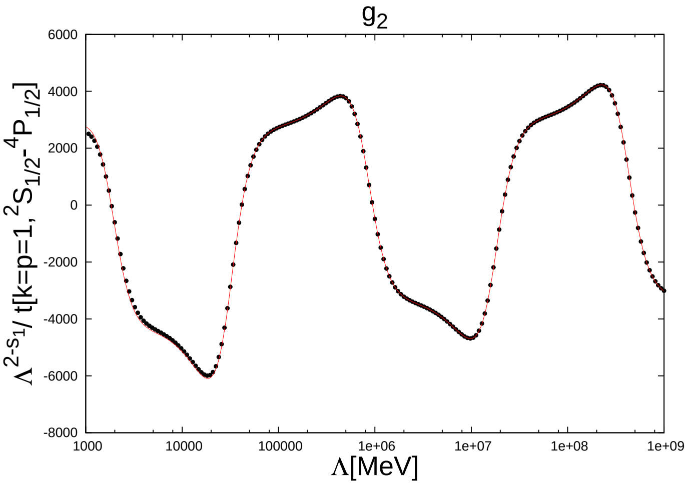
$$g_3 \to g_{(\Delta I=0)}^{^1S_0-^3P_0}$$



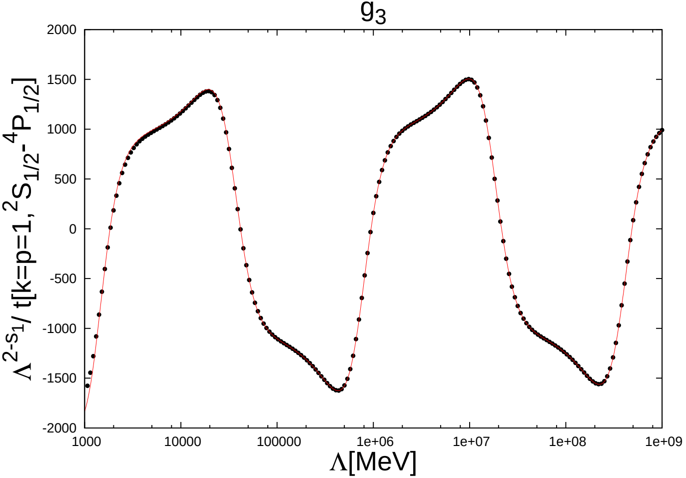
$g_1 \to g^{{}^3S_1-{}^1P_1}$



$$g_2 \rightarrow g^{^3S_1-^3P_1}$$



$$g_3 \rightarrow g_{(\Delta I=0)}^{{}^1S_0-{}^3P_0}$$



The value of $H^{^2P_{1/2}}$ and $H^{^4P_{1/2}}$ in terms of the PV LECs is

$$H^{^XP_{1/2}}=\frac{\frac{4}{\pi}\sqrt{\frac{2}{3}}C\left(2\mathcal{M}[0,is_0]+\mathcal{M}[1,is_0+1]\right)}{1+\frac{4}{\sqrt{3}\pi}\mathcal{M}[1,is_0+1]}g_{H^{^XP_{1/2}}}$$

where

$$g_{H^{^2P_{1/2}}}=g^{^1S_0-^3P_0}_{(\Delta I=0)}-\frac{2}{3}g^{^1S_0-^3P_0}_{(\Delta I=1)}-g^{^3S_1-^1P_1}+\frac{2}{3}g^{^3S_1-^3P_1}$$

and

$$g_{H^{^4P_{1/2}}}=g^{^1S_0-^3P_0}_{(\Delta I=0)}-\frac{2}{3}g^{^1S_0-^3P_0}_{(\Delta I=1)}-\frac{1}{3}g^{^3S_1-^3P_1}$$

The value of $D^{{}^2P_{1/2}}$ and $D^{{}^4P_{1/2}}$ in terms of the PV LECs is

$$D^{{}^XP_{1/2}} = \frac{\frac{4}{\sqrt{3}\pi}\left(2\mathcal{M}[0,1-S_1]+\mathcal{M}[1,-s_1]\right)B^{{}^XP_{1/2}}}{1-I(1-s_1)}g_{D^{{}^XP_{1/2}}}$$

where

$$g_{D^{{}^2P_{1/2}}} = g_{(\Delta I=0)}^{{}^1S_0-{}^3P_0} - \frac{2}{3}g_{(\Delta I=1)}^{{}^1S_0-{}^3P_0} - \frac{1}{3}g^{{}^3S_1-{}^3P_1}$$

and

$$g_{D^{{}^2P_{1/2}}} = g_{(\Delta I=0)}^{{}^1S_0-{}^3P_0} - \frac{2}{3}g_{(\Delta I=1)}^{{}^1S_0-{}^3P_0} - g^{{}^3S_1-{}^1P_1} + \frac{2}{3}g^{{}^3S_1-{}^3P_1}$$

Conclusions and Future Directions

- Any PV observable of interest can be calculated for nd scattering at LO in $\mathrm{EFT}_{\not\pi}$.
- PV three-body force needed at NLO
- What does Large N_C say about PV three-body force?
- New perturbative technique can be used with external currents making calculations of PV in $nd \to {}^3H + \gamma$ and $pd \to {}^3He + \gamma$ feasible.
- Need to add Coulomb to investigate *pd*