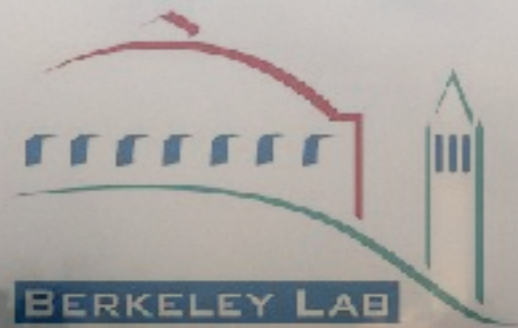


Lattice QCD, the Isotensor Amplitude and Beyond

Hadronic Parity Nonconservation
15-16 March 2018
KITP

André Walker-Loud
LBNL



Lattice QCD for Hadronic Parity Nonconservation

- Introduction to Lattice QCD (LQCD)
- Lattice QCD Challenges for Nuclear Physics
- Lattice QCD Challenges for Parity Nonconservation (PNC)
- g_A - a success story
 - convergence of SU(2) baryon chiral perturbation theory
- Inspiration for Lattice QCD Calculations of I=2 PNC
 - New method for 4-quark operators
 - New method for two-nucleon calculations

Introduction to LQCD

$$\begin{aligned} C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle &= \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t) \mathcal{O}^\dagger(0) e^{-S[\bar{\psi}, \psi, U]} \\ &= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \det(\not{D} + M) e^{-S[U]} \mathcal{O}(t) \mathcal{O}^\dagger(0) \end{aligned}$$

Introduction to LQCD

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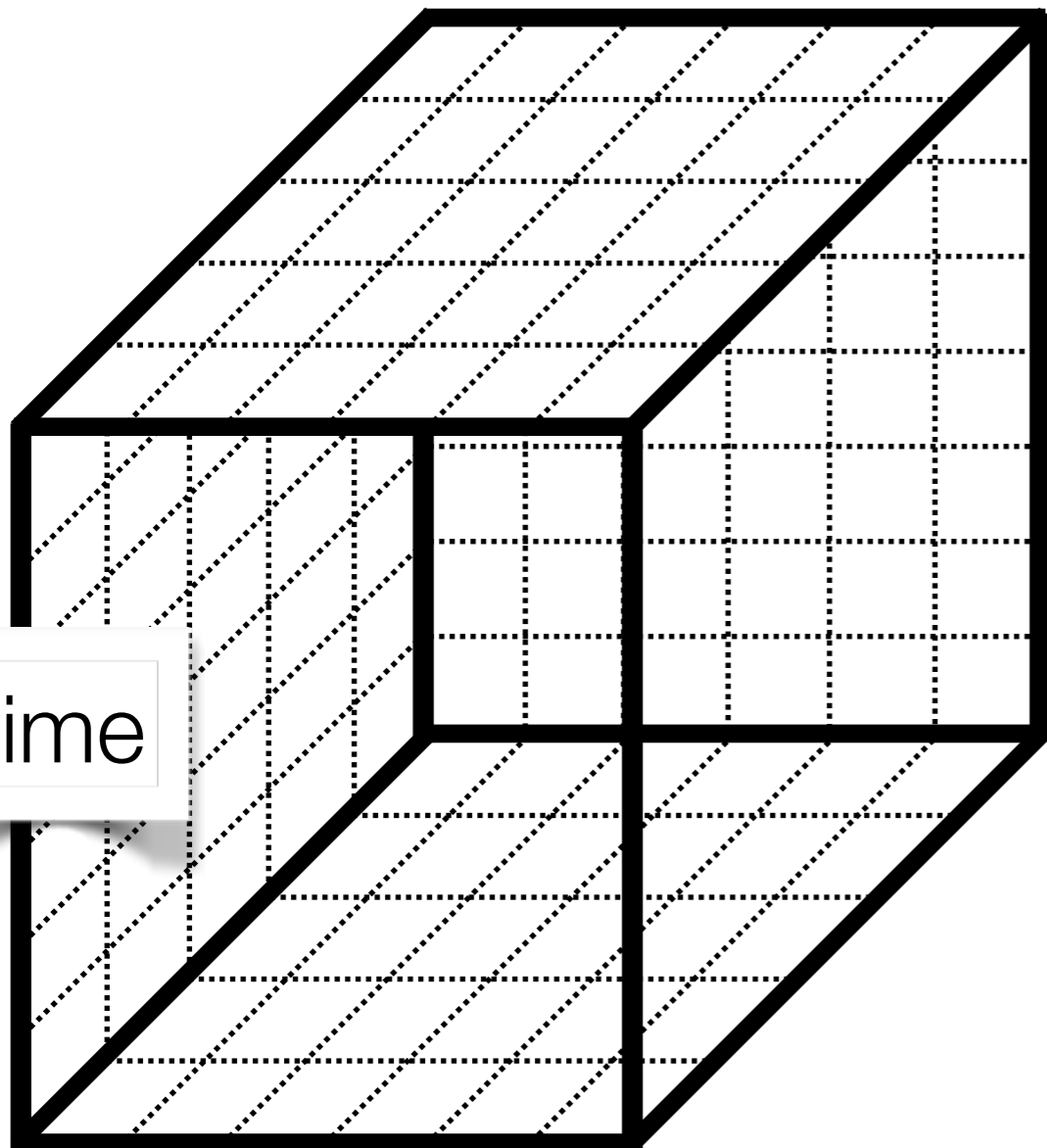
lattice
finite volume

Introduction to LQCD

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t) \mathcal{O}^\dagger(0) e^{-S[\bar{\psi}, \psi, U]}$$

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lattice
finite volume

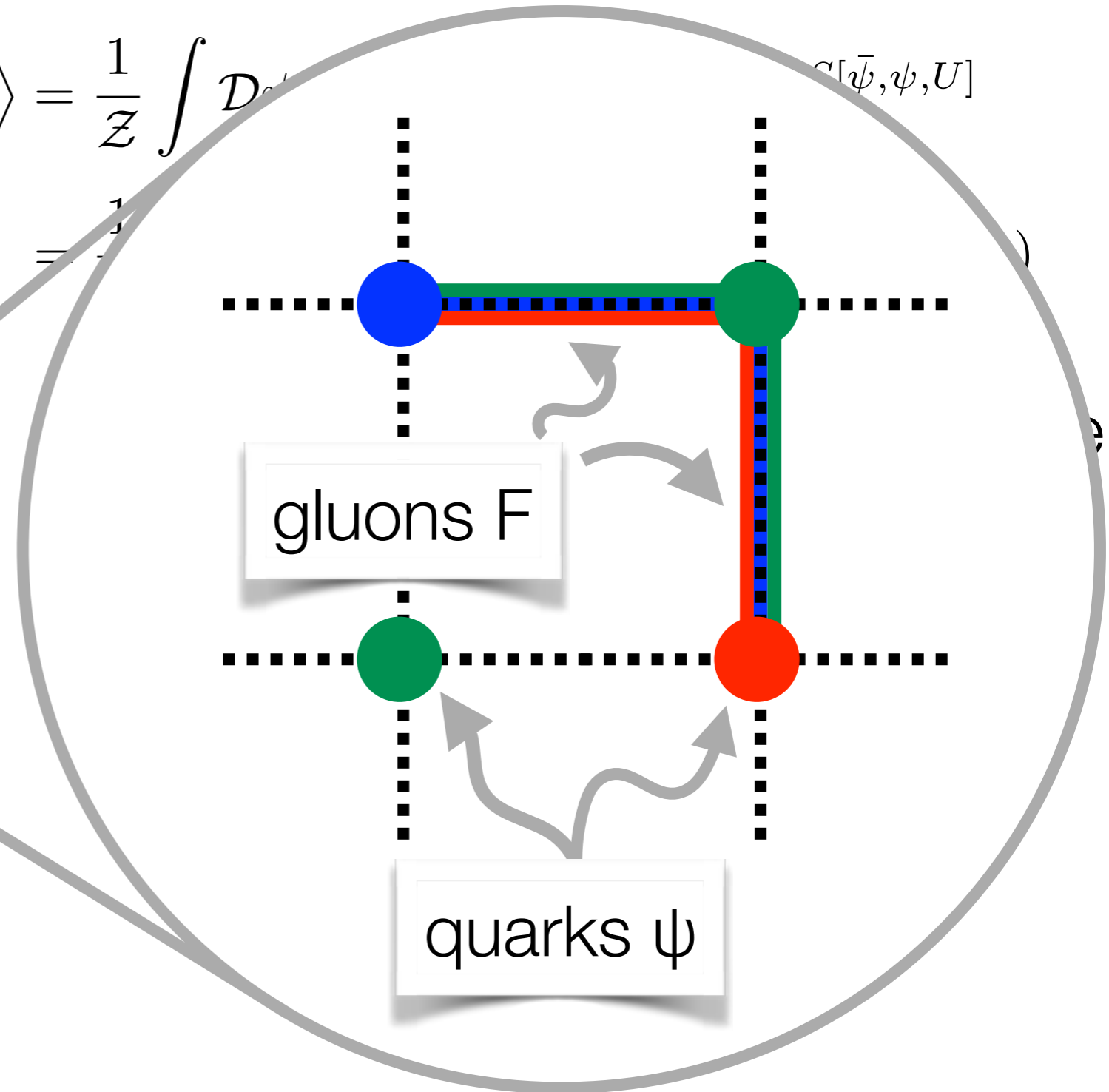
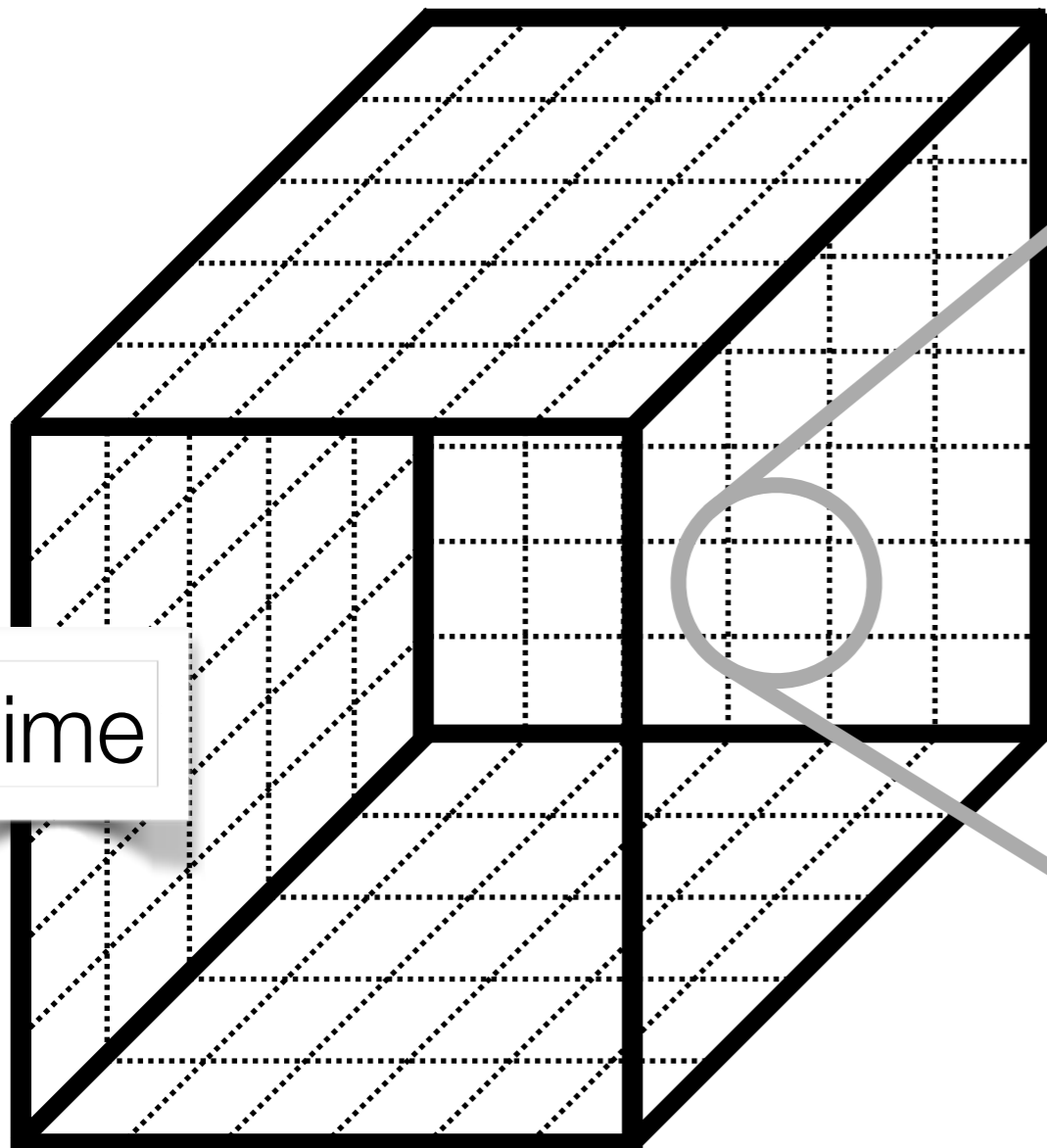


time

space

Introduction to LQCD

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \frac{1}{Z} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U e^{iS[\bar{\psi}, \psi, U]}$$

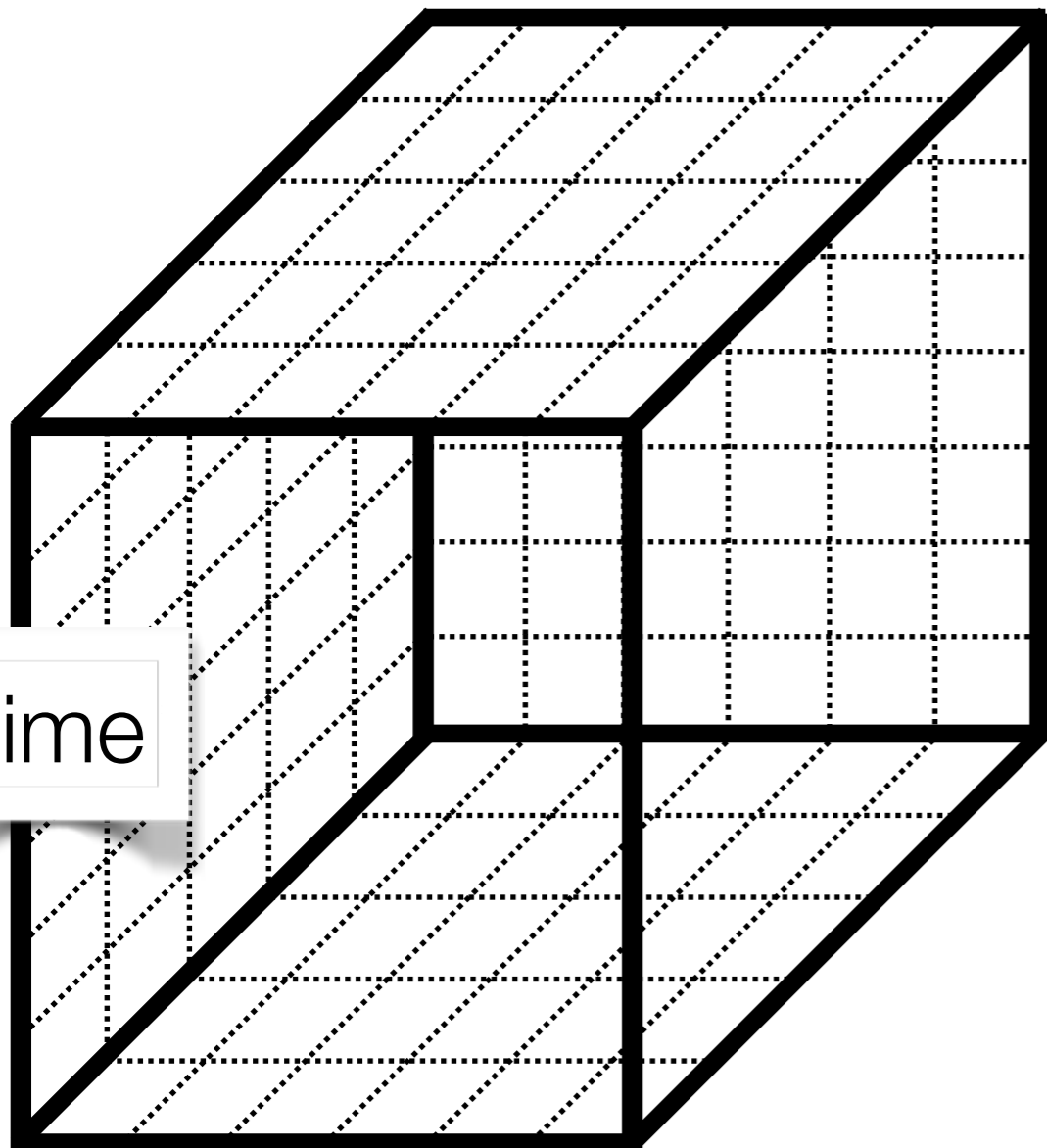


Introduction to LQCD

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lattice
finite volume



time

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Introduction to LQCD

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$$= \frac{1}{\mathcal{Z}} \int \mathcal{D}U \underbrace{\det(\not{D} + M) e^{-S[U]}}_{\text{Probability}} \mathcal{O}(t) \mathcal{O}^\dagger(0)$$

Probability



time

space

Introduction to LQCD

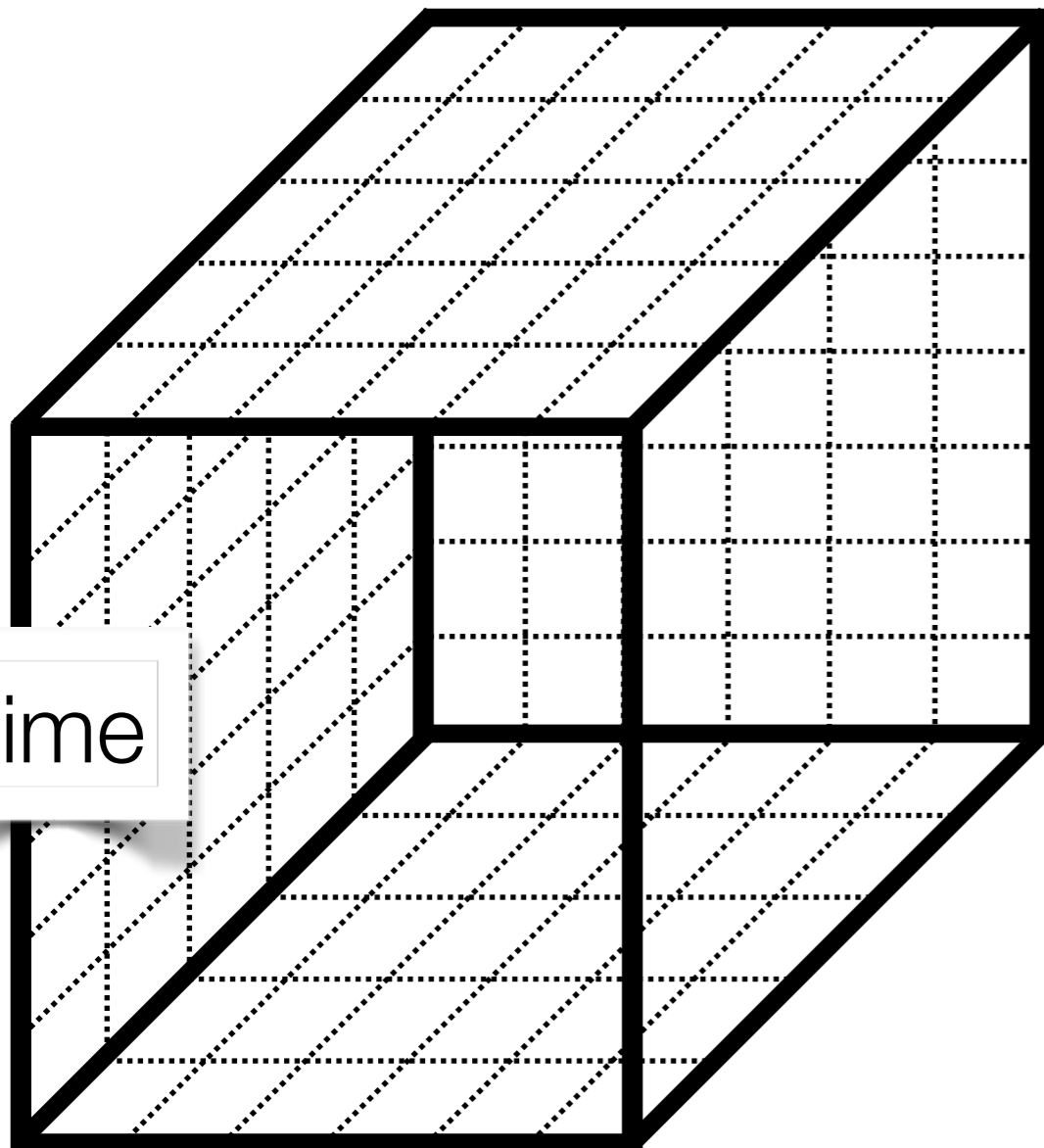
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Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

Markov Chain Monte Carlo



time

space

Introduction to LQCD

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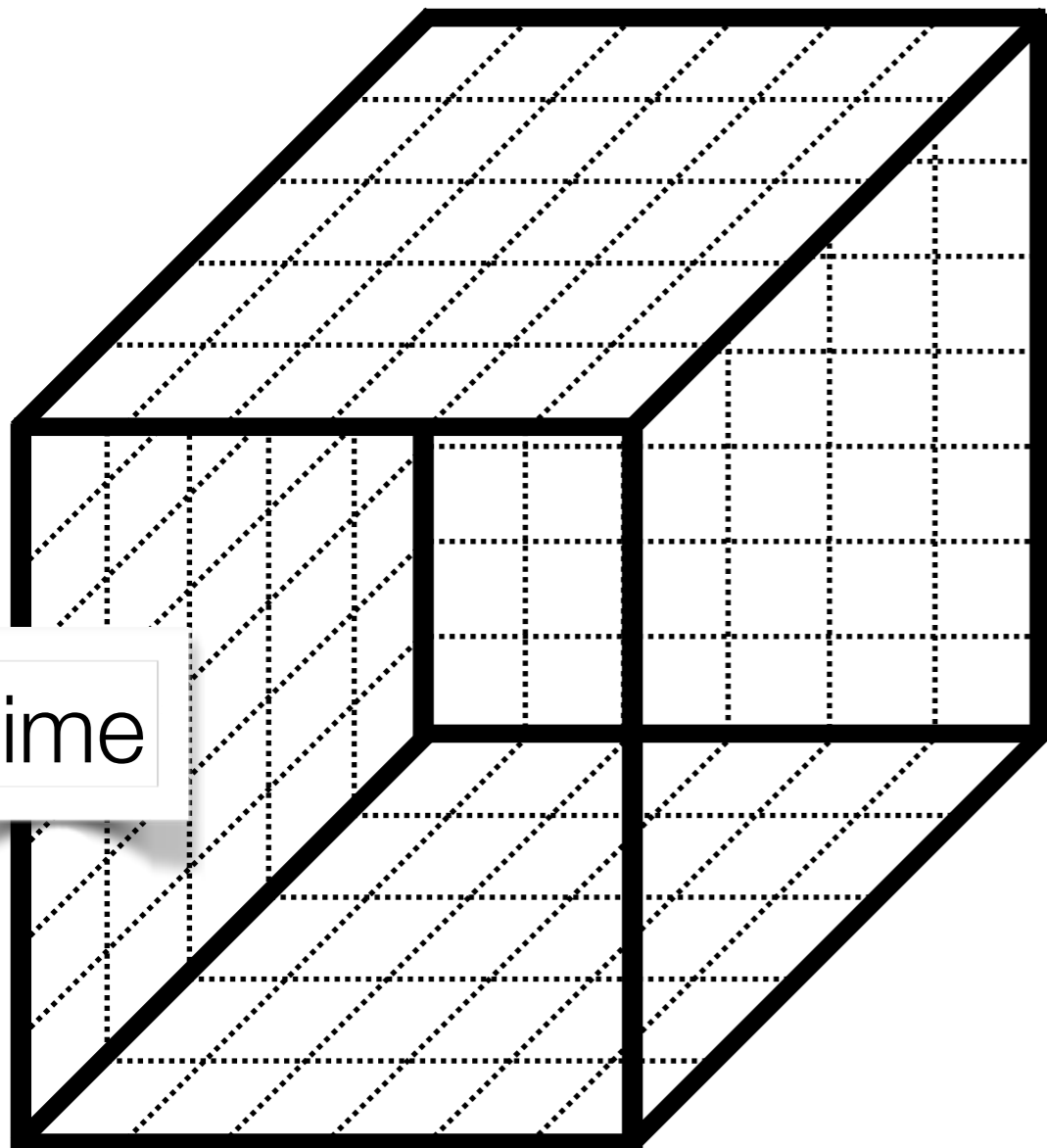
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Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^N \mathcal{O}(t) \mathcal{O}^\dagger(0)[U_i]$$



time

space

Introduction to LQCD

$$C(t) = \langle \mathcal{O}(t) \mathcal{O}^\dagger(0) \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}U \mathcal{O}(t) \mathcal{O}^\dagger(0) e^{-S[\bar{\psi}, \psi, U]}$$

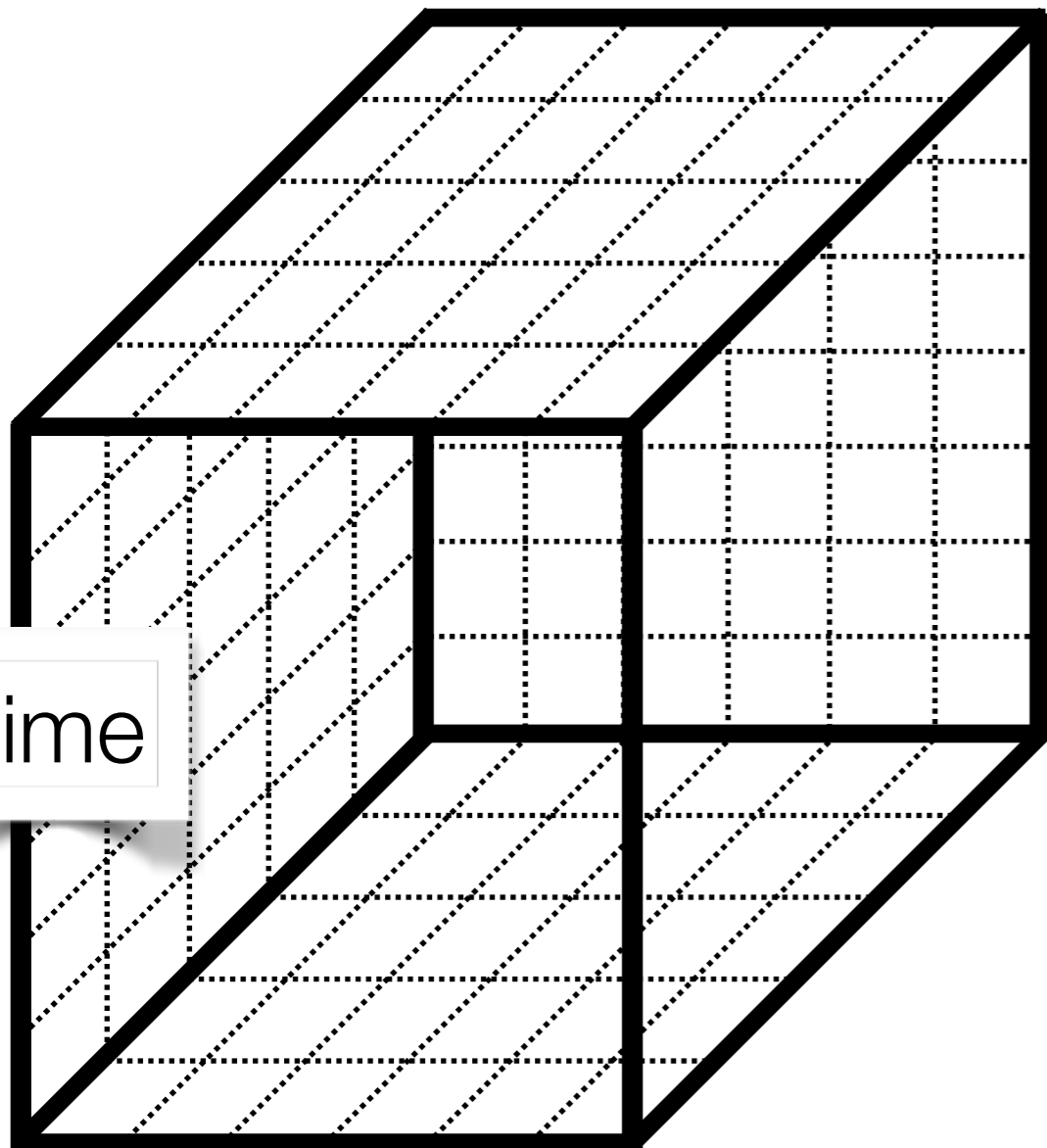
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Probability

$$\{U_1, U_2, U_3, \dots, U_N\}$$

Markov Chain Monte Carlo

$$\approx \frac{1}{N} \sum_{i=1}^N \mathcal{O}(t) \mathcal{O}^\dagger(0)[U_i] + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$



time

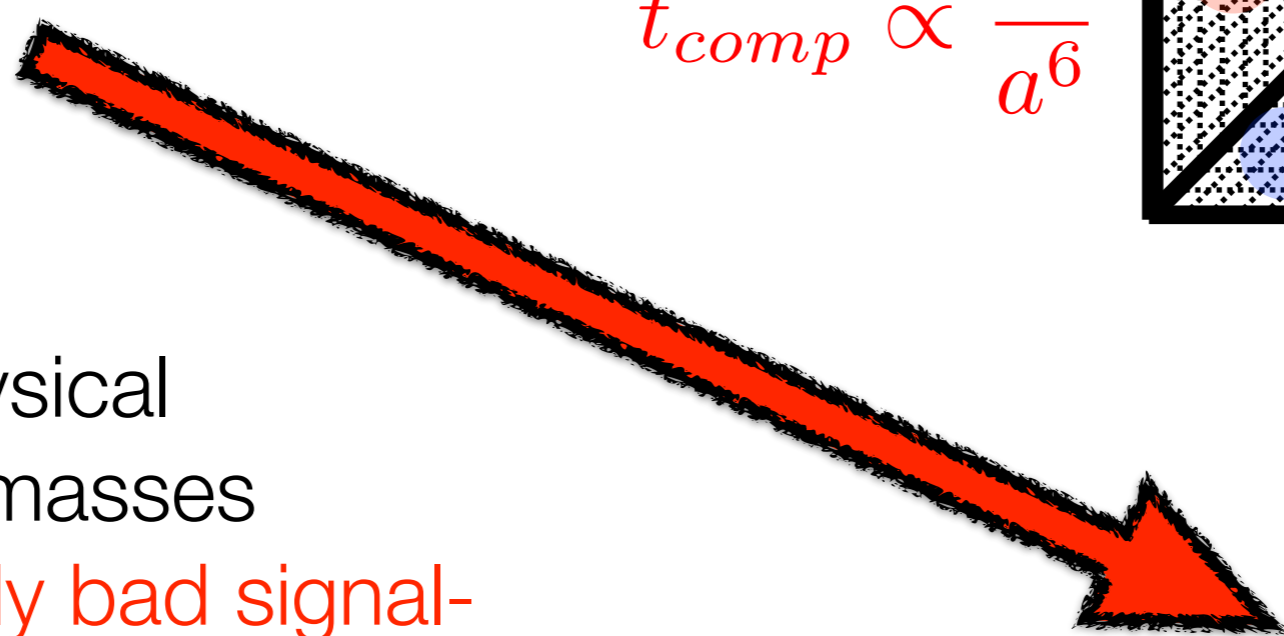
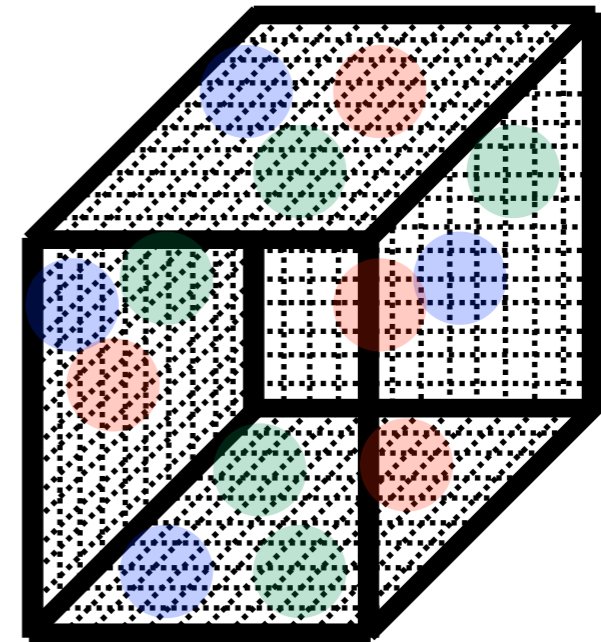
space

