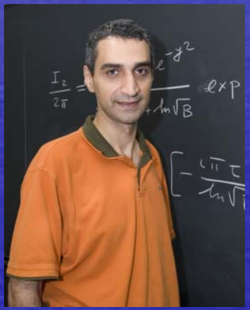
Superconductivity due to massless boson exchange.

Andrey Chubukov University of Wisconsin

Consultations



Artem Abanov
Texas A&M

Mike Norman
ANL

Joerg Schmalian
ISU

Boris Altshuler
Columbia

Emil Yuzbashyan
Rutgers

Critical issues related to high temperature superconductors, Santa Barbara, June 2009

1. In the case of electron-electron interactions, is the concept of a pairing "glue" even meaningful?

2. If your theory advocates an instantaneous interaction, does this mean the pairs have no dynamics, or just that the theory has not developed to the extent to address this question?

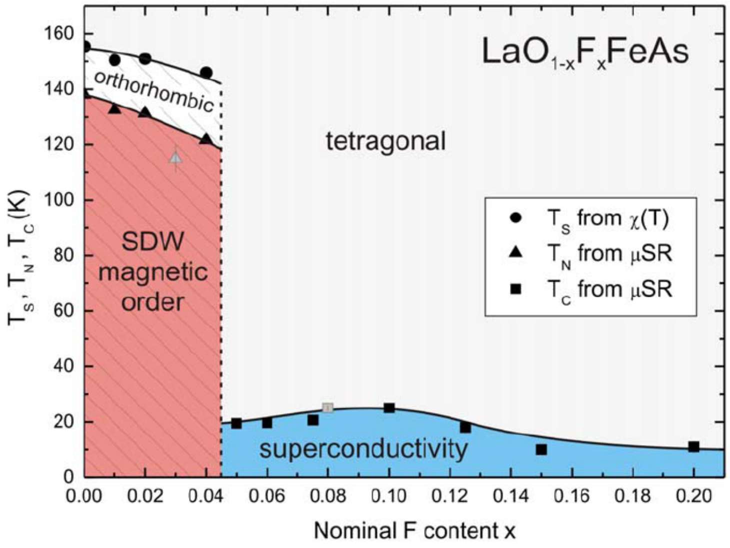
3. If your theory ignores phonons, can you really get away with that? Do you think phonons are even relevant?

4. What is the spectroscopic signatures predicted for your theory? Is a McMillan-Rowell inversion or related procedure possible for your theory? Is this question meaningful?

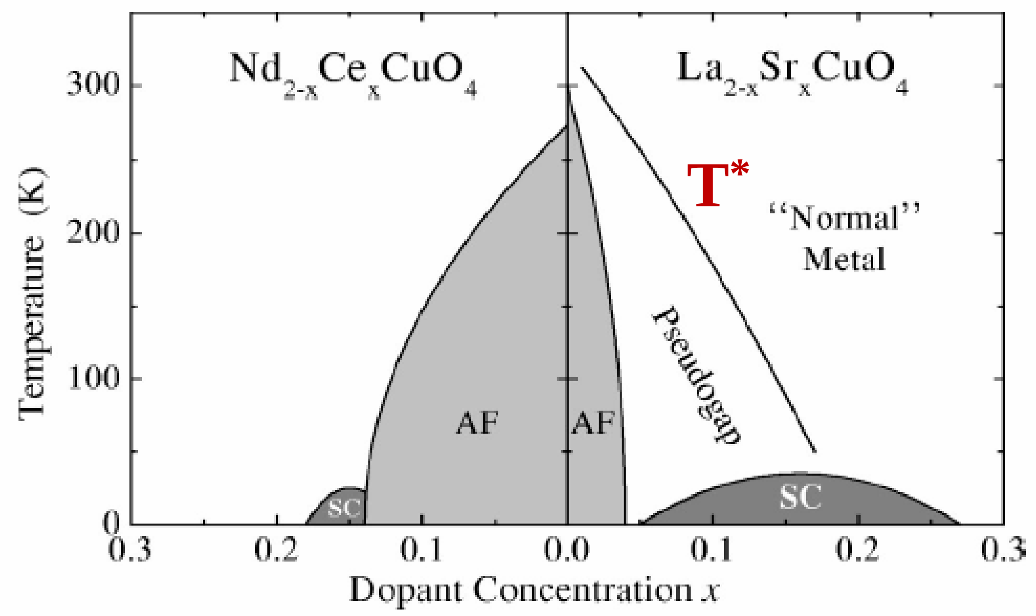
5. What would your theory predict in regards to collective modes? Is this even an important question?

Pairing glue

Pnictides



Cuprates



Is the pairing due to electron-electron interaction mediated by collective spin fluctuations?

Can T* line be understood in the context of a Fermi liquid, as the result of a collective mode exchange?

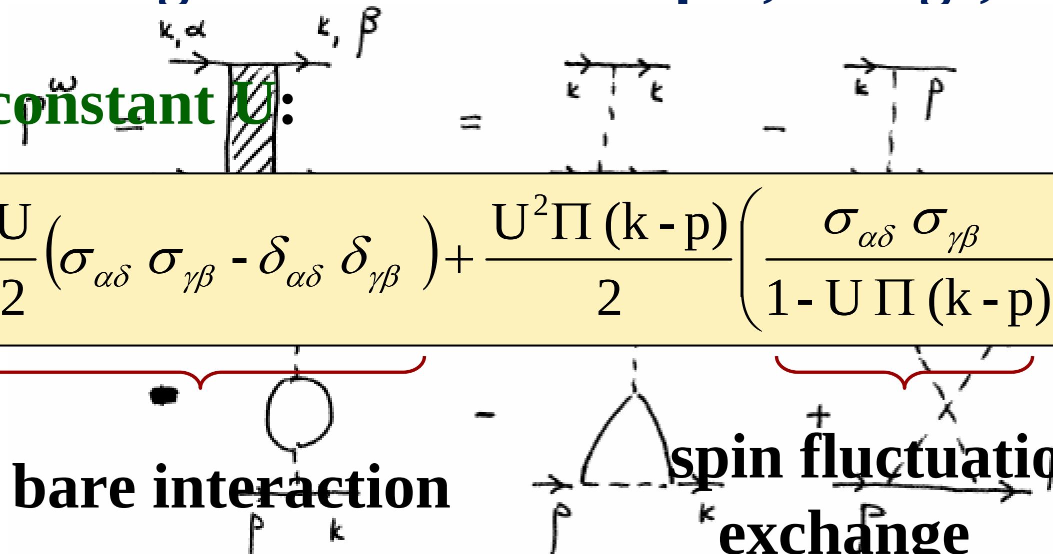
1. In the case of electron-electron interactions, is the concept of a pairing "glue" even meaningful?

Yes, but the exact prove is lacking

The goal is to re-write electron-electron interaction as the exchange of collective degrees of freedom: spin, charge, or pairing fluctuations

RPA, constant U:

Fermi



$$\Gamma^{\omega}_{\alpha\beta,\gamma\delta} = \frac{U}{2}\left(\sigma_{\alpha\delta}\,\sigma_{\gamma\beta} - \delta_{\alpha\delta}\,\delta_{\gamma\beta}\right) + \frac{U^2\Pi(k-p)}{2}\left(\frac{\sigma_{\alpha\delta}\,\sigma_{\gamma\beta}}{1-U\,\Pi(k-p)} + \frac{\delta_{\alpha\delta}\,\delta_{\gamma\beta}}{1+U\,\Pi(k-p)}\right)$$

