Origin of the (Un?)bearable Smallness of f_π

(or h^1_π or $\Lambda_1^{{}^3S_1 - {}^3P_1}$)

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[based on SG, Haxton, & Holstein, 1704.02617]

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Perspective

PNC at low energies arises
from H_{str} x H_{weak}
ergo one-pion exchange can mediate PNC

The size of "f_π" quantifies the role of this long-range (cf. to $1/M_W$) component to the parity-violating force between nucleons

Since $m_\pi >> m_\rho$, this mechanism seemingly ought dominate the PNC (isovector) NN interaction at very low energies

Today we consider why this is not so.

Analysis Framework

In a theory of meson-mediated forces between nucleons

$$\mathcal{H}_{\rm st} \supset i g_{\pi NN} \bar{N} \gamma_5 \boldsymbol{\tau} \cdot \boldsymbol{\pi} N$$

"f_π"

$$\mathcal{H}_{\rm wk} \supset i \frac{h^1_\pi}{\sqrt{2}} \bar{N} (\boldsymbol{\tau} \times \boldsymbol{\pi})_z N$$

with estimates of h^1_π (in units of 10^{-7})

0 - 11 (reasonable range)

4.6 (best value) ★

[DDH, 1980]

At extremely low energies this description can be transcribed into contact interactions (Danilov partial wave analysis ⟷ pionless EFT) [Zhu et al., 2005...]

Analysis Framework

Enter Danilov parameters as per DDH "best values"

$$\Lambda_0^{{}^1S_0-{}^3P_0} = -g_\rho(2+\chi_\rho)h_\rho^0 - g_\omega(2+\chi_\omega)h_\omega^0 \qquad \text{DDH}\ \Lambda_0^{{}^1S_0-{}^3P_0} = 210,$$

$$\Lambda_0^{{}^3S_1-{}^1P_1} = -3g_\rho\chi_\rho h_\rho^0 + g_\omega\chi_\omega h_\omega^0 \qquad \text{DDH}\ \Lambda_0^{{}^3S_1-{}^1P_1} = 360,$$

$$\Lambda_1^{{}^1S_0-{}^3P_0} = -g_\rho(2+\chi_\rho)h_\rho^1 - g_\omega(2+\chi_\omega)h_\omega^1 \qquad \text{DDH}\ \Lambda_1^{{}^1S_0-{}^3P_0} = 21,$$

$$\Lambda_1^{{}^3S_1-{}^3P_1} = \sqrt{\tfrac{1}{2}}\, g_{\pi NN}\left(\frac{m_\rho}{m_\pi}\right)^2 h_\pi^1 + g_\rho(h_\rho^1 - h_\rho^{1\prime}) - g_\omega h_\omega^1 \qquad \text{DDH}\ \Lambda_1^{{}^3S_1-{}^3P_1} = 1340,$$

$$\Lambda_2^{{}^1S_0-{}^3P_0} = -g_\rho(2+\chi_\rho)h_\rho^2 \qquad \text{DDH}\ \Lambda_2^{{}^1S_0-{}^3P_0} = 160,$$

Note!

But large N_c predicts a very different hierarchy

[Phillips, Samart, Schat, 2015; Schindler, Springer, Vanasse,2016]

$$\Lambda_0^+ \equiv \frac{3}{4}\Lambda_0^{{}^3S_1-{}^1P_1} + \frac{1}{4}\Lambda_0^{{}^1S_0-{}^3P_0} \sim N_c$$

Schindler, 2018

$$\Lambda_2^{{}^1S_0-{}^3P_0} \sim N_c \sin^2\theta_w \sim 1$$

Analysis Framework

The large N_c hierarchy also predicts

$$\Lambda_0^- \equiv \frac{1}{4}\Lambda_0^{^3S_1-^1P_1} - \frac{3}{4}\Lambda_0^{^1S_0-^3P_0} \sim 1/N_c$$

$$\Lambda_1^{^1S_0-^3P_0} \sim \sin^2\theta_w$$

$$\Lambda_1^{^3S_1-^3P_1} \sim \sin^2\theta_w$$

cf. with DDH "best values" & leading N_c fits

$$\left\{\begin{array}{c} {}^{\rm DDH}\Lambda_0^+ \\ {}^{\rm DDH}\Lambda_2^{^1S_0-^3P_0} \end{array}\right\} = \left\{\begin{array}{c} 319 \\ 151 \end{array}\right\} \begin{array}{c} (717) \\ (324) \end{array} \left\{\begin{array}{c} {}^{\rm DDH}\Lambda_0^- \\ {}^{\rm DDH}\Lambda_1^{^1S_0-^3P_0} \\ {}^{\rm DDH}\Lambda_1^{^3S_1-^3P_1} \end{array}\right\} = \left\{\begin{array}{c} -70 \\ 21 \\ 1340 \end{array}\right\}$$

glaringly off…

explanation must lie in h^1_π itself!