LQCD Systematics

continuum limit



$$t_{comp} \propto \frac{1}{a^6}$$



physical
pion masses

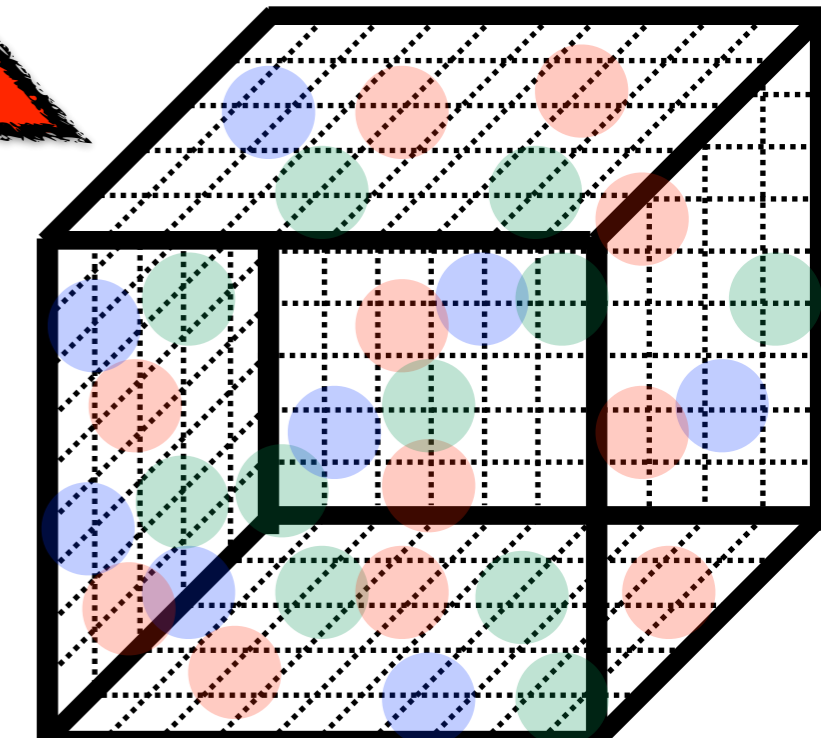
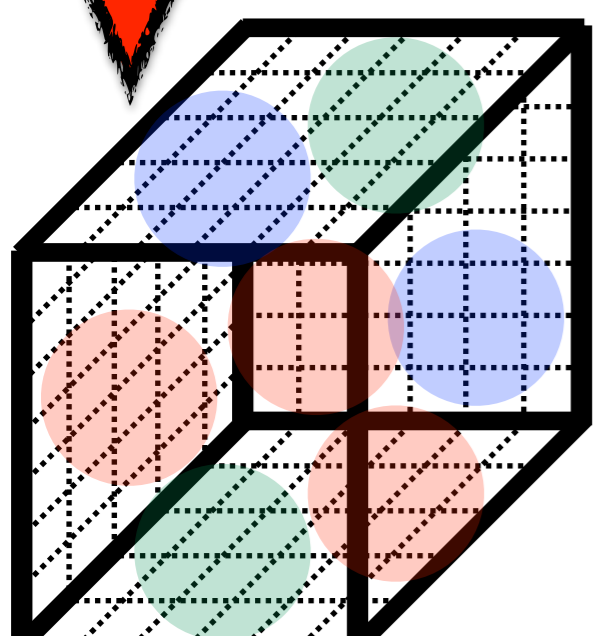
exponentially bad signal-
to-noise problem



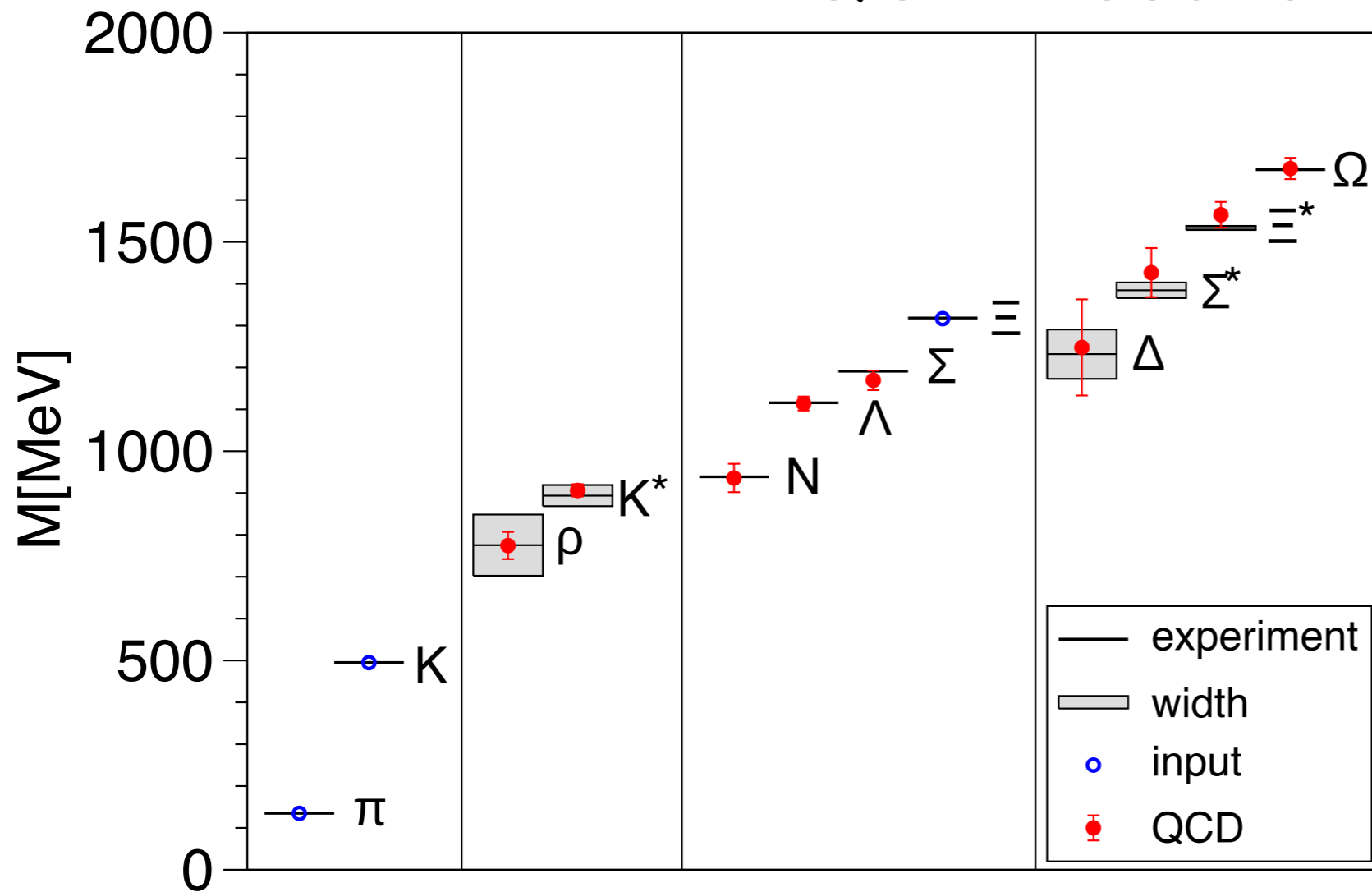
infinite volume limit

$$t_{comp} \propto V^{5/4}$$

$$V = N_L^3 \times N_T$$



LQCD Results



Durr et al., Science 322
(2008) [arXiv:0906.3599]

Pion mass is used to fix the physical **light** quark mass

$$M_\pi^2 = 2B_0\hat{m}_l + \dots$$

Kaon mass is used to fix the physical **strange** quark mass

$$M_K^2 = B_0(m_s + \hat{m}_l) + \dots$$

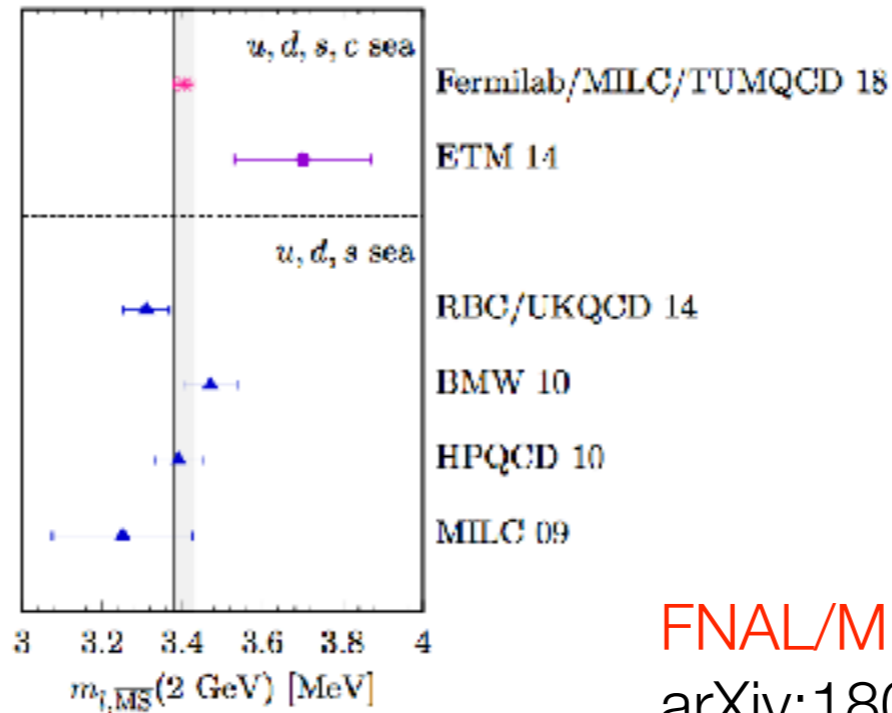
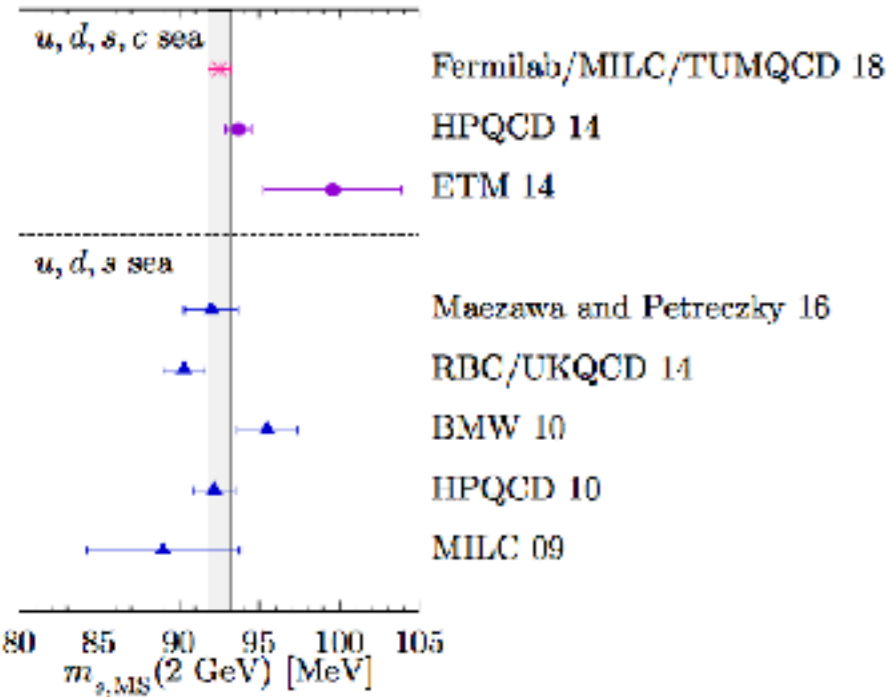
Hadronic quantity is used to set the **scale** in MeV

$$M_\Sigma [\text{MeV}] \leftrightarrow a [\text{fm}]$$

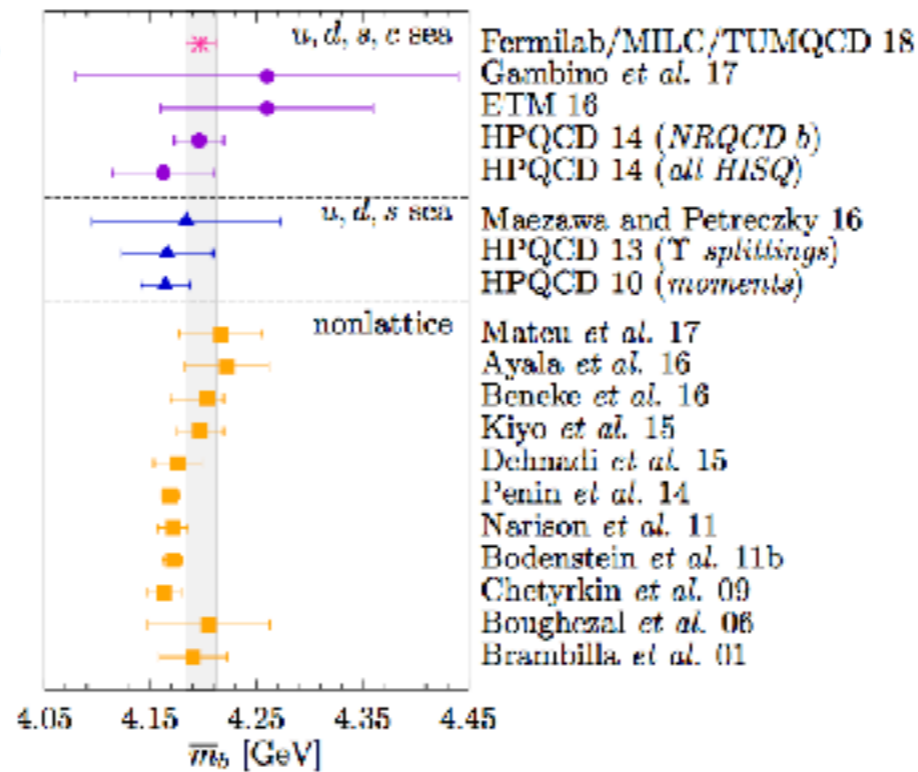
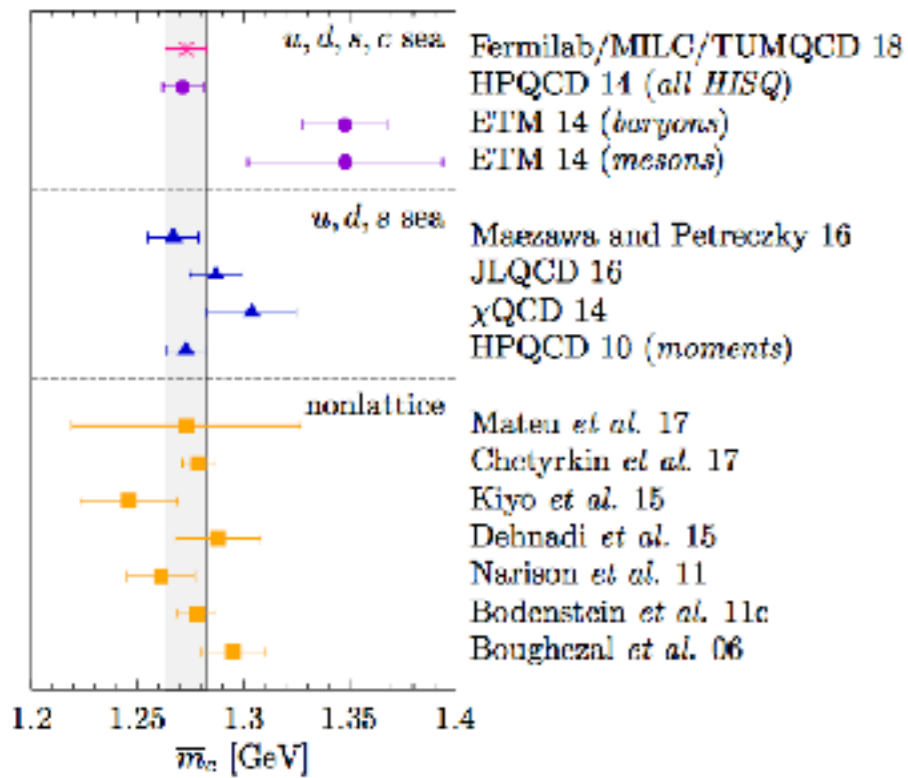
Everything else is a prediction!

(need 1 new quantity for each new input)

LQCD Results



FNAL/MILC/TUMQCD Collaborations
arXiv:1802.04248



Most precise determination
of quark masses now
comes from LQCD

LQCD Challenges for NP

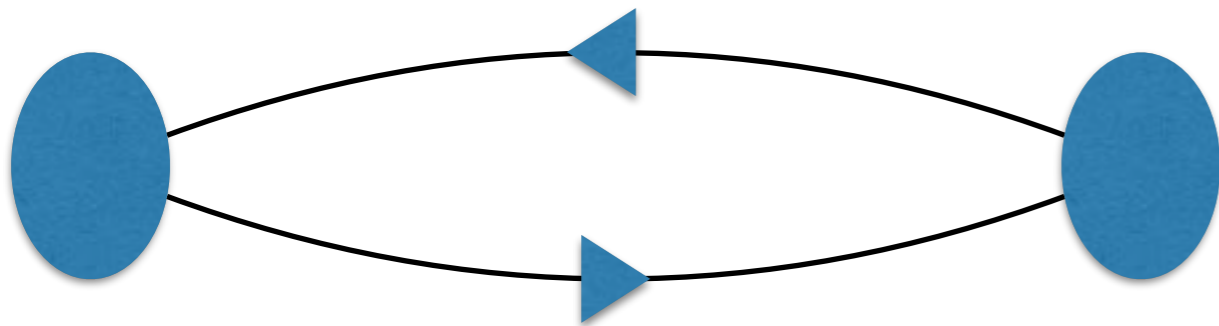
- The most difficult challenge in applying LQCD to NP is an **exponentially bad signal-to-noise** problem for nucleons



$$\sim e^{-\frac{1}{2}m_\pi t} + e^{-\frac{1}{3}m_N t} + \dots$$

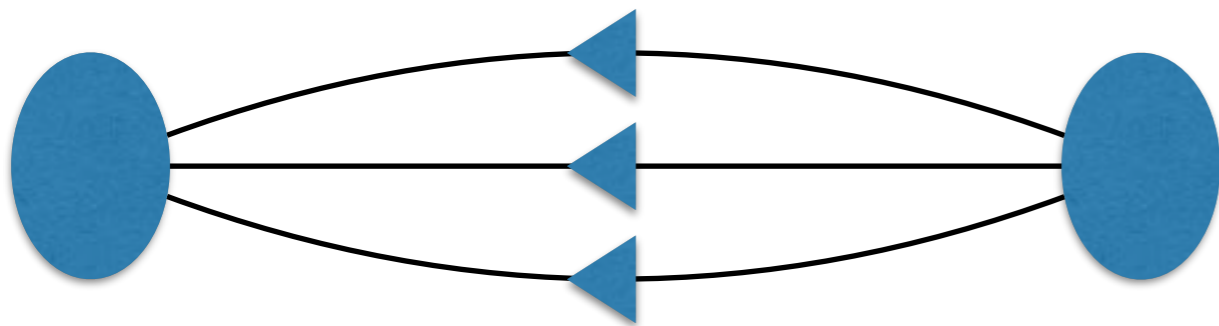
each quark carries information about pions and nucleons

$$\lambda_\pi(t) \gg \lambda_N(t)$$



$$\bar{d}\gamma_5 u : C(t) = A_\pi e^{-m_\pi t} + \dots$$

For the nucleon - the large **pion eigenvalues** must cancel to expose the small nucleon eigenvalues



$$(u^T C \gamma_5 d)u : C(t) = A_N e^{-m_N t} + \dots$$

- The most difficult challenge in applying LQCD to NP is an **exponentially bad signal-to-noise** problem for nucleons

Lepage Argument

- The most difficult challenge in applying LQCD to NP is an **exponentially bad signal-to-noise** problem for nucleons

Lepage Argument


- signal-to-noise ratio of correlation functions

$$\text{SNR} \sim \frac{\langle N \bar{N} \rangle}{\sqrt{\langle (N \bar{N})(N \bar{N})^\dagger \rangle - \langle N \bar{N} \rangle^2}}$$

○ The most difficult challenge in applying LQCD to NP is an **exponentially bad signal-to-noise** problem for nucleons

Lepage Argument

- signal-to-noise ratio of correlation functions
- numerator
 $\sim \exp(-m_N t)$

$$\text{SNR} \sim \frac{\langle N \bar{N} \rangle}{\sqrt{\langle (N \bar{N})(N \bar{N})^\dagger \rangle - \langle N \bar{N} \rangle^2}}$$


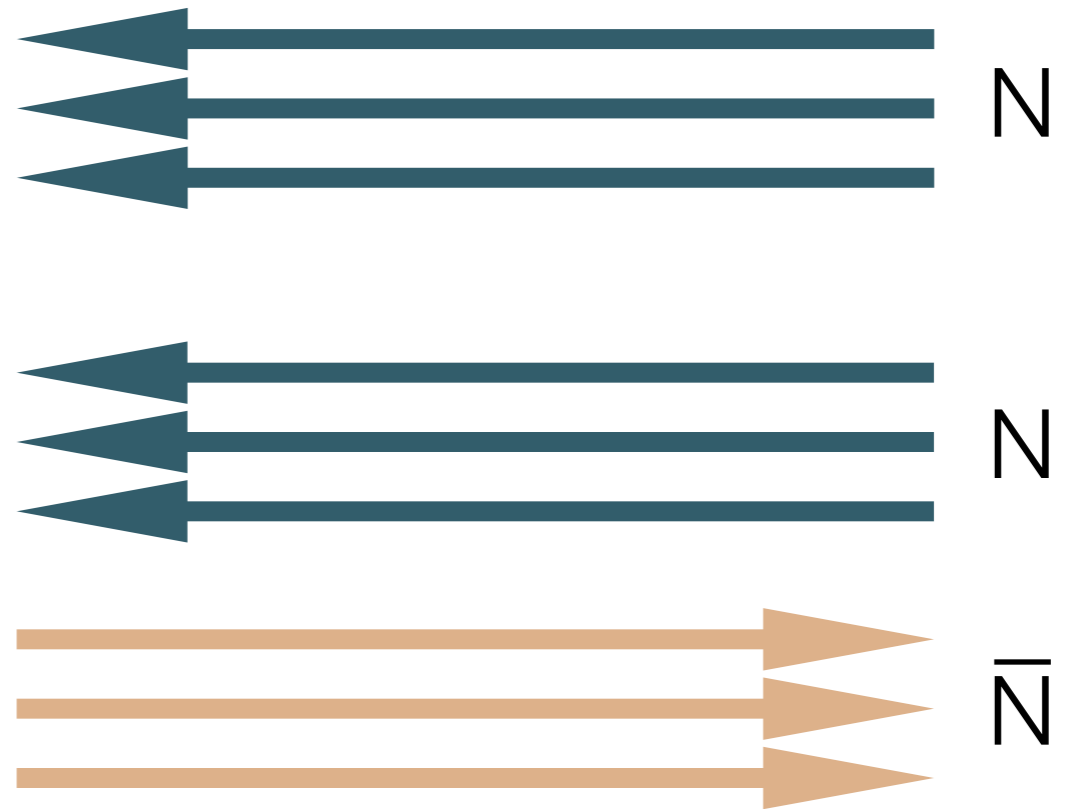
N

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Lepage Argument

- signal-to-noise ratio of correlation functions
- numerator
 $\sim \exp(-m_N t)$
- denominator

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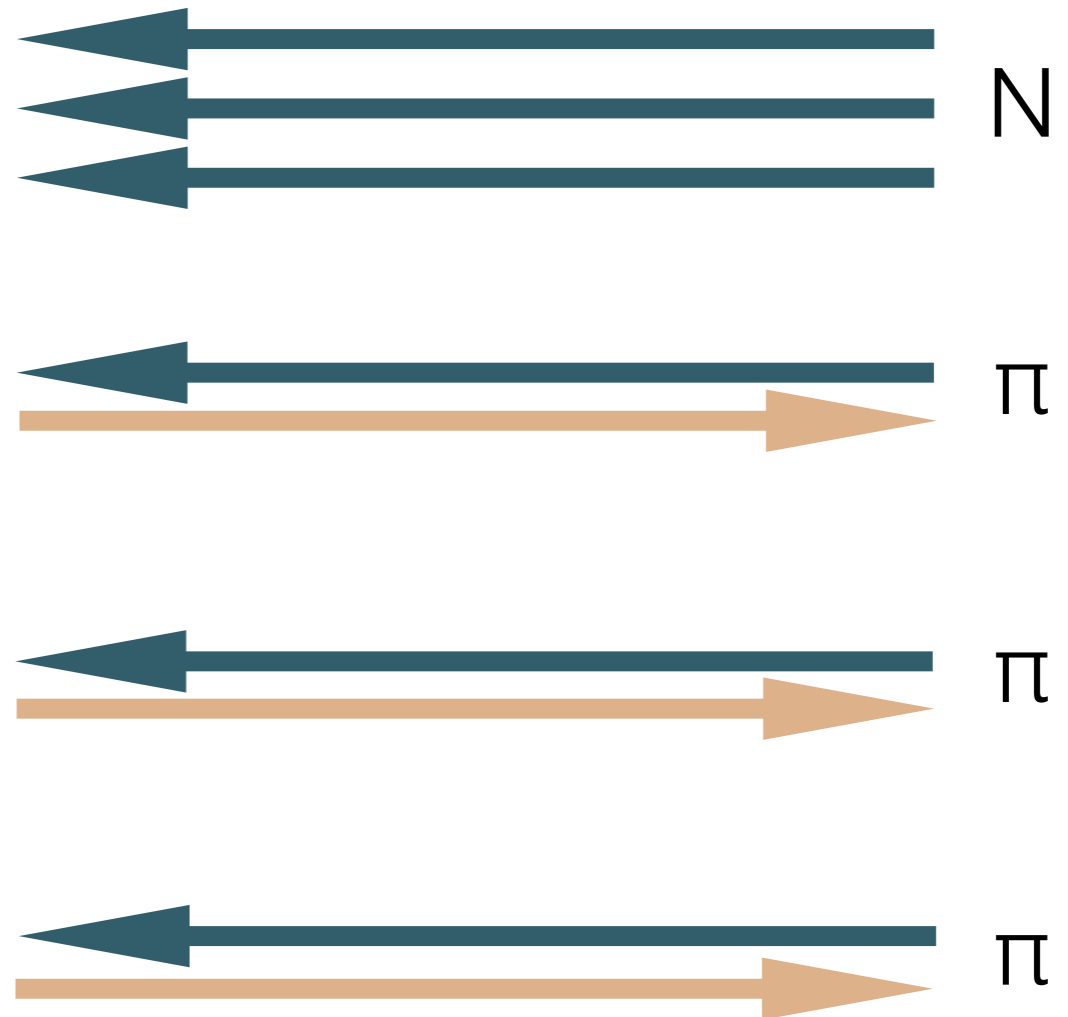


- The most difficult challenge in applying LQCD to NP is an **exponentially bad signal-to-noise** problem for nucleons

Lepage Argument

- signal-to-noise ratio of correlation functions
- numerator
 $\sim \exp(-m_N t)$
- denominator
 $\sim \exp\left(-\frac{3}{2}m_\pi t\right)$

$$\text{SNR} \sim \frac{\langle N \bar{N} \rangle}{\sqrt{\langle (N \bar{N})(N \bar{N})^\dagger \rangle - \langle N \bar{N} \rangle^2}}$$



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Lepage Argument

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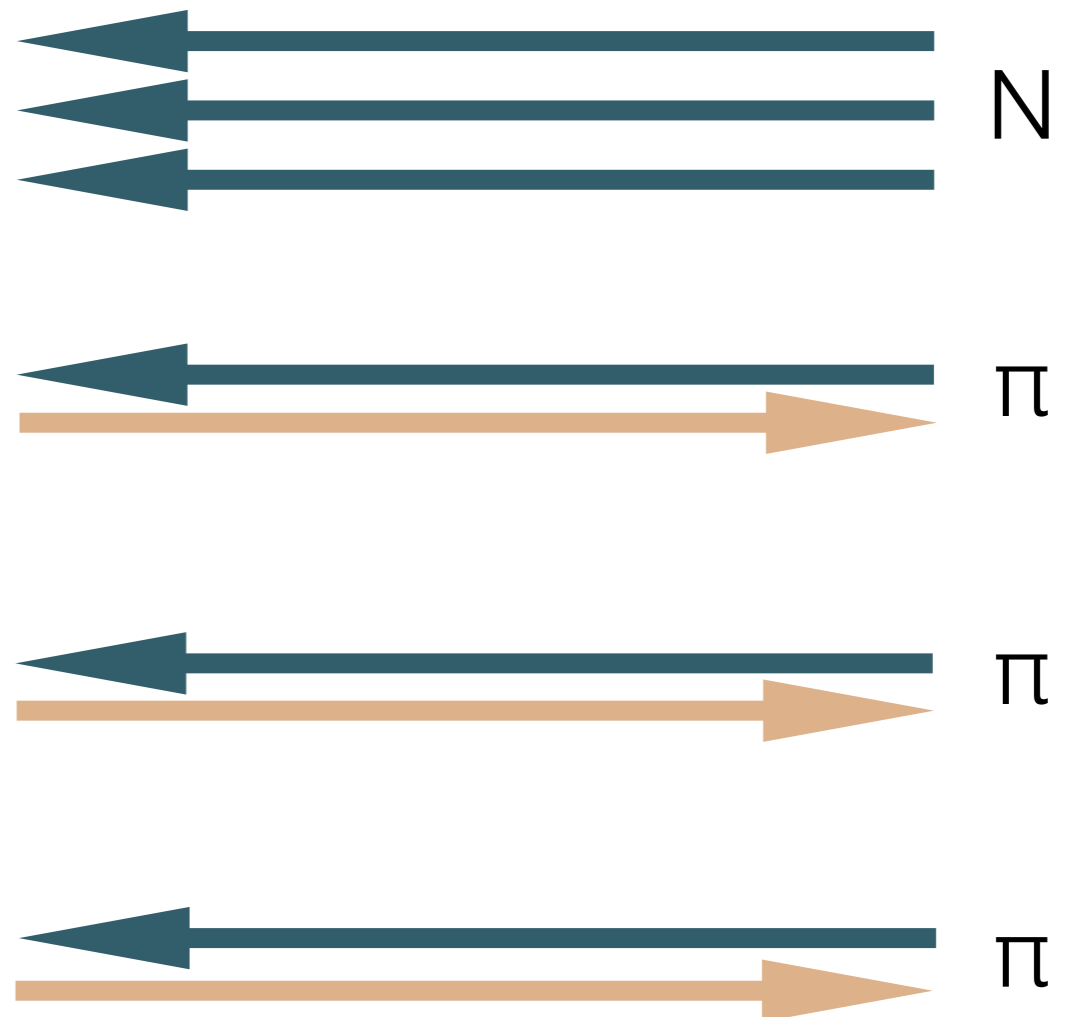
- numerator
 $\sim \exp(-m_N t)$