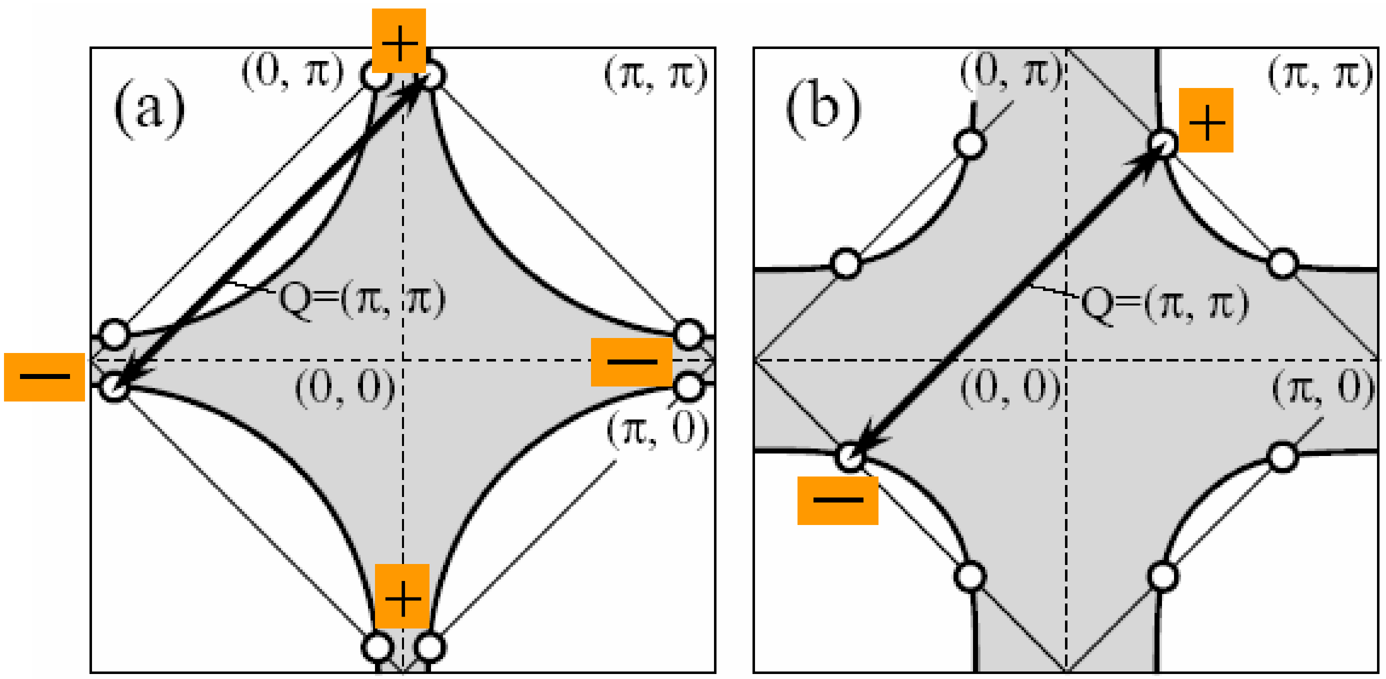
bare interaction | **spin fluctuation exchange** | **charge fluctuation exchange**

Near a magnetic instability with momentum Q

$$\Gamma^{\mathrm{eff}}(q) \approx \frac{U}{2}\frac{\sigma_{\alpha\delta}\,\sigma_{\gamma\beta}}{1-U\,\Pi(q+Q)}, \quad U\,\Pi(Q) \approx 1$$

$$\Gamma^{\text{eff}}(q) \approx \frac{U}{2} \frac{\sigma_{\alpha\delta}\,\sigma_{\gamma\beta}}{1 - U\,\Pi\,(q+Q)}, \quad U\,\Pi\,(Q) \approx 1$$

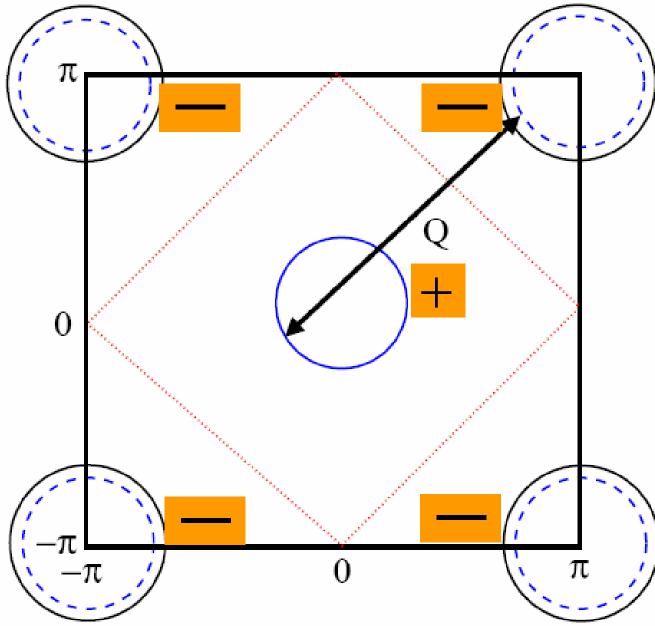
Advantage: the effective interaction is momentum dependent and contributes to non-s-wave channels.



$$\Delta(\theta) = \Delta_0\,(\cos k_x - \cos k_y)$$

Scalapino, Loh, Hirsh
Bickers, Scalapino, Scalettar…

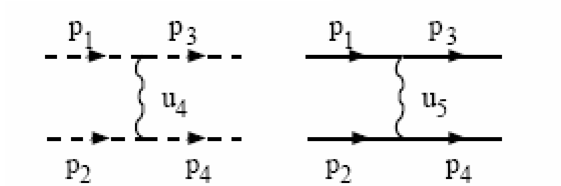
Pnictides



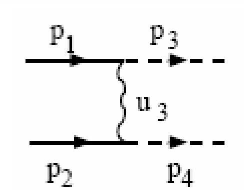
$$\Delta(\theta) = \Delta_0 (\cos k_x + \cos k_y)$$

sign-changing , extended s-wave gap

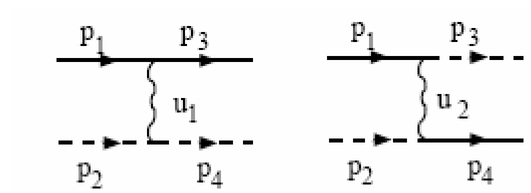
Mazin et al, Graser et al,
Gorkov & Barzykin, Kuroki et al….



Intra-band repulsion



Pair hopping



Inter-band forward and "back-scattering"

$$\Gamma^{s^+}_{\alpha\beta,\gamma\delta} = \frac{u_4 - u_3}{2}\left(\sigma_{\alpha\delta}\,\sigma_{\gamma\beta} - \delta_{\alpha\delta}\,\delta_{\gamma\beta}\right) - \frac{1}{2}\sigma_{\alpha\delta}\,\sigma_{\gamma\beta}\,\frac{(u_1+u_3)^2\,\Pi(k-p)}{1-(u_1+u_3)\Pi(k-p)}$$

s-wave bare interaction

spin fluctuation exchange

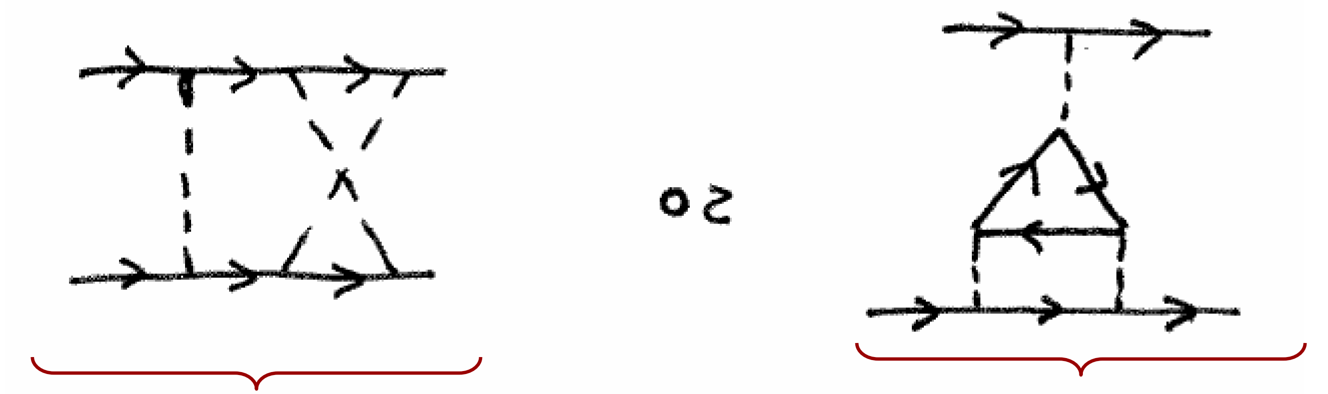
Problem:

$$\Gamma^{\mathrm{eff}}(q) \approx \frac{U}{2}\frac{\sigma_{\alpha\delta}\,\sigma_{\gamma\beta}}{1-U\,\Pi(q+Q)}$$

In general, $\Pi(Q) \sim 1/E_F$, hence $U_{cr} \sim E_F$

No small parameter

Third order



Coupling between ph and pp channels

Non-RPA diagrams In the ph channel

$$\Gamma^{\mathrm{eff}}(q) \approx \frac{U}{2}\frac{\sigma_{\alpha\delta}\,\sigma_{\gamma\beta}}{1-U\,\Pi(q+Q)}, \quad U\,\Pi(Q) \approx 1$$

Obtained within RPA, but no proof beyond RPA (or 1 loop RG)

2. If your theory advocates an instantaneous interaction, does this mean the pairs have no dynamics, or just that the theory has not developed to the extent to address this question?

