

Boson Induced Pairing: Horses and Zebras

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CLASSES OF SUPERCONDUCTORS

[1] BCS: conventional metals, C60,
some organics, doped
semiconductors, MgB₂, ...

[2] EXOTIC: copper oxides, heavy
fermion metals, some organics,
(Fe,Ni) oxypnictides ?, ...

CURRENT SITUATION

CLASS [1]--- ELECTRON PAIRS
PAIRING INDUCED BY EL-PH
INTERACTIONS

ALL EXPERIMENTS EXPLAINED

CLASS [2]---ELECTRON PAIRS
MANY THEORIES

BUT NO CONSENSUS ON THE
MECHANISM

OUTLINE

[1] CLASS 1: PREDICTING NEW SUPERCONDUCTORS AND EXPLAINING PROPERTIES.

[2] POSSIBILITIES OF HIGHER TRANSITION TEMPERATURES IN CLASS 1.

[3] CLASS 2: HORSES AND ZEBRAS.

[4] A FEW CONCLUSIONS/COMMENTS.

HOW WELL CAN WE
PREDICT THE EXISTENCE
OF CLASS 1
SUPERCONDUCTORS AND
EXPLAIN THEIR
PROPERTIES?

A FEW EXAMPLES

PREDICTING

SEMIEMPIRICAL

GeTe, SnTe, SrTiO₃

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GeTe, SnTe, SrTiO₃

AB INITIO
high pressure:
sh Si, hcp Si, Ge(?), GaAs,
Nb₃Nb (SEMI-AB INITIO)

PREDICTIONS FOR Si

(Si at high pressures: sh and hcp)

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existence, stability, lattice constants,
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phonon properties, electron-phonon coupling,
superconducting T_c 's and their pressure
dependences

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ONLY INPUTS ARE: ATOMIC NUMBER &
ATOMIC MASS, ESTIMATE OF μ^* , AND
CANDIDATE STRUCTURES

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CANDIDATE STRUCTURES

silicon may be the best understood
superconductor!

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Schematic pictures of alkali and alkaline-earth doped C₆₀ fullerides, (a) face-centered cubic phase observed in alkali fullerides A₃C₆₀ (A $\frac{1}{4}$ K, Rb, or mixture of Na, K, Rb, and Cs), (b) body-centered orthorhombic phase observed in Cs₄C₆₀, Sr₄C₆₀, and Ba₄C₆₀, (c) body-centered cubic phase observed in alkali fullerides A₆C₆₀ (A $\frac{1}{4}$ K, Rb, or Cs) and alkaline-earth fullerides Sr₆C₆₀ and Ba₆C₆₀, and (d) A₁₅ phase observed in Cs₃C₆₀ and Ba₃C₆₀.

[S.Saito, K.Umemoto, S.G.Loiiue, and M.L. Cohen (2004)]

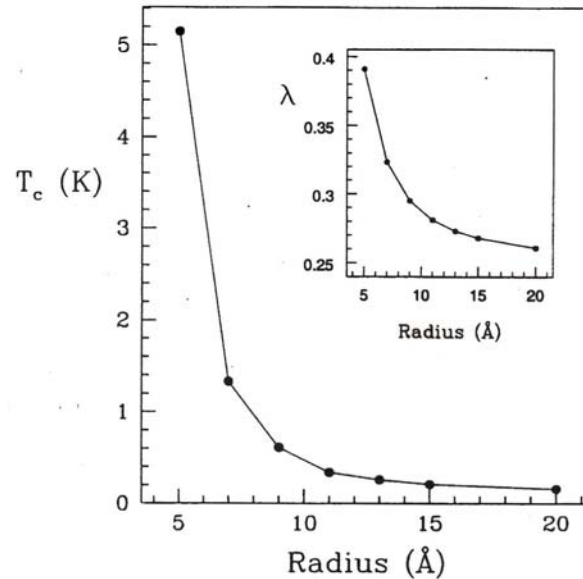
QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

(2008)

Electron-Phonon Coupling and Superconductivity in Carbon Nanotubes

THEORY:

[L. X. Benedict, V. H. Crespi, S. G. Louie and M. L. Cohen, PRB 52, 14935 (1995)]



- Large curvature of small diameter tubes leads to enhanced λ .
- T_c can be in the range of 10-20 degrees Kelvin for diameter $d < 5$ Å.

EXP:

Z.K. TANG *et al*, Science 292, 2462 (2001)

EXPLAINING

“AFTER THE FACT”

(WITH SOME PREDICTIONS OF
PROPERTIES)

Good example: MgB₂

Computational Methods

Atomic numbers of Mg and B



Ab initio pseudopotentials for Mg and B



Density functional theory
(electronic structures of MgB₂)



Frozen-phonon method (phonon structure) ←

Atomic masses
of Mg and B



Electron-phonon interactions
(normal-state specific heat)



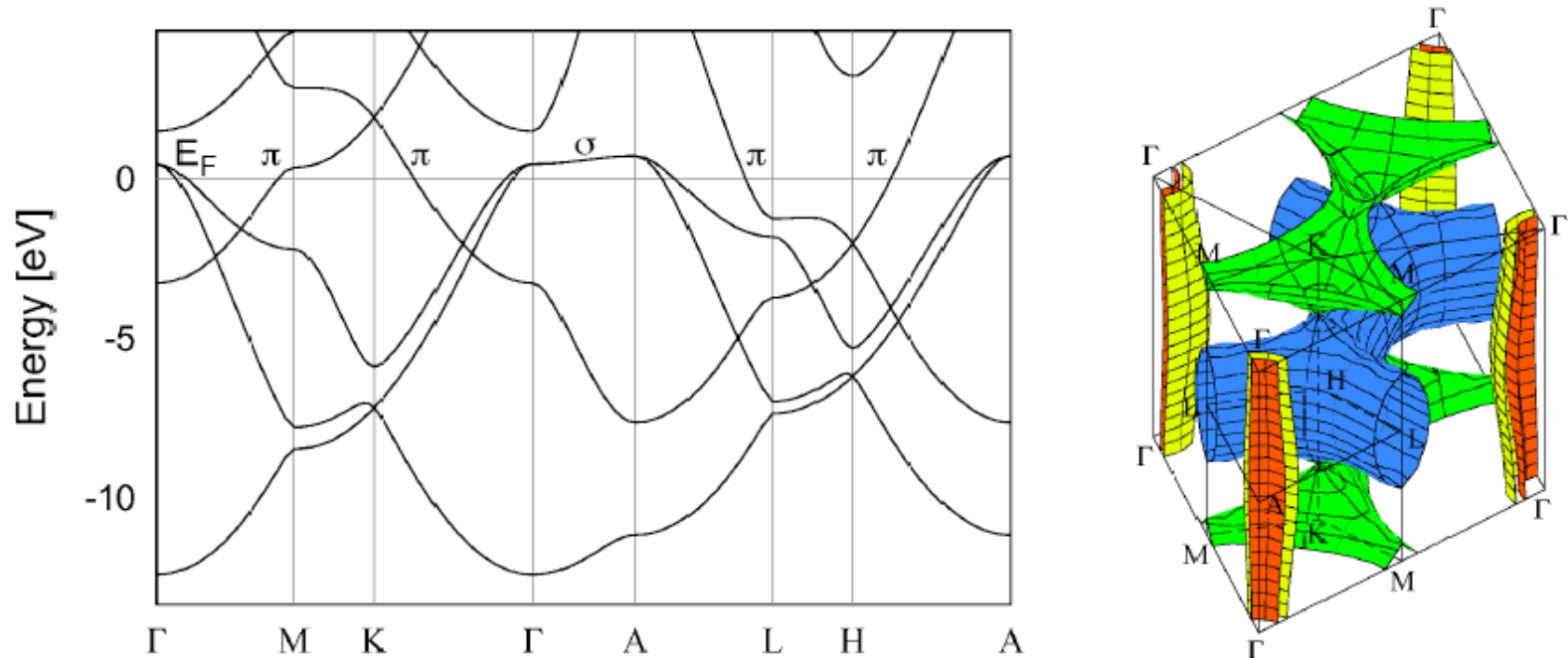
Eliashberg formalism ←

μ^*

(T_c , isotope effect, superconducting gap, specific heat)

Pristine MgB₂: Electronic structures and Fermi surface

- Band structure

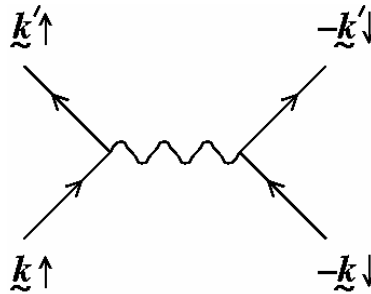


- Fermi surface consists of four sheets:
two from σ -bonding boron $p_{x,y}$ orbitals
and two from π -bonding boron p_z orbitals.