- denominator
 $\sim \exp\left(-\frac{3}{2}m_\pi t\right)$

- time-dependence of SNR

$$\sim \sqrt{N} \exp\left[-A \left(m_N - \frac{3}{2}m_\pi\right) t\right] \rightarrow$$

$$\text{SNR} \sim \frac{\langle N \bar{N} \rangle}{\sqrt{\langle (N \bar{N})(N \bar{N})^\dagger \rangle - \langle N \bar{N} \rangle^2}}$$




exponential noise
power-law statistics

LQCD Challenges for NP

○ Consider a 2-point correlation function

$$C(t) = \sum_{\mathbf{x}} \langle \Omega | \mathcal{O}(t, \mathbf{x}) \mathcal{O}^\dagger(0, \mathbf{0}) | \Omega \rangle$$

all we do with LQCD is compute 2, 3, 4 point functions


$$1 = \sum_n |n\rangle \langle n|$$

multiply by 1

$$C(t) = \sum_n z_n z_n^\dagger e^{-E_n t}$$

$$z_n = \langle \Omega | \mathcal{O} | n \rangle$$

overlap factor

$$C(t) = z_0 z_0^\dagger e^{-E_0 t} [1 + \delta_{10}^z e^{-\Delta_{10} t} + \dots] \quad \Delta_{10} = E_1 - E_0$$

$$\lim_{t \rightarrow \infty} C(t) = z_0 z_0^\dagger e^{-E_0 t}$$

$$\delta_{10}^z = \frac{z_1 z_1^\dagger}{z_0 z_0^\dagger}$$

The ground state spectrum is what we can compute with better control than any other quantity

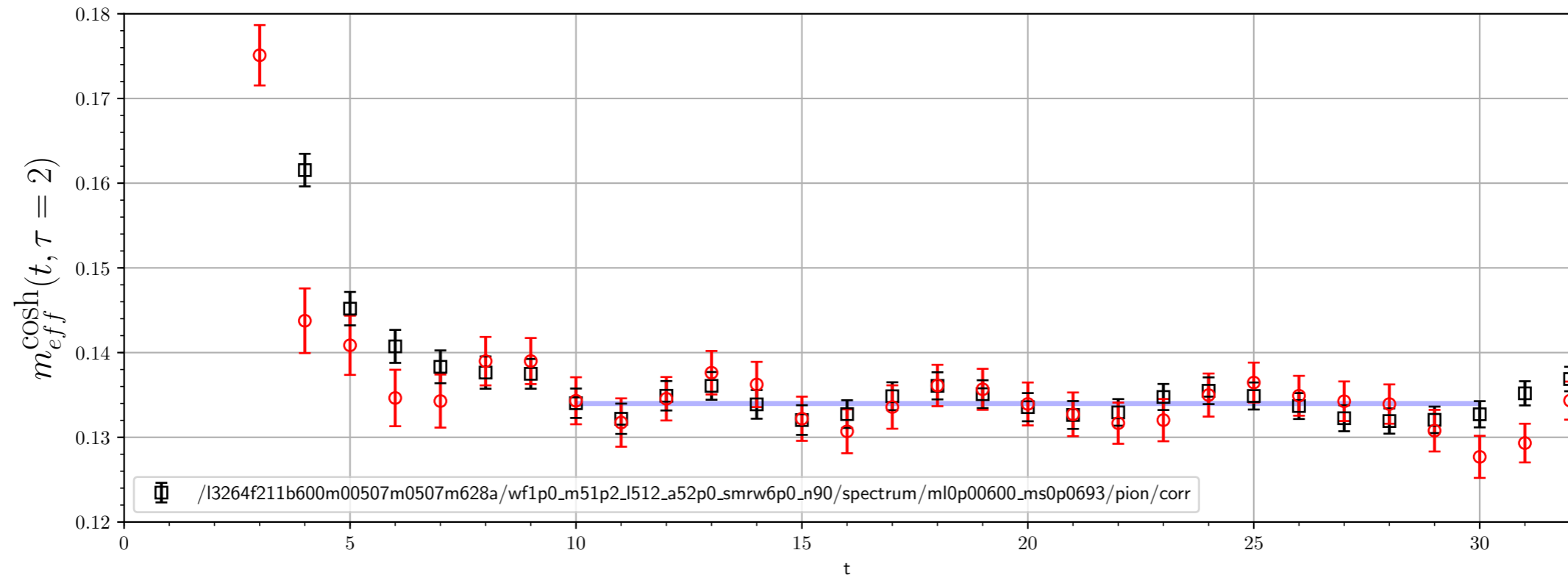
$$m_{eff}(t) = \frac{1}{\tau} \ln \left(\frac{C(t)}{C(t + \tau)} \right)$$

A useful quantity is the effective mass

$$\lim_{t \rightarrow \infty} m_{eff}(t) = E_0$$

LQCD Challenges for NP

○ Consider a 2-point correlation function



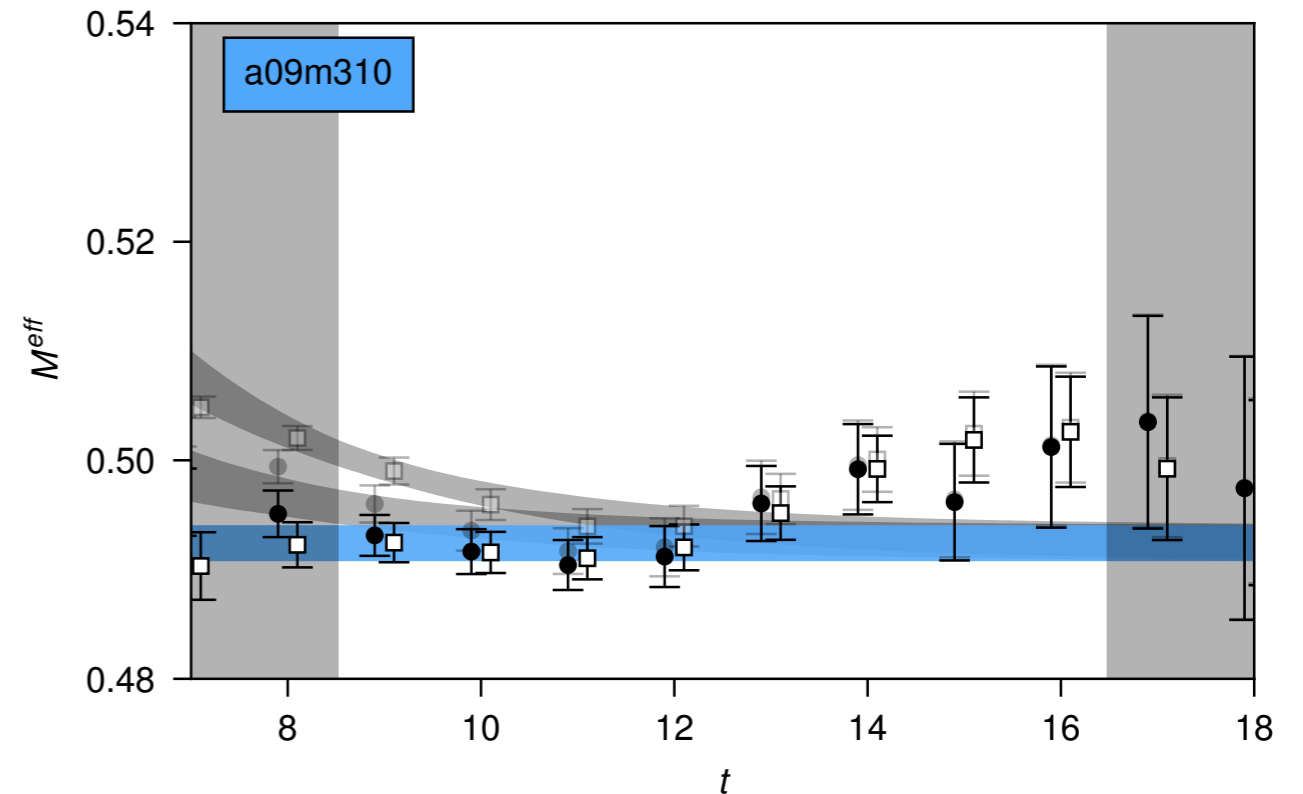
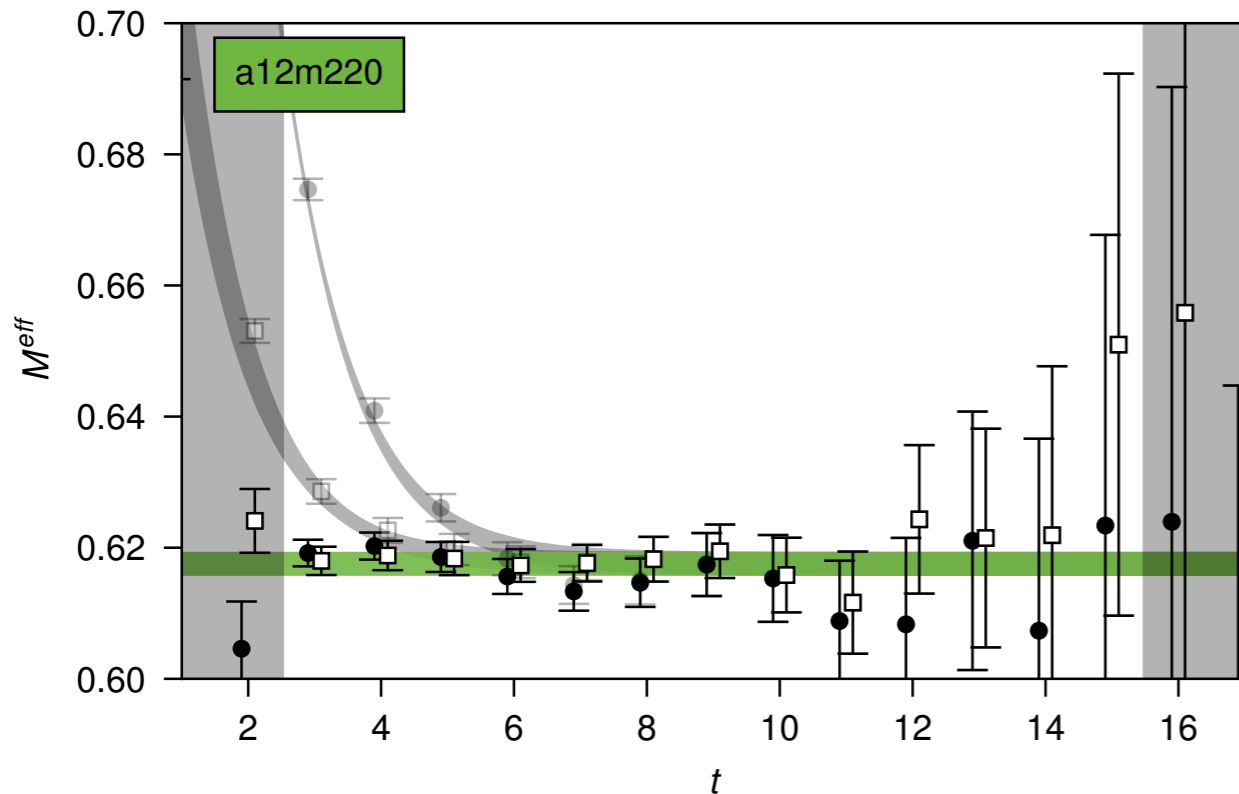
Effective mass of **Pion** 2-point correlation function

red and black “data” are from different choices of overlap operators

Noise is constant in time - can determine very clean ground state (blue band)

LQCD Challenges for NP

○ Consider a 2-point correlation function



Two examples of **nucleon** effective mass

Noise is growing in time - can not simply go to the long-time limit without exponentially increasing the amount of statistics needed

$$\frac{\text{Signal}}{\text{Noise}} \rightarrow \sqrt{N_{\text{stat}}} e^{-(m_N - \frac{3}{2}m_\pi)t}$$

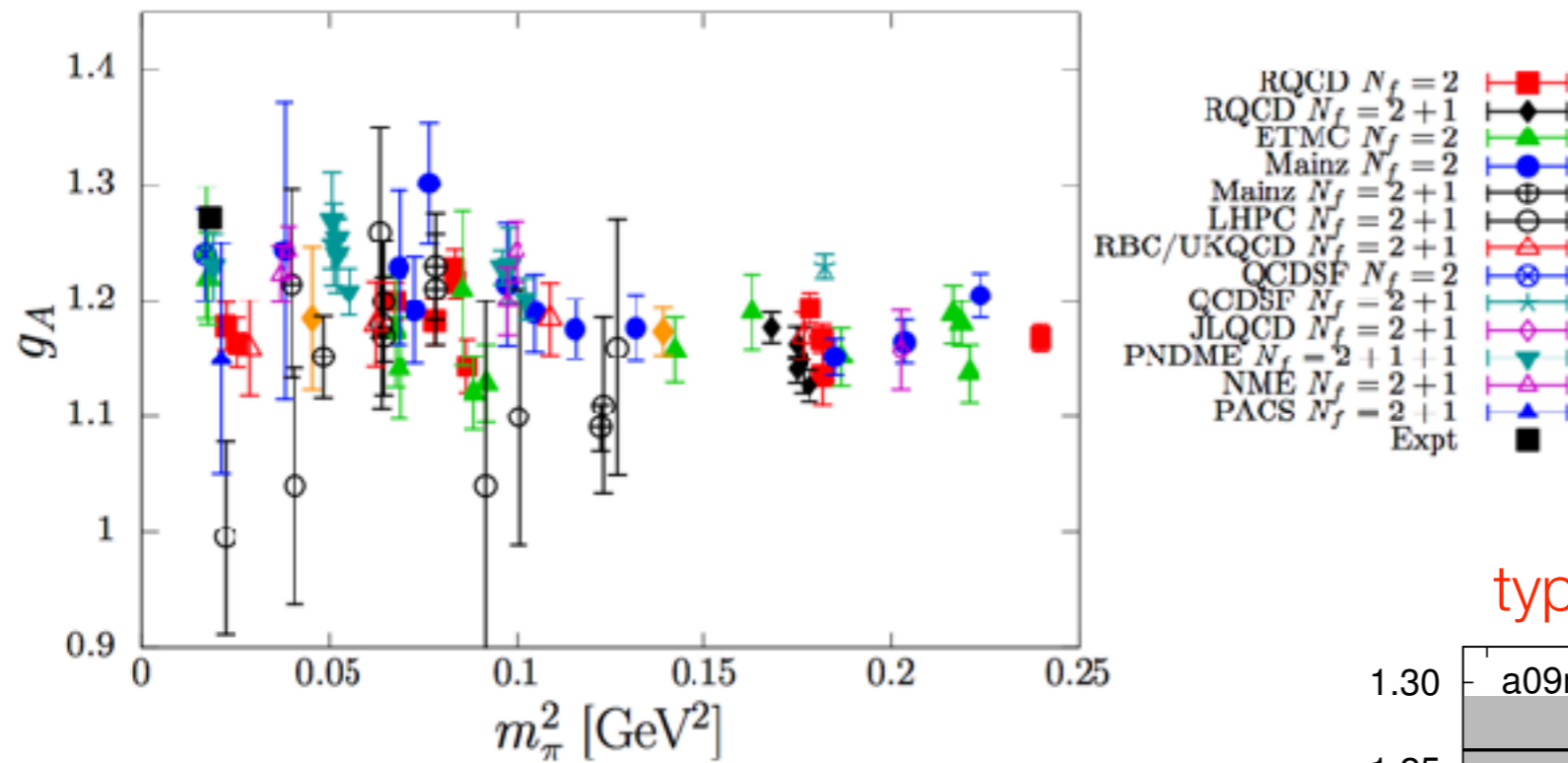
Correlated late-time fluctuations... what is the ground state?

Need sophisticated analysis to ensure you are not susceptible to correlated fluctuations

This problem is exacerbated with 2+ nucleons and form-factor calculations (g_A)

LQCD Challenges for NP

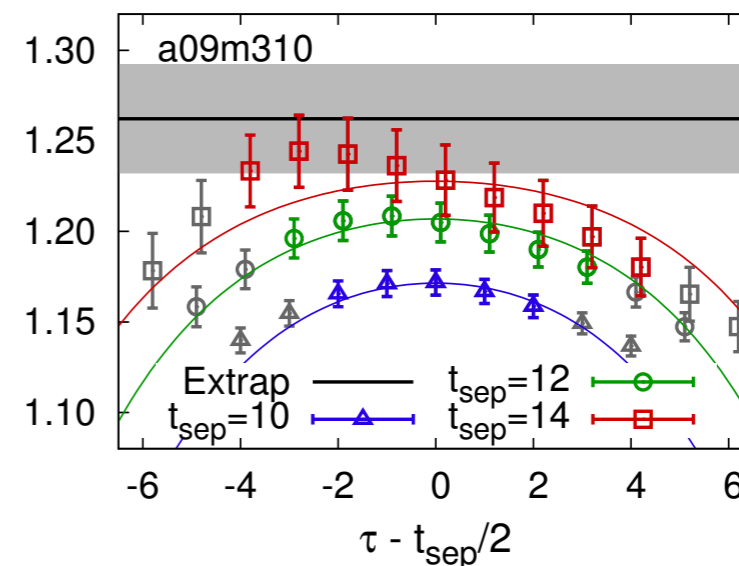
Nucleon axial charge - S. Collins - Lattice 2016 Plenary
 Benchmark quantity sensitive to systematics.



g_A is an important benchmark quantity - very challenging for LQCD to get under control

Presented 2016:
 PNDME, NME, Mainz, RQCD, ETMC, PACS, χ QCD, QCDSF, ...

typical calculation



in long-time limit - should be flat

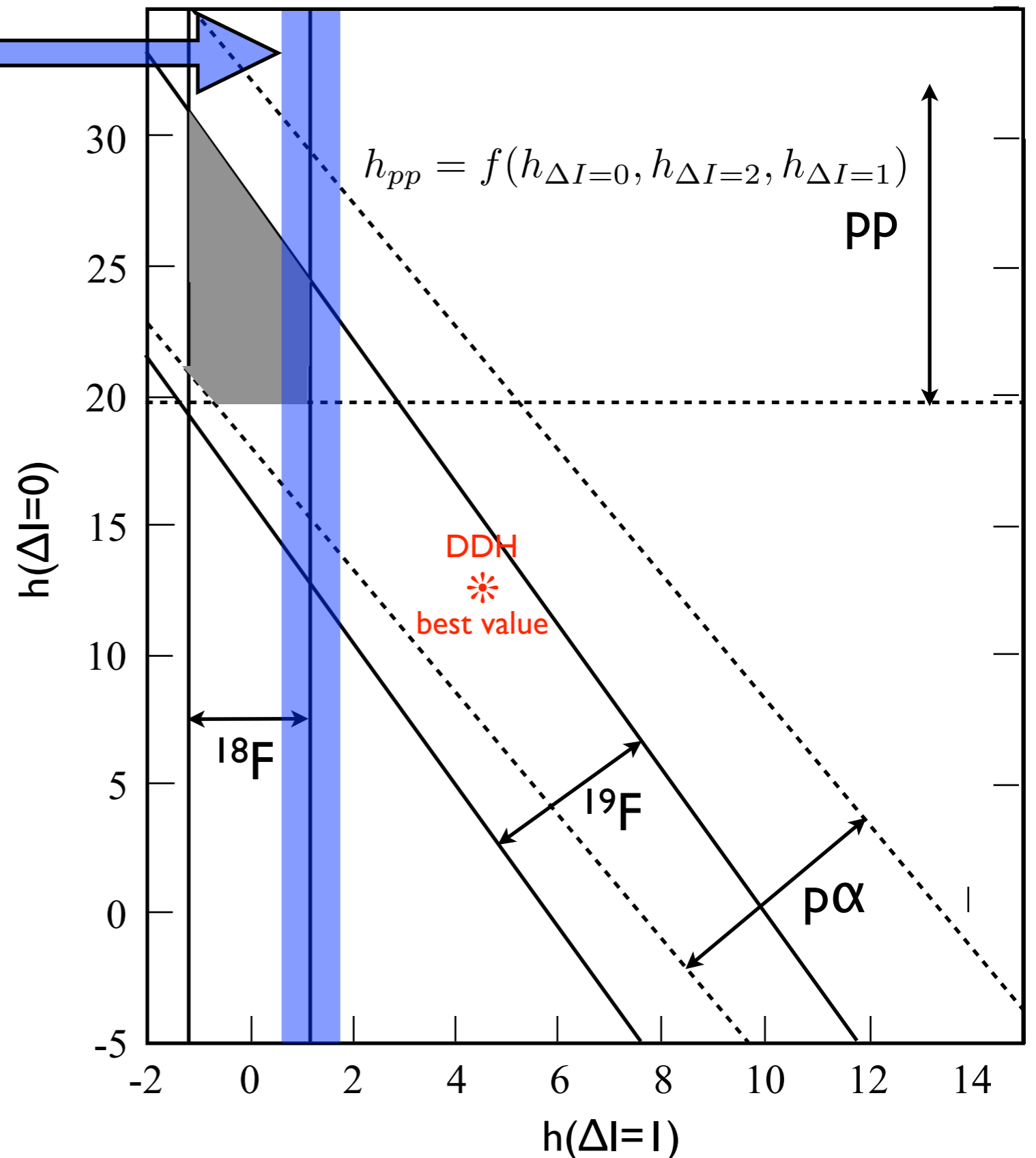
LQCD Challenges for Parity Nonconservation

first LQCD calculation of h_{π}^1 for
 $L=2.5$ f $a=0.123$ f $m_{\pi}=389$ MeV

J. Wasem Phys. Rev. C85 (2012) 022501

Several unquantified approximations

- assumption about coupling of “wave function” used to $N\pi$ state in LQCD calculations
- “disconnected” quark loops neglected
- single lattice spacing
- single pion mass
- single volume
- no renormalization
- This was a tour-de-force calculation carried out single handedly by Joe Wasem



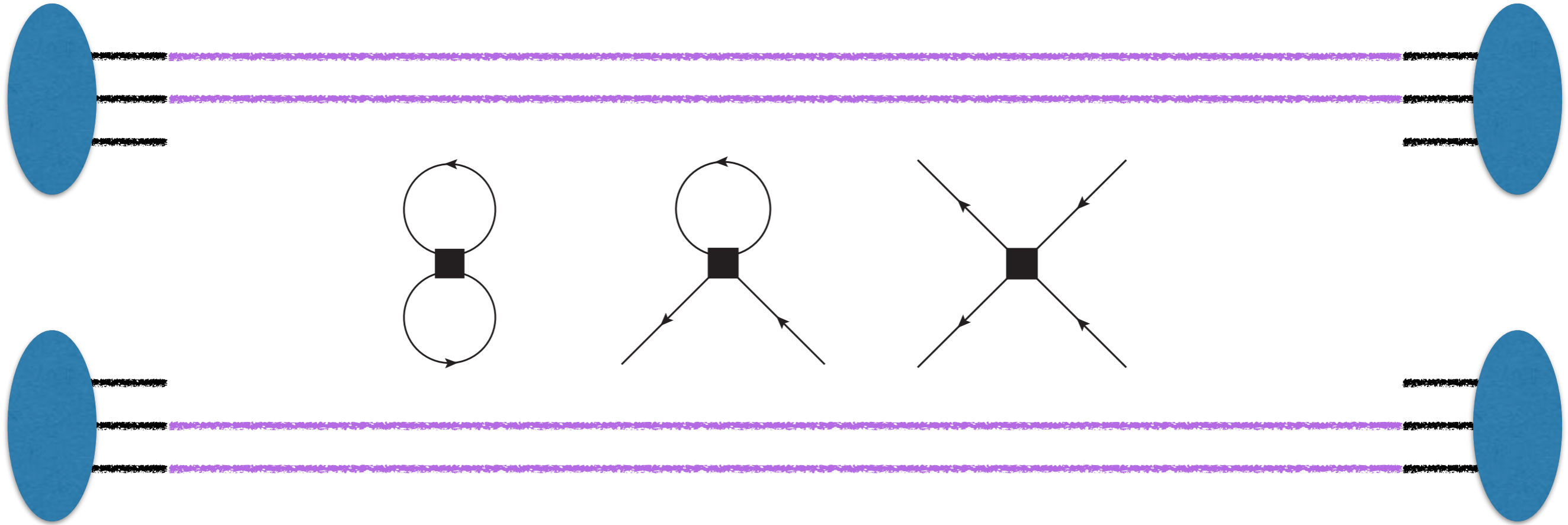
LQCD Challenges for Parity Nonconservation

- Signal-to-noise is exponentially worse than single nucleon

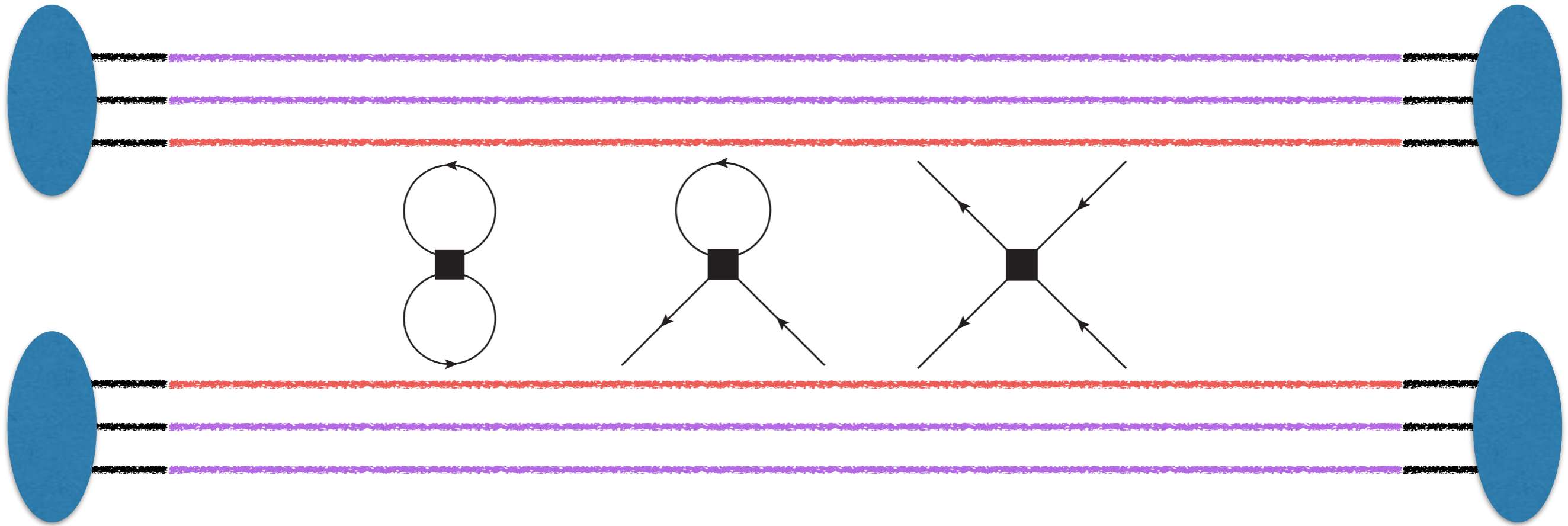
$$\frac{S_{NN}}{\sigma_{NN}} \sim \left(\frac{S_N}{\sigma_N} \right)^2 = \left(\sqrt{N_{samples}} e^{(m_N - \frac{3}{2}m_\pi)t} \right)^2$$

- Either need **all-to-all quark** propagators (1 or more orders of magnitude more expensive) or
 - can not do $l=0$ and $l=1$ PNC amplitudes
 - lose a Volume factor in statistics in $l=2$
- Wick contraction cost of connecting all quark lines is ~ 100 times more than for two-nucleons