The bare interaction is instantaneous

$$H^{eff} = g\,(c^{+}_{\alpha}\,\vec{\sigma}_{\alpha\delta}\,c_{\delta})\,(c^{+}_{\gamma}\,\vec{\sigma}_{\gamma\beta}\,c_{\beta})\,\chi\,(q+Q)$$

$$\chi\,(q+Q) = \frac{1}{q^2+\xi^{-2}} \quad \text{or} \ldots.$$

The dynamics of χ (the Landau damping) is generated within the theory and in turn affects the dynamics of fermions

2D

0 → E

Fermi liquid

$$\omega_{sf} \sim \frac{v_F^2\,\xi^{-2}}{g}$$

$$\Sigma'(\omega) = \lambda\omega,$$

$$\Sigma''(\omega) \propto \omega^2$$

$$\chi_L(\omega) \propto \xi$$

$$\chi_L = \oint_{FL} \chi\,(q+Q)\,d^2q$$

Non-Fermi liquid

$$\Sigma'(\omega) \propto \omega^{1-\gamma} > \omega,$$

$$\Sigma''(\omega) \propto \omega^{1-\gamma} > \omega$$

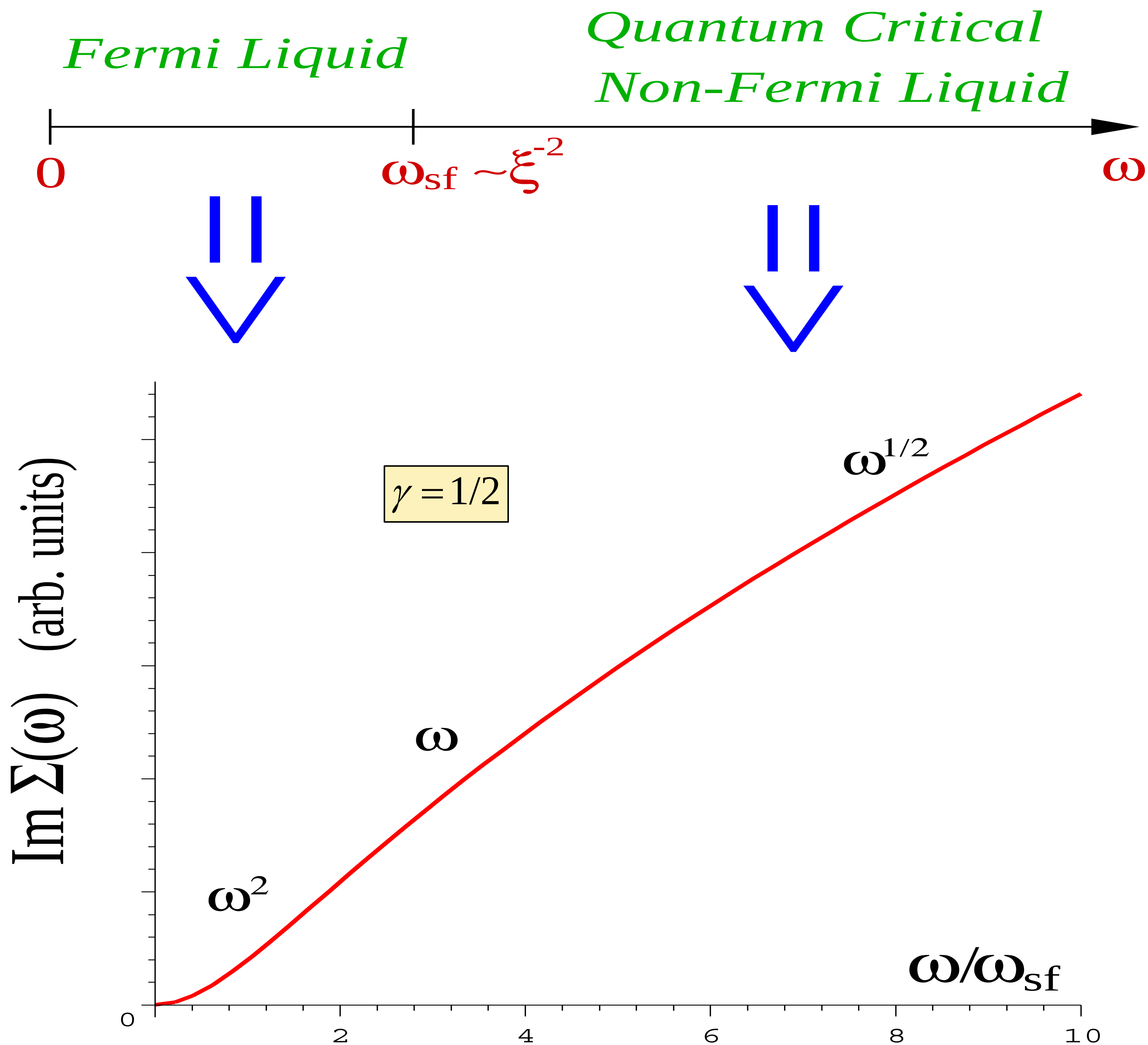
$$\chi_L(\omega) \propto \omega^{-\gamma}$$

$$\overline{\omega} \sim g$$

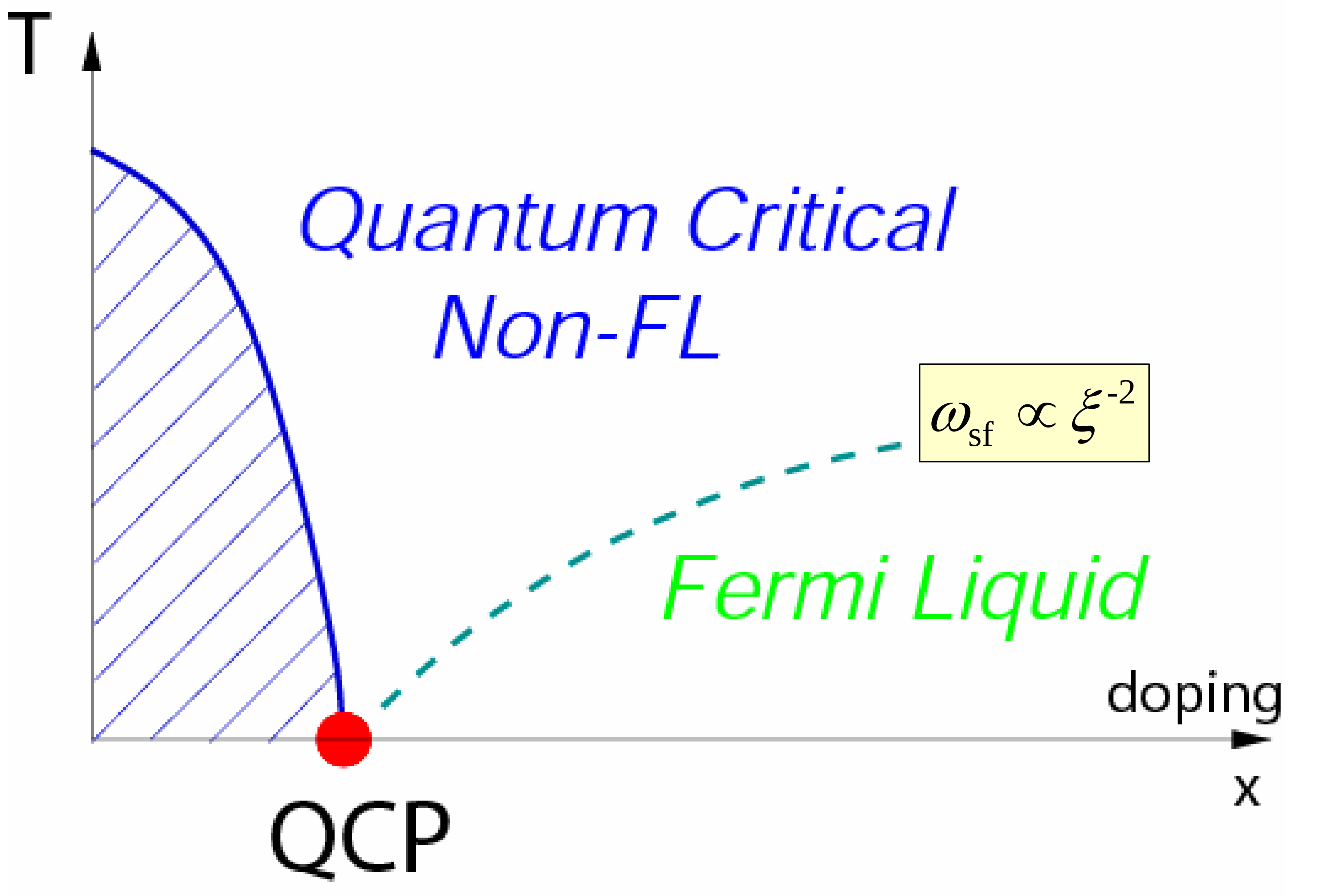
Fermi gas

$$\Sigma(\omega) < \omega,$$

$$\chi_L(\omega) \propto \omega^{-\gamma}$$



Fermi Liquid
Quantum Critical
Non-Fermi Liquid
0
ωsf ~ξ-2
ω
γ = 1/2
Im Σ(ω) (arb. units)
ω2
ω
ω1/2
ω/ωsf
0
2
4
6
8
10


