Hadronic parity non-conservation in the large-N_c expansion

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- D.R. Phillips, D. Samart, C. Schat, PRL 114 (2015)
- MRS, R.P. Springer, J. Vanasse, PRC 93 (2016)

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Large-N_c QCD

QCD in limit $N_c \to \infty$

- Taken with $g^2 N_c$ fixed
- Simplifications
  - Color-singlet physical states
  - Mesons, glueballs: Weakly interacting $\sim 1/\sqrt{N_c}$
- Systematic expansion in $1/N_c$
- Seems to work well phenomenologically

't Hooft (1974); Witten (1979)

Baryons in the large-N_c limit

- Bound state of N_c quarks
- Completely antisymmetric in color: $\epsilon_{i_1 i_2 \cdots i_{N_c}} q^{i_1} q^{i_2} \cdots q^{i_{N_c}}$
- Baryon mass $M \sim N_c$
- lim $N_c \to \infty$: SU(4) spin-flavor symmetry: $u\uparrow$, $u\downarrow$, $d\uparrow$, $d\downarrow$

Witten (1979); Dashen, Jenkins, Manohar (1994)

NN potential in large-N_c expansion

$$V(\vec{p}_-, \vec{p}_+) = \langle (\vec{p}_1', C), (\vec{p}_2', D) | H | (\vec{p}_1, A), (\vec{p}_2, B) \rangle$$

with $\vec{p}_\pm = \vec{p}\,' \pm \vec{p}$

- Effective Hamiltonian

$$H = N_c \sum_{s,t,u} v_{stu} \left(\frac{S}{N_c}\right)^s \left(\frac{I}{N_c}\right)^t \left(\frac{G}{N_c}\right)^u$$

- Building blocks

$$S^i = q^\dagger \frac{\sigma^i}{2} q\,, \quad I^a = q^\dagger \frac{\tau^a}{2} q\,, \quad G^{ia} = q^\dagger \frac{\sigma^i \tau^a}{4} q$$

- Coefficients v_{stu}
  - Momentum dependent
  - Constrained by symmetries

Witten (1979); Kaplan, Savage (1996); Kaplan, Manohar (1997); Cohen et al. (2002)

Large-N_c scaling

- Nucleon matrix elements

$$\langle N'|S^i|N\rangle \sim \langle N'|I^a|N\rangle \sim 1,$$
$$\langle N'|G^{ia}|N\rangle \sim \langle N'|\mathbb{1}|N\rangle \sim N_c$$

- Momenta

$$\vec{p}_- \sim 1$$
$$\vec{p}_+ \sim 1/M_N \sim 1/N_c$$

- Coefficients excluding momenta

$$\tilde{v}_{stu} \sim 1$$

Example: Central potential

- General form

$$V_c = V_0^0 + V_\sigma^0\,\vec{\sigma}_1\cdot\vec{\sigma}_2 + V_0^1\,\vec{\tau}_1\cdot\vec{\tau}_2 + V_\sigma^1\,\vec{\sigma}_1\cdot\vec{\sigma}_2\,\vec{\tau}_1\cdot\vec{\tau}_2$$

- Scaling

$$\vec{\sigma}_1\cdot\vec{\sigma}_2 \sim \hat{S}_1\cdot\hat{S}_2$$
$$\vec{\tau}_1\cdot\vec{\tau}_2 \sim \hat{I}_1\cdot\hat{I}_2$$
$$\vec{\sigma}_1\cdot\vec{\sigma}_2\,\vec{\tau}_1\cdot\vec{\tau}_2 \sim \hat{G}_1\cdot\hat{G}_2$$