LQCD Challenges for Parity Nonconservation

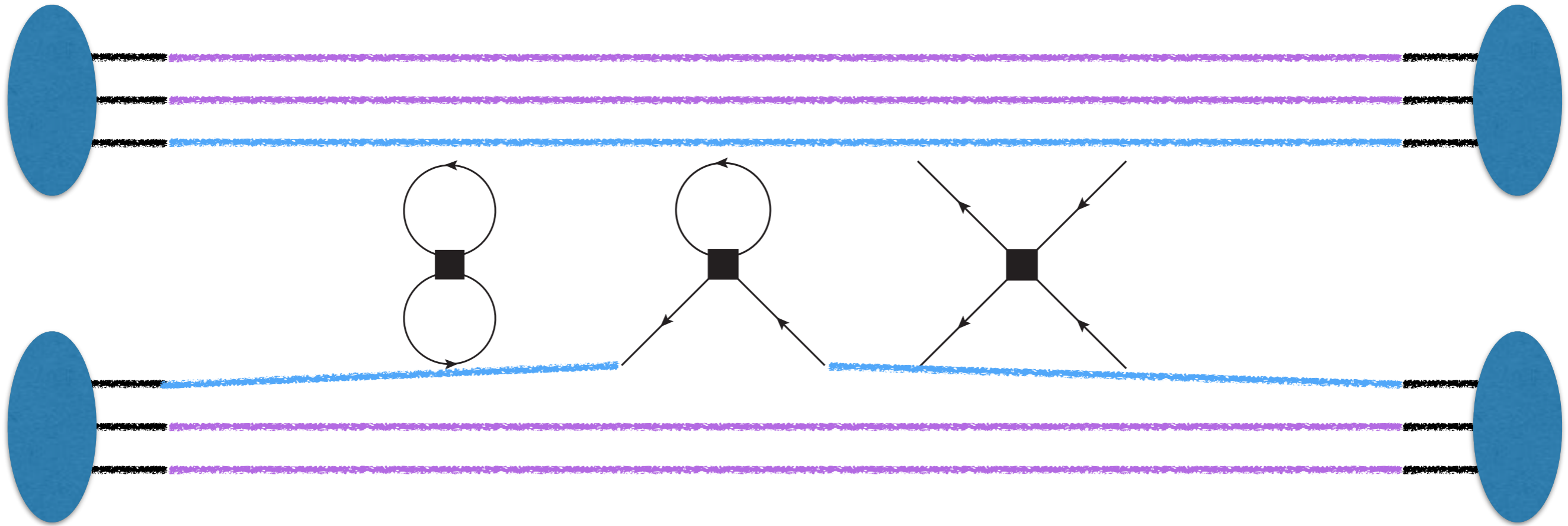


LQCD Challenges for Parity Nonconservation



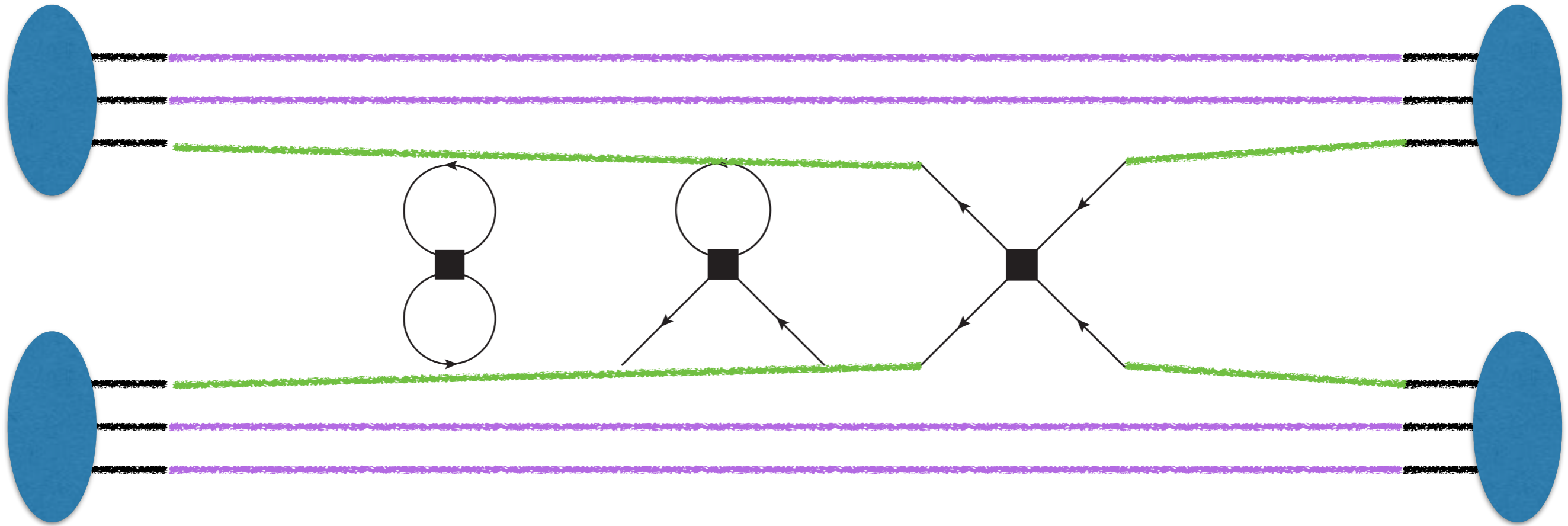
$$\Delta I = 0$$

LQCD Challenges for Parity Nonconservation



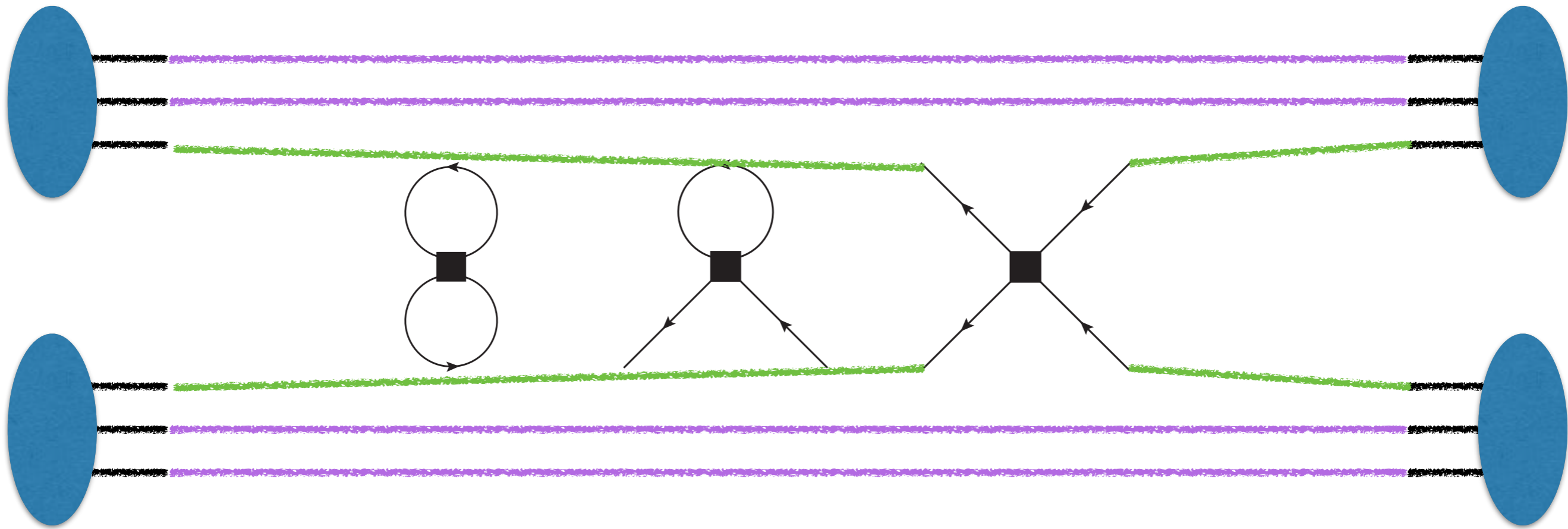
$$\Delta I = 0, 1$$

LQCD Challenges for Parity Nonconservation



$\Delta I=0,1,2$

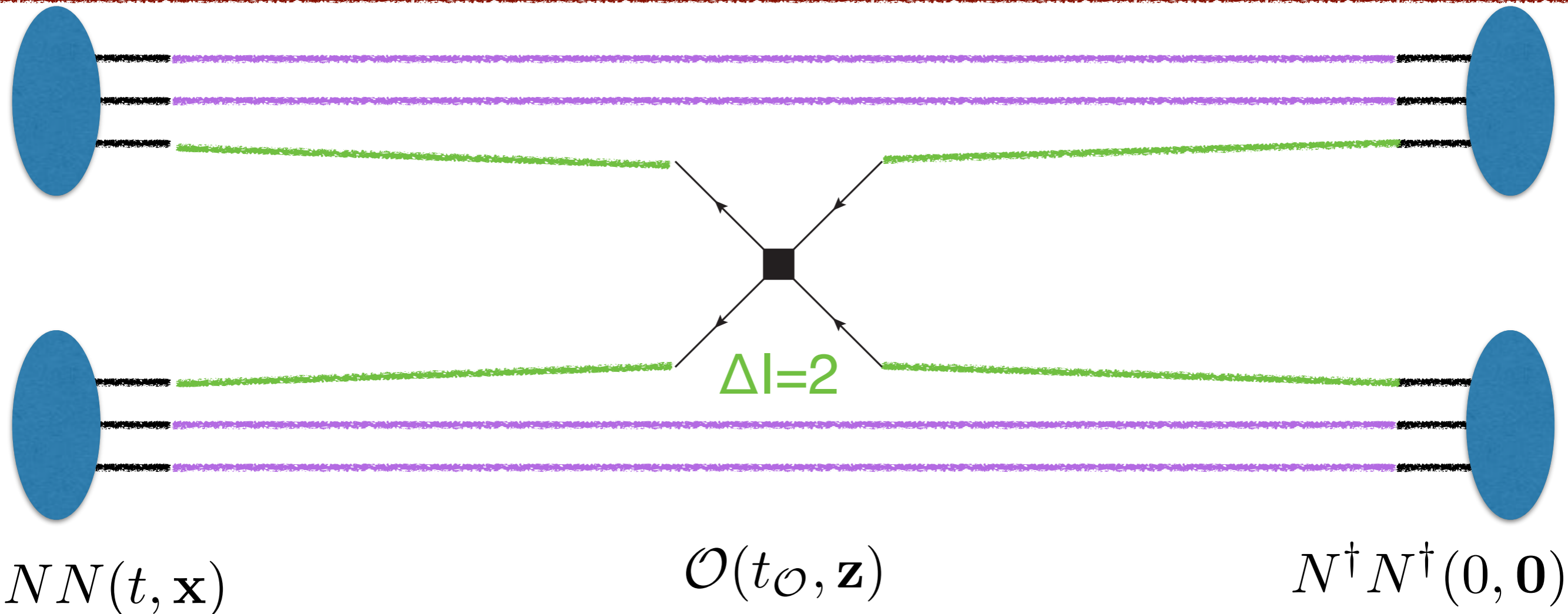
LQCD Challenges for Parity Nonconservation



$$\Delta I = 0, 1, 2$$

- The “disconnected” quark loops are numerically more expensive, and stochastically noisier
- LQCD calculations can project onto definite ΔI

LQCD Challenges for Parity Nonconservation



- To project the operator, O , onto definite momentum, and to project the final NN state onto definite momentum, we need all-to-all propagators (expensive): $\sum_{\mathbf{x}}, \sum_{\mathbf{z}}$
- For now - fix source-sink separation (t) and do NOT sum over \mathbf{x} , loss of spatial volume in statistics

Hadronic Parity Violation



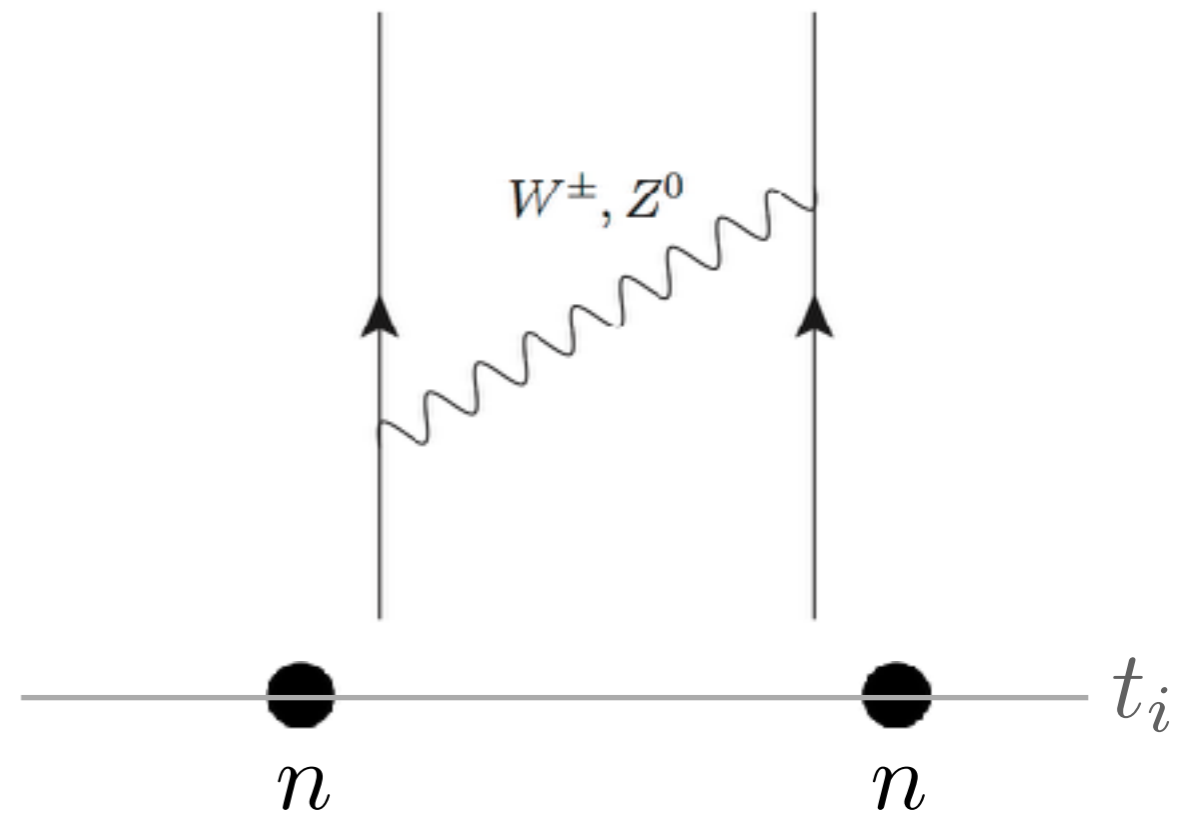
- 2 Baryon „s-wave“ source



Hadronic Parity Violation



- 2 Baryon „s-wave“ source
- EW vertices \Rightarrow 4-quark operator insertion



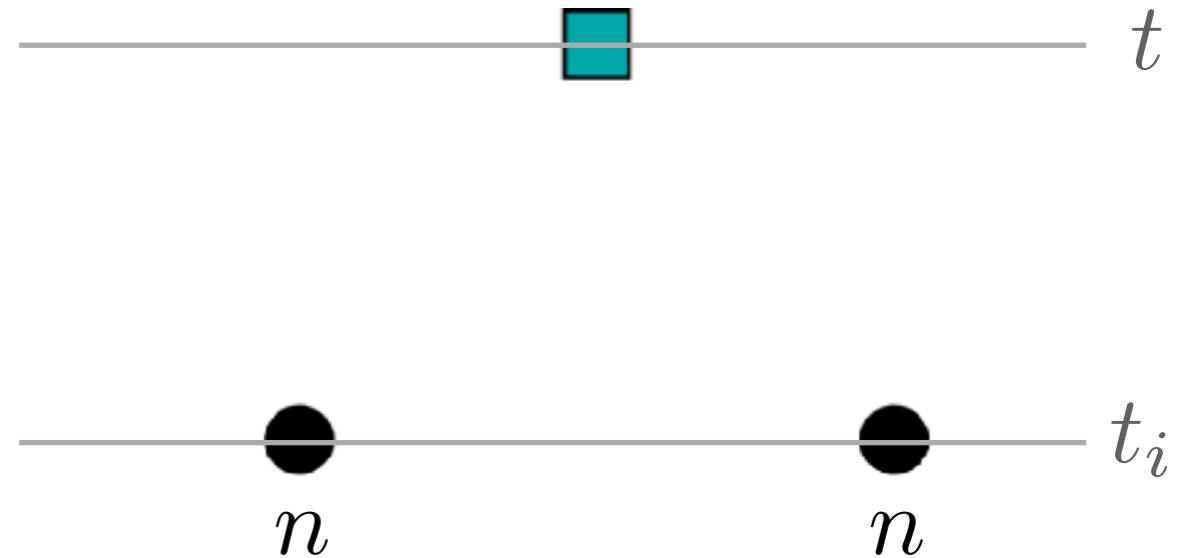
Hadronic Parity Violation



- 2 Baryon „s-wave“ source

$$\mathcal{O}^{\Delta I=2} \equiv \sum_{\mathbf{x}} \left[(\bar{q}\tau^3 q)_A (\bar{q}\tau^3 q)_V - \frac{1}{3} (\bar{q}\vec{\tau}q)_A (\bar{q}\vec{\tau}q)_V \right] (\mathbf{x})$$

- EW vertices \Rightarrow 4-quark operator insertion

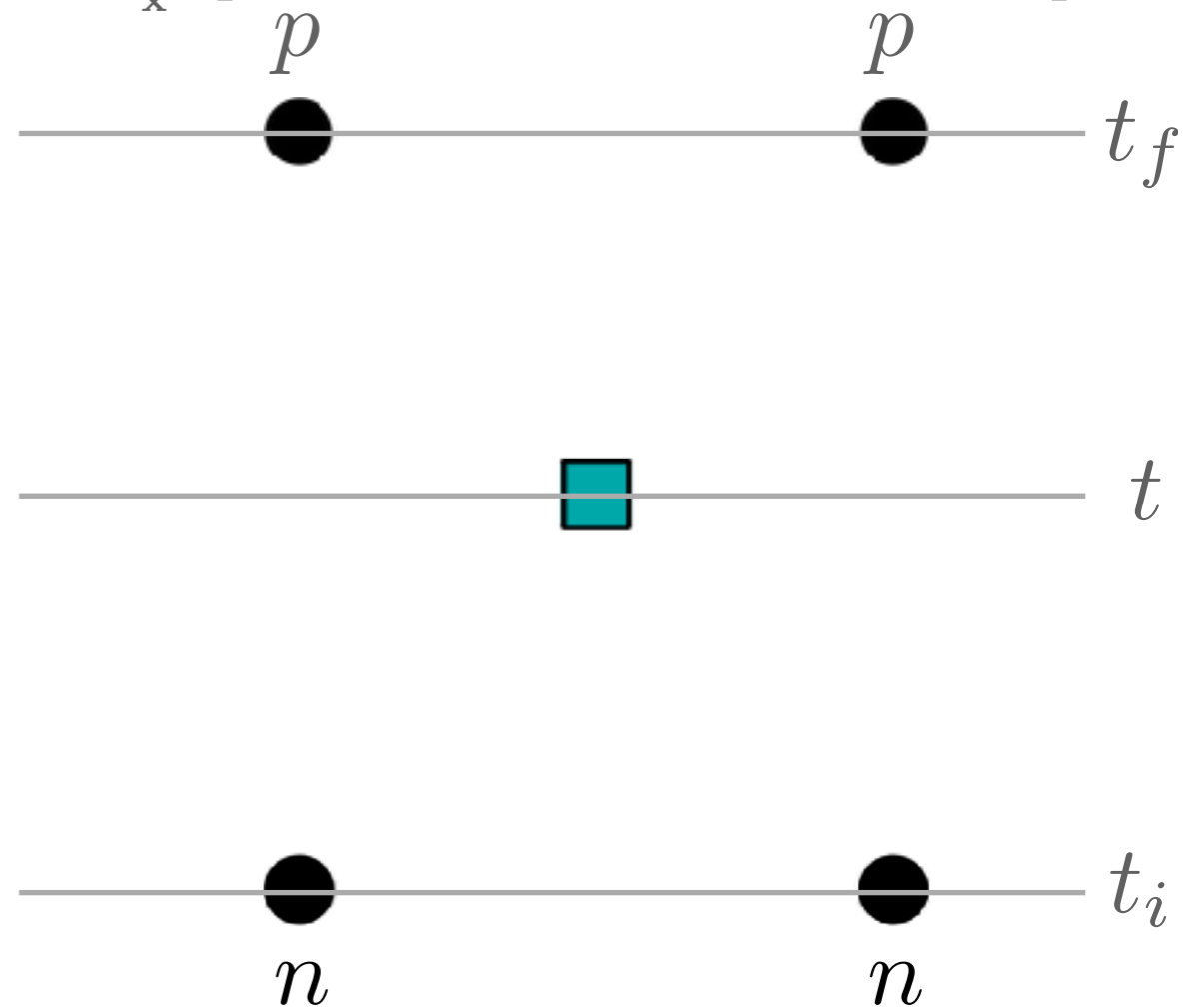


Hadronic Parity Violation



- 2 Baryon „s-wave“ source
- EW vertices \Rightarrow 4-quark operator insertion
- 2 Baryon „p-wave“ sink
- In total there are 4896 contractions
- isospin limit reduces this number to 2208

$$\mathcal{O}^{\Delta I=2} \equiv \sum_{\mathbf{x}} \left[(\bar{q}\tau^3 q)_A (\bar{q}\tau^3 q)_V - \frac{1}{3} (\bar{q}\vec{\tau}q)_A (\bar{q}\vec{\tau}q)_V \right] (\mathbf{x})$$

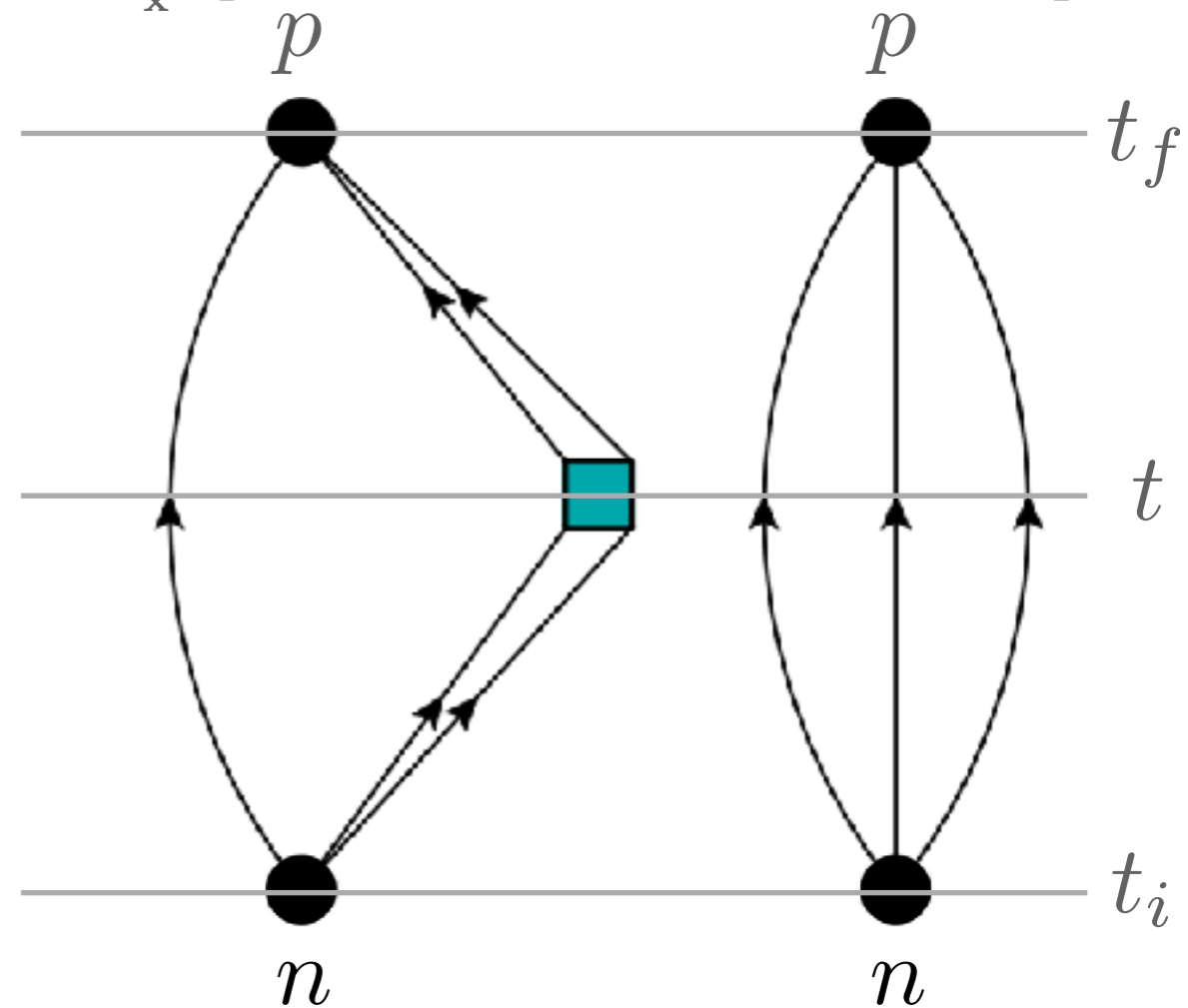


Hadronic Parity Violation



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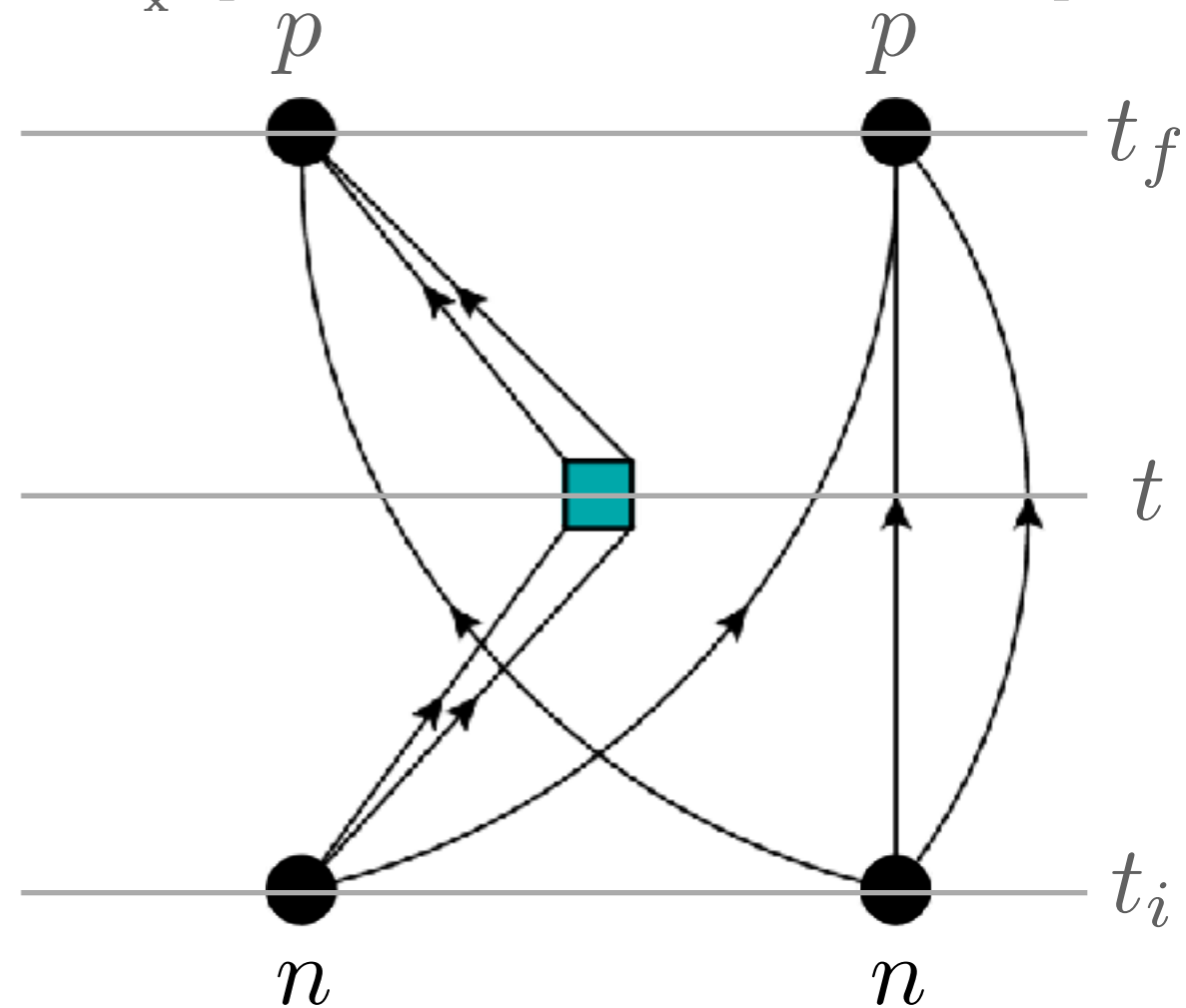


Hadronic Parity Violation



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$$\mathcal{O}^{\Delta I=2} \equiv \sum_{\mathbf{x}} \left[(\bar{q}\tau^3 q)_A (\bar{q}\tau^3 q)_V - \frac{1}{3} (\bar{q}\vec{\tau}q)_A (\bar{q}\vec{\tau}q)_V \right] (\mathbf{x})$$

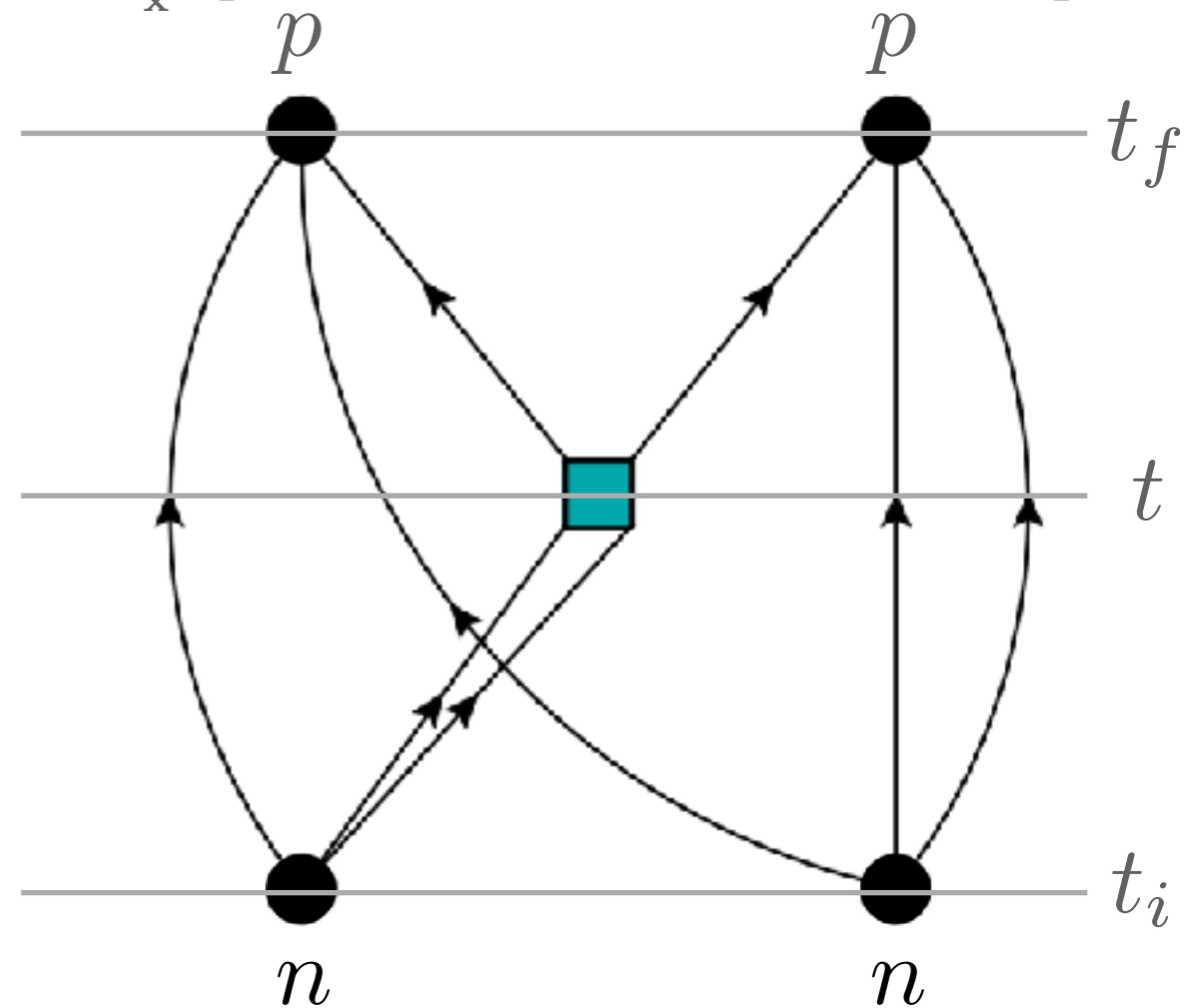


Hadronic Parity Violation



- 2 Baryon „s-wave“ source
- EW vertices \Rightarrow 4-quark operator insertion
- 2 Baryon „p-wave“ sink
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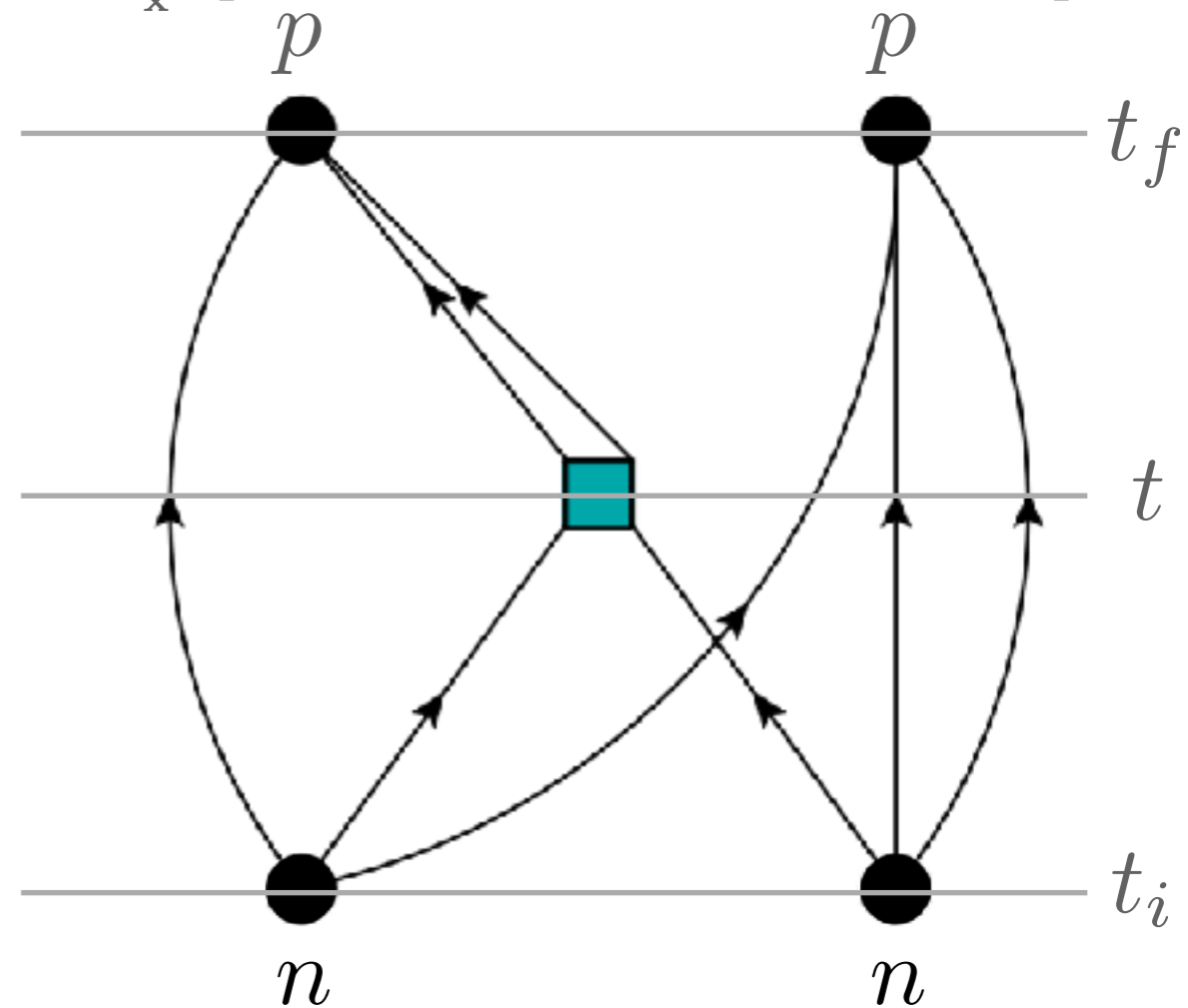