T
Quantum Critical
Non-FL
$\omega_{sf} \propto \xi^{-2}$
Fermi Liquid
doping
x
QCP

Fermi liquid $\omega_{sf} \sim \frac{v_F^2 \xi^{-2}}{g}$ **Non-Fermi liquid** $\overline{\omega} \sim g$ **Fermi gas**

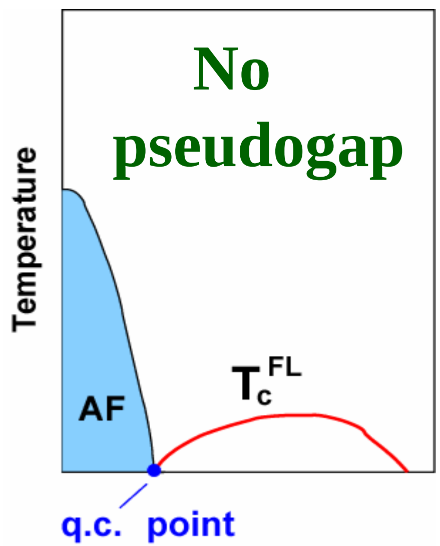
0 → E

Pairing in the Fermi liquid regime is essentially d-wave BCS

$$Tc \propto \xi^{-2} \exp\left(-\frac{\xi + \xi_0}{\xi}\right)$$

In analogy with McMillan formula for phonons

$$T_c \sim \omega_D \exp[-(1+\lambda)/\lambda]$$



Fermi liquid $\omega_{sf} \sim \frac{v_F^2 \xi^{-2}}{g}$ **Non-Fermi liquid** $\bar{\omega} \sim g$

0 → **E**

Pairing in non-Fermi liquid regime is a new phenomenon

Pairing vertex Φ becomes frequency dependent $\Phi(\Omega)$

Gap equation has non-BCS form

$$\Phi(\Omega) = \bar{\lambda}\, \pi T \sum_{\omega} \frac{\Phi(\omega)}{|\omega|^{1-\gamma}\, |\Omega - \omega|^{\gamma}} \frac{1}{1 + (|\omega|/\bar{\omega})^{\gamma}}, \quad \bar{\lambda} = \frac{1-\gamma}{2}$$

$\Sigma(\omega) \propto \omega^{1-\gamma}$ $\quad$ $\chi_L(\omega) \propto \omega^{-\gamma}$ $\quad$ $1 + \omega/\Sigma(\omega)$, soft cutoff

This is NOT BCS pairing – summing up logarithms leads nowhere

There exists a threshold $\bar{\lambda}_{cr}(\gamma)$

This problem is quite generic (not only cuprates)

$$\Phi(\Omega) = \frac{1-\gamma}{2} \int_0^g d\omega \, \frac{\Phi(\omega)}{|\omega|^{1-\gamma} \, |\Omega - \omega|^{\gamma}}$$

She, Zaanen

$\gamma = 1/2$ — **Antiferromagnetic QCP** — Abanov, A.C., Finkelstein, hot spots

$\gamma = 1/3$ — **Ferromagnetic QCP** — Haslinger et al, Millis et al, Bedel et al…
$\Omega^{2/3}$ problem: gauge field, nematic ….

$\gamma = 1/4$ — **$2k_F$ QCP** — Krotkov et al, electron-doped

$\gamma = 2$ — **Pairing by near-gapless phonons**

$T_c^{ad} = 0.1827\, g$

Allen, Dynes, Carbotte, Marsiglio, Scalapino, Combescot, Maksimov, Bulaevskii, Dolgov, …..

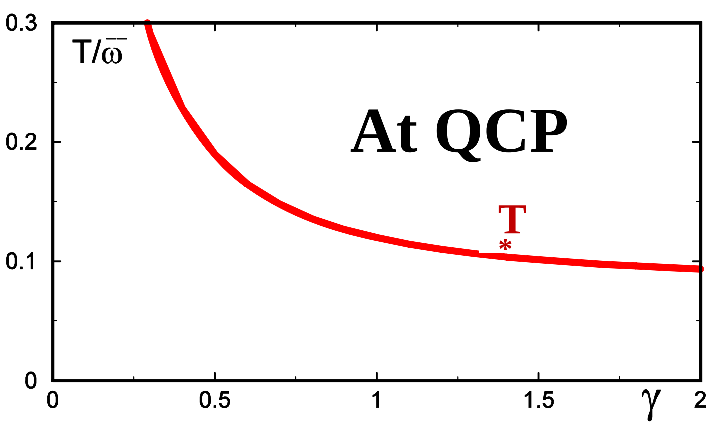
$\gamma = +0 \;\; (\log \omega)$ — **3D QCP, Color superconductivity** — Son, Schmalian, A.C….

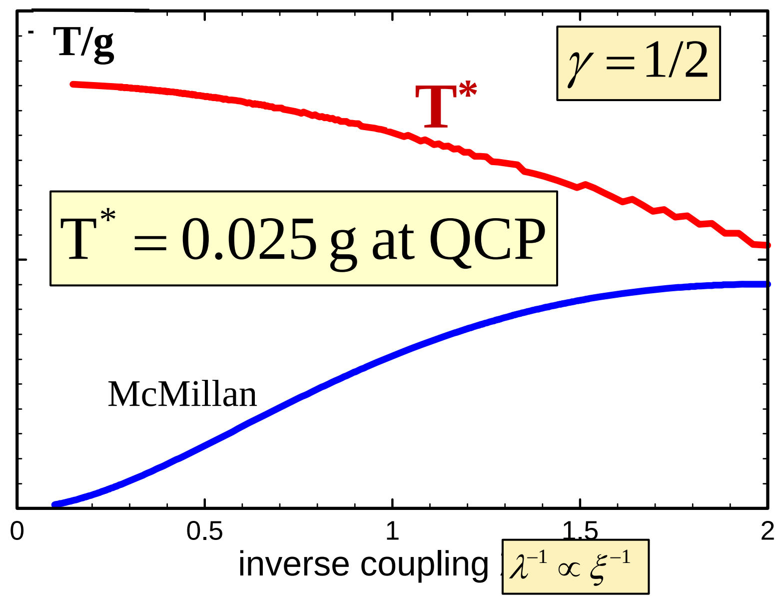
$\gamma = 1$ — **Z=1 pairing problem**

$\gamma = +0 \;\rightarrow \gamma = 1$ — **pairing in the presence of SDW** — Moon, Sachdev