Anatomy of ΔS=0 H_{wk}

In the Standard Model

$$SU(2)_L \times U(1)$$

$$\mathcal{H}^{\Delta S=0} = \frac{G_F}{\sqrt{2}}[\cos^2\theta_c J_W^{0\,\dagger} J_W^0 + \sin^2\theta_c J_W^{1\,\dagger} J_W^1$$
$$+ J_Z^{0\,\dagger} J_Z^0 + J_Z^{1\,\dagger} J_Z^1 + J_Z^{0\,\dagger} J_Z^1 + J_Z^{1\,\dagger} J_Z^0]$$

$J_W^{|\Delta S|}$

$J_Z^{|\Delta I|}$

ΔI=0,2

ΔI=1

N.B. $\tan^2\theta_c \sim 0.04$

Suggests neutral current dominates ΔI=1

Anatomy of $\Delta S=0$ H_{wk}

Assuming u,d quark operators and $\theta_c=0$

At the W scale [Dai et al., 1991; DDH, 1980]

$$H^{pv} = H^{pv}_{\Delta I=0} + H^{pv}_{\Delta I=1} + H^{pv}_{\Delta I=2}$$

$$H^{\rm pv}_{\Delta I=0} = -\frac{G_F}{2\sqrt{2}}(1-\frac{2}{3}\sin^2\theta_w)(\bar{q}\gamma_\mu\sigma^a q)(\bar{q}\gamma^\mu\gamma_5\sigma^a q)$$

$$H^{\rm pv}_{\Delta I=1} = \frac{G_F\sin^2\theta_w}{3\sqrt{2}}(\bar{q}\gamma_\mu q)(\bar{q}\gamma^\mu\gamma_5\sigma^3 q)$$ (neutral curr. only)

$$H^{\rm pv}_{\Delta I=2} = \frac{G_F\sin^2\theta_w}{\sqrt{2}}[(\bar{q}\gamma_\mu\sigma^3 q)(\bar{q}\gamma^\mu\gamma_5\sigma^3 q) - \frac{1}{3}(\bar{q}\gamma_\mu\sigma^a q)(\bar{q}\gamma^\mu\gamma_5\sigma^a q)]$$

N.B. $(\sin^2\theta_w)/3 \sim 0.08$ (!)

Anatomy of h^1_π

Must include QCD evolution effects to understand the strength of the various operators at low E

For the isovector part [Dai et al., 1991]

$$H^{\rm pv}_{\Delta I=1} = \frac{G_F \sin^2\theta_w}{3\sqrt{2}} \sum_i C_i(\mu) O_i^\lambda(\mu)$$

$$\theta_1^W = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V + (\bar{c}c)_V + (\bar{b}b)_V]_{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]_{\beta\beta},$$

$$\theta_2^W = [(\bar{u}u)_V + (\bar{d}d)_V + (\bar{s}s)_V + (\bar{c}c)_V + (\bar{b}b)_V]_{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]_{\beta\alpha},$$

$$\theta_3^W = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A + (\bar{c}c)_A + (\bar{b}b)_A]_{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V]_{\beta\beta},$$

$$\theta_4^W = [(\bar{u}u)_A + (\bar{d}d)_A + (\bar{s}s)_A + (\bar{c}c)_A + (\bar{b}b)_A]_{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V]_{\beta\alpha},$$

$$\theta_5^W = [(\bar{s}s)_V - (\bar{c}c)_V + (\bar{b}b)_V]_{\alpha\alpha}[(\bar{u}u)_A - (\bar{d}d)_A]_{\beta\beta},$$

$$\theta_6^W = [(\bar{s}s)_V - (\bar{c}c)_V + (\bar{b}b)_V]_{\alpha\beta}[(\bar{u}u)_A - (\bar{d}d)_A]_{\beta\alpha},$$

$$\theta_7^W = [(\bar{s}s)_A - (\bar{c}c)_A + (\bar{b}b)_A]_{\alpha\alpha}[(\bar{u}u)_V - (\bar{d}d)_V]_{\beta\beta},$$

$$\theta_8^W = [(\bar{s}s)_A - (\bar{c}c)_A + (\bar{b}b)_A]_{\alpha\beta}[(\bar{u}u)_V - (\bar{d}d)_V]_{\beta\alpha},$$

from QCD!