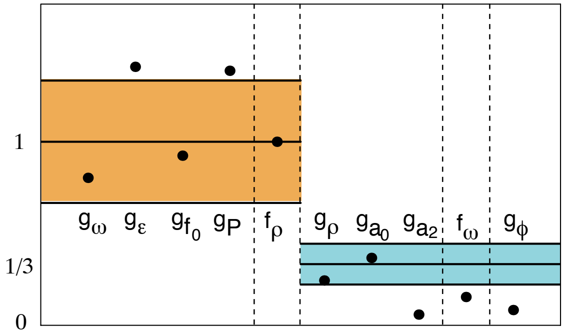
- Coefficients

$$V_0^0 \sim N_c \qquad V_\sigma^0 \sim 1/N_c$$
$$V_0^1 \sim 1/N_c \qquad V_\sigma^1 \sim N_c$$

Kaplan, Savage (1996); Kaplan, Manohar (1997)

$1/N_c$ expansion of NN potential

Comparison large-N_c scaling vs Nijmegen potential



Kaplan, Manohar (1997)

PV operators in $1/N_c$ expansion

- Leading order $[\mathcal{O}(N_c)]$

$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2$$
$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, [\tau_1\, \tau_2]_2^{zz}$$

- Next-to-leading order $[\mathcal{O}(N_c^0) \sin^2\theta_W]$

$$\vec{p}_+ \cdot (\vec{\sigma}_1 \tau_1^3 - \vec{\sigma}_2 \tau_2^3)$$
$$\vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)\, (\vec{\tau}_1 \times \vec{\tau}_2)^3$$
$$\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, (\vec{\tau}_1 + \vec{\tau}_2)^3$$
$$\left[(\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_1\, \vec{p}_- \cdot \vec{\sigma}_2 + (\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_2\, \vec{p}_- \cdot \vec{\sigma}_1\right]\; (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

Phillips, Samart, Schat (2015), MRS, Springer, Vanasse (2016)

$1/N_c$ expansion of PV potential

Next-to-next-to-leading order $[\mathcal{O}(N_c^{-1})]$

$$\begin{aligned}
&\vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\\
&\vec{p}_+^2\, \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2\\
&\vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\\
&\vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\, \vec{\tau}_1 \cdot \vec{\tau}_2\\
&\vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\, [\tau_1\, \tau_2]_2^{zz}\\
&\vec{p}_+^2\, \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\, [\tau_1\, \tau_2]_2^{zz}
\end{aligned}$$

In general potential

- Multiplied by independent functions $U_i(\vec{p}_-^2) \sim \mathcal{O}(1)$

Comparison with meson exchange potential

- Single-meson exchange with strong and weak vertex
- Scaling of strong meson-nucleon couplings known
- Structure not in DDH:

$$\left[(\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_1 \, \vec{p}_- \cdot \vec{\sigma}_2 + (\vec{p}_+ \times \vec{p}_-) \cdot \vec{\sigma}_2 \, \vec{p}_- \cdot \vec{\sigma}_1\right] \; (\vec{\tau}_1 \times \vec{\tau}_2)^3$$

- Isovector and isotensor currents $\sim \sin^2\theta_W \approx 0.23 \approx 1/3$

Comparison with meson exchange potential

- Large-N_c scaling of weak meson-nucleon couplings

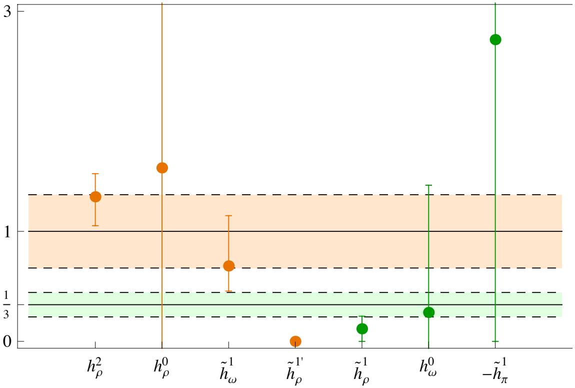
$$
\begin{array}{lll}
h_\rho^0 \sim \sqrt{N_c} & h_\rho^2 \sim \sqrt{N_c}\,[\sin^2\theta_W] & \\
h_\rho^{1\prime} \lesssim \sqrt{N_c}\sin^2\theta_W & h_\omega^1 \sim \sqrt{N_c}\sin^2\theta_W & \\
h_\rho^1 \lesssim \frac{1}{\sqrt{N_c}}\sin^2\theta_W & h_\pi^1 \lesssim \frac{1}{\sqrt{N_c}}\sin^2\theta_W & h_\omega^0 \sim \frac{1}{\sqrt{N_c}}
\end{array}
$$