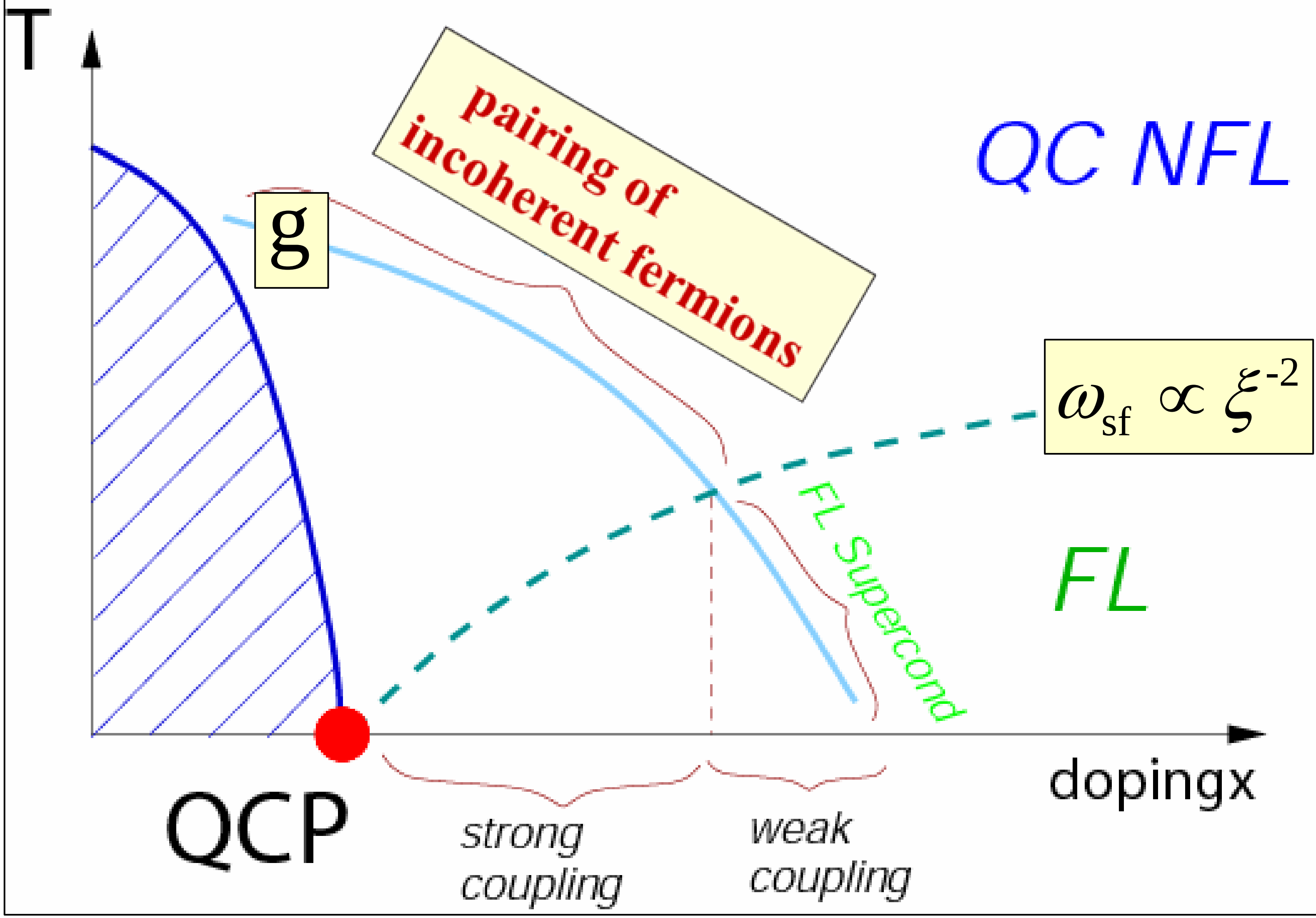
$\gamma \approx 0.7$ — **fermions with Dirac cone dispersion** — Metzner et al

It turns out that for all γ, the coupling (1 + γ)/2 is larger than the threshold





Dome of a pairing instability above QCP



Problems:

1. Calculations are performed within Eliashberg approximation, $\Sigma(k,\omega) \approx \Sigma(\omega)$

Valid when bosons are slow compared to fermions

Bosons (spin fluctuations) are Landau-damped $q_{typ} \sim \sqrt{\omega}$

Free fermions have $q_{typ} \sim \omega$ **, i.e are fast compared to bosons**

Dressed fermions have $q_{typ} \sim \Sigma(\omega) \sim \sqrt{\omega}$ **, i.e. are comparable to bosons**

Large N (actual N=2, N = infinity makes Eliashberg approximation exact)

First order in 1/N: $\chi(q,\omega) \propto \frac{1}{(q^2 + |\omega|)^{\eta}}$ $\eta = 1 - \frac{1}{2N}$

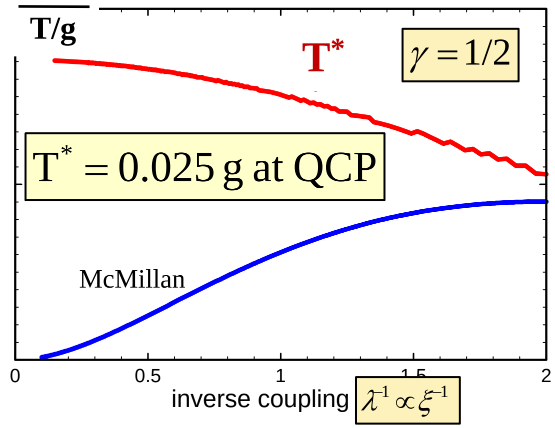
Then, the only change is $\gamma \rightarrow \frac{1}{2} - \frac{1}{2N}$

Apparently, Eliashberg approximation is OK

Problems:

2. The coupling g is assumed to be smaller than $E_F \sim v_F/a$

$$T^* = g\,\Psi_\xi$$

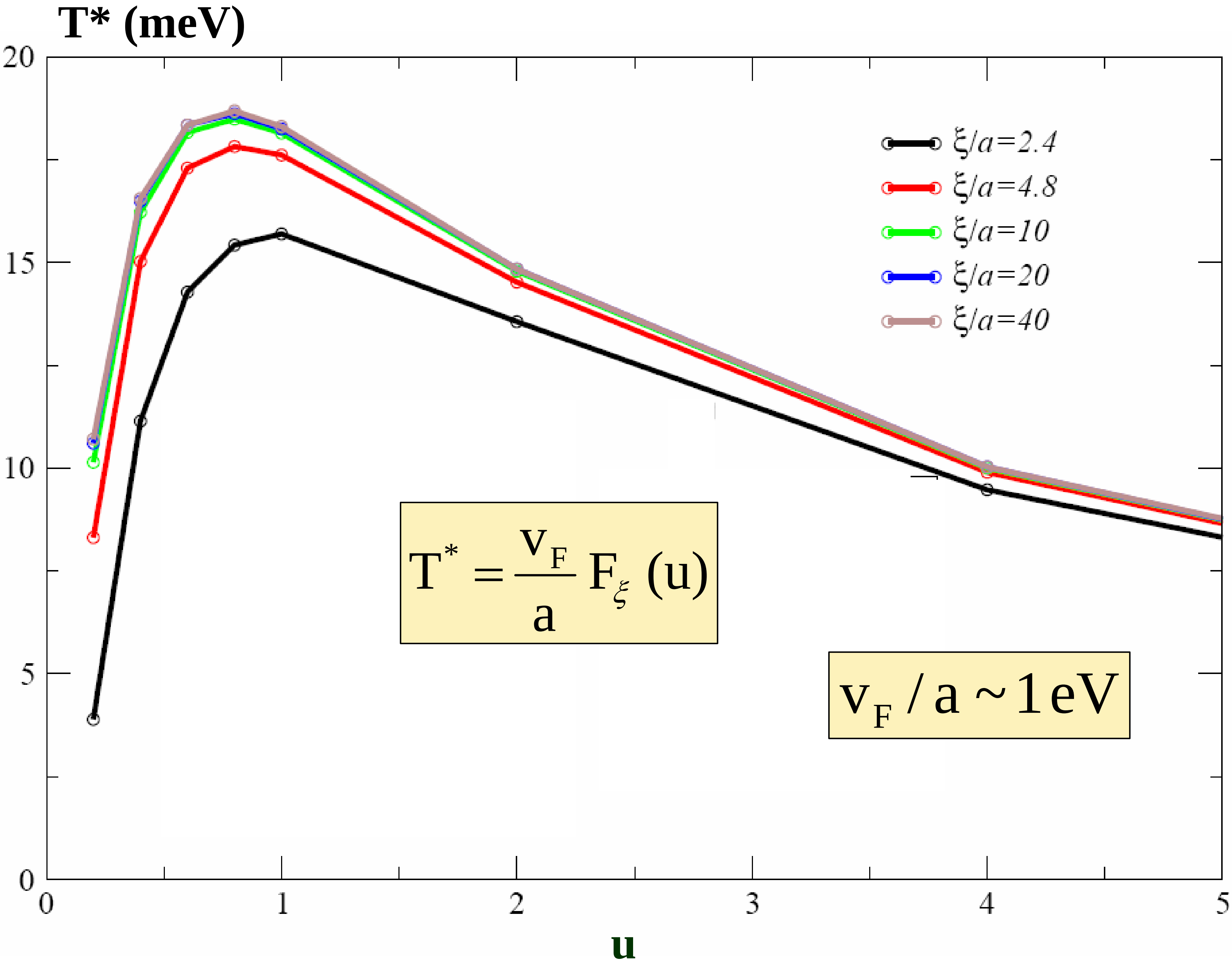


The larger is g, the larger is T*

In general, we have two parameters,

$$u = \frac{g\,a}{v_F} \quad \text{and} \quad \lambda = u\,\xi$$

$$T^* = \frac{v_F}{a} F_\xi(u) \quad \left(\Rightarrow g\,\Psi_\xi \text{ when u is small}\right)$$