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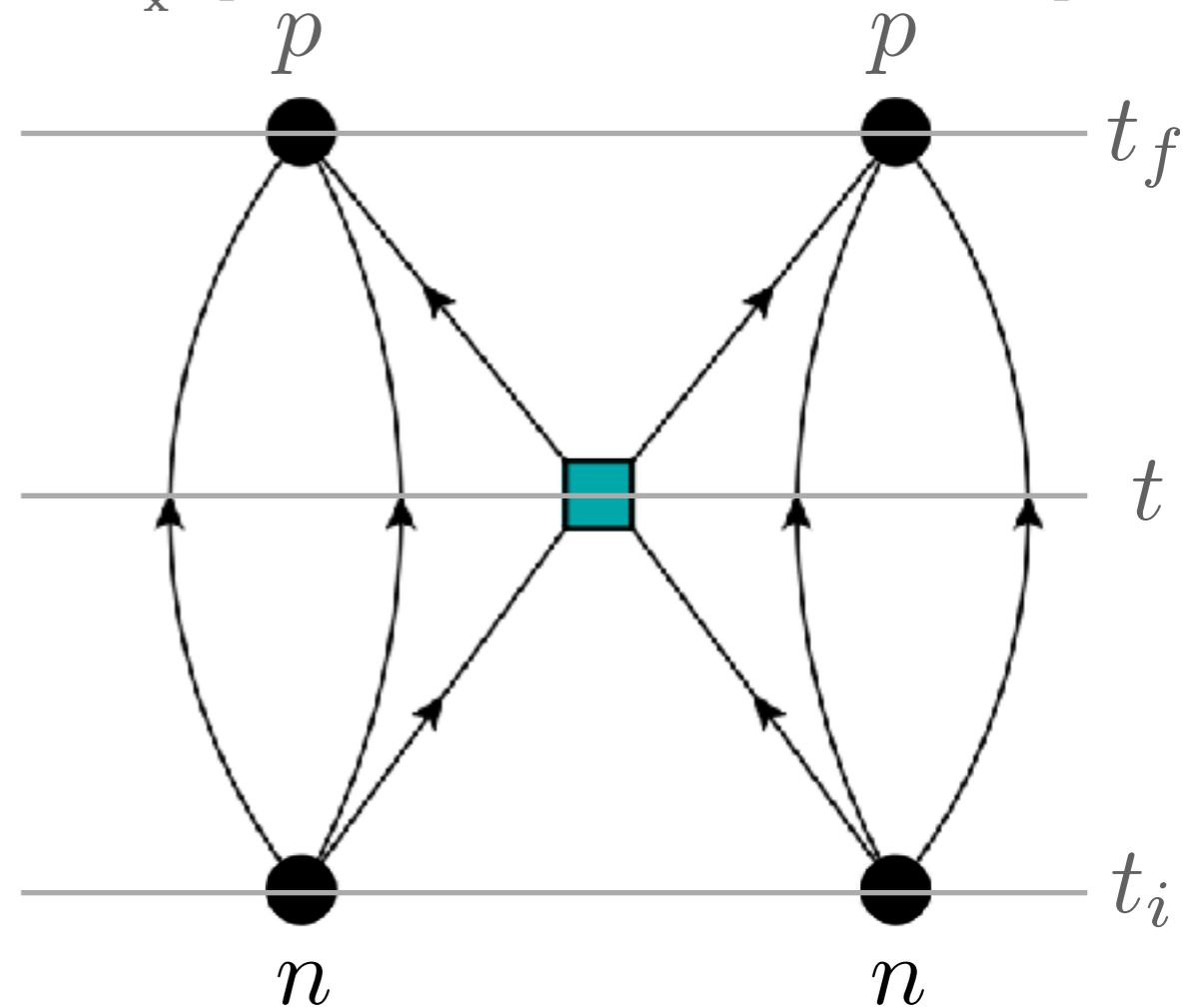


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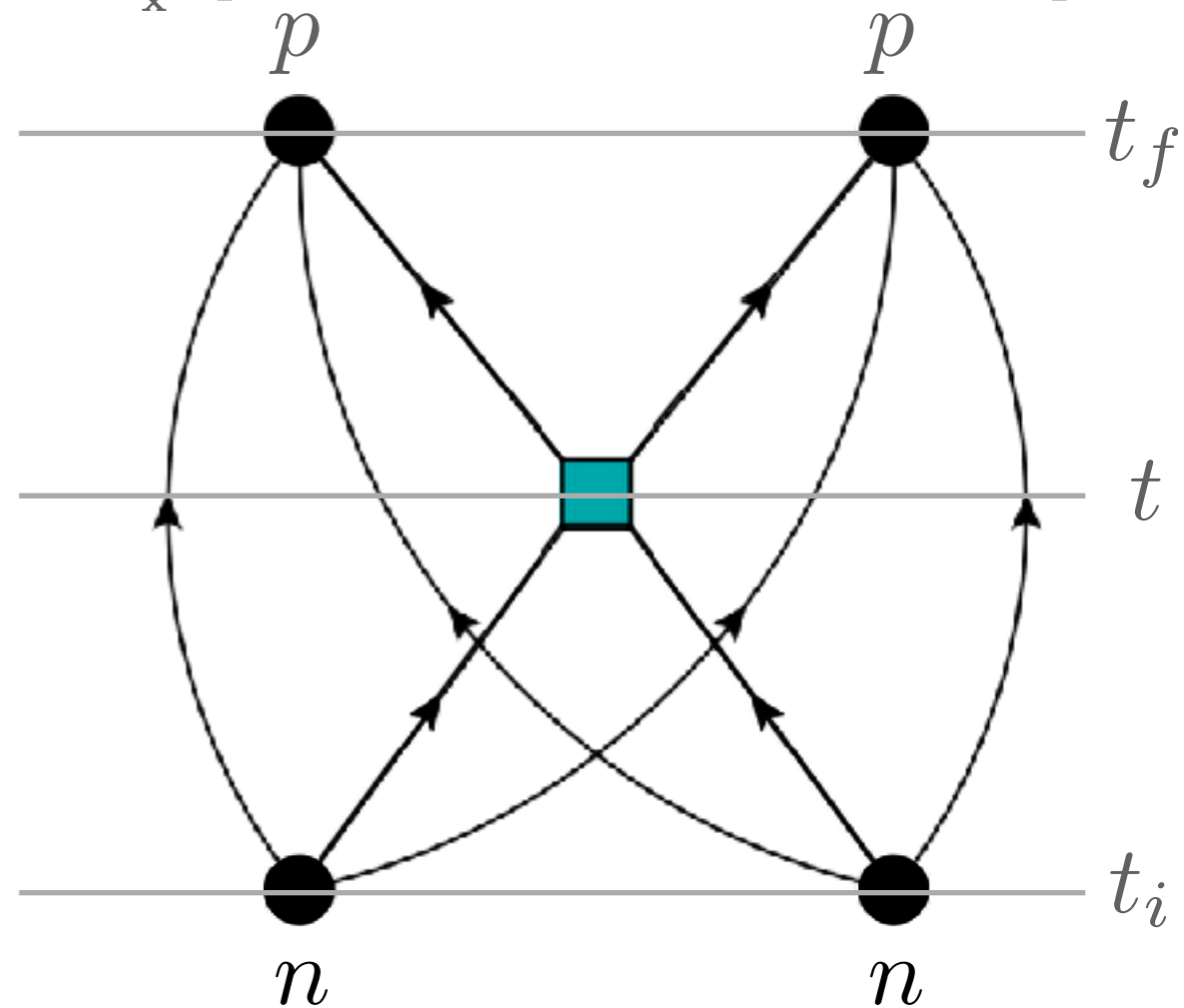


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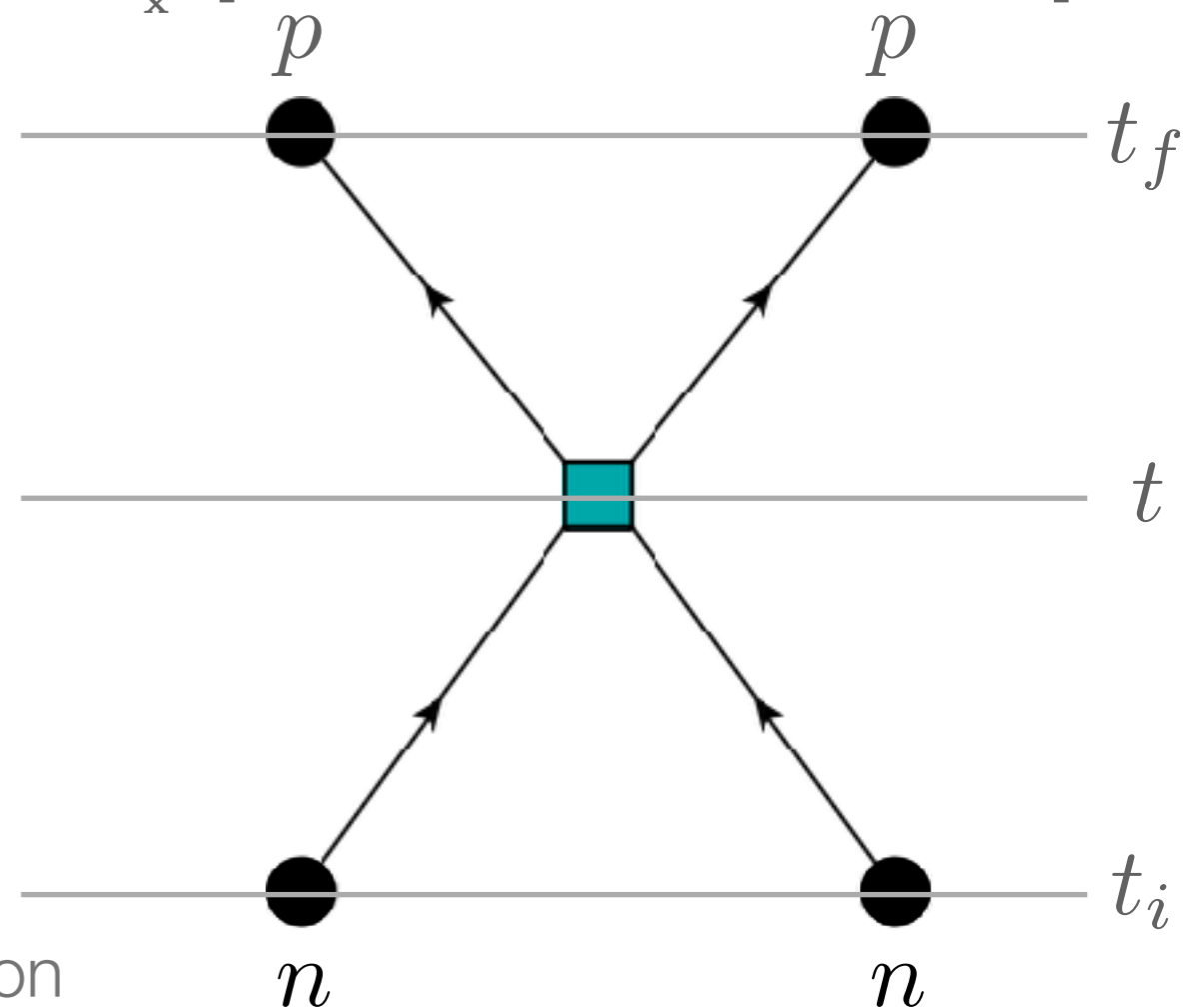


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- Extension of UCM, automatic code generation using Mathematica

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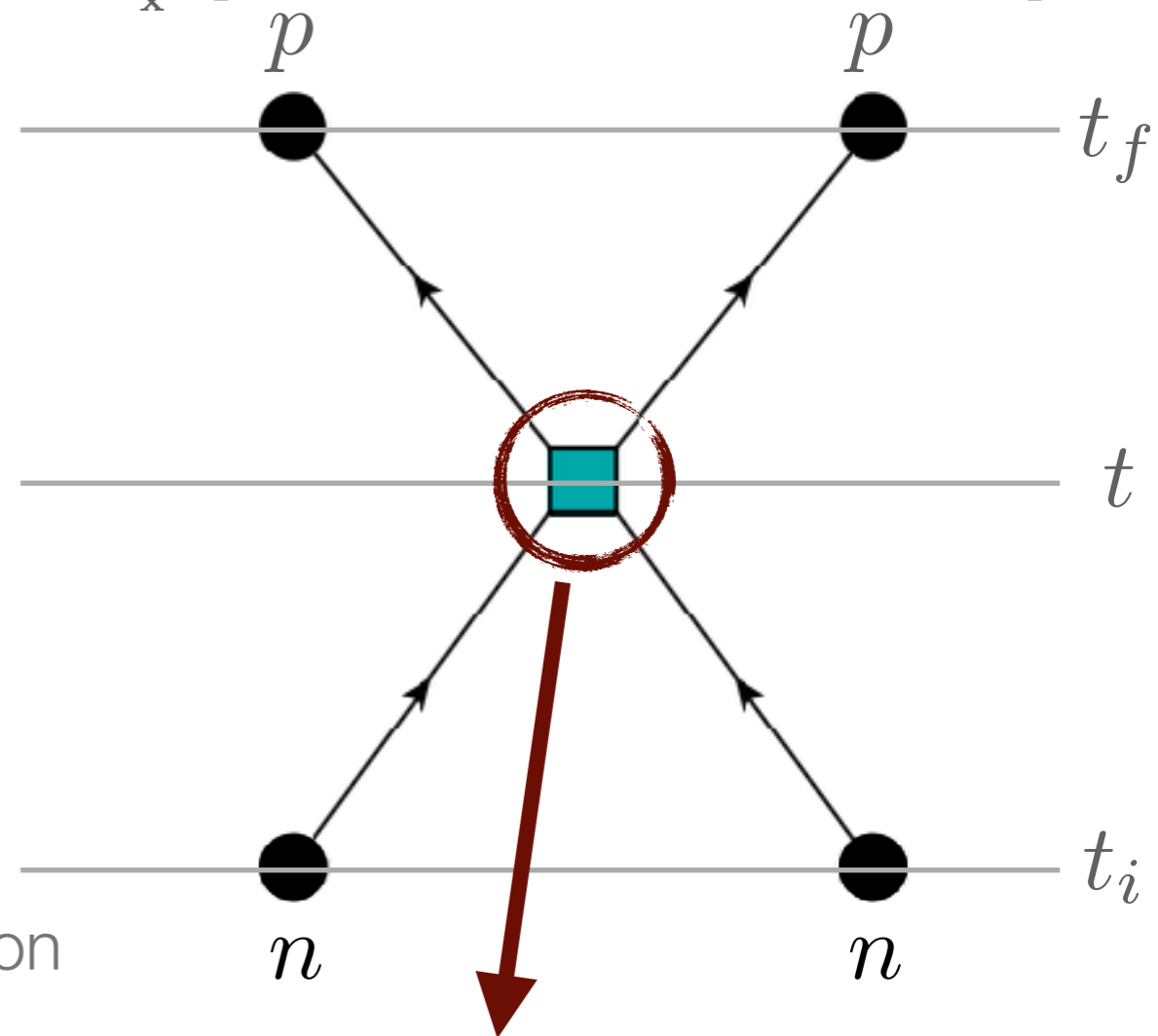
Doi & Endres, Originos et. al., Günther et. al.

Hadronic Parity Violation



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Doi & Endres, Originos et. al., Günther et. al.
- Partial wave scattering needed as well

$$\mathcal{O}^{\Delta I=2} \equiv \sum_{\mathbf{x}} \left[(\bar{q}\tau^3 q)_A (\bar{q}\tau^3 q)_V - \frac{1}{3} (\bar{q}\vec{\tau}q)_A (\bar{q}\vec{\tau}q)_V \right] (\mathbf{x})$$

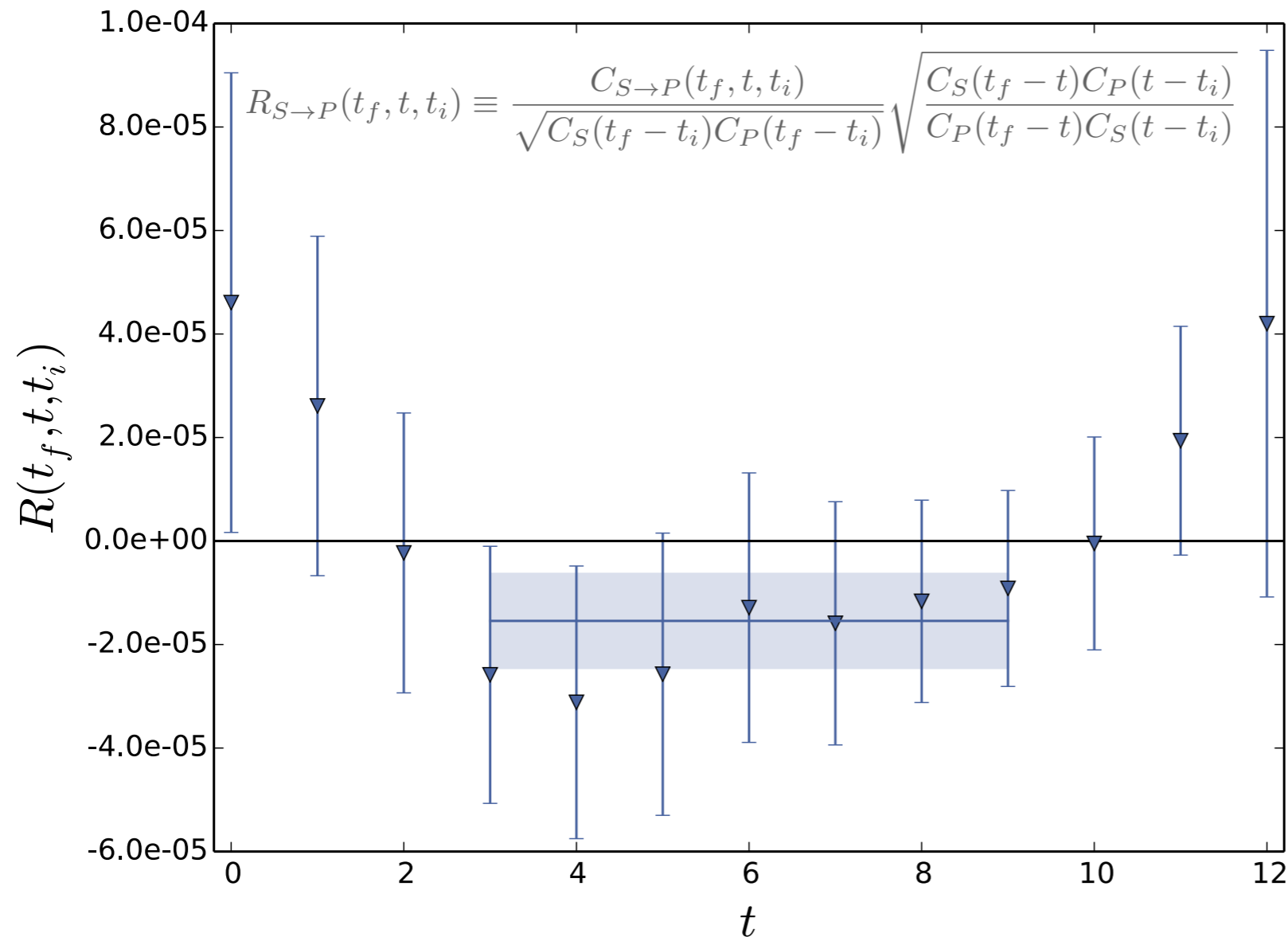


$$\langle {}^3P_0 | H_{EW} | {}^1S_0 \rangle_{\infty} = f(\delta^{(S)}, \partial_E \delta^{(S)}, \delta^{(P)}, \partial_E \delta^{(P)}) \times \langle {}^3P_0 | H_{EW} | {}^1S_0 \rangle_{FV}$$

Hadronic Parity Violation



Normalized Ratio



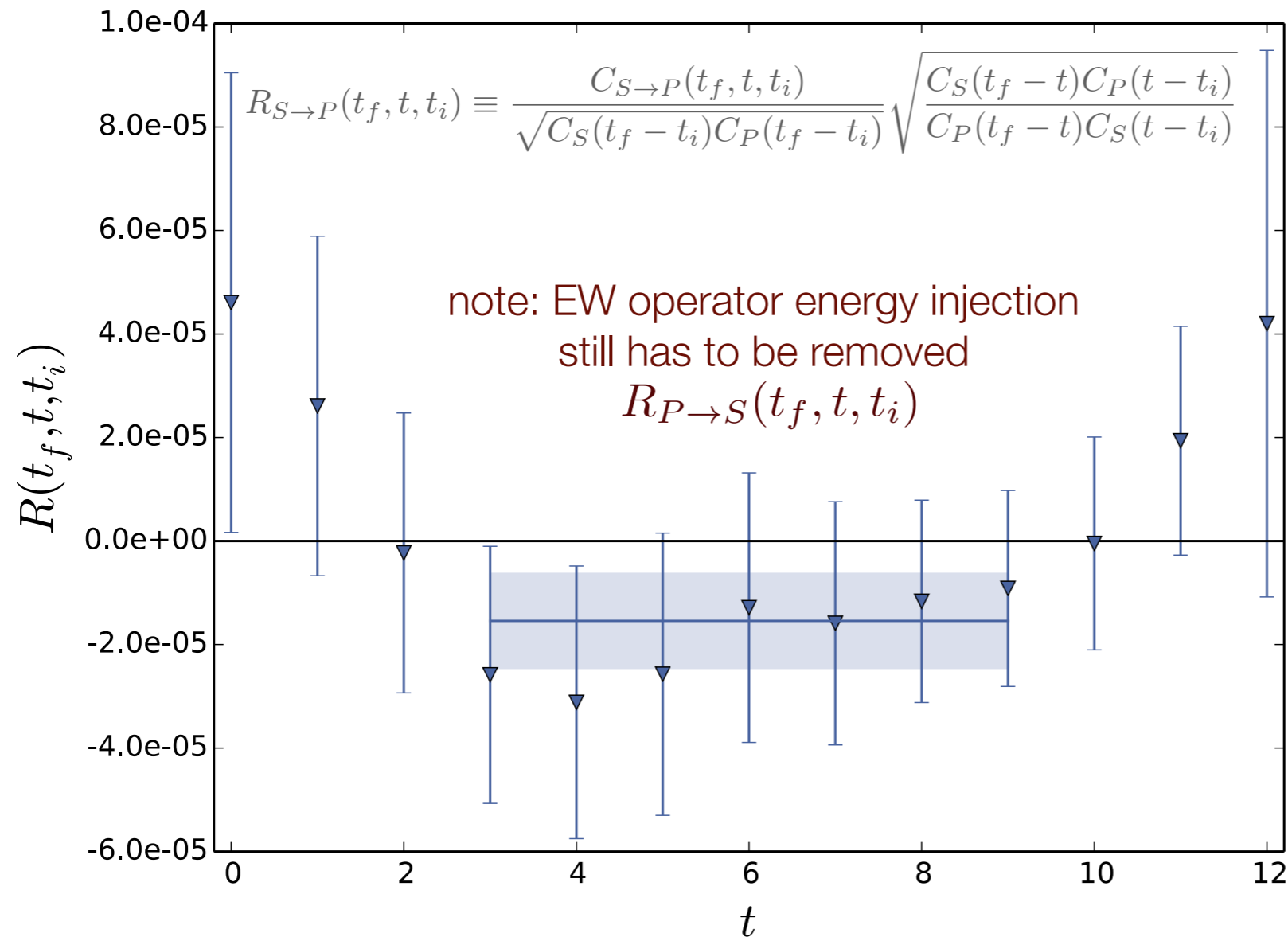
Preliminary!
 $m_\pi = 800$ MeV
only 200 samples

The most challenging aspect of this calculation is the NN interaction

Hadronic Parity Violation



Normalized Ratio



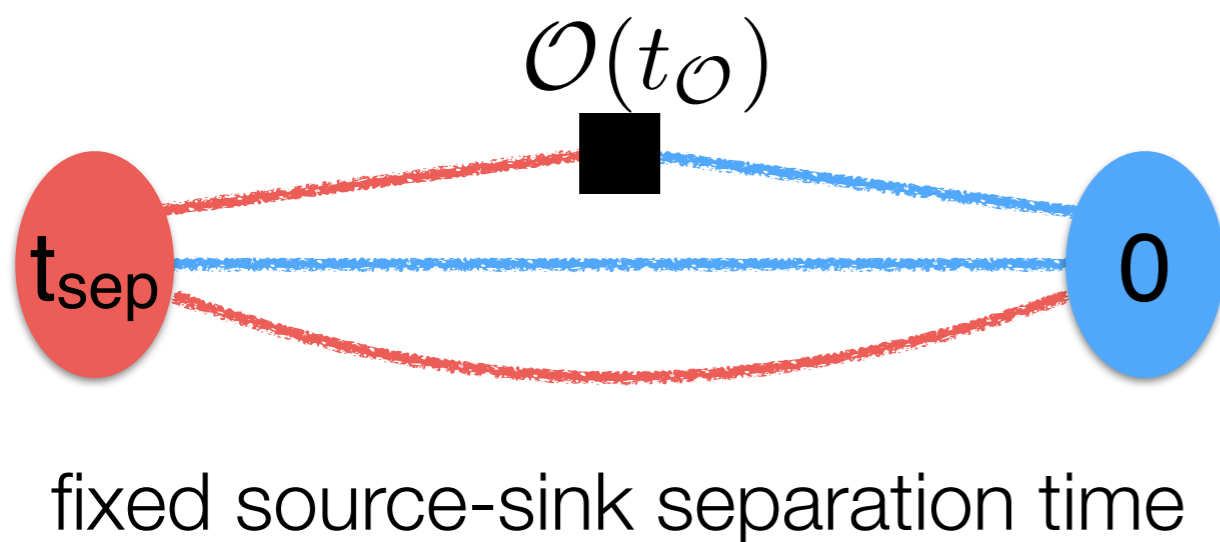
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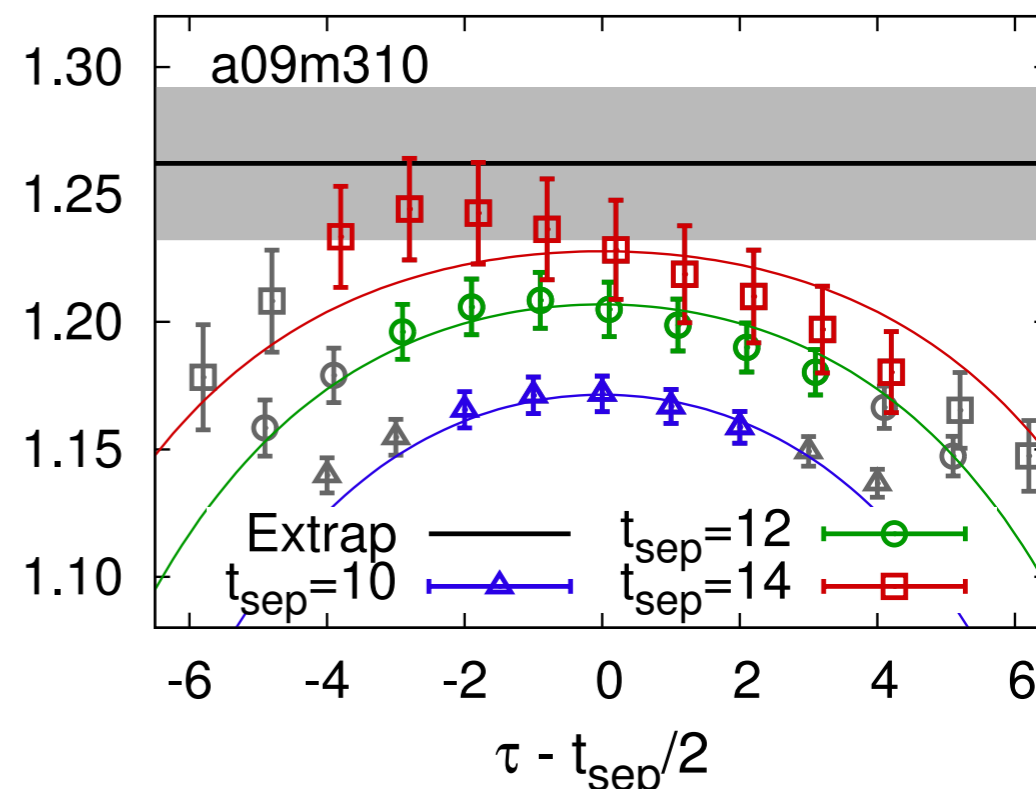
g_A - a success story