Anatomy of h^1_π

Not suppressed!

$$C_1(M_W)=1\,,\quad C_5(M_W)=C_7(M_W)=3\left(\frac{1}{2s_W^2}-1\right) \sim 3.47$$

$$C_2(M_W)=C_3(M_W)=C_4(M_W)=C_6(M_W)=C_8(M_W)=0\,.$$

Running below the charm scale yields

[Dai et al., 1991]

$$C(\Lambda)=\begin{pmatrix}1.24\\0.45\\0.46\\-1.55\\5.75\\-3.64\\5.75\\-3.64\end{pmatrix}$$

But numerically incomplete (θ_c=0)....

presumably suppressed by light hadron strangeness content

Numerical Estimates of h^1_π

Survey of existing results

Impact of operators of (V-A)x(V-A) form can be assessed using PCAC techniques and hyperon decay data assuming $SU(3)_f$ symmetry

This yields the numerical value $g_\pi = 0.376 \times 10^{-7}$

The anatomy of the DDH estimate is $(0.5 + 11.5)g_\pi$

But including charm effects & QCD running yields $4g_\pi$

[Dubovik & Zenkin, 1986]

Skyrme model assessments also exist…

Summary

- HPNC can in principle be mediated by one-pion exchange, but the DDH analysis fails to assess its strength, cf. 1340 & 270ε for a loose idea
- Possible disappointment may be transmuted by theoretical understanding, in that its smallness arises from its appearance as a suppressed term in a large N_c analysis
- Dynamical calculations of h^1_π vary particularly in their assessment of the effect of strange quark contributions, though “better” calculations find smaller results
- It is important to test the new LEC hierarchy of π-less EFT however we can

Backup Slides

Analysis Framework

$$
\begin{aligned}
V_{\mathrm{LO}}^{\mathrm{PNC}}(\boldsymbol{r}) = \Lambda_0^{{}^1\mathrm{S}_0-{}^3\mathrm{P}_0} & \left(\frac{1}{i} \frac{\overleftrightarrow{\boldsymbol{\nabla}}_A}{2m_N} \frac{\delta^3(\boldsymbol{r})}{m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) - \frac{1}{i} \frac{\overleftrightarrow{\boldsymbol{\nabla}}_S}{2m_N} \frac{\delta^3(\boldsymbol{r})}{m_\rho^2} \cdot i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \right) \\
+ \Lambda_0^{{}^3\mathrm{S}_1-{}^1\mathrm{P}_1} & \left(\frac{1}{i} \frac{\overleftrightarrow{\boldsymbol{\nabla}}_A}{2m_N} \frac{\delta^3(\boldsymbol{r})}{m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) + \frac{1}{i} \frac{\overleftrightarrow{\boldsymbol{\nabla}}_S}{2m_N} \frac{\delta^3(\boldsymbol{r})}{m_\rho^2} \cdot i(\boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2) \right) \\
+ \Lambda_1^{{}^1\mathrm{S}_0-{}^3\mathrm{P}_0} & \left(\frac{1}{i} \frac{\overleftrightarrow{\boldsymbol{\nabla}}_A}{2m_N} \frac{\delta^3(\boldsymbol{r})}{m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)(\tau_{1z} + \tau_{2z}) \right) \\
+ \Lambda_1^{{}^3\mathrm{S}_1-{}^3\mathrm{P}_1} & \left(\frac{1}{i} \frac{\overleftrightarrow{\boldsymbol{\nabla}}_A}{2m_N} \frac{\delta^3(\boldsymbol{r})}{m_\rho^2} \cdot (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2)(\tau_{1z} - \tau_{2z}) \right) \\
+ \Lambda_2^{{}^1\mathrm{S}_0-{}^3\mathrm{P}_0} & \left(\frac{1}{i} \frac{\overleftrightarrow{\boldsymbol{\nabla}}_A}{2m_N} \frac{\delta^3(\boldsymbol{r})}{m_\rho^2} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2)(\boldsymbol{\tau}_1 \otimes \boldsymbol{\tau}_2)_{20} \right),
\end{aligned}
$$