Also: An & Pickett, PRL 2001; Kortus, et al, PRL 2001; Liu, et al, PRL 2001; Kong, et al, PRB 2001; ...

Superconductivity in the Eliashberg Formalism

BCS Theory



Electron pairing
via phonon exchange

Main ingredient: momentum- and frequency-dependent
Eliashberg function

$$\alpha^2 F(\vec{k}, \vec{k}', \omega) \equiv N(\varepsilon_F) \sum_j \left| g_{\vec{k}\vec{k}'}^j \right|^2 \delta(\omega - \omega_{j\vec{q}})$$

where $N(\varepsilon_F)$ = density of states per spin at Fermi level

$g_{\vec{k}\vec{k}'}$ = electron-phonon matrix element

$\omega_{j\vec{q}}$ = frequency of phonon in j th branch with $\vec{q} = \vec{k} - \vec{k}'$

Equivalently:

$$\lambda(\vec{k}, \vec{k}', n) \equiv \int_0^\infty d\omega \alpha^2 F(\vec{k}, \vec{k}', \omega) \frac{2\omega}{\omega^2 + (2n\pi T)^2}$$

$$\lambda = \langle \lambda(\vec{k}, \vec{k}', 0) \rangle$$

Transition Temperature and Isotope Effect

	harmonic		anharmonic		experiment
	isotropic	anisotropic	isotropic	anisotropic	
T_c	28 K	55 K	19 K	39 K	39 K
α_B	0.42	0.46	0.25	0.32	0.26, 0.30
α_{Mg}	0.04	0.02	0.05	0.03	0.02
λ	0.73		0.61		0.58, 0.62
ω_{ph}	62.7 meV		75.9 meV		75.9, 76.9

$$\mu^*(\omega_c) = 0.12.$$

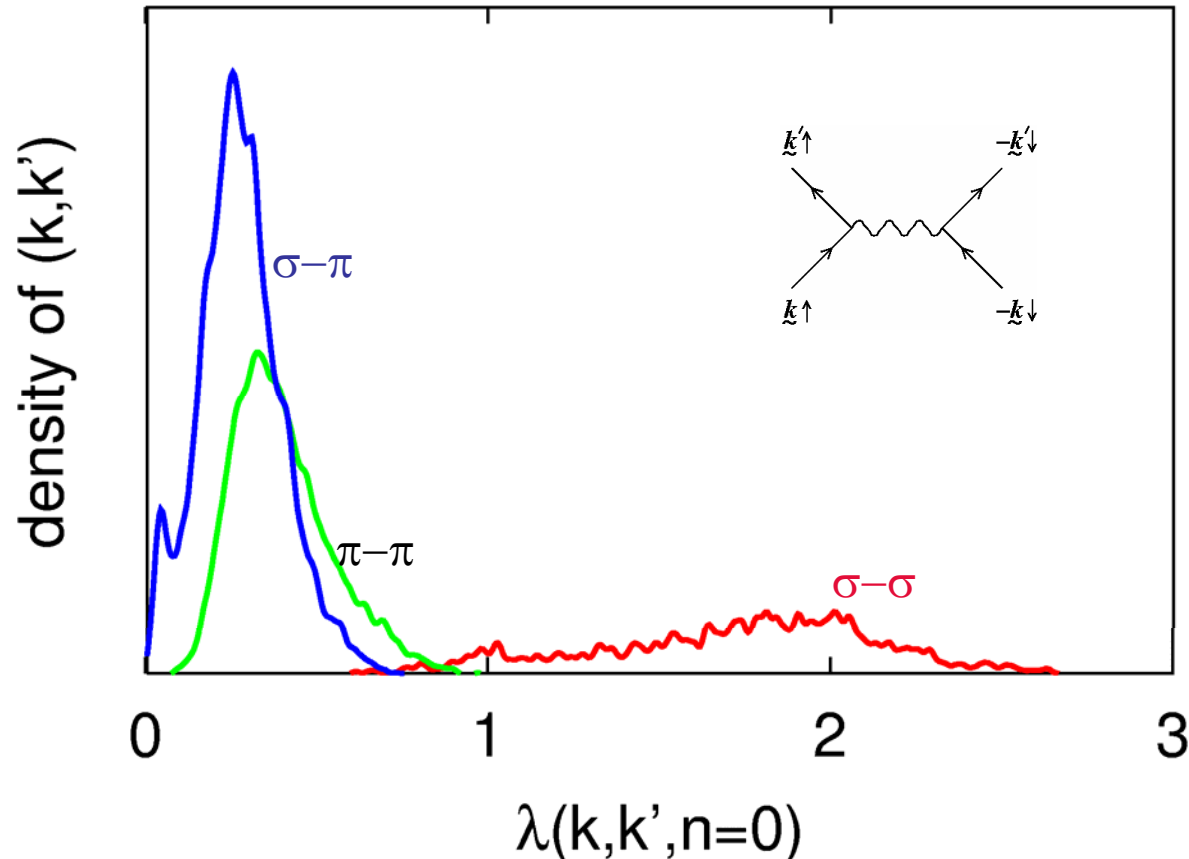
Anharmonicity --> small α_B

λ : averaged electron-phonon coupling.

ω_{ph} : frequency of the in-plane B-B stretching modes (E_{2g}) at Γ .

$$\text{For } 0.10 \leq \mu^*(\omega_c) \leq 0.14, 41 \text{ K} \geq T_c \geq 37 \text{ K}$$

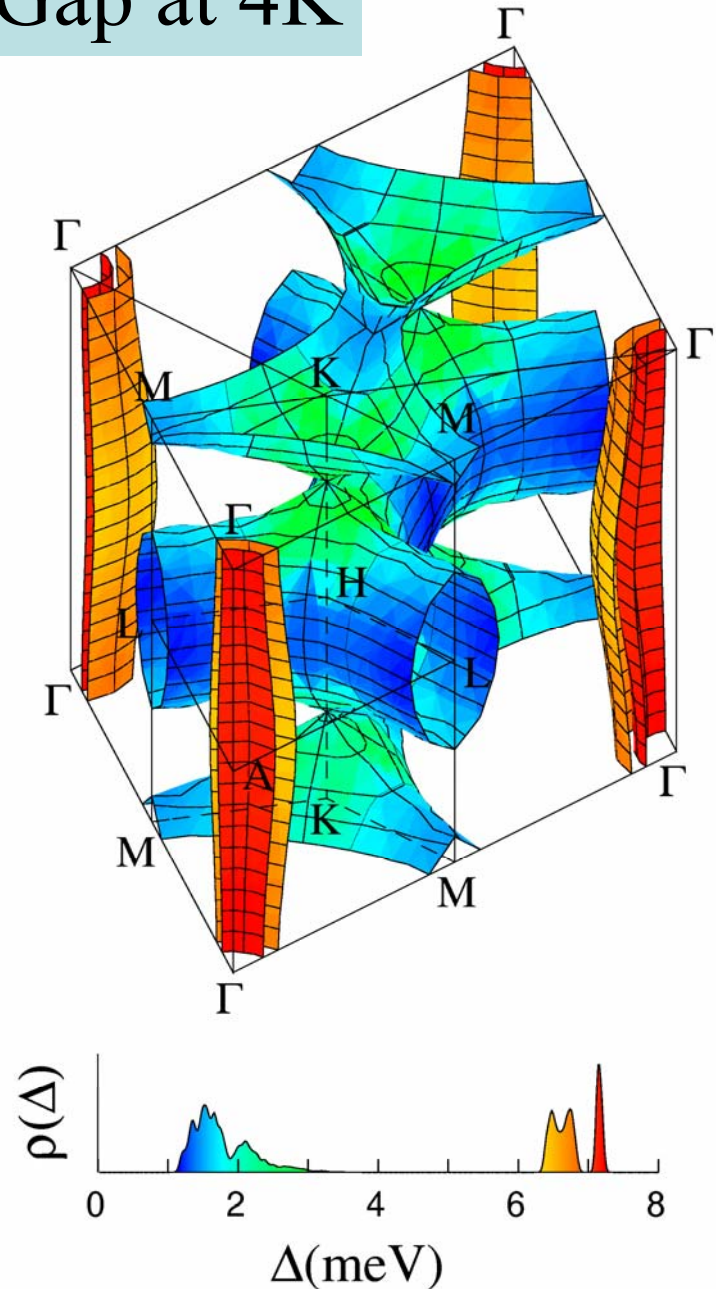
Distribution of Electron-Phonon Couplings



- Notes:
- 1) most metals: $\lambda \sim 0.3 - 0.5$
 - 2) MgB_2 : $\langle \lambda \rangle = \mathbf{0.61}$; specific heat data $\lambda = 0.58, 0.62$