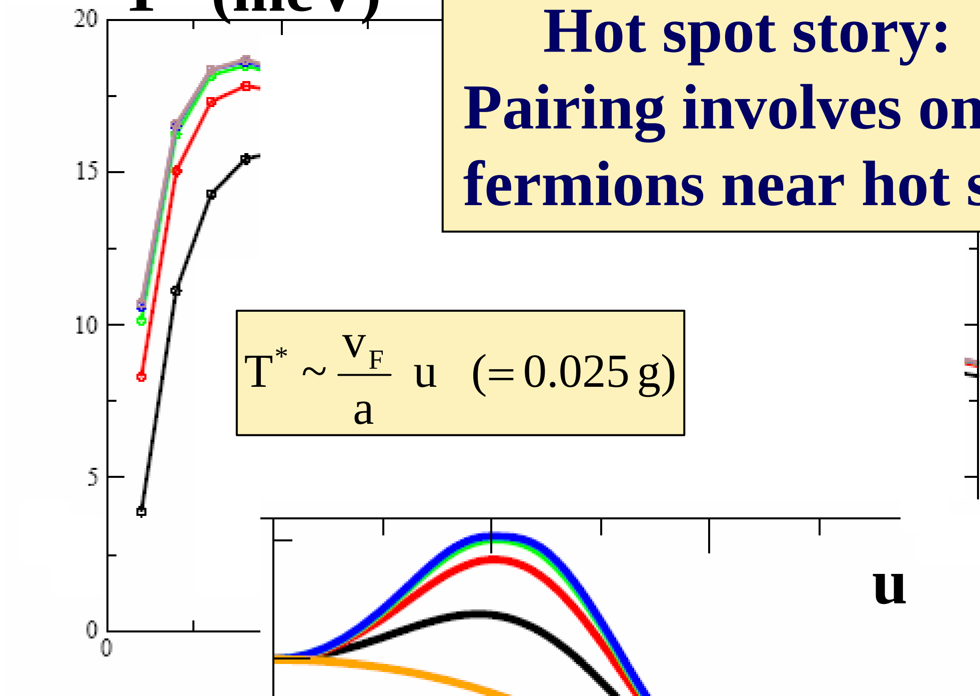


T* (meV)
20
15
10
5
0
0
1
2
3
4
5
u
ξ/a=2.4
ξ/a=4.8
ξ/a=10
ξ/a=20
ξ/a=40
$T^* = \frac{v_F}{a} F_\xi (u)$
$v_F / a \sim 1 eV$

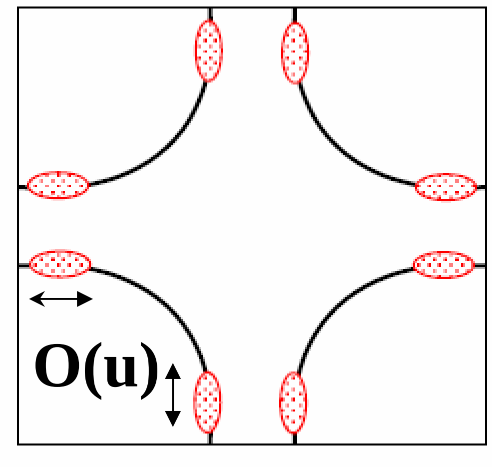
$u < 1, \lambda = u\,\xi > 1$

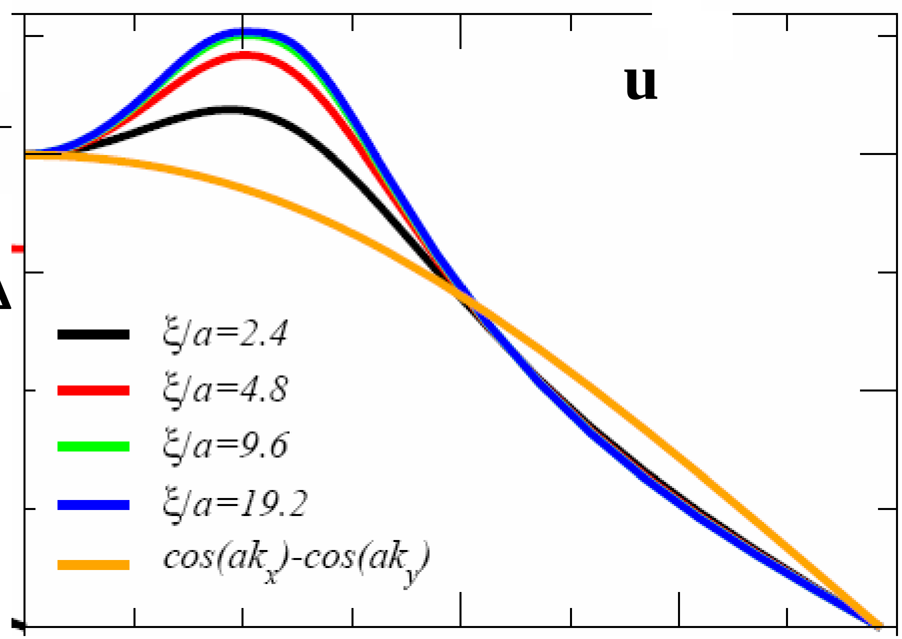
$u \sim g\, a/v_F$

Hot spot story: Pairing involves only fermions near hot spots



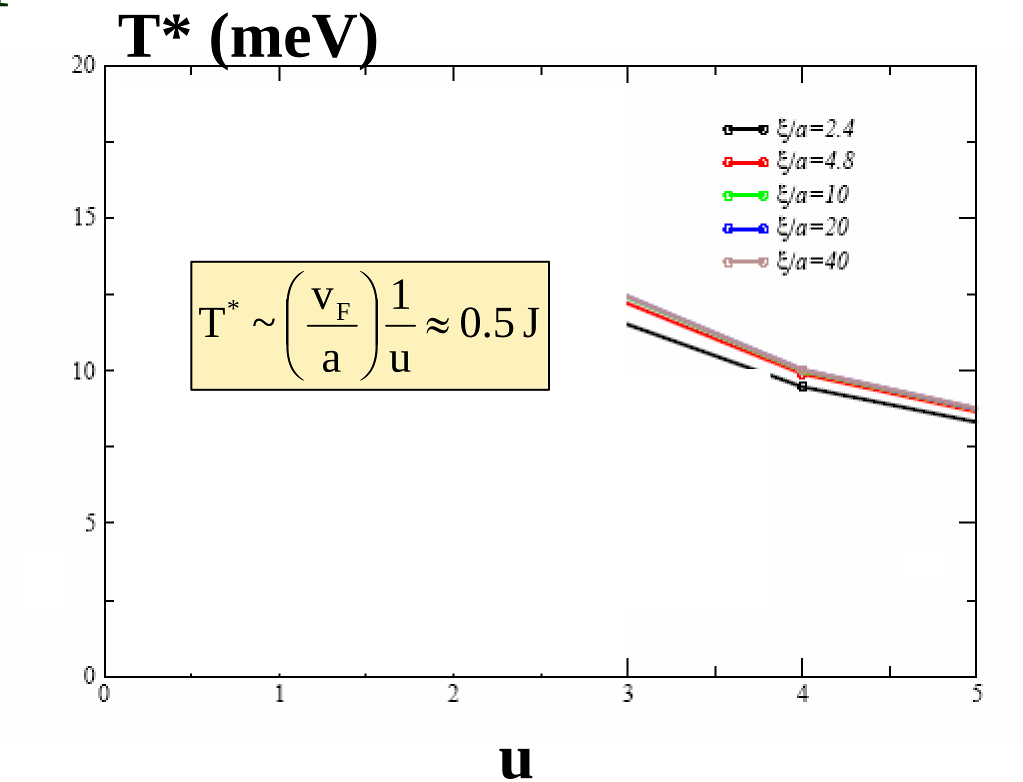
$$T^* \sim \frac{v_F}{a}\, u \quad (= 0.025\, g)$$