- Our success required a few key components:
 - Access to **publicly available** lattice QCD gauge configurations ([MILC Collaboration](#)) with multiple **lattice spacings**, multiple **volumes**, near physical **pion masses**
 - ***ludicrously fast*** GPU code (Quda library)
 - access to leadership class computing (Titan via INCITE)
 - a new strategy motivated by the Feynman-Hellmann Theorem

standard method



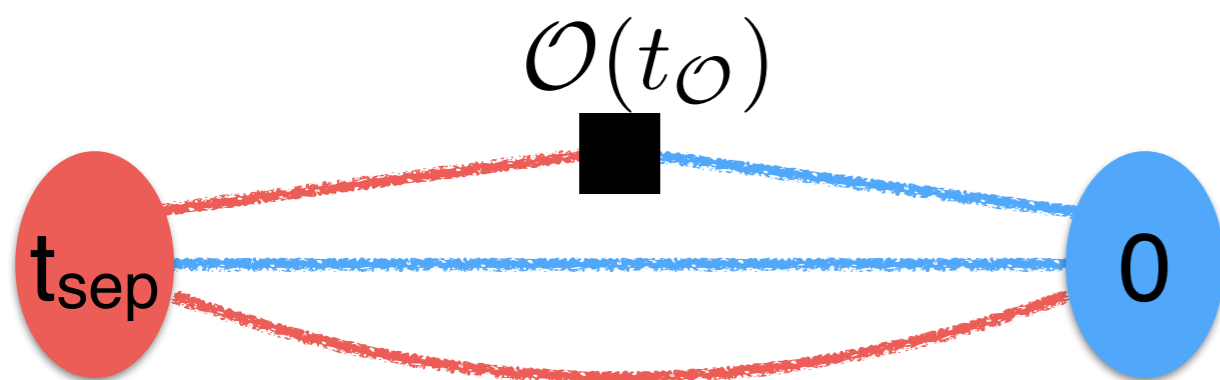
PNDME arXiv:1606.07049



g_A - a success story

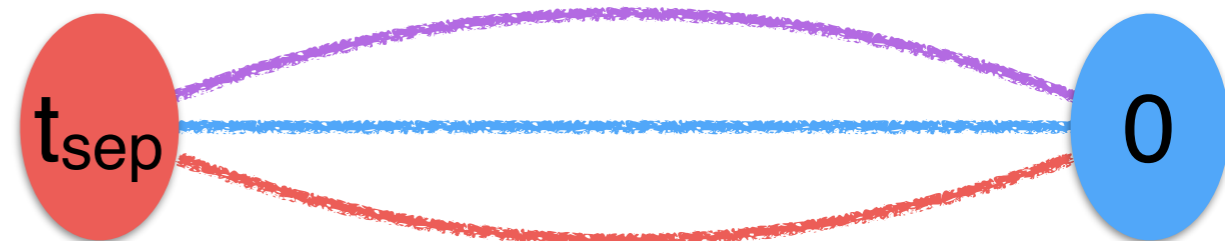
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standard method



fixed source-sink separation time

our new method

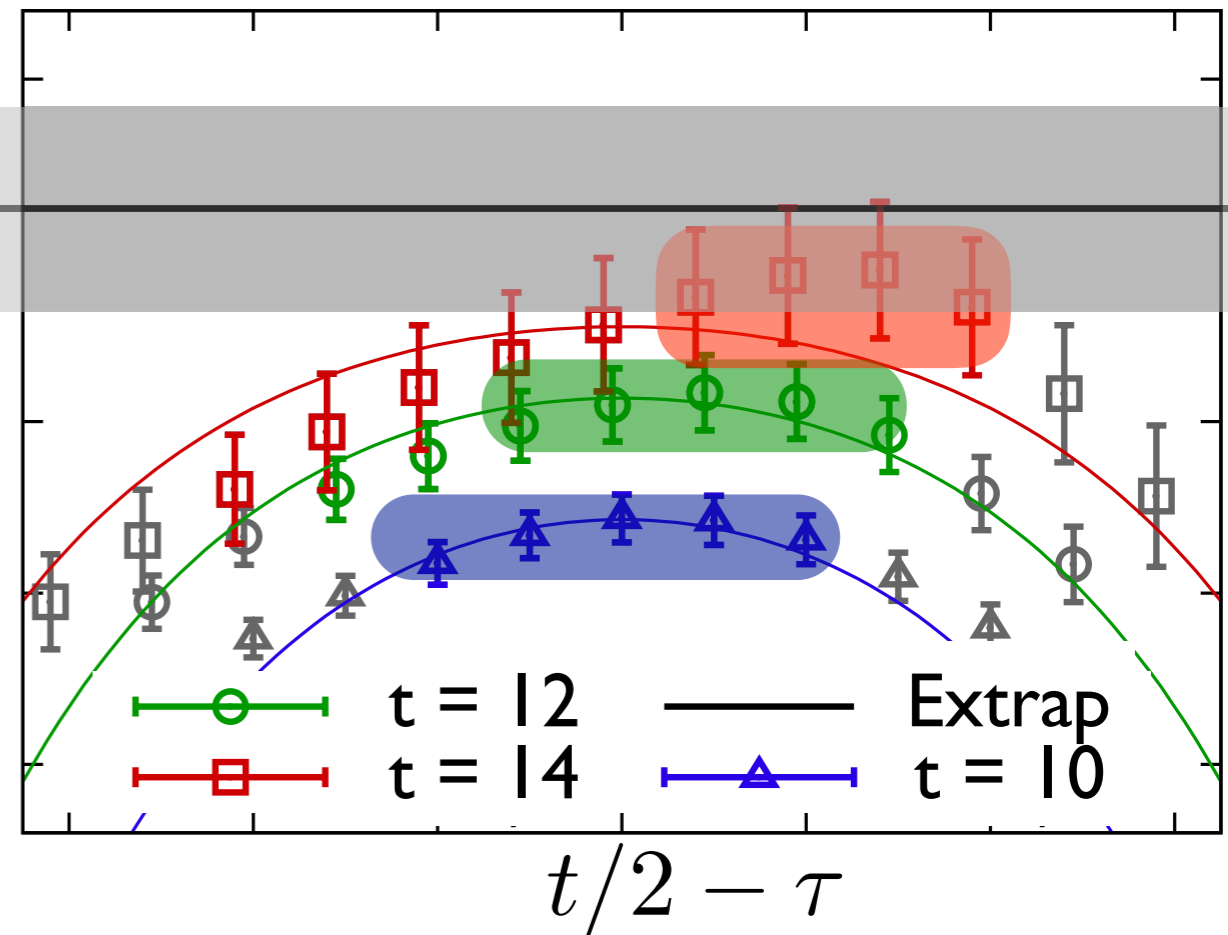


$$\text{--- purple ---} = \int dt_O \text{--- red ---} \text{--- blue ---}$$

$O(t_O)$

Comparison with a Standard Method

PNDME Phys. Rev. D94 (2016) arXiv:1606.07049



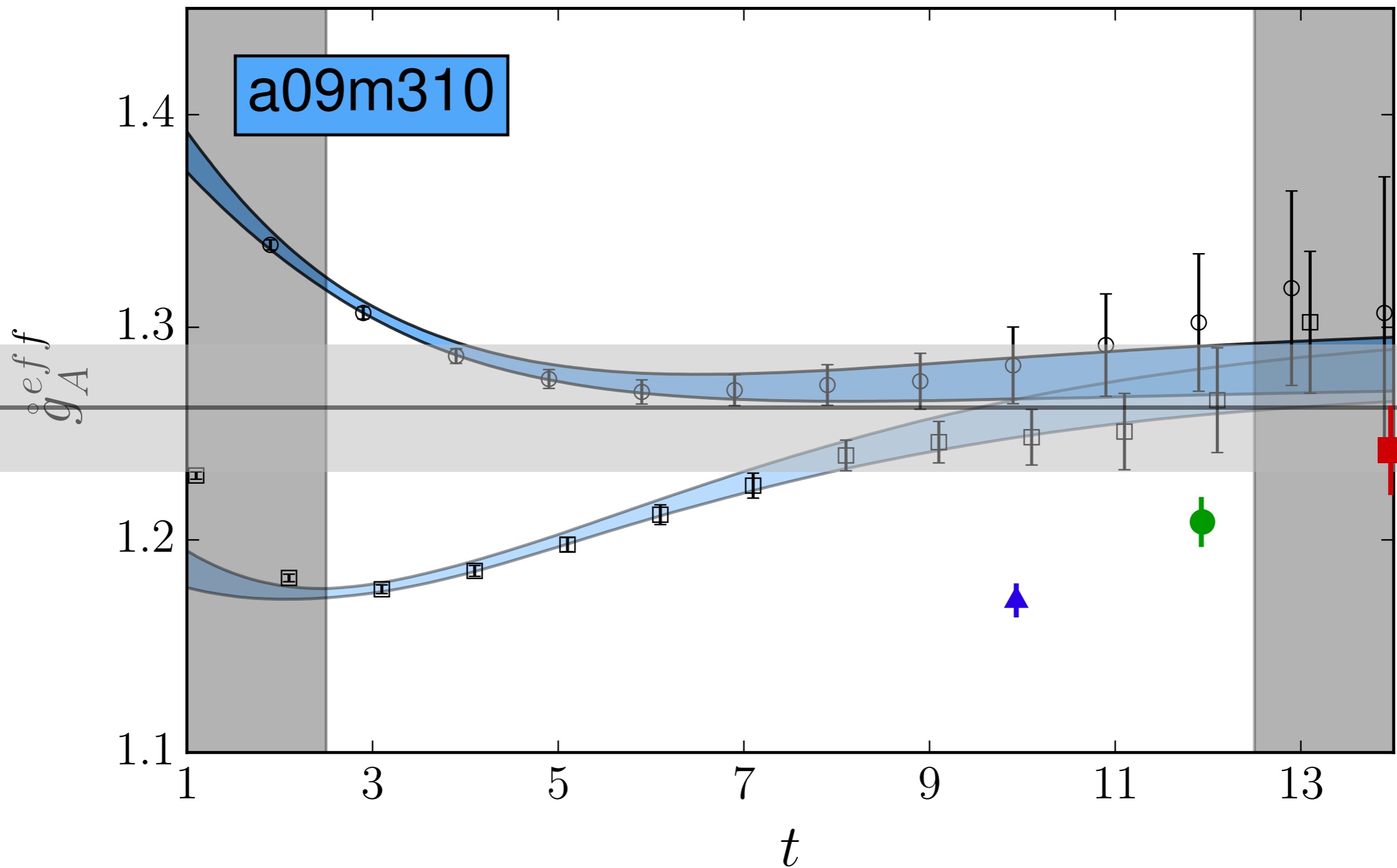
Comparison with a Standard Method

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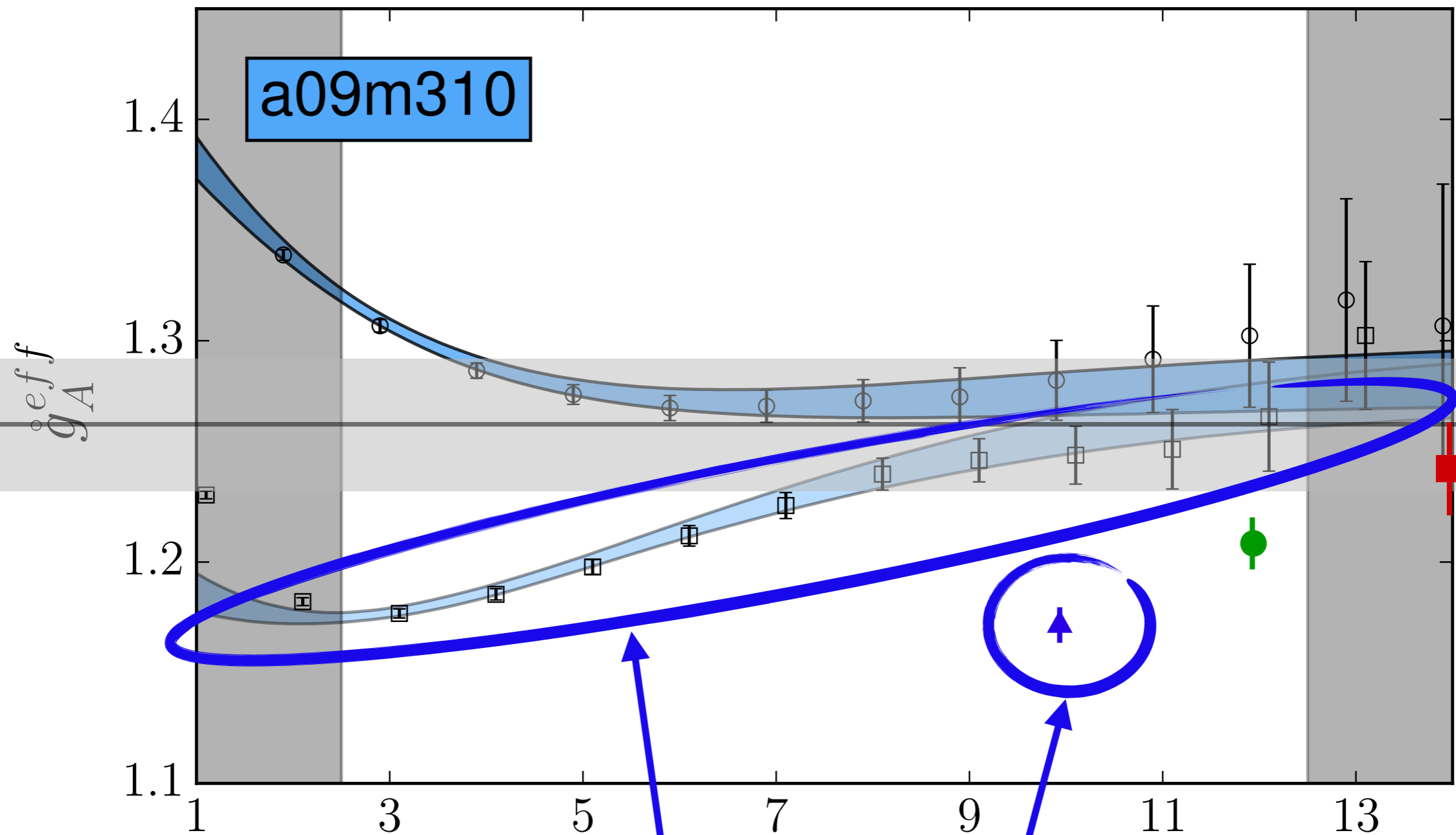
$t \longrightarrow \infty$

Comparison with a Standard Method



Slides adapted from A. Nicholson
adapted from E. Berkowitz

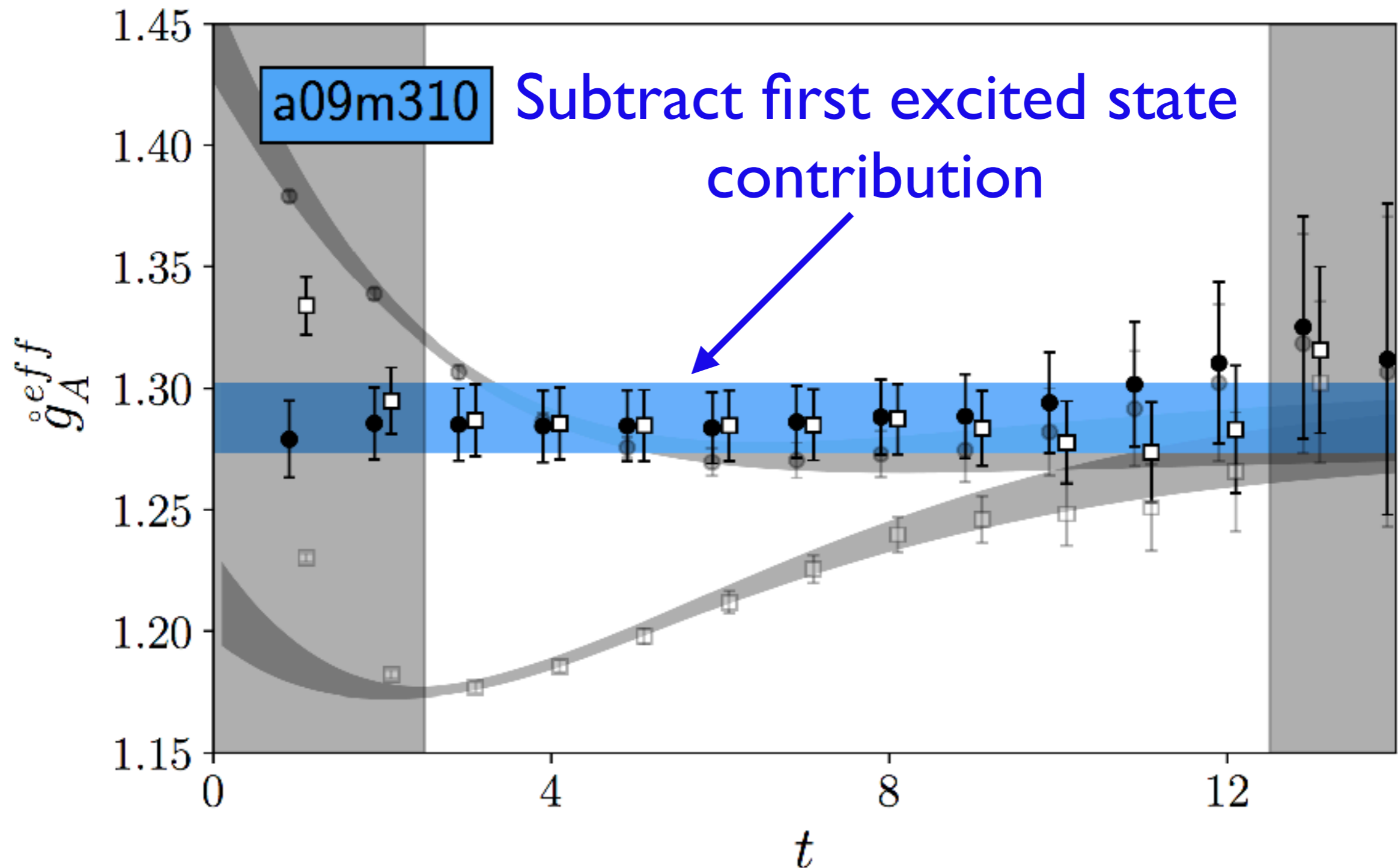
Comparison with a Standard Method



Each one of these points costs the same as all times for one source in our calc

Slides adapted from A. Nicholson adapted from E. Berkowitz

Comparison with a Standard Method



A percent-level determination of the nucleon axial coupling from QCD

Lattice QCD Team

Glasgow: Chris Bouchard
 INT: Chris Monahan
 JLab: Balint Joo
 Jülich: Evan Berkowitz
 LBL/UCB: David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
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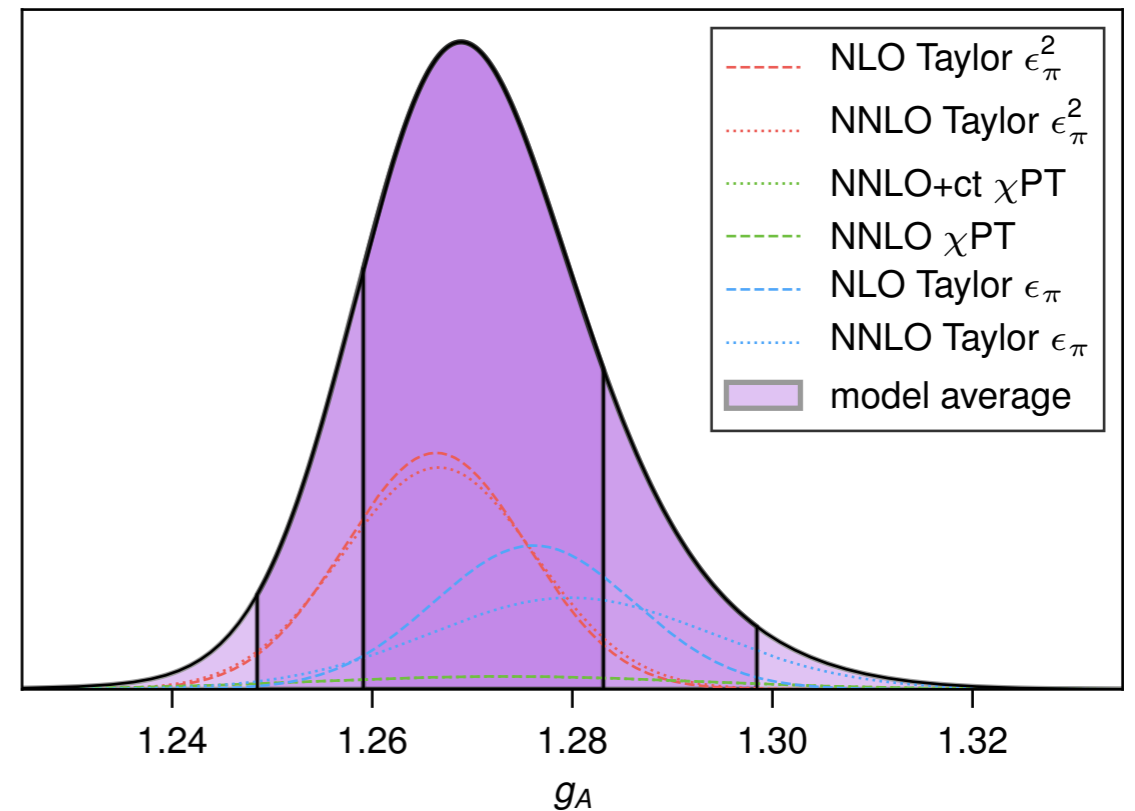
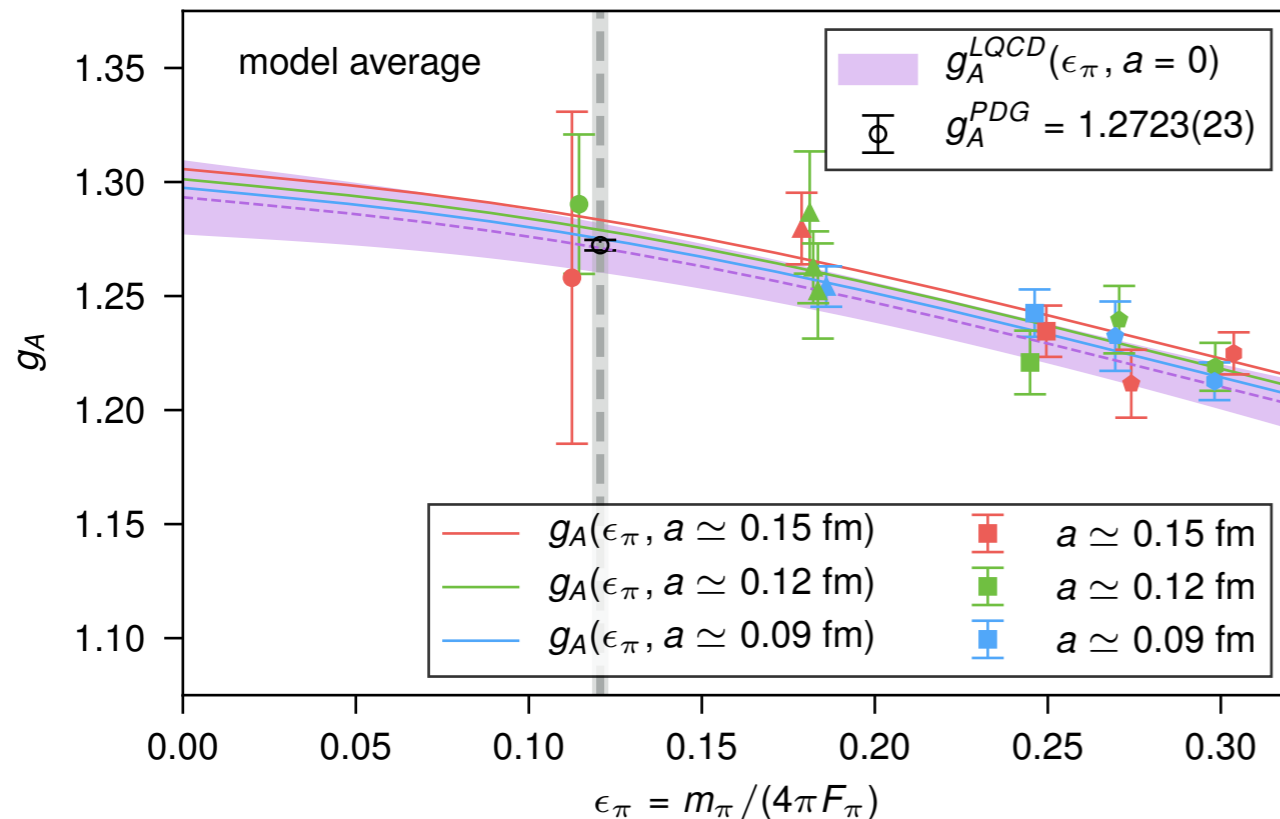
plus a few others



$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^x(15)^a(19)^V(04)^I(55)^M$$

$$= 1.2711(126)$$

$$g_A^{\text{UCNA}} = 1.2772(020) \quad \text{experiment factor of 6 more precise}$$



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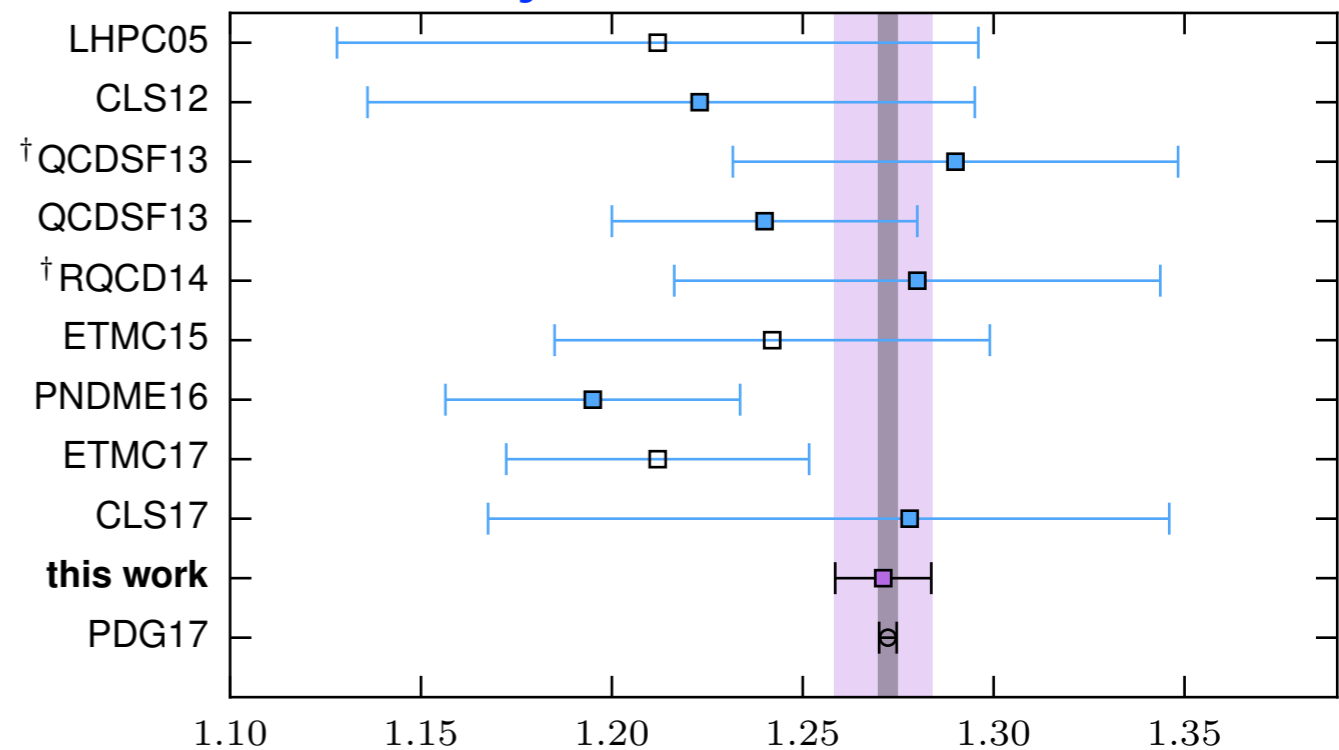
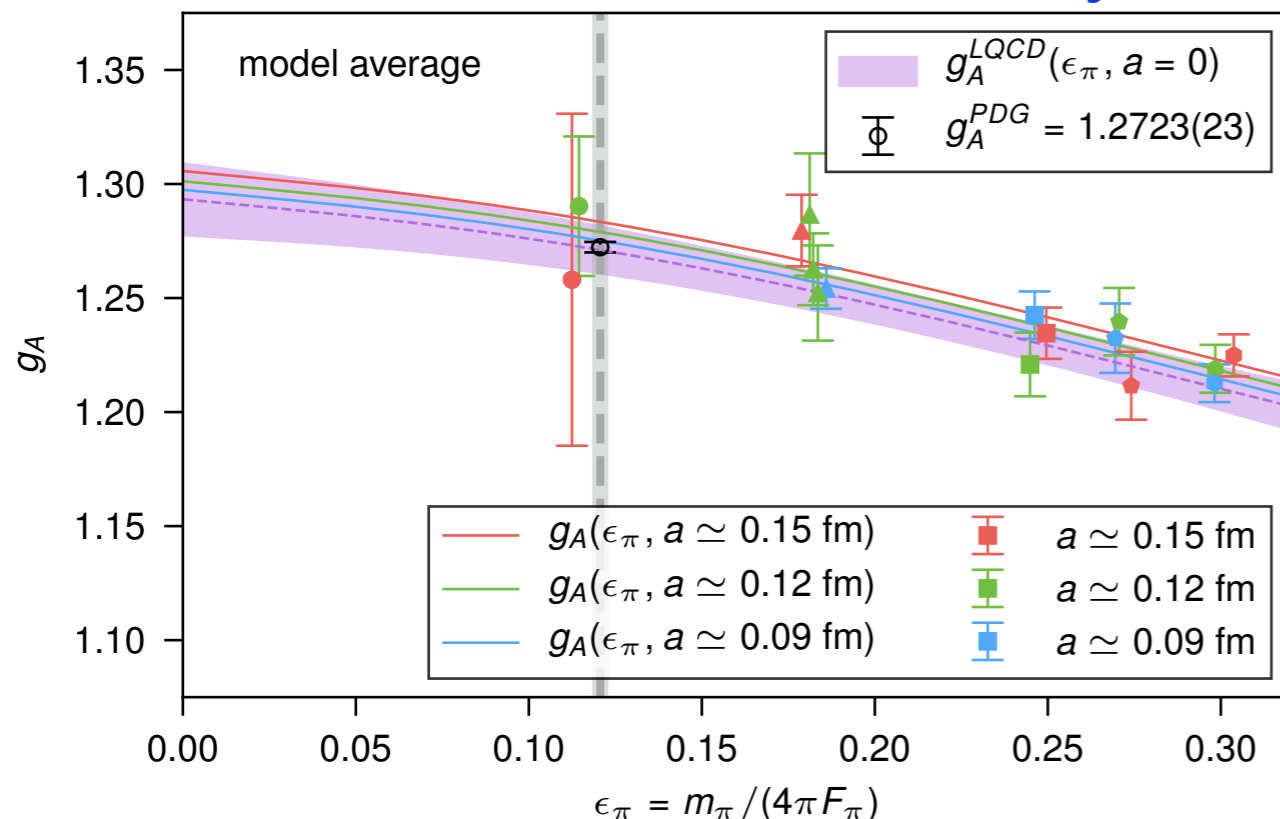
plus a few others



$$g_A^{\text{QCD}} = 1.2711(103)^s(39)^x(15)^a(19)^V(04)^I(55)^M$$

$$= 1.2711(126)$$

Lattice community estimated 2% by 2020 LQCD results



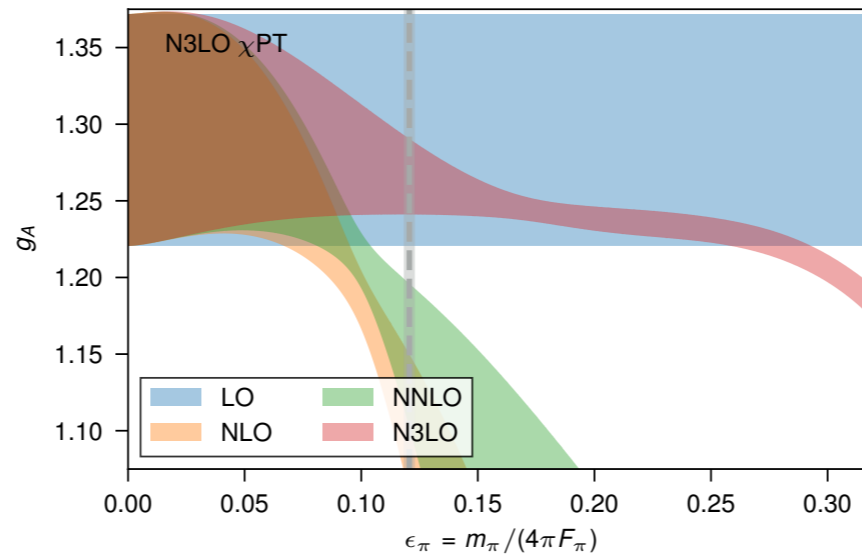
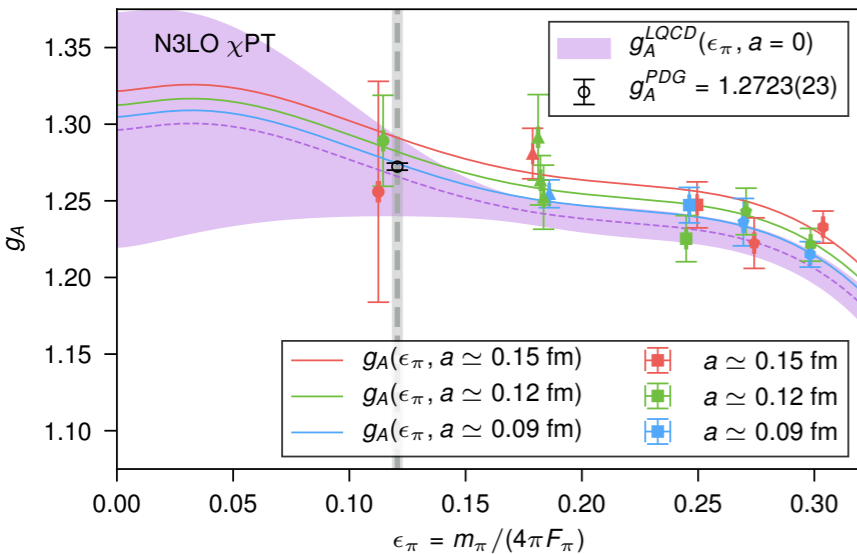
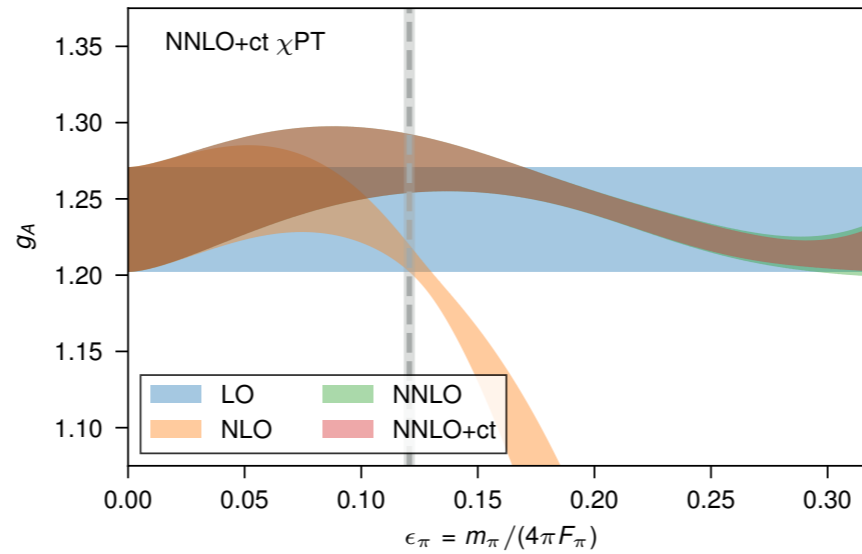
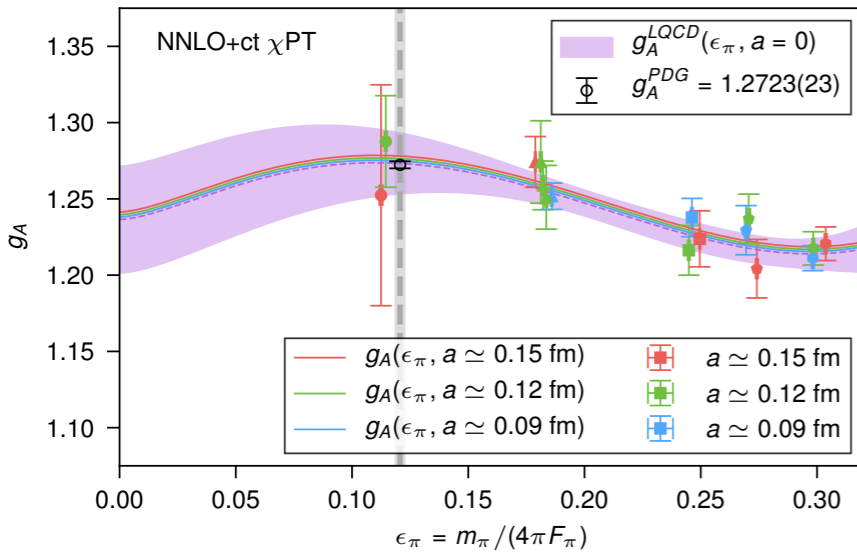
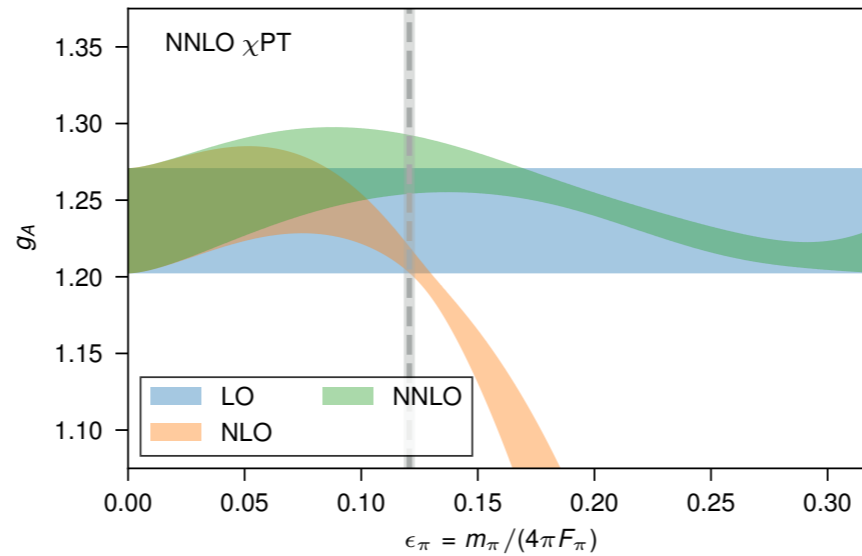
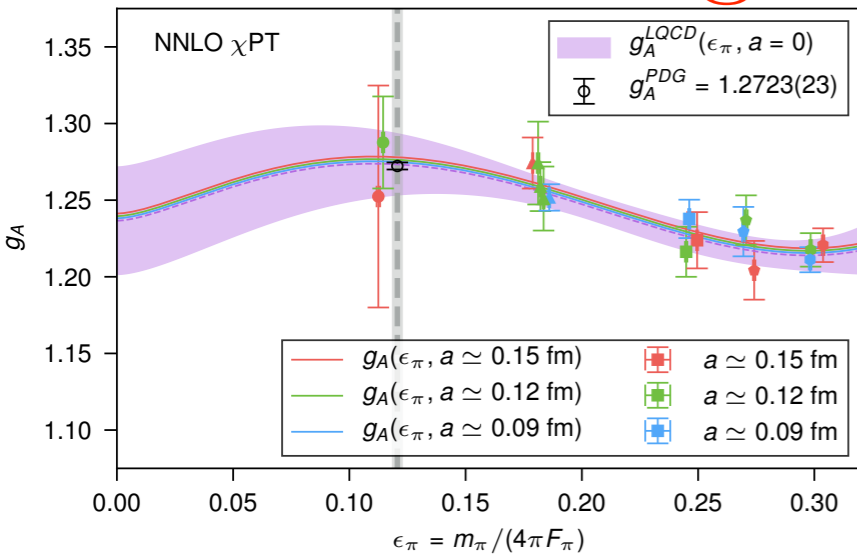
convergence of the chiral expansion...