Superconducting Gap at 4K

- $\Delta(\mathbf{k})$ on Fermi surface at $T=4$ K
- Large gap on cylindrical σ -sheets
- 2 dominant sets of gap values



CLASS [1]

ACHIEVING HIGHER
TRANSITION TEMPERATURES

T_c formulas

$$T_c = 1.14 \omega_{ph} \exp\left(\frac{-1}{N(0)V}\right) \quad \text{BCS, 1957}$$

$$T_c = \frac{T_D}{1.45} \exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right) \quad \text{McMillan, 1968}$$

$$T_c = \frac{\langle \omega \rangle_{\log}}{1.20} \exp\left(-\frac{1.04(1+\lambda)}{\lambda - \mu^*(1+0.62\lambda)}\right) \quad \text{for } \lambda < 1.5$$

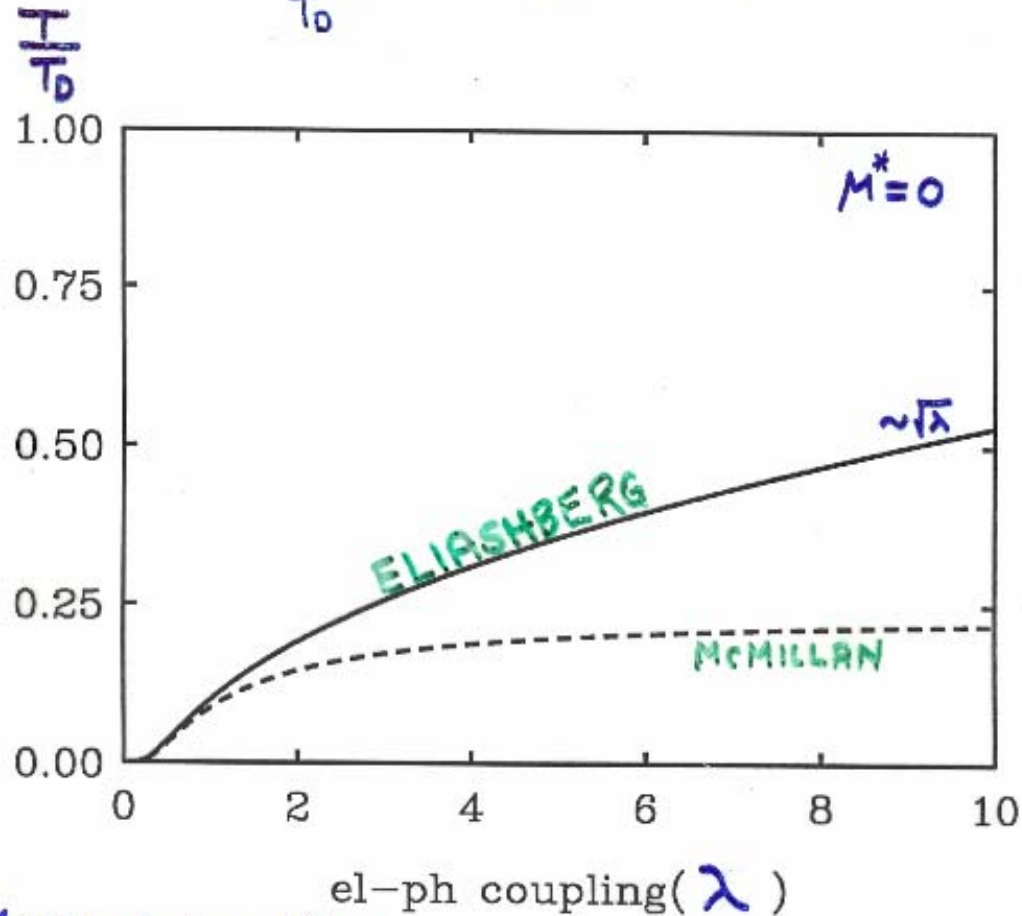
$$T_c = 0.183 \sqrt{\lambda \langle \omega^2 \rangle} \quad \text{for } \lambda > 10, \mu^* = 0 \quad \text{Allen and Dynes, 1975}$$

$$\lambda \equiv 2 \int_0^{\omega_{\max}} \alpha^2 F(\omega) \omega^{-1} d\omega$$

+ anisotropic electrons, anharmonic phonons, etc...

McMILLAN 1968

$$\frac{T_c}{T_0} \approx 0.69 e^{-\frac{1}{\lambda^2 - \mu^*}}$$



Kresin-Barbee-Cohen

$$\frac{T_c}{\langle \omega \rangle} = 0.26 (e^{2\lambda} - 1)^{-1}$$

AT THIS POINT, WE WANT..,

ROUGHLY SPEAKING

1] LARGE LAMBDA

2] BUT NOT TOO LARGE TO AVOID
INSTABILITIES, THEREFORE WE WANT

“CAGED” STRUCTURES OR OTHERS WITH


LATTICE INTEGRITY ASSURED BY
“OTHER BONDS”




TO GO FURTHER, WE NEEDED A MORE
QUANTITATIVE ASSESSMENT

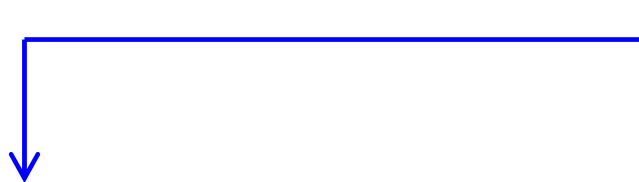
Bloch to Wannier Representation

$$g(\mathbf{k}, \mathbf{q}) = \sum_{\mathbf{R}_e, \mathbf{R}_p} e^{i\mathbf{k} \cdot \mathbf{R}_e} e^{i\mathbf{q} \cdot \mathbf{R}_p} u_{\mathbf{q}} U_{\mathbf{k}+\mathbf{q}} g(\mathbf{R}_e, \mathbf{R}_p) U_{\mathbf{k}}^\dagger$$

 Bloch

 Wannier

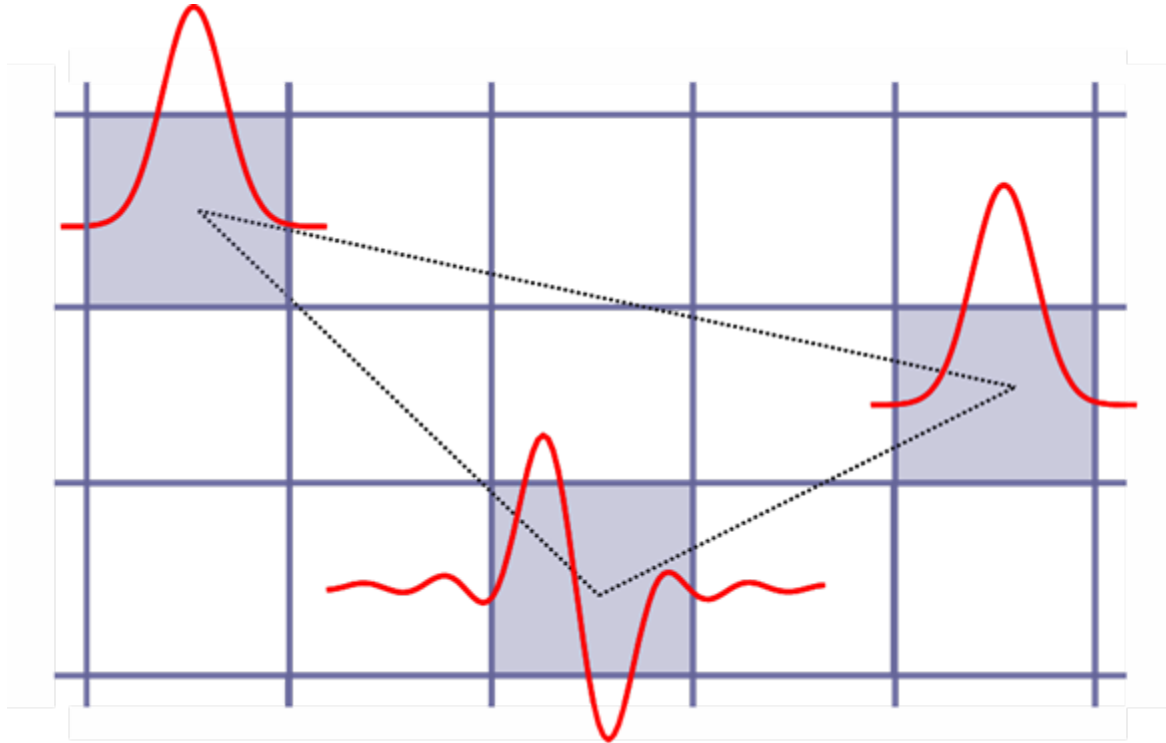
$$\underbrace{g(\mathbf{k}, \mathbf{q})}_{\text{Bloch}} = \sum_{\mathbf{R}_e, \mathbf{R}_p} e^{i\mathbf{k} \cdot \mathbf{R}_e} e^{i\mathbf{q} \cdot \mathbf{R}_p} u_{\mathbf{q}} U_{\mathbf{k}+\mathbf{q}} \underbrace{g(\mathbf{R}_e, \mathbf{R}_p)}_{\text{Wannier}} U_{\mathbf{k}}^\dagger$$