Large-N_c suppression of PV πN coupling h_π^1

- $h_\rho^{1\prime}$ not necessarily small

Phillips, Samart, Schat (2015)

Comparison with DDH best guesses



$$\tilde{h}_m^i = \frac{h_m^i}{\sin^2 \theta_W}$$

Phillips, Samart, Schat (2015)

Large-N_c expansion and pionless EFT

- Leading-order EFT($\not\pi$) interactions

$$\mathcal{L} = -\frac{1}{2}C_S(N^\dagger N)(N^\dagger N) - \frac{1}{2}C_T(N^\dagger \sigma^i N)(N^\dagger \sigma^i N)$$

- Large-N_c scaling

$$C_S \sim N_c, \quad C_T \sim 1/N_c$$

- In partial-wave basis

$$\mathcal{C}^{(^1S_0)} = (C_S - 3C_T), \quad \mathcal{C}^{(^3S_1)} = (C_S + C_T)$$

In large-N_c limit

$$\mathcal{C}^{(^1S_0)} = \mathcal{C}^{(^3S_1)}$$

Kaplan, Savage (1996)

Parity-conserving S-wave couplings

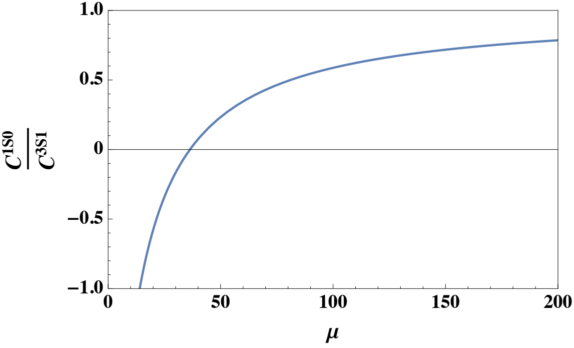
- In field theory LECs renormalization-scale dependent
- In PDS renormalization

$$\frac{\mathcal{C}^{(^1S_0)}}{\mathcal{C}^{(^3S_1)}} = \frac{\frac{1}{a^{(^3S_1)}} - \mu}{\frac{1}{a^{(^1S_0)}} - \mu} \xrightarrow{\mu\to 0} \frac{a^{(^1S_0)}}{a^{(^3S_1)}} \approx -4.4$$

Kaplan, Savage (1996); Kaplan, Savage, Wise (1998)

Parity-conserving S-wave couplings

- Large-N_c + EFT($\not\pi$) requires suitable renormalization scale
- Agreement with large-N_c predicted errors for $\mu \gtrsim m_\pi$



Parity violation in pionless EFT

- In 'Girlanda basis'

$$
\begin{aligned}
\mathcal{L}_{PV}^{\min} = & \mathcal{G}_1 (N^\dagger \vec{\sigma} N \cdot N^\dagger i \overleftrightarrow{\nabla} N - N^\dagger N N^\dagger i \overleftrightarrow{\nabla} \cdot \ \vec{\sigma} N) \\
& - \tilde{\mathcal{G}}_1 \epsilon_{ijk} N^\dagger \sigma_i N \nabla_j (N^\dagger \sigma_k N) \\
& - \mathcal{G}_2 \epsilon_{ijk} \left[N^\dagger \tau_3 \sigma_i N \nabla_j (N^\dagger \sigma_k N) + N^\dagger \sigma_i N \nabla_j (N^\dagger \tau_3 \sigma_k N) \right] \\
& - \tilde{\mathcal{G}}_5 \mathcal{I}_{ab} \epsilon_{ijk} N^\dagger \tau_a \sigma_i N \nabla_j (N^\dagger \tau_b \sigma_k N) \\
& + \mathcal{G}_6 \epsilon_{ab3} \vec{\nabla} (N^\dagger \tau_a N) \cdot N^\dagger \tau_b \vec{\sigma} N
\end{aligned}
$$