The gap is anisortopic, not $\cos k_x - \cos k_y$

Strong coupling, u >1



$$T^* \sim \left(\frac{v_F}{a}\right)\frac{1}{u} \approx 0.5\,J$$

$$\chi_q(\Omega) = \frac{1}{(q-\pi)^2 + \xi^{-2} + \frac{16}{3}u\frac{|\Omega|}{v_F\,a}}$$

d-wave attraction

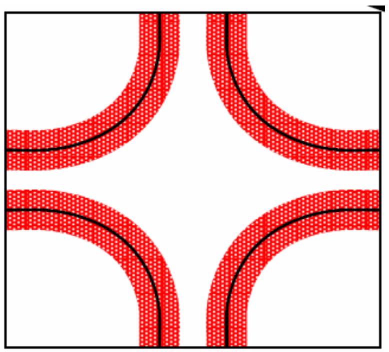
A tendency towards $\chi(\Omega) \sim 1/\Omega$

Balance when $T\,u \sim v_F/a$

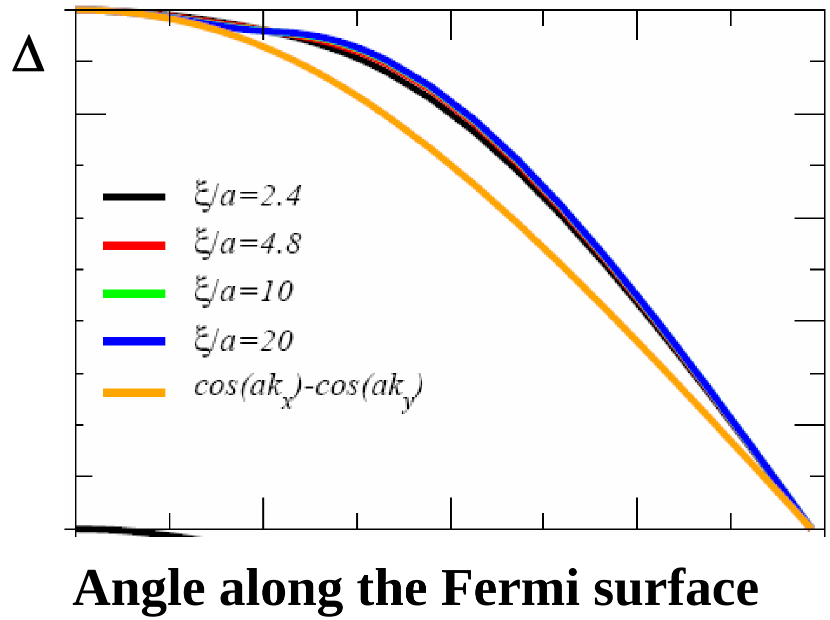
At strong coupling, T* scales with the magnetic exchange J

Strong coupling, u >1

The whole Fermi surface is involved in the pairing

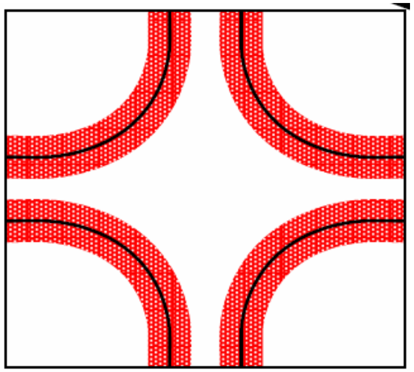


Gap variation along the Fermi surface



Almost cos k_x – cos k_y d-wave gap
(as if the pairing is between nearest neighbors)

Strong coupling, u >1



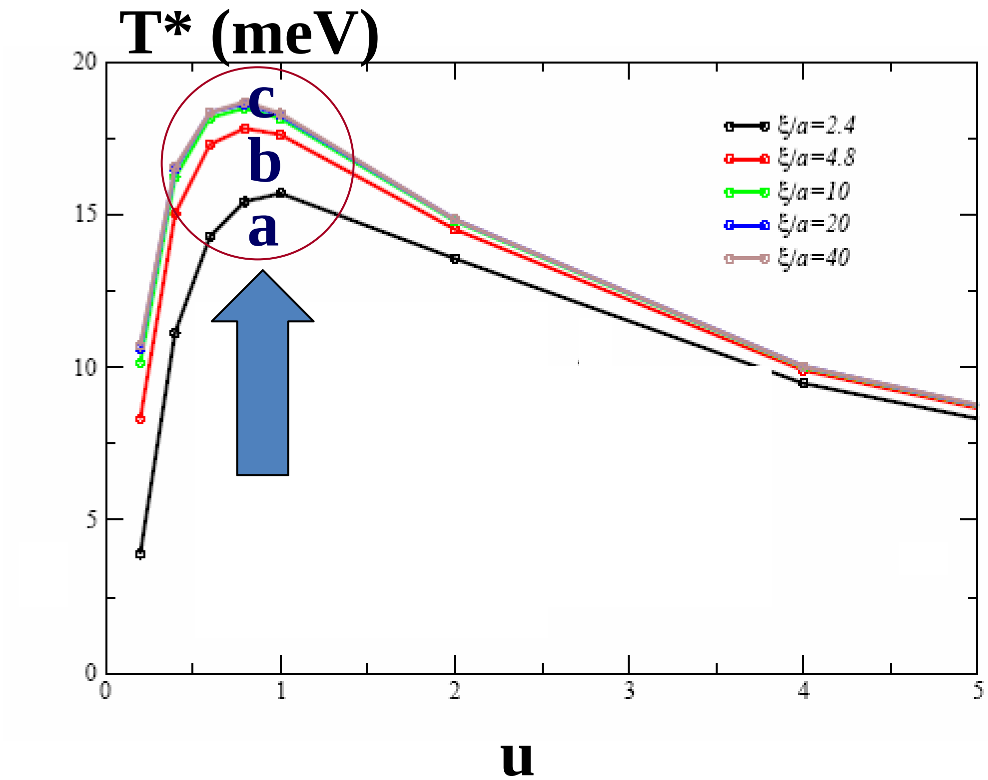
On one hand, the whole Fermi surface is involved in the pairing

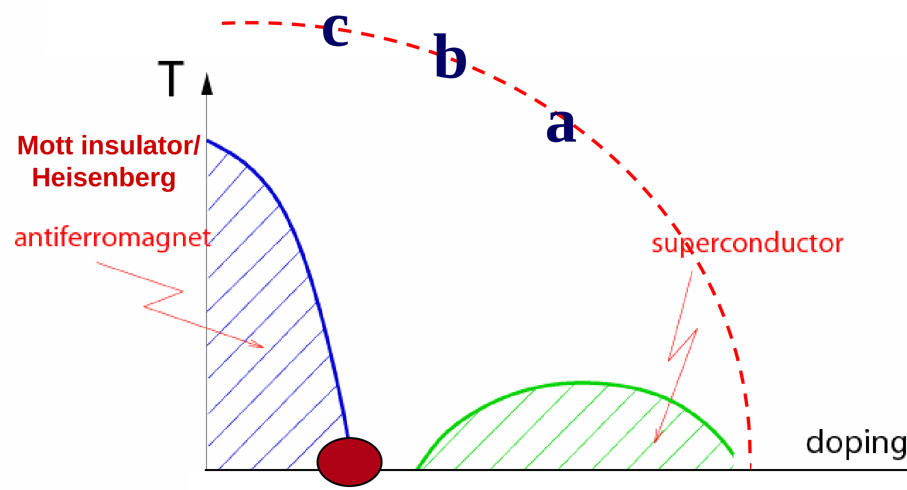
On the other, the fact that T* does not grow with u restricts relevant fermionic states: $\varepsilon_k \approx v_F (k - k_F) \sim J << v_F/a$

$$| k - k_F | \sim \frac{(3-4)\, J}{v_F} \sim 0.1 \frac{\pi}{a} << \frac{\pi}{a}$$

d-wave pairing at strong coupling involves fermions in the near vicinity of the Fermi surface

Intermediate u = O(1)



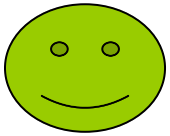


$$T^*_{max} \sim 200 - 250\,K$$

a

Universal pairing scale

Robustness of T^*_{max}



FLEX (similar but not identical to Eliashberg)

Monthoux, Scalapino
Monthoux, Pines,
Eremin, Manske,
Bennemann, Schmalian,
Dahm, Tewordt ….

$$T^* \sim (0.01 - 0.015)\ \frac{v_F}{a} \sim 100 - 150\ \mathrm{K}\ \ \text{for } u \sim 0.25$$

CDA, cluster DMFT

Maier, Jarrell, …
Haule, Kotliar, Capone …
Tremblay, Senechal, ….

$$T^* \sim 0.01 \frac{v_F}{a}\ \ \text{for } u \sim 0.25,\ \ T^* \sim 0.015\ \frac{v_F}{a}\ \ \text{for } u = 0.75$$