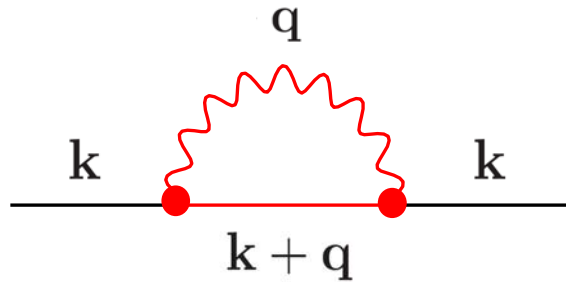
$$\langle m\mathbf{0}_e | \Delta_{\kappa\alpha, \mathbf{R}_p} V(\mathbf{r}) | n\mathbf{R}_e \rangle$$

Wannier Representation

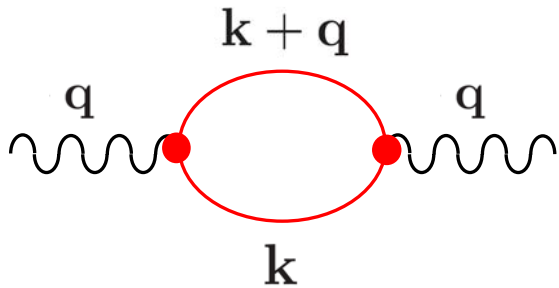
$$\langle m\mathbf{0}_e | \Delta_{\kappa\alpha, \mathbf{R}_p} V(\mathbf{r}) | n\mathbf{R}_e \rangle$$



Electron and Phonon Self-energy



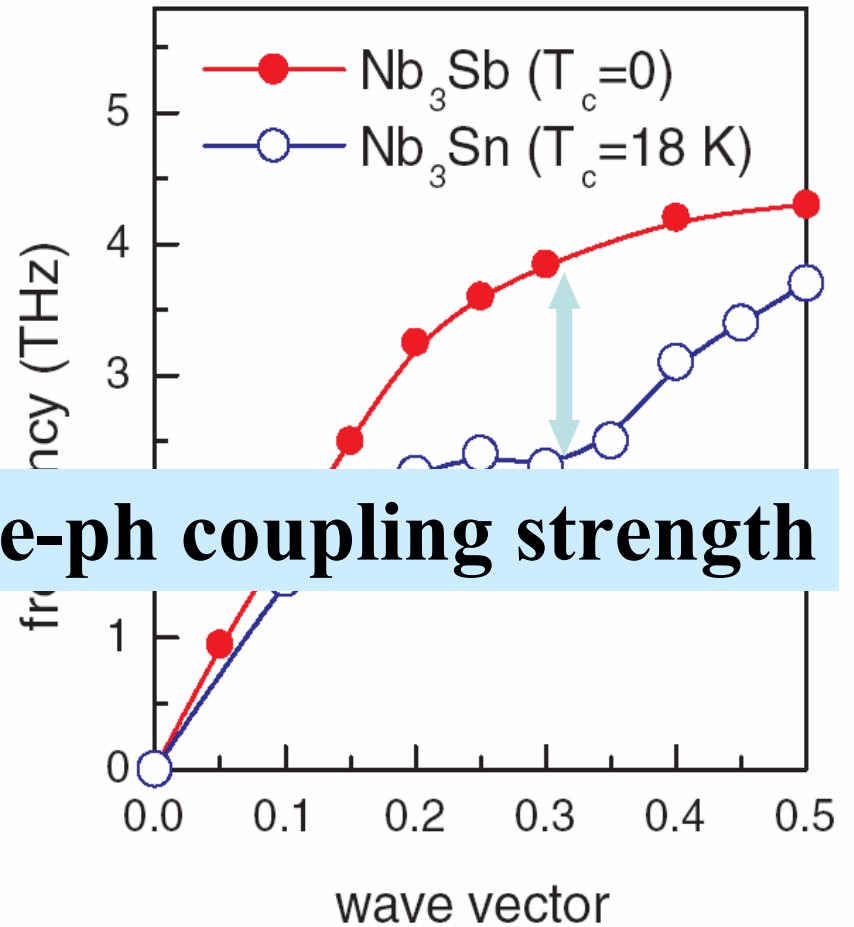
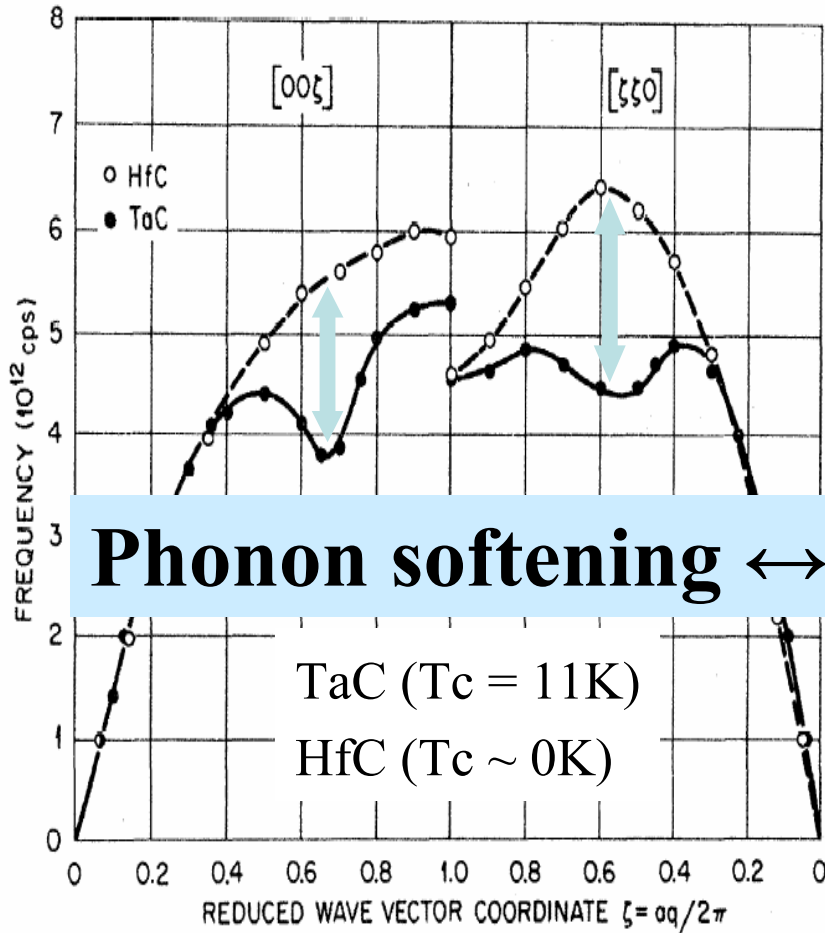
$$\Sigma = i \int \frac{d^2}{(2\pi)^4} |g(1, 2)|^2 D(1 - 2) G(2)$$