- 5 independent contact terms at LO

Girlanda (2008)

Large-N_c scaling of LECs?

$$\begin{aligned} V^{\min} = &- \mathcal{G}_1 \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) - i\tilde{\mathcal{G}}_1 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2) \\ &- i\mathcal{G}_2 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)(\tau_1 + \tau_2)^3 \\ &- i\tilde{\mathcal{G}}_5 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)\mathcal{I}_{ab}\tau_1^a \tau_2^b \\ &+ \frac{i}{2}\mathcal{G}_6 \vec{p}_- \cdot (\vec{\sigma}_1 + \vec{\sigma}_2)(\tau_1 \times \tau_2)^3 \end{aligned}$$

Extracted N_c scaling?

$$\tilde{\mathcal{G}}_5 \sim N_c\, [\sin^2\theta_W],$$
$$\mathcal{G}_2 \sim \mathcal{G}_6 \sim N_c^0 \sin^2\theta_W,$$
$$\mathcal{G}_1 \sim \tilde{\mathcal{G}}_1 \sim N_c^{-1}$$

- Only one term at LO in N_c?
- Isoscalar coupling suppressed?

Vanasse, MRS, Springer (2016)

Fierz identities and large-N_c scaling

- Minimal form of Lagrangian derived using Fierz identities
- Fierz identities
  - Relate different (iso-)spin and momentum structures
  - Do not change EFT power counting
  - Change large-N_c counting
- Identify large-N_c scaling from non-minimal form of $\mathcal{L}$
- Example:

$$\mathcal{A}_1^+ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)$$
$$\mathcal{A}_3^+ \ \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2$$
$$\mathcal{A}_3^- \ \vec{p}_- \cdot i(\vec{\sigma}_1 \times \vec{\sigma}_2)\vec{\tau}_1 \cdot \vec{\tau}_2$$

- After Fierz transformation contribute to

$$-\mathcal{G}_1 \vec{p}_+ \cdot (\vec{\sigma}_1 - \vec{\sigma}_2) - i\tilde{\mathcal{G}}_1 \vec{p}_- \cdot (\vec{\sigma}_1 \times \vec{\sigma}_2)$$

Non-minimal potential

- Extract N_c-scaling

$$\mathcal{A}_1^+ \sim N_c^{-1}, \quad \mathcal{A}_3^+ \sim N_c^{-1}, \quad \mathcal{A}_3^- \sim N_c$$

- Relations

$$\mathcal{G}_1 = -\mathcal{A}_1^+ + \mathcal{A}_3^+ - 2\mathcal{A}_3^- \ ,$$
$$\tilde{\mathcal{G}}_1 = -\mathcal{A}_1^- - 2\mathcal{A}_3^+ + \mathcal{A}_3^- \ ,$$

- Maintain most dominant scaling

$$\mathcal{G}_1 \sim N_c \ , \qquad \tilde{\mathcal{G}}_1 \sim N_c \ ,$$
$$\mathcal{G}_2 \sim N_c^0 \sin^2\theta_W \ , \qquad \tilde{\mathcal{G}}_5 \sim N_c \, [\sin^2\theta_W] \ ,$$
$$\mathcal{G}_6 \sim N_c^0 \sin^2\theta_W$$

Large-N_c relation

$$\mathcal{G}_1 = -2\tilde{\mathcal{G}}_1[1 + \mathcal{O}(1/N_c^2)]$$

Large-N_c scaling of partial-wave LECs

$$\mathcal{C}^{({}^3S_1-{}^1P_1)} \sim N_c$$

$$\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=0)} \sim N_c$$

$$\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=2)} \sim N_c\ [\sin^2\theta_W]$$

$$\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=1)} \sim N_c^0 \sin^2\theta_W$$

$$\mathcal{C}^{({}^3S_1-{}^3P_1)} \sim N_c^0 \sin^2\theta_W$$

$$\mathcal{C}^{({}^3S_1-{}^1P_1)} = 3\,\mathcal{C}^{({}^1S_0-{}^3P_0)}_{(\Delta I=0)}[1+\mathcal{O}(1/N_c^2)]$$