FLEX with experimental inputs

Scalapino, Dahn, Hinkov,
Hanke, Keimer, Fink, Borisenko,
Kordyuk, Zabolotny, Buechner

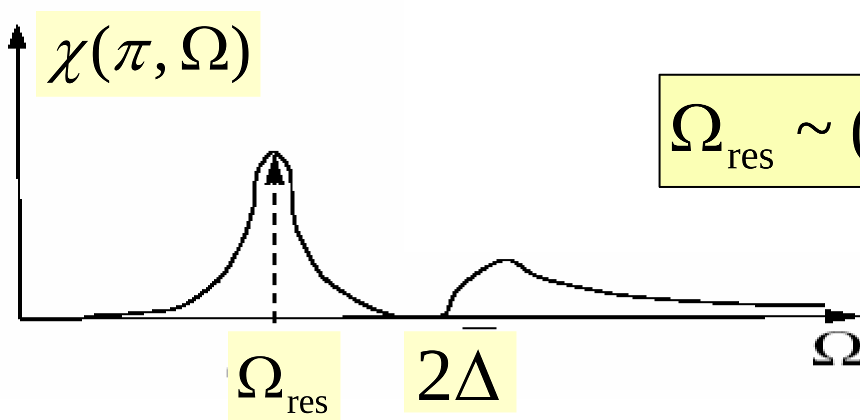
$$T^* \sim 170\ \mathrm{K}$$

5. What would your theory predict in regards to collective modes? Is this even an important question?

Spin resonance, B1g Raman resonance, …

Most important is the spin resonance

Feedback from a d-wave superconductivity on the pairing boson



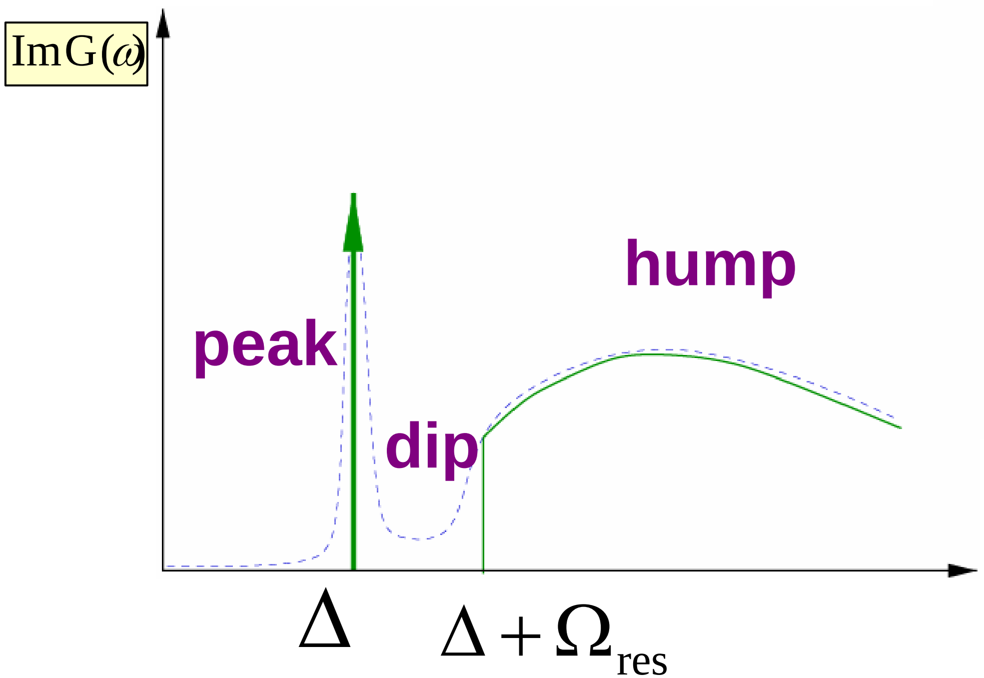
$$\Omega_{res} \sim (g\,\omega_{sf})^{1/2} \sim \xi^{-1}$$

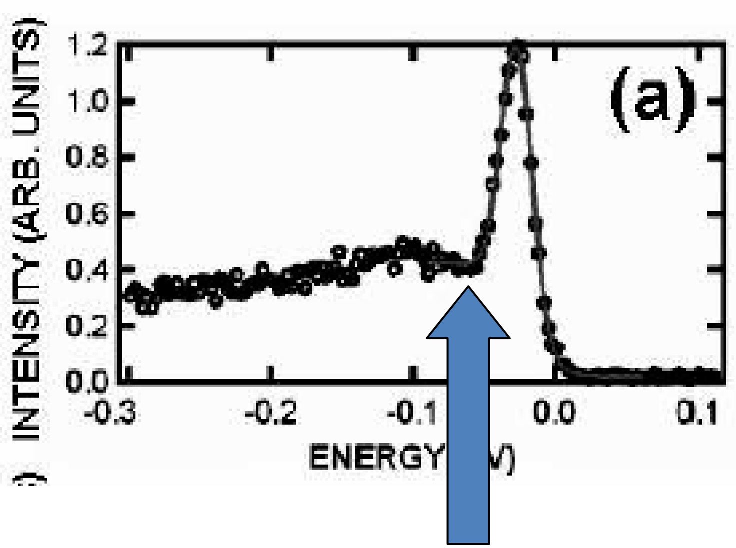
$$\chi(\Omega) \sim \frac{1}{\Omega^2 - \Omega_{res}^2}$$

The pairing boson becomes a mode (+ a gapped continuum)

By itself, the resonance is NOT a fingerprint of spin-mediated pairing, nor it is a glue to a superconductivity –

What must be observable at strong coupling is how the resonance peak affects the electronic behavior, if the spin-fermion interaction is the dominant one





$\Omega_{mode} \sim 38 - 40$ meV

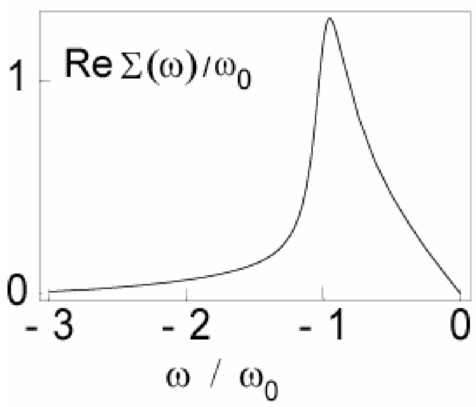
Dispersion anomalies along the Fermi surface

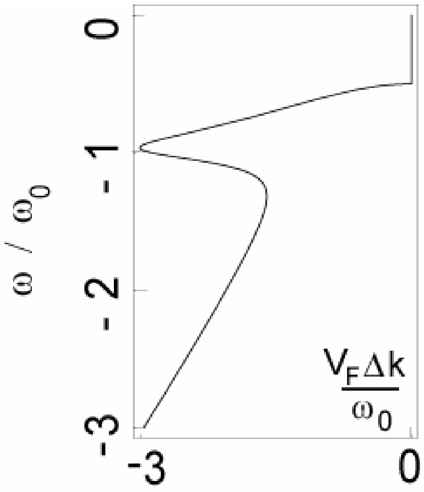
Norman, AC

The self-energy

The dispersion

Antinodal direction

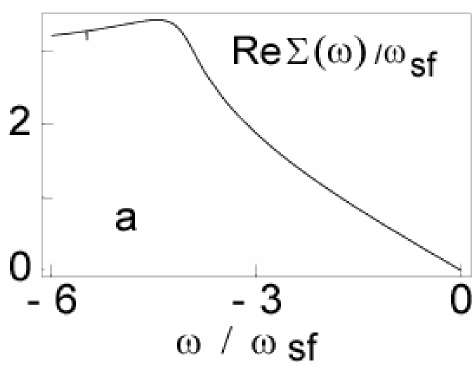


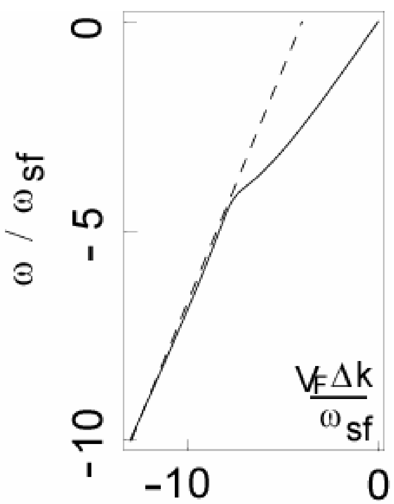


the S-shape dispersion