$$\Pi = -2i \int \frac{d^1}{(2\pi)^4} |g(1, 2)|^2 G(1) G(2)$$

TaC and HfC

Phonon softening in superconductors

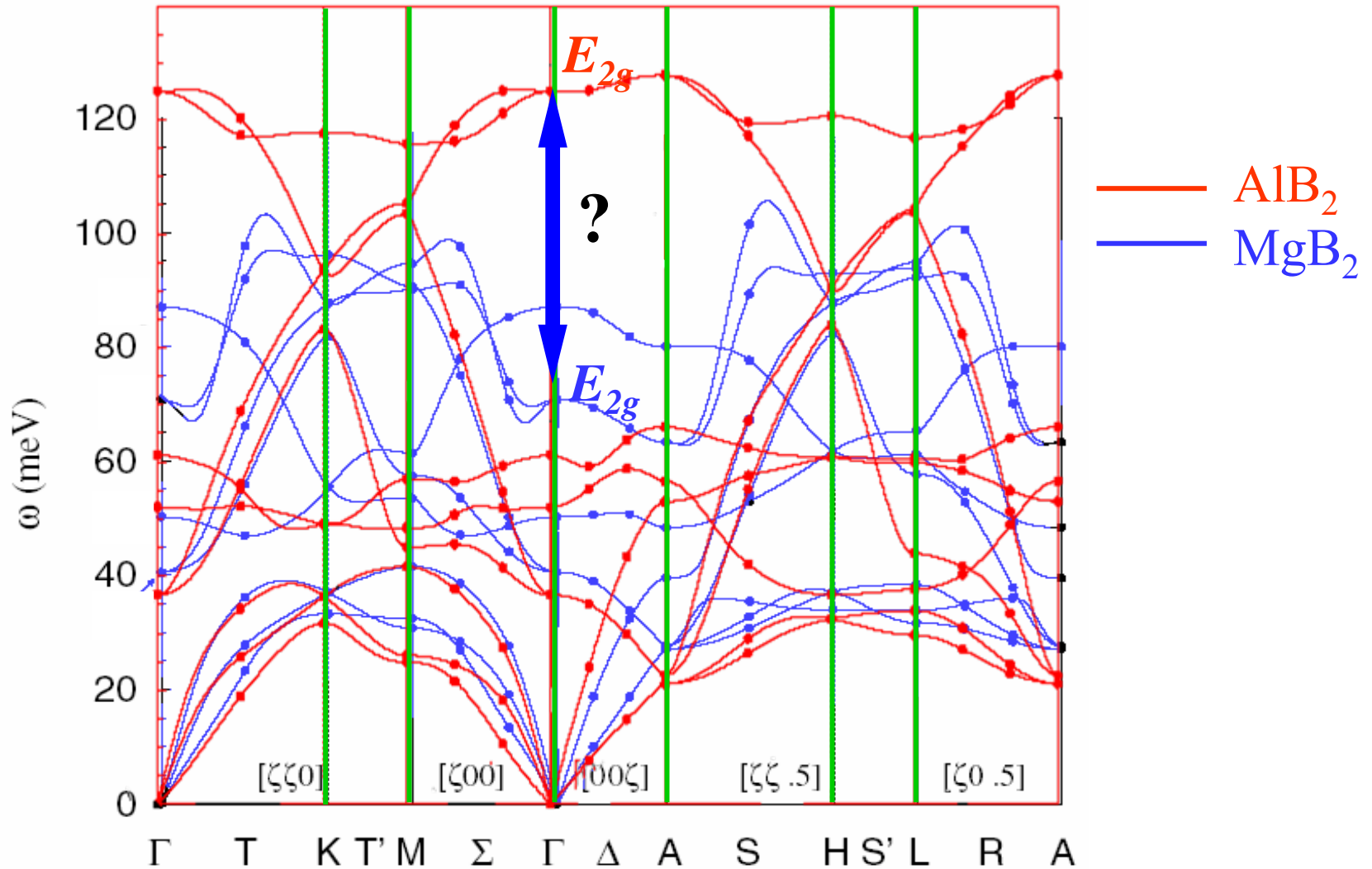


Phonon softening \leftrightarrow e-ph coupling strength

WHAT ABOUT MgB₂?

DOPING, PRESSURE, ETC.

E_{2g} phonon in MgB_2 and AlB_2



RAISING T_c

WE TRIED TO USE THEORY TO SUGGEST
HOW TO INCREASE THE TRANSITION
TEMPERATURE OF MAGNESIUM DIBORIDE
SIGNIFICANTLY BUT FAILED!

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THIS RESULT IS CONSISTENT WITH
EXPERIMENTS UP TO NOW.

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BUT WE ARE STILL TRYING--RECENTLY WE
FIND ABOUT A 10K INCREASE FOR CaB_2 .

CLASS 1

MORE GENERAL
CONSIDERATIONS FOR
INCREASING T_c

ELECTRON-PHONON COUPLING

$$\lambda \langle \omega^2 \rangle = \sum_i \frac{\eta_i}{M_i}$$

SO λ CAN BE VIEWED AS THE RATIO OF AN ELECTRONIC SPRING CONSTANT η AND A LATTICE SPRING CONSTANT

Values of η

	η (eV/Å ²)	$0.183\sqrt{\lambda\langle\omega^2\rangle}$ (K)	EXP
C (diamond)*	54	290	~ 10
C (graphite)*	48	270	?
BN*	36	240	?
Si*	10	82	~10

*at peak of $\eta(E)$

Superconductivity in diamond

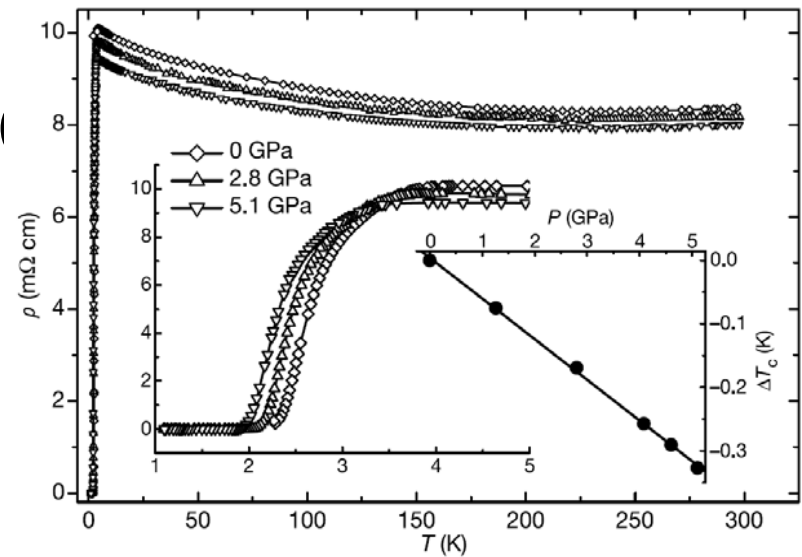
E. A. Ekimov¹, V. A. Sidorov¹, E. D. Bauer², N. N. Mel'nik³, N. J. Curro²,
J. D. Thompson² & S. M. Stishov¹

¹Vereshchagin Institute for High Pressure Physics, Russian Academy of Sciences,
142190 Troitsk, Moscow region, Russia

²Los Alamos National Laboratory, Los Alamos, New Mexico 87545, USA

³Lebedev Physics Institute, Russian Academy of Sciences, 117924 Moscow, Russia

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CLASS 2

HORSES AND ZEBRAS

WHEN YOU HEAR
HOOFBEATS ON THE
BRIDGE,

WHEN YOU HEAR
HOOFBEATS ON THE
BRIDGE,

DON'T THINK OF ZEBRAS

IT IS IMPORTANT TO TEST FOR
CLASS [1] SIGNATURES (HORSES)