PV LECs

- Renormalization-scale dependent
- Scale dependence driven by S-wave interactions
- “Wrong” choice of scale can hide large-N_c scaling

Conclusion & Outlook

Large-N_c analysis

- Effects of embedding PV quark interactions in hadrons
- Establishes hierarchy of couplings
- Important constraints in absence of experimental data
- Gives trends, not exact predictions

Parity violation

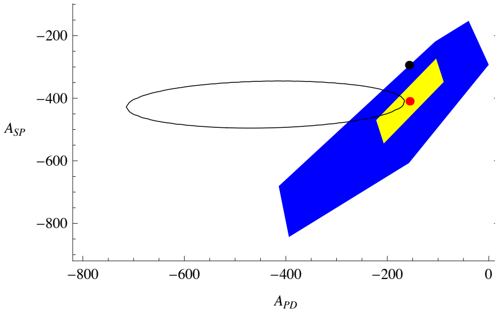
- Two LECs at LO in combined EFT/large-N_c expansion [[$\sin^2\theta_W$] for $\Delta I = 2$]
- Relation between two isoscalar LECs
- Isovector coupling suppressed by $\sin^2\theta_W/N_c$

pp scattering in meson-exchange picture

- DDH contributions

$$A_{SP} \equiv g_\rho h_\rho^{pp}(2+\chi_V) + g_\omega h_\omega^{pp}(2+\chi_S)$$
$$A_{PD} \equiv g_\rho h_\rho^{pp}\chi_V + g_\omega h_\omega^{pp}\chi_S$$

- $G_F f_\pi \Lambda_\chi \sim 1.0 \times 10^{-6}$



Haxton, Holstein (2013); Carlson et al. (2002); Phillips, Samart, Schat (2015)

Application to measurements

Longitudinal asymmetry in $\vec{p}p$ scattering

- Experimental result

$$A_L^{\vec{p}p}(E = 13.6\ \text{MeV}) = (-0.93 \pm 0.21) \times 10^{-7}$$

- Constraint on LECs in large-N_c limit

$$(-1.5 \pm 0.3) \times 10^{-10}\ \text{MeV}^{-1} = \frac{4\left[\mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=0)} + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=1)} + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}\right]}{\mathcal{C}^{(^1S_0)}}$$

$$\to \frac{4\left[\mathcal{C}^{(^3S_1-^1P_1)}/3 + \mathcal{C}^{(^1S_0-^3P_0)}_{(\Delta I=2)}\right]}{\mathcal{C}}$$

Eversheim et al (1991); Phillips, MRS, Springer (2009)

- Induced circular polarization in $np \to d\vec{\gamma}$

$$
\begin{aligned}
P_\gamma &= (1.8 \pm 1.8) \times 10^{-7} \\
&\to -\frac{16 M_N}{\mathcal{C}} \frac{1}{\kappa_1 (1 - \gamma a^{^1S_0})} \left(\mathcal{C}^{(^3S_1 - ^1P_1)} (1 - \frac{5}{9} \gamma a^{^1S_0}) \right. \\
&\qquad \left. - \frac{2}{3} \gamma a^{^1S_0} \mathcal{C}^{(^1S_0 - ^3P_0)}_{(\Delta I = 2)} \right)
\end{aligned}
$$

- In large-N_c limit $\mathcal{C}^{(^1S_0 - ^3P_0)}_{(\Delta I = 2)} \sim \mathcal{C}^{(^3S_1 - ^1P_1)}$
- If $\mathcal{C}^{(^1S_0 - ^3P_0)}_{(\Delta I = 2)} \ll \mathcal{C}^{(^3S_1 - ^1P_1)}$ predict P_γ outside of current bound

Knyaz'kov et al (1983/84); Vanasse, MRS, Springer (2016)