$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3$$

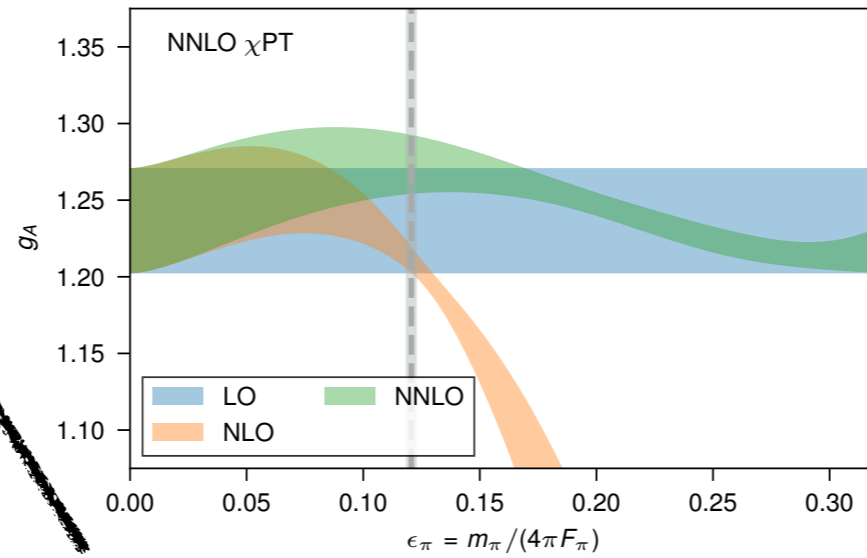
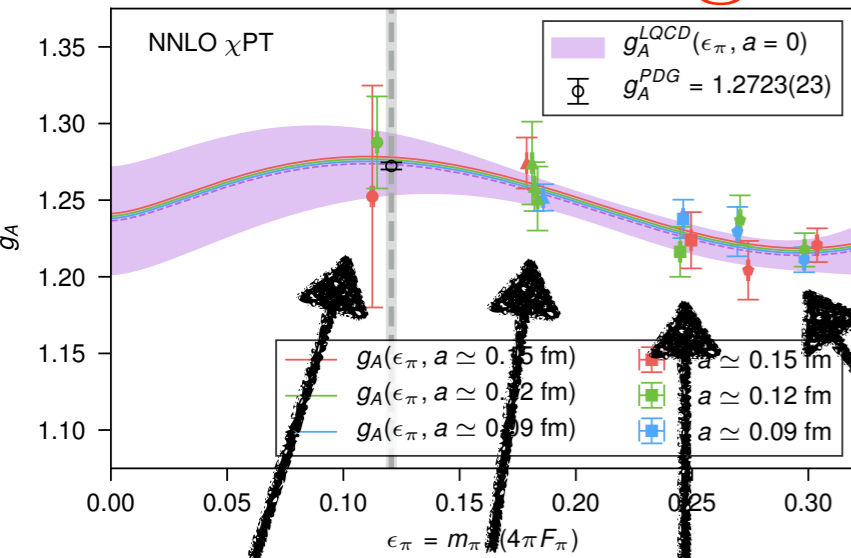
$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + c_4 \epsilon_\pi^4$$

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convergence of the chiral expansion...

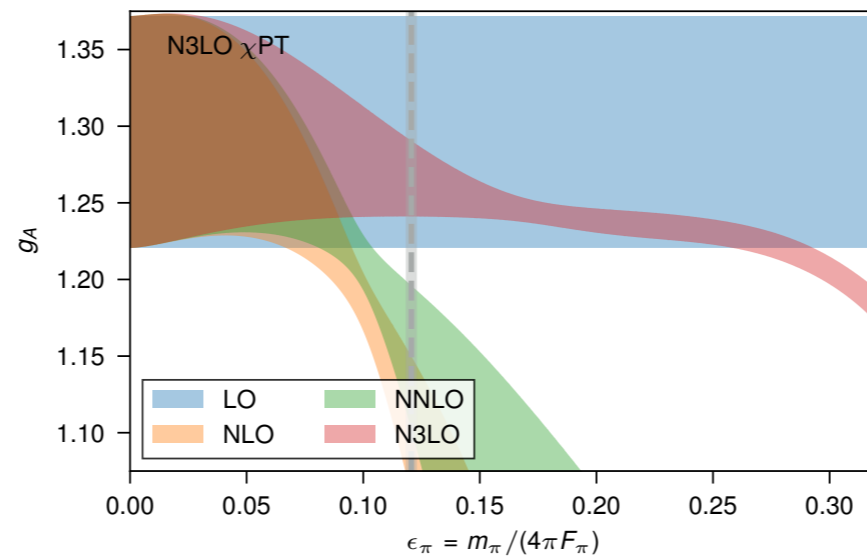
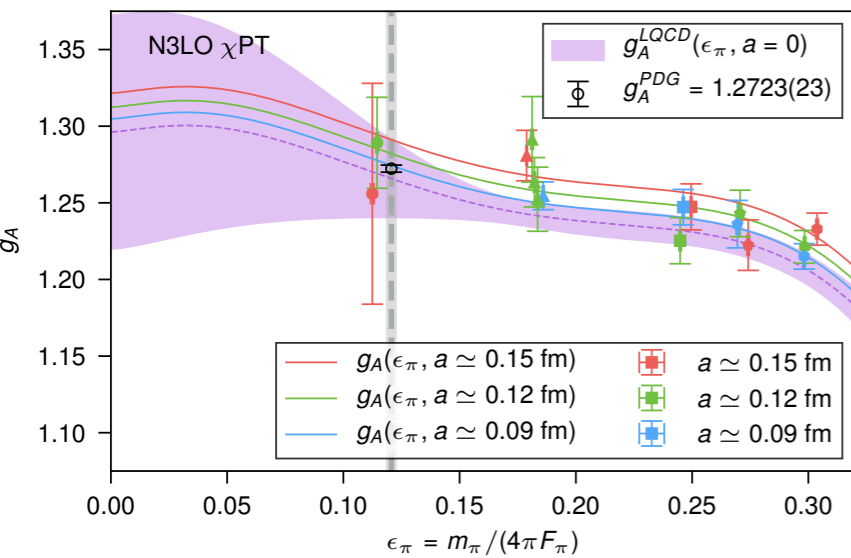


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$$\epsilon_\pi = \frac{m_\pi}{4\pi F_\pi}$$

$m_\pi \sim 130 \text{ MeV}$ 220 310 400

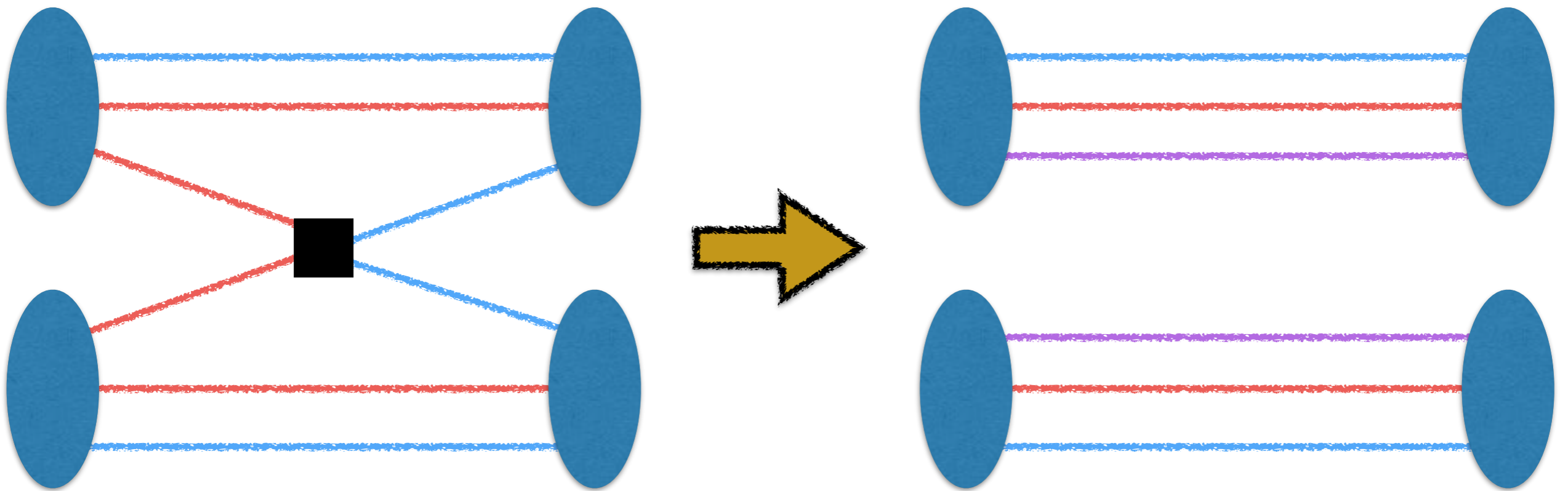
if the single nucleon is not converging, would you trust chiral extrapolations of two or more nucleons?



$$g_A = g_0 - \epsilon_\pi^2 (g_0 + 2g_0^3) \ln(\epsilon_\pi^2) + c_2 \epsilon_\pi^2 + g_0 c_3 \epsilon_\pi^3 + \epsilon_\pi^4 \left[c_4 + \tilde{\gamma}_4 \ln(\epsilon_\pi^2) \right] + \left(\frac{2}{3} g_0 + \frac{37}{12} g_0^3 + 4g_0^5 \right) \ln^2(\epsilon_\pi^2)$$

Inspiration for LQCD calculations of $\Delta I=2$ PNC

- The method we developed to compute g_A is only applicable to quark bi-linear currents $\bar{q}\Gamma q$
- This has inspired us - we believe we know how to generalize this method to 4-quark operators - if successful, it will substantially simplify the calculations



- We will hopefully know soon if this new idea works

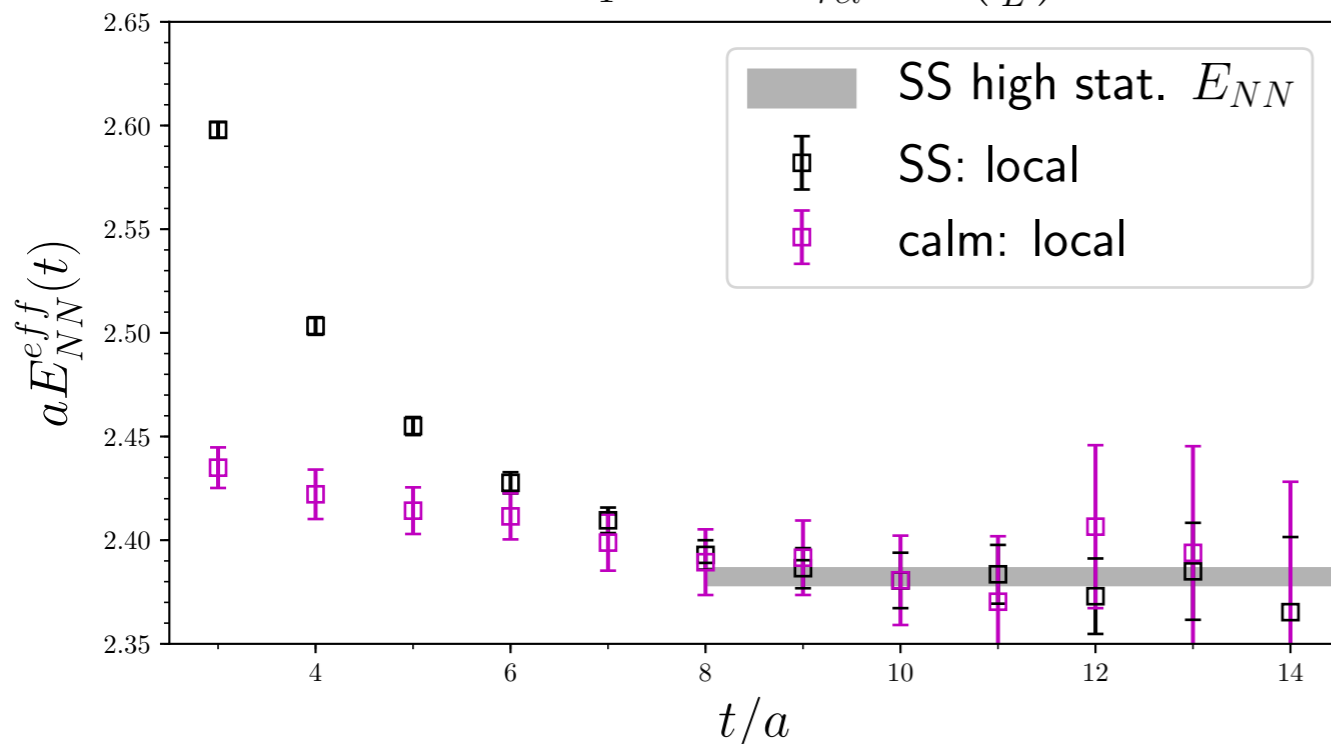
Inspiration for LQCD calculations of $\Delta I=2$ PNC

- Two-nucleon LQCD calculations are the most substantial challenge to making progress in two-nucleon matrix element calculations
- We have cooked up a simple idea that offers the promise of exponentially improving the calculations

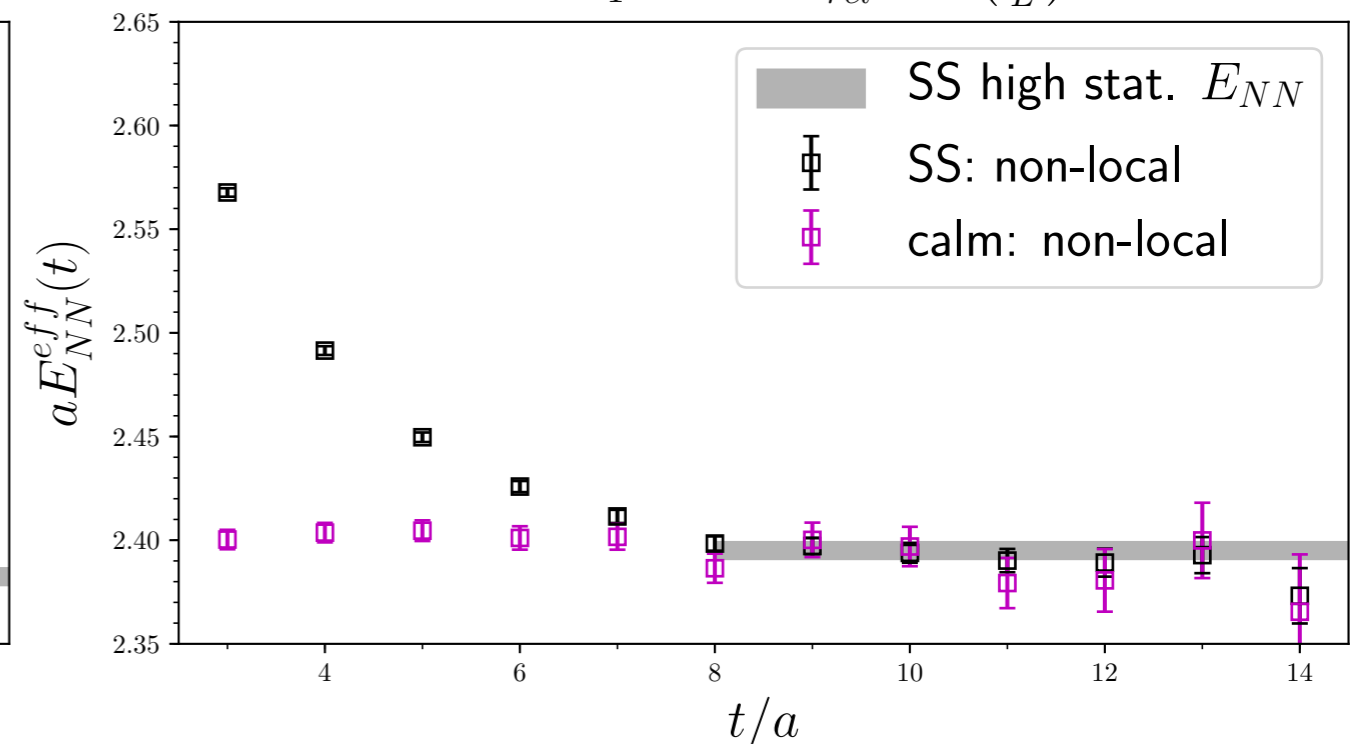
Calm Multi-Baryon Operators

E. Berkowitz, A. Nicholson, C.C. Chang, E. Rinaldi, M.A. Clark, B. Joo, T. Kurth, P. Vranas, AWL
arXiv:1710.05642

$$NN : T_1^+ : {}^3S_1 : p_{rel}^2 = 0 \left(\frac{2\pi}{L}\right)^2$$



$$NN : T_1^+ : {}^3S_1 : p_{rel}^2 = 0 \left(\frac{2\pi}{L}\right)^2$$



Matrix Prony

arXiv.org > hep-lat > arXiv:0903.2990

Search or Art

(Help | Advanced)

High Energy Physics - Lattice

High Statistics Analysis using Anisotropic Clover Lattices: (I) Single Hadron Correlation Functions

Silas R. Beane, William Detmold, Thomas C. Luu, Kostas Orginos, Assumpta Parreno, Martin J. Savage, Aaron Torok, Andre Walker-Loud

- In this work, we applied for the first time, the Matrix Prony method for analyzing two-point correlation functions.
- Idea: construct linear combination of correlation functions to remove excited state contamination

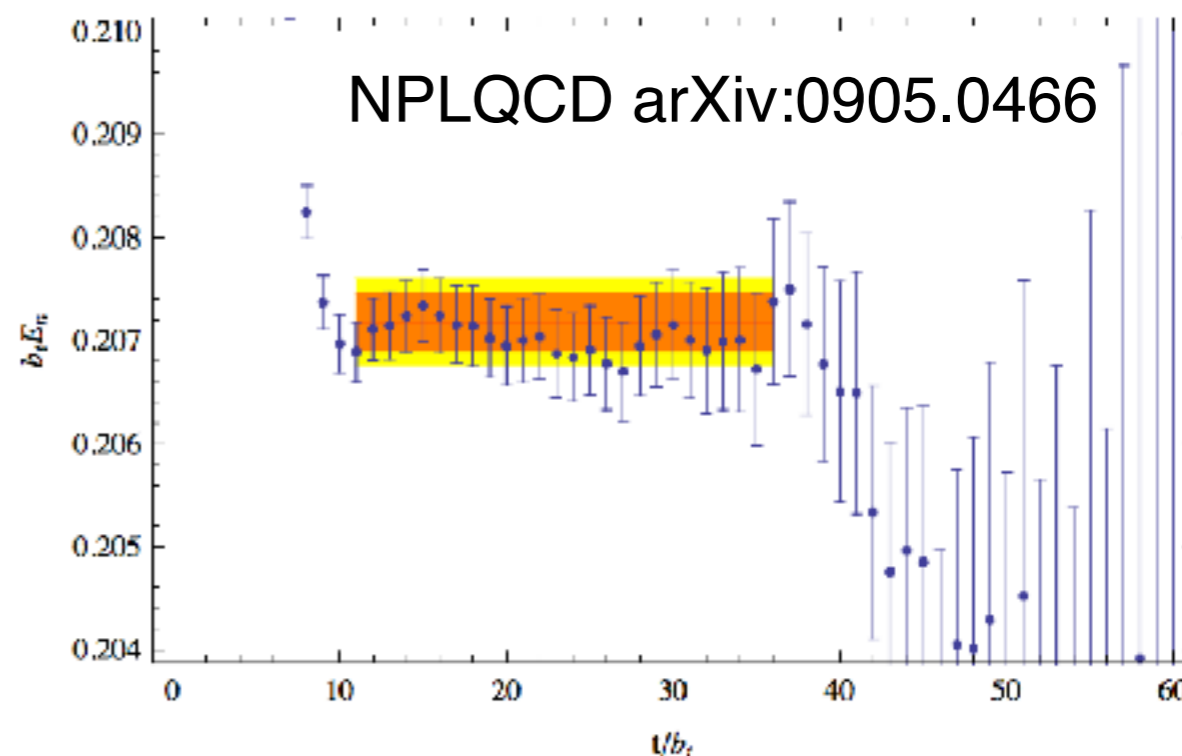
$$C_A(t) = A_0 e^{-E_0 t} + A_1 e^{-E_1 t} + \dots$$

$$C_B(t) = B_0 e^{-E_0 t} + B_1 e^{-E_1 t} + \dots$$

$$C_0 = \alpha_0 C_A + \beta_0 C_B$$

$$\alpha_0 A_1 + \beta_0 B_1 = 0$$

Find α_0 and β_0 with black-box method
(Matrix Prony)



Matrix Prony: Improved NN

○ What is the new idea?

- Previously, Matrix Prony has been used to analyze linear combinations of correlation functions in B, BB, BBB, BBBB systems after they were generated
- We realized we could instead use Matrix Prony to form an optimal linear combination of single-nucleon correlation sinks, that could then be inserted into the two(multi)-nucleon contraction code

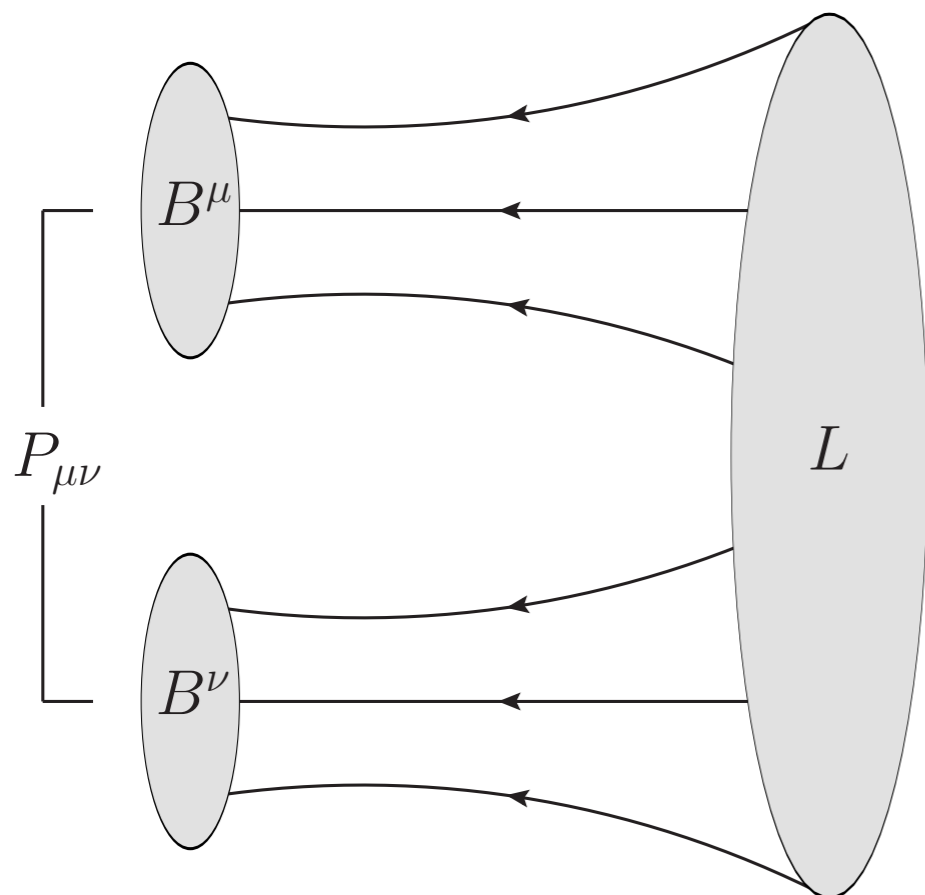
$$B^\mu = \alpha_0 B_{SS}^\mu + \beta_0 B_{PS}^\mu$$

- This is a “poor man’s” version of the more sophisticated variational methods used by Hadspec, Bulava et. al., etc.
- The numerical cost is less than the standard method in which both SS and PS two-nucleon correlation functions are generated
 - only single set of contractions/FFT required

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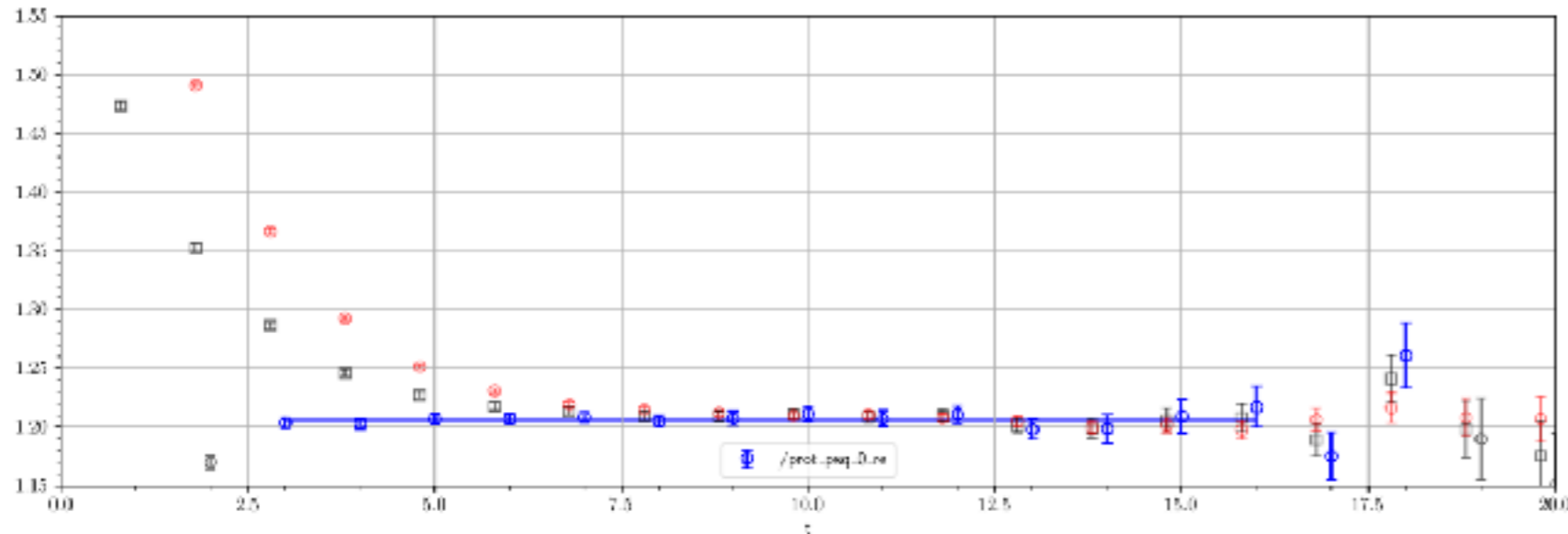
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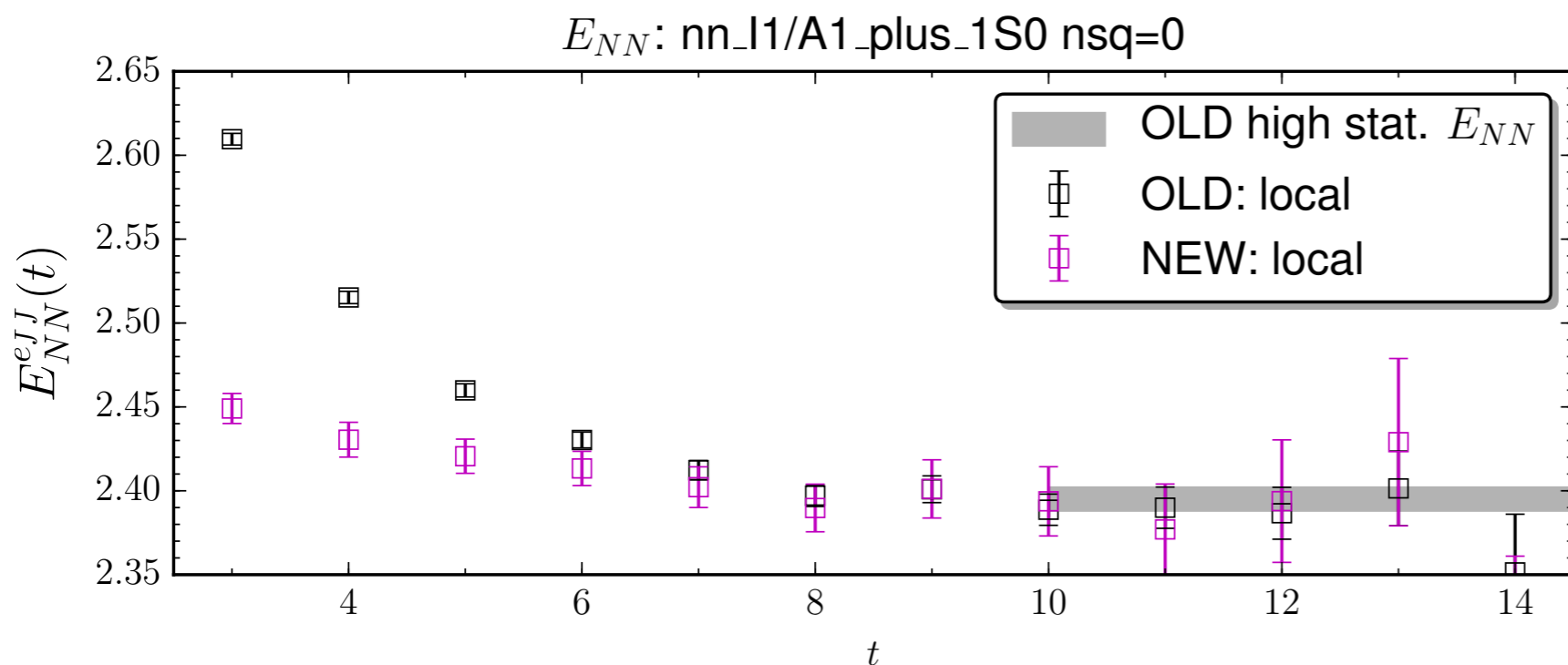
Matrix Prony: Improved NN

How well does it work?