CUPRATES

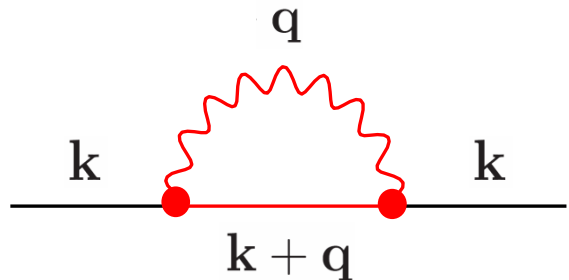
ELECTRONIC STRUCTURE

PHONON ENERGIES

ELECTRON-PHONON COUPLINGS

KINKS

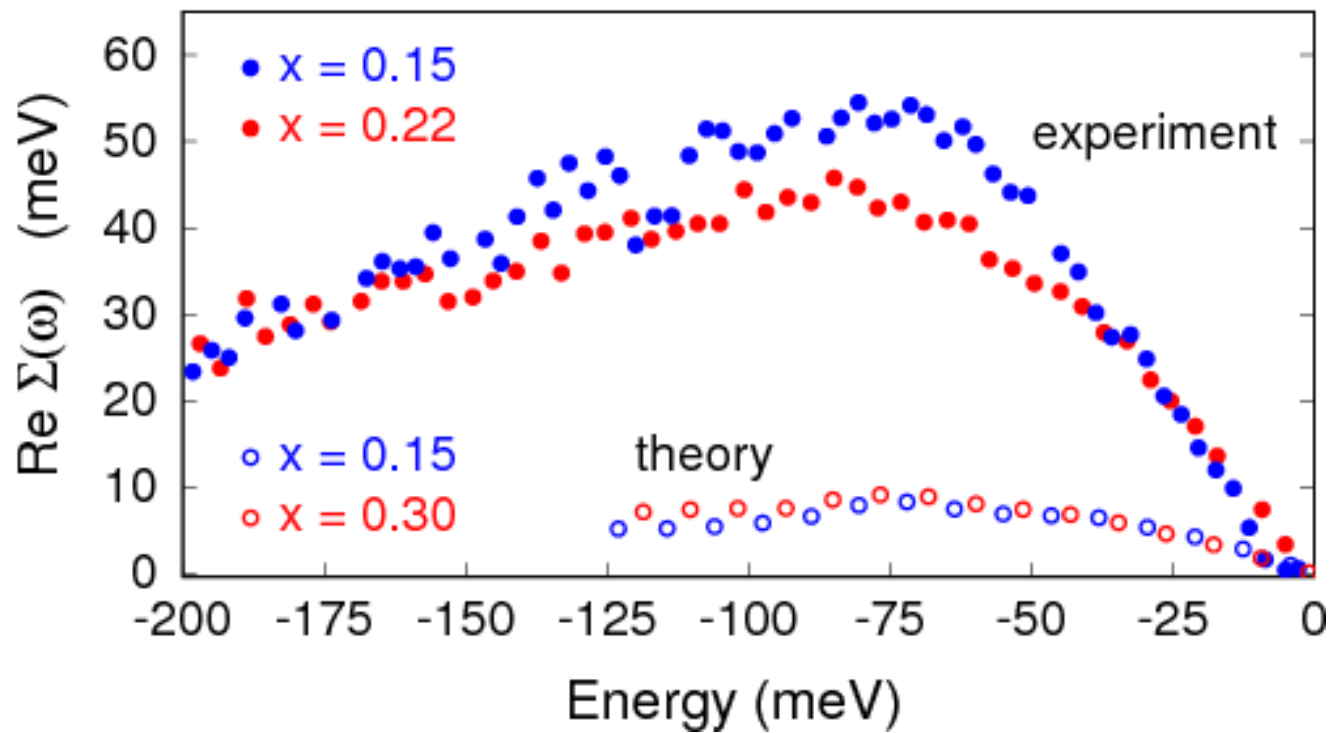
Electron Self-energy



The diagram shows a horizontal black line representing an electron propagator. Two red dots are placed on this line, connected by a red wavy line representing a phonon loop. The left dot is labeled with momentum k above it. The right dot is also labeled with momentum k above it. The wavy line is labeled with momentum q above it. Below the line, between the two dots, is the label $k + q$.

$$\Sigma = i \int \frac{d^2}{(2\pi)^4} |g(1, 2)|^2 D(1 - 2) G(2)$$

YIELDS A MASS ENHANCEMENT AND ASSOCIATED “KINK” AT THE FERMI SURFACE.”KINKS” HAVE BEEN OBSERVED IN ARPES DATA AND INTERPRETETED AS SIGNATURES OF STRONG ELECTRON-PHONON COUPLING.



CONCLUSION

BASED ON THE WANNIER
FORMALISM FOR CALCULATING
ELECTRON-PHONON SELF-
ENERGIES, THE COUPLING IS 1/7
OF WHAT IS NEEDED TO
REPRODUCE THE OBSERVED
ARPES “KINKS”

IT IS IMPORTANT TO TEST FOR
CLASS [1] SIGNATURES (HORSES)

PNICTIDES

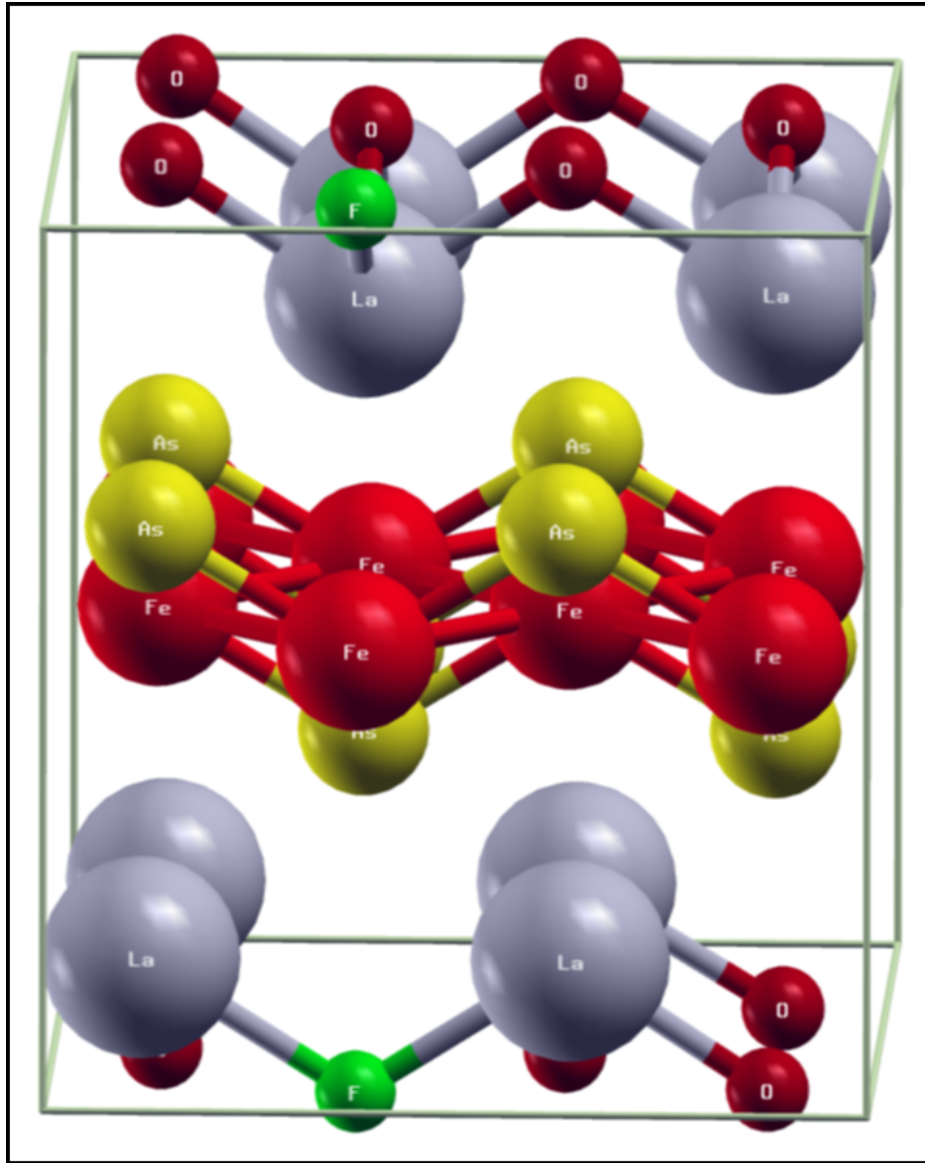
ELECTRONIC STRUCTURE

(LDA, SPIN POL GGA)

PHONON ENERGIES

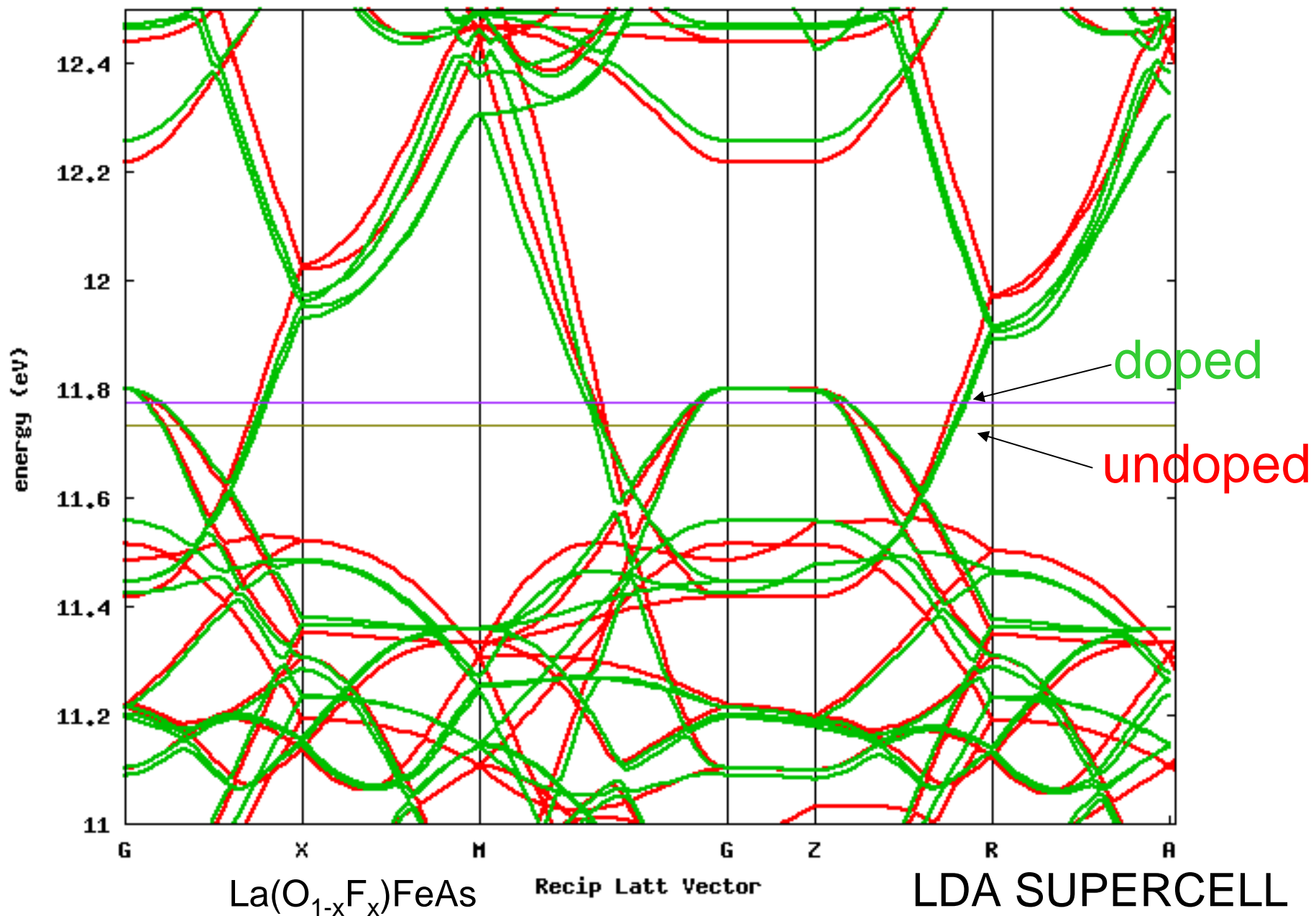
(BOTH CALCULATED WITH AND WITHOUT
DOPING USING A VIRTUAL XTAL AND A
SUPERCELL APPROACH)

ELECTRON-PHONON COUPLINGS



$$x = 0.125$$

32 ATOM CELL



Difference in Planar Integrated Charge Density Along z-axis When Doped

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

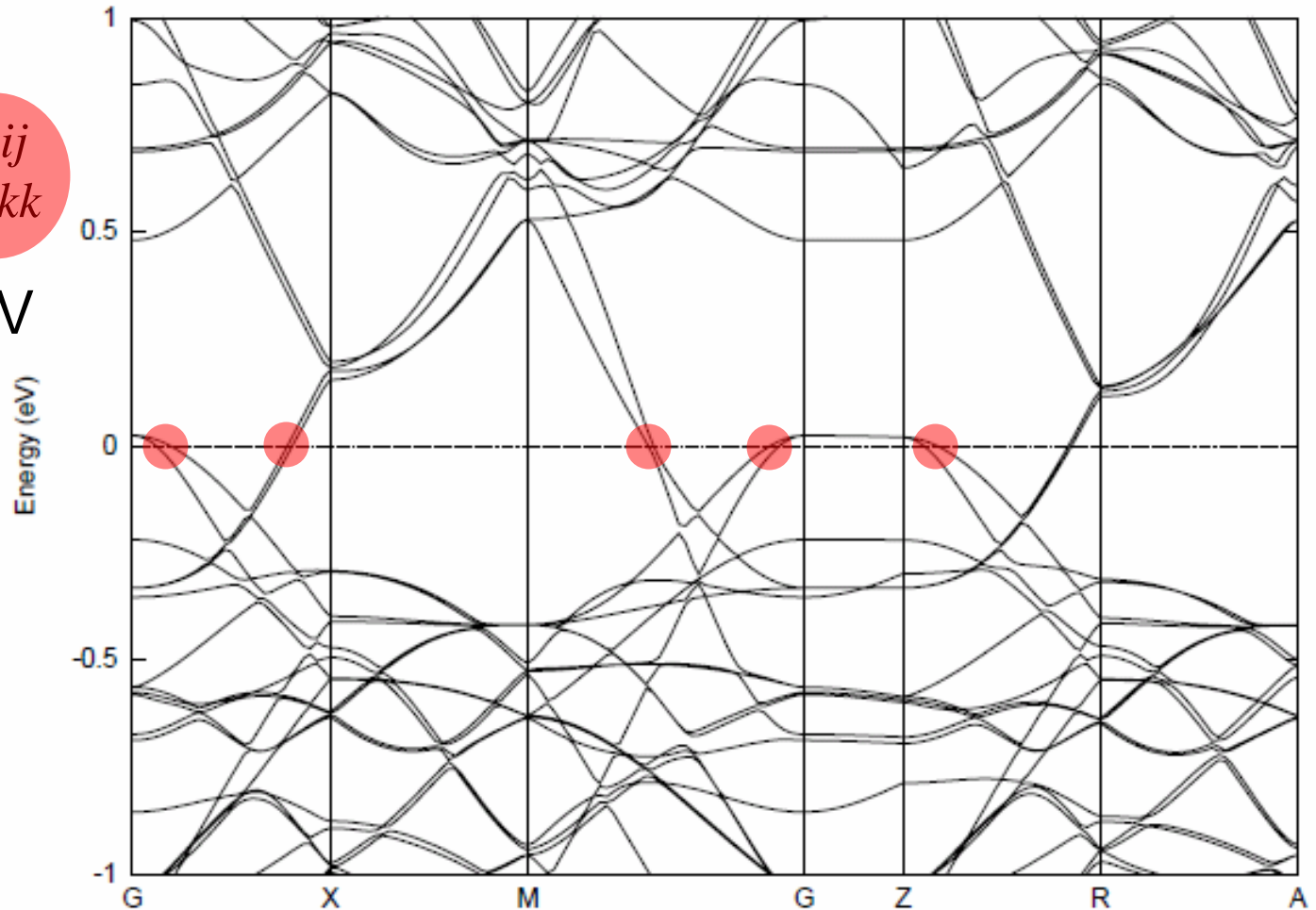
Noffsinger et al

ELECTRONIC STATES COUPLED TO PHONONS

Maximum

$$g_{kk}^{ij}$$

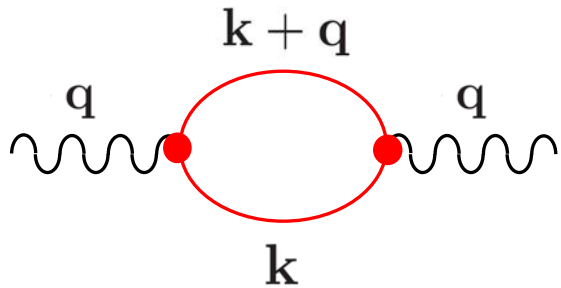
$$= 20 \pm 7 \text{ meV}$$



TENTATIVE CONCLUSIONS FOR LAMBDA

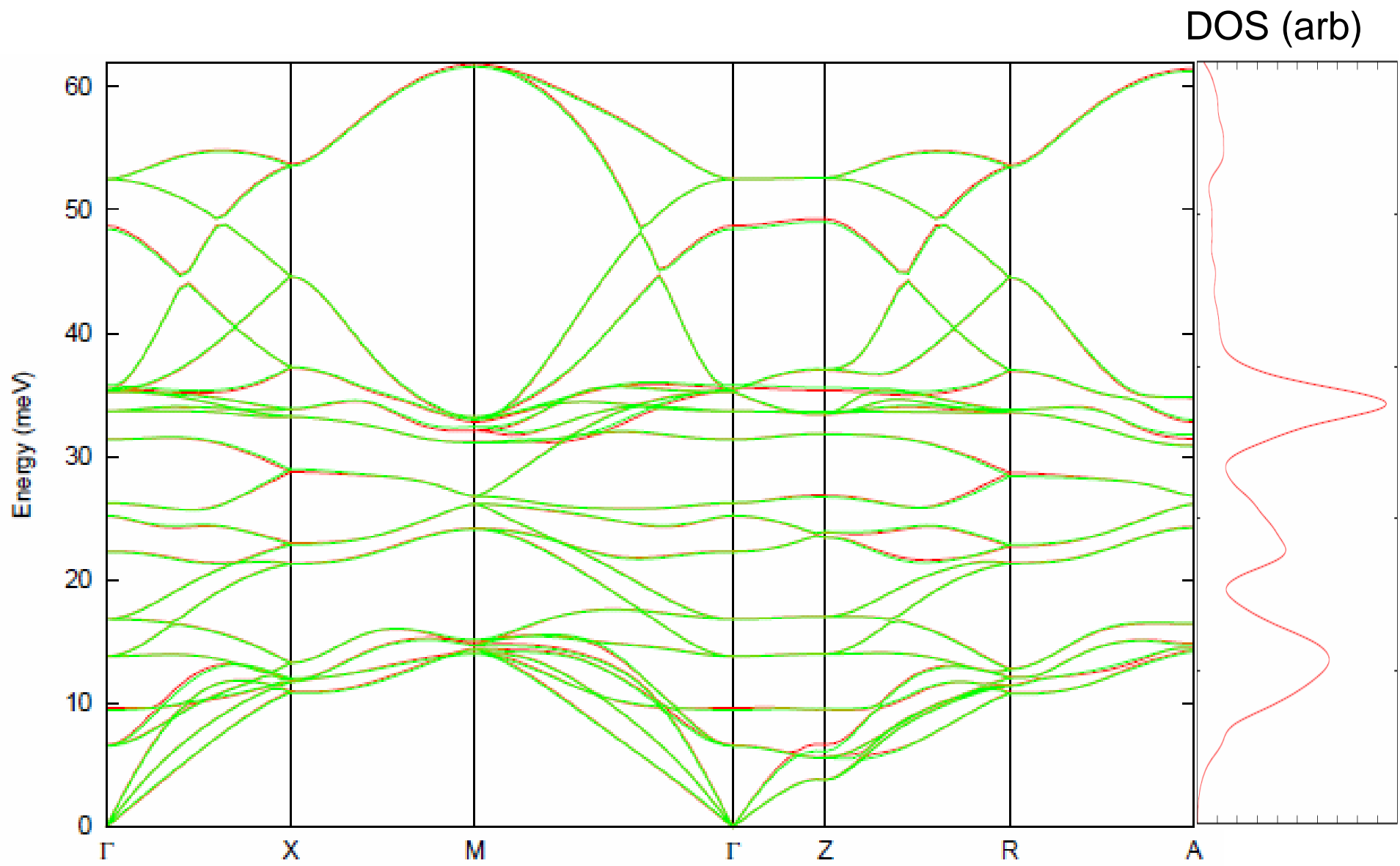
BASED ON THESE CALCULATIONS
OF THE ELECTRON-PHONON g 's,
OUR ESTIMATES OF LAMBDA (~ 0.18)
ARE IN LINE WITH THOSE
CALCULATED BY OTHERS USING A
VIRTUAL CRYSTAL MODEL.