- Test the idea out on $m_\pi \sim 800$ MeV data - iso-clover WM/JLab cfgs



single nucleon pulled in
6 time slices



used with local NN
interpolating fields

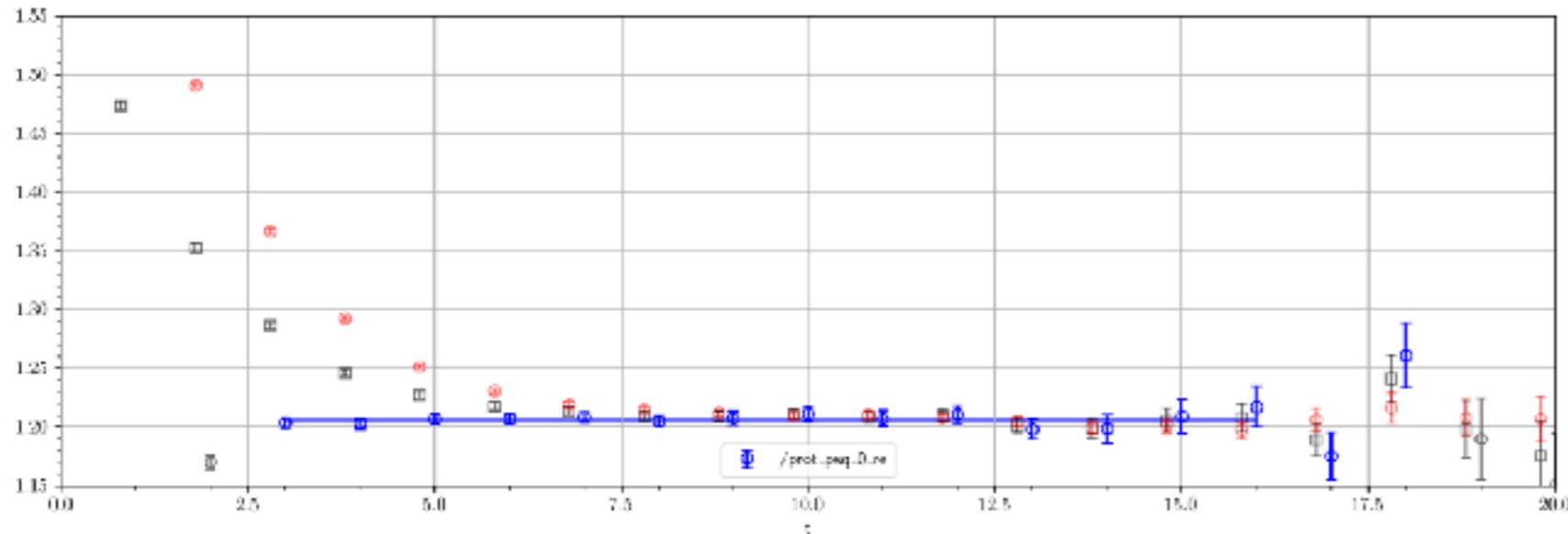
$$N^\dagger(0, x_i) N^\dagger(0, x_i)$$

we observe a reduction
in the excited state
contamination

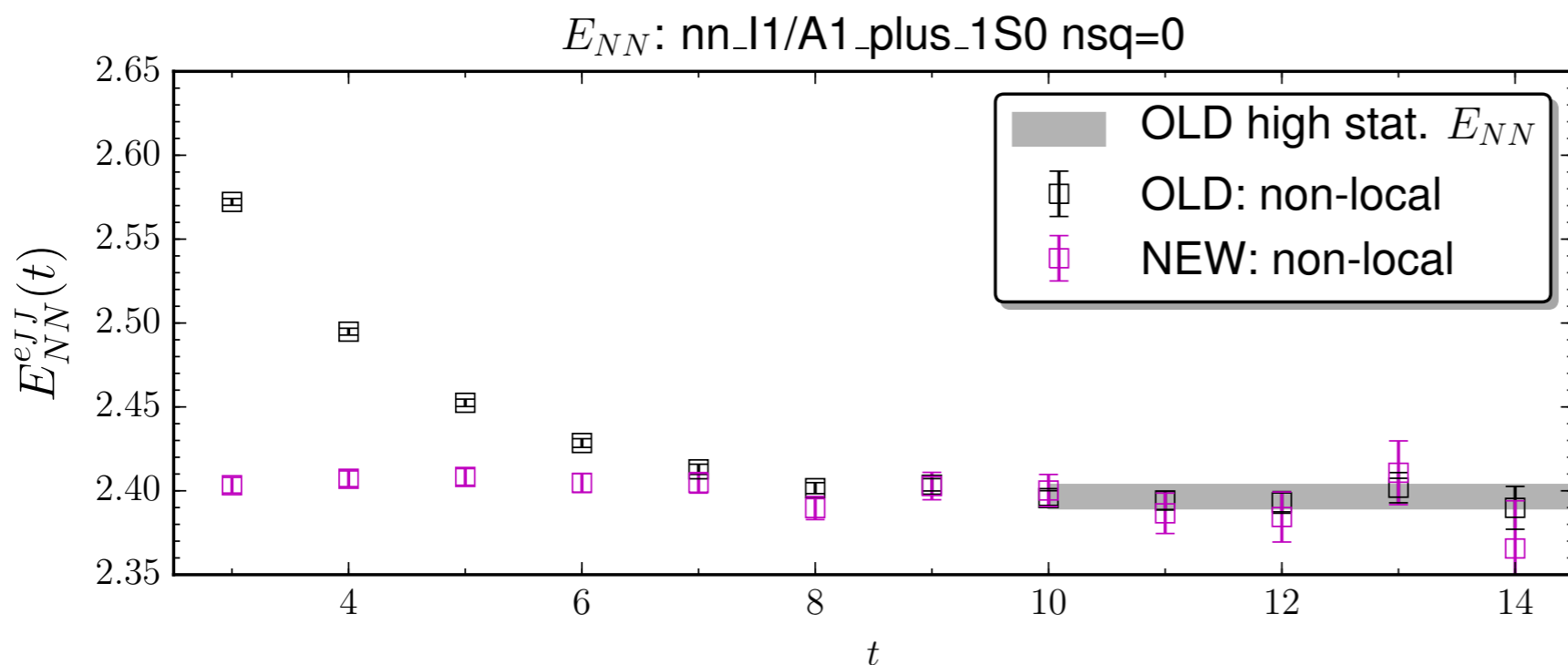
Matrix Prony: Improved NN

How well does it work?

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single nucleon pulled in
6 time slices



used with non-local NN
interpolating fields

$$N^\dagger(0, x_i) N^\dagger(0, x_i + \Delta)$$

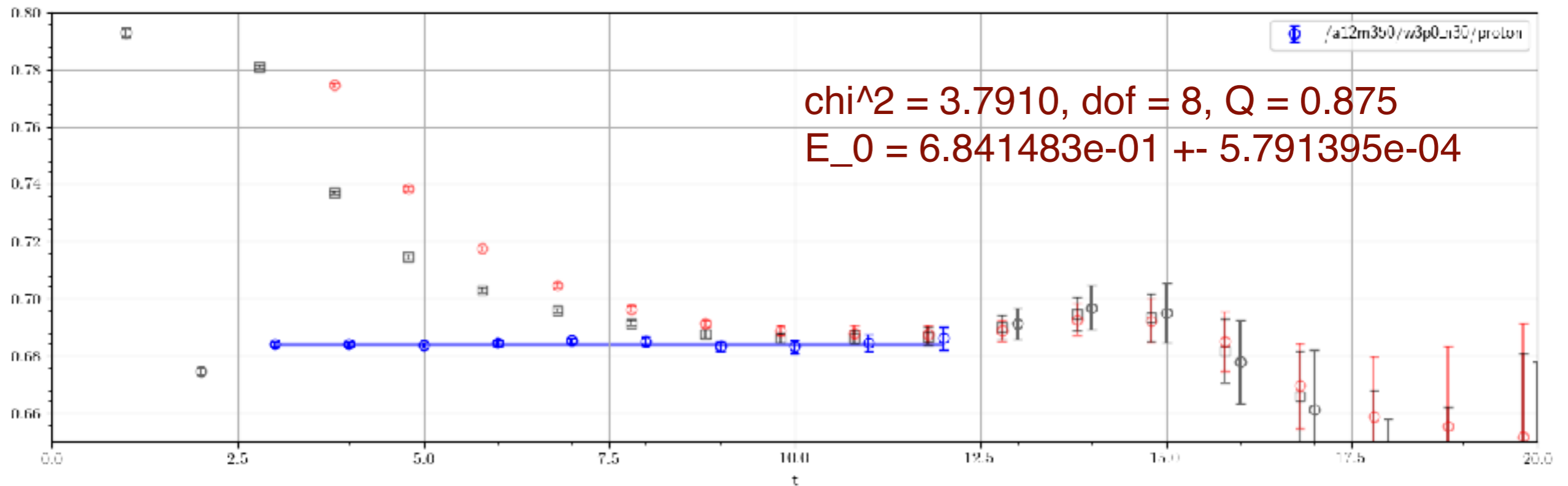
we observe a more
significant reduction in
the excited state
contamination

Matrix Prony: Improved NN

○ How well does it work with an interesting pion mass?

○ Application with MDWF on gradient-flowed HISQ

$m_\pi \sim 350$ MeV, $N_{\text{src}} = 20,000$ (2 srcs/cfg, 10K cfgs)



$am_N = 0.68417(58) \sim 0.085\%$, g.s. pulled in 8 time slices ~ 0.96 fm

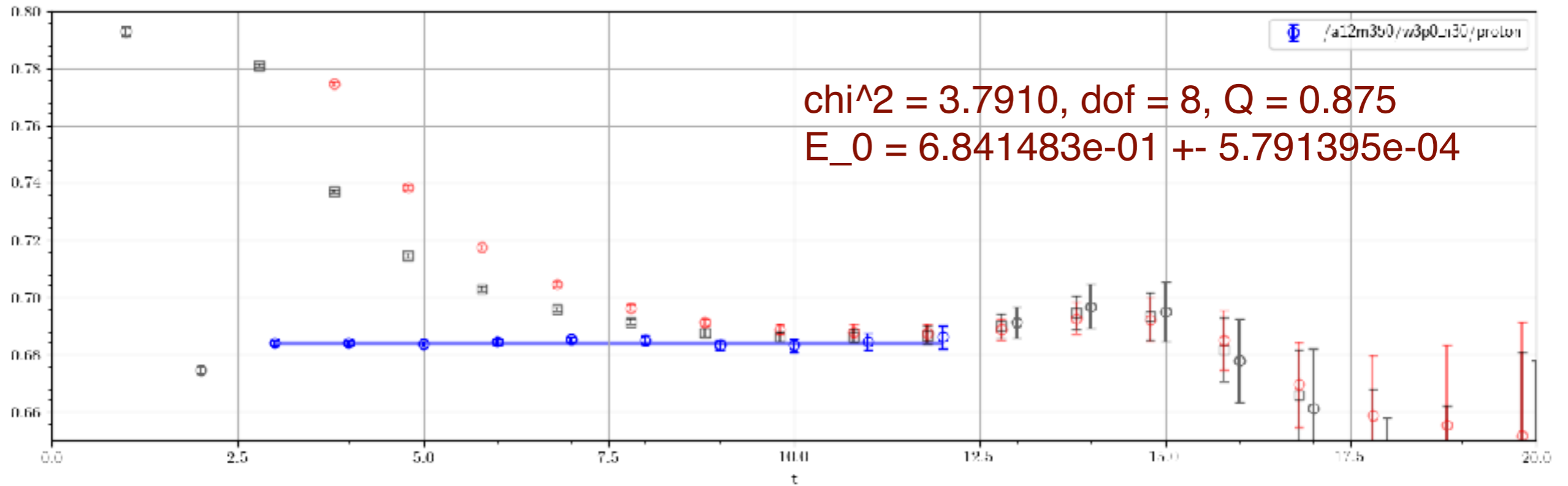
NOTE: this is one of the ensembles used for our gA calculation - this demonstrates from the numerical data (no fits) that there are only 2 states meaningfully contributing to the correlation function all the way down to $t = 3$ (0.36 fm)

Matrix Prony: Improved NN

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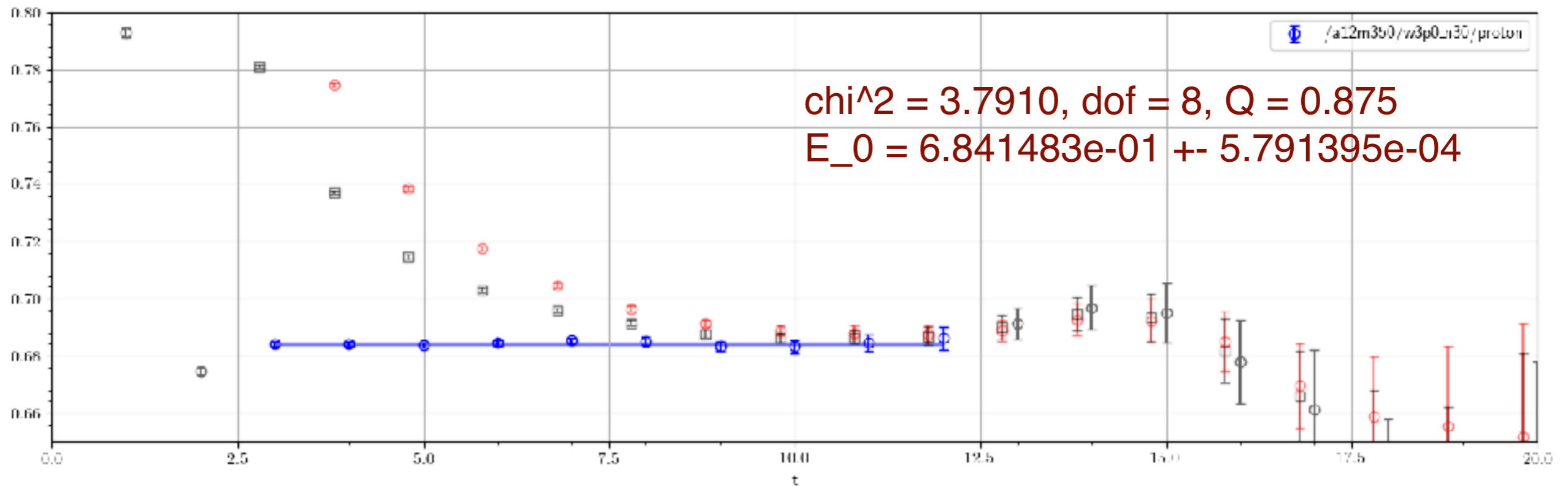
NPLQCD [arXiv:0903.2990] $m_\pi \sim 400$ MeV to achieve 0.16% precision with
 $a_t m_N = 0.20693(33)$ aniso-clover, needed 300K srcs

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 $a_t m_N = 0.20693(33)$ aniso-clover, needed 300K srcs

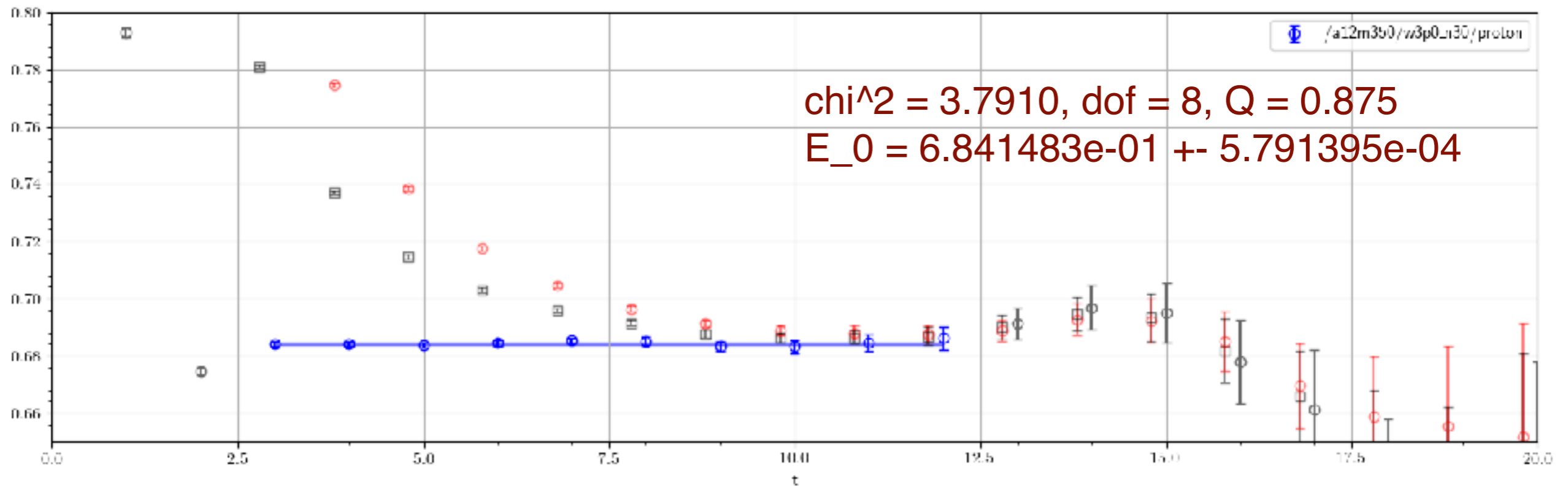
We are very optimistic this idea will allow for a good LQCD calculations of NN at this interesting pion mass!

Matrix Prony: Improved NN

○ How well does it work with an interesting pion mass?

○ Application with MDWF on gradient-flowed HISQ

$m_\pi \sim 350$ MeV, $N_{\text{src}} = 20,000$ (2 srcs/cfg, 10K cfgs)



$am_N = 0.68417(58) \sim 0.085\%$, g.s. pulled in 8 time slices ~ 0.96 fm

These calculations are so expensive - it is imperative to extract as much information from them as possible, which is optimally achieved early in Euclidean time where we have a clear theoretical understanding of the correlation functions and they are clean - before the noise sets in

Summary

- Applications of Lattice QCD to Nuclear Physics is Difficult
- With current generation of computers - we have finally made our first “nuclear physics” prediction of a benchmark quantity g_A - with a clear path to sub-percent precision
- The next generation of computers (appearing now at ORNL and LLNL) - plus new ideas - will enable us to begin computing *real* nuclear physics quantities ($A=2$ (3? 4?)), including $\Delta I=2$ Hadronic Parity Nonconservation
- What do we need?
 - people! (students, postdocs, ...) **we are personnel-power limited**
 - more computing time! **tell your friends to review our proposals positively and support computational NP**

Thank You

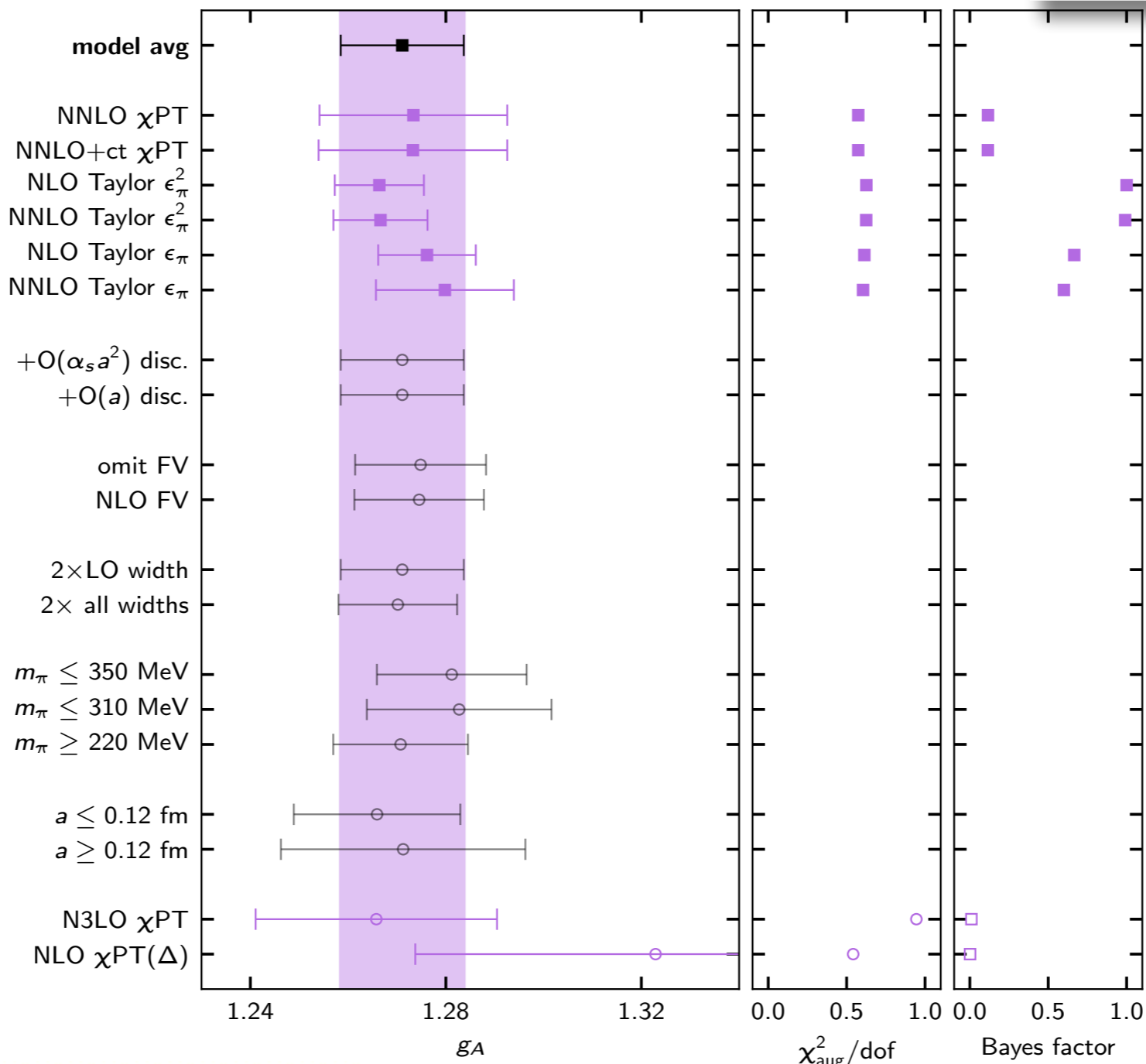
Lattice QCD Team

Glasgow: Chris Bouchard
 INT: Chris Monahan
 JLab: Balint Joo
 Jülich: Evan Berkowitz
 LBL/UCB: David Brantley, Chia Cheng (Jason) Chang, T. Kurth (NERSC), Henry Monge-Camacho, AWL
 LLNL: Pavlos Vranas
 Liverpool: Nicolas Garron
 NVIDIA: Kate Clark
 RIKEN/BNL: Enrico Rinaldi
 UNC: Amy Nicholson
 William and Mary: Kostas Orginos

red = postdoc
 blue = grad student



plus a few others



Lattice QCD Team

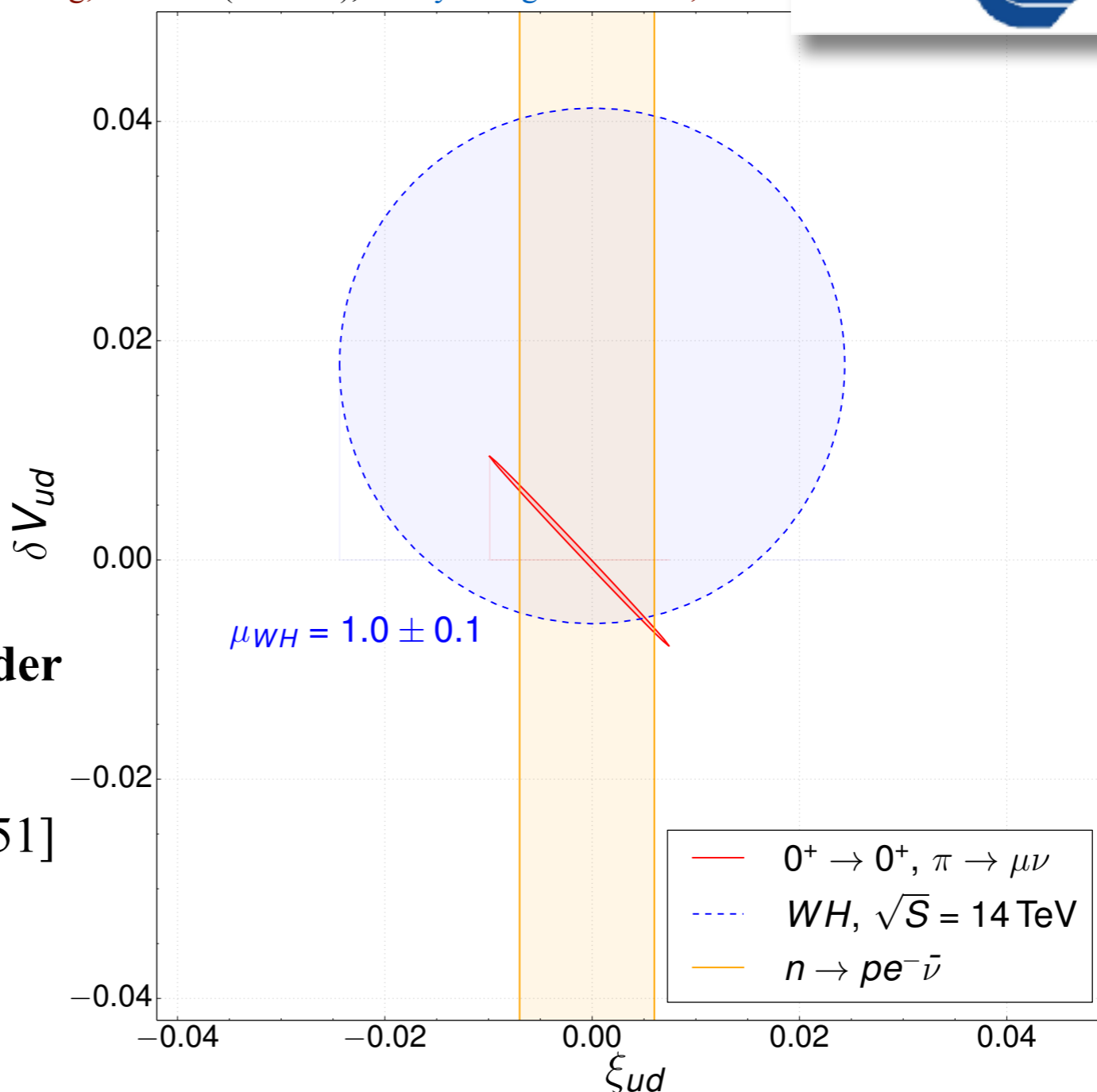
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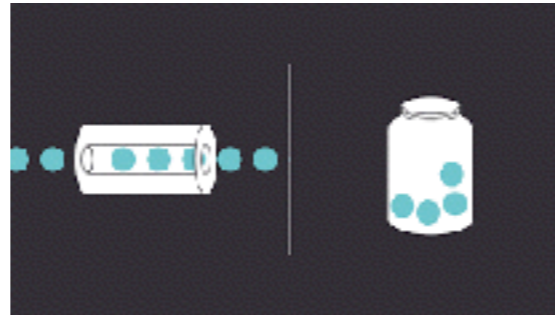
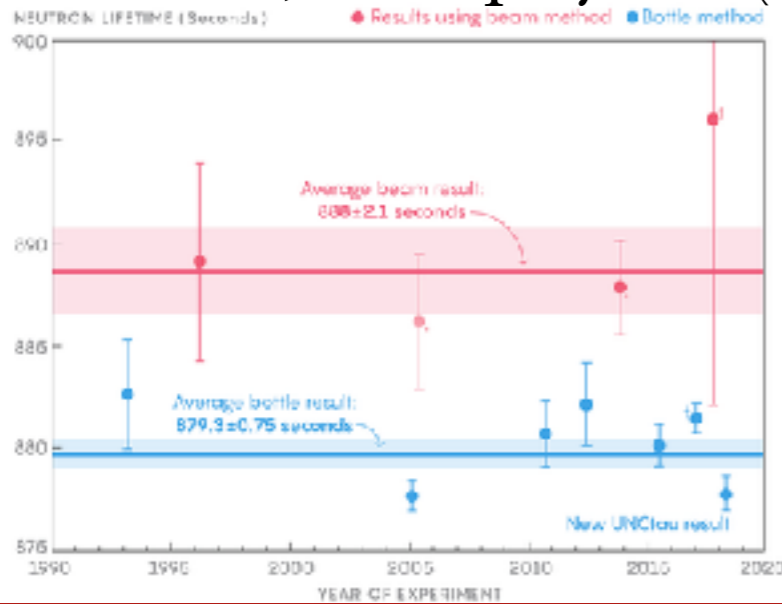
plus a few others

updated figure from
Right-handed charged currents in the era of the Large Hadron Collider
Alioli, Cirigliano, Dekens, de Vries and Mereghetti
JHEP 1705 (2017) [arXiv:1703.04751]



Neutron Lifetime...

- there is a 4-sigma discrepancy: **beam** $\tau_n^{\text{beam}} = 888.0(2.0)s$ and **bottle** $\tau_n^{\text{bottle}} = 879.4(0.6)s$ measurements of the neutron lifetime, new physics (dark matter) or unknown systematic?



Czarnecki, Marciano, Sirlin
arXiv:1802.01804

$$\tau_n = \frac{5172.0(1.0) s}{1 + 3g_A^2}$$

arXiv.org > hep-ph > arXiv:1802.01804

Search on
(Help | Adv)

High Energy Physics – Phenomenology

The Neutron Lifetime and Axial Coupling Connection

Andrzej Czarnecki, William J. Marciano, Alberto Sirlin

(Submitted on 6 Feb 2018 (v1), last revised 22 Feb 2018 (this version, v2))

Experimental studies of neutron decay, $n \rightarrow pe\bar{\nu}$, exhibit two anomalies. The first is a $8.6(2.1)s$, roughly 4σ difference between the average beam measured neutron lifetime, $\tau_n^{\text{beam}} = 888.0(2.0)s$, and the more precise average trapped ultra cold neutron determination, $\tau_n^{\text{trap}} = 879.4(6)s$. The second is a 5σ difference between the pre2002 average axial coupling, g_A , as measured in neutron decay asymmetries $g_A^{\text{pre2002}} = 1.2637(21)$, and the more recent, post2002, average $g_A^{\text{post2002}} = 1.2755(11)$, where, following the UCNA collaboration division, experiments are classified by the date of their most recent result. In this study, we correlate those τ_n and g_A values using a (slightly) updated relation $\tau_n(1 + 3g_A^2) = 5172.0(1.1)s$. Consistency with that relation and better precision suggest $\tau_n^{\text{favored}} = 879.4(6)s$ and $g_A^{\text{favored}} = 1.2755(11)$ as preferred values for those parameters. Comparisons of g_A^{favored} with recent lattice QCD and muonic hydrogen capture results are made. A general constraint on exotic neutron decay branching ratios is discussed and applied to a recently proposed solution to the neutron lifetime puzzle.