What about Phonon Self-energies?



$$\Pi = -2i \int \frac{d^1}{(2\pi)^4} |g(1, 2)|^2 G(1) G(2)$$

Phonon Dispersion: Red = undoped, green = 12% rigid band doped



EXPERIMENT

M. Le Tacon, M. Krisch, A. Bosak, J.-W. G. Bos, S. Margadonna, arXiv:0809.2898

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

THEORY

Noffsinger et al

TENTATIVE CONCLUSIONS FOR DOPING SHIFTS IN PHONONS

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1] EXPERIMENT AND SUPERCELL THEORY ARE IN GOOD AGREEMENT FOR THE EFFECTS OF F DOPING, WHEREAS A VIRTUAL CRYSTAL MODEL SHOWS LITTLE CHANGE AND THEREFORE DOESN'T AGREE.

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2] IT IS INTERESTING THAT SOME OF THE DOPING SHIFTS (e.g. SOFTENING) OCCUR FOR THE LOWER FREQUENCY PHONONS (BY INFLUENCING THE BONDS TO THE HEAVIER ELEMENTS) IN ADDITION TO THE MORE OR LESS EXPECTED CHANGES SEEN FOR THE HIGHER OXYGEN FREQUENCIES.

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3] THE SHIFTS DO NOT APPEAR TO ARISE FROM STANDARD EL-PH SELF-ENERGY THEORY VIA INCREASED ELECTRON CONCENTRATION.

GENERAL CONCLUSIONS/COMMENTS

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- 2] BE CAREFUL IF YOUR MODE IS TRANSVERSE (e.g. EXCITONS).
- 3] LOCAL FIELDS ARE IMPORTANT.
- 4] THE **Q-DEPENDENCE** IS CRUCIAL FOR AN ATTRACTIVE INTERACTION TO WORK (e.g. DEMONS).

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3) CAUTIONS FOR BOSON MECHANISMS

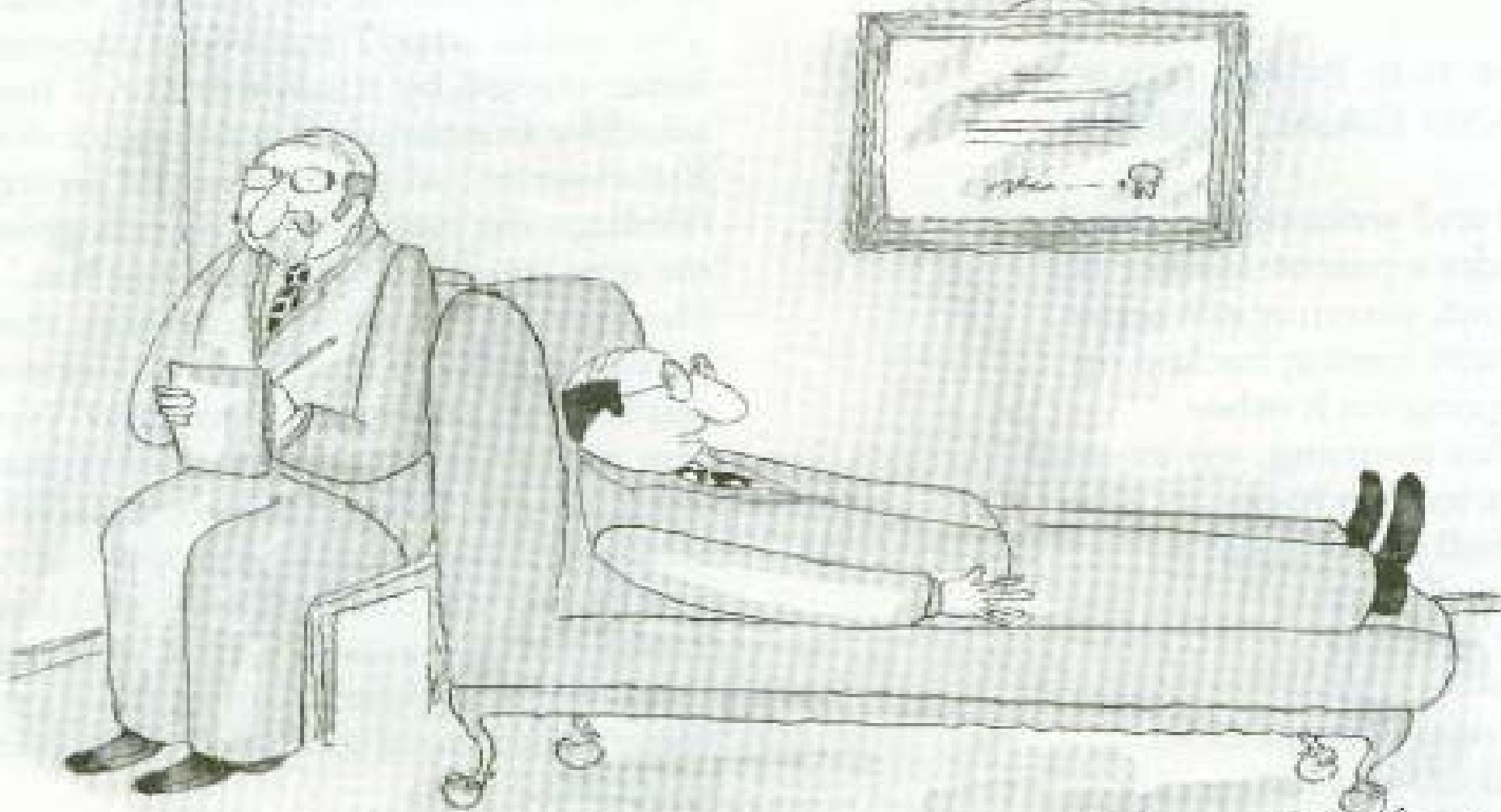
GENERAL CONCLUSIONS/COMMENTS

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2) THE Fe SYSTEMS PROBABLY BELONG IN CLASS [2].

3) CAUTIONS FOR BOSON MECHANISMS

4) CLASS [2] HAS INTRODUCED INTERESTING NEW PHYSICS, BUT THERE IS NO AGREEMENT ON **THE** THEORY AS YET. **ZEBRAS** APPEAR LIKELY, BUT **MULES (HYBRIDS)** MAY BE POSSIBLE.



Victoria Roberts

"Can we start over? Like, from 1987, say?"



THE END

1. In the case of electron-electron interactions, is the concept of a pairing "glue" even meaningful? YES
2. If your theory advocates an instantaneous interaction, does this mean the pairs have no dynamics, or just that the theory has not developed to the extent to address this question? RETARDED
3. If your theory ignores phonons, can you really get away with that? Do you think phonons are even relevant? NO, YES
4. What is the spectroscopic signatures predicted for your theory? Is a McMillan-Rowell inversion or related procedure possible for your theory? Is this question meaningful? YES
5. What would your theory predict in regards to collective modes? Is this even an important question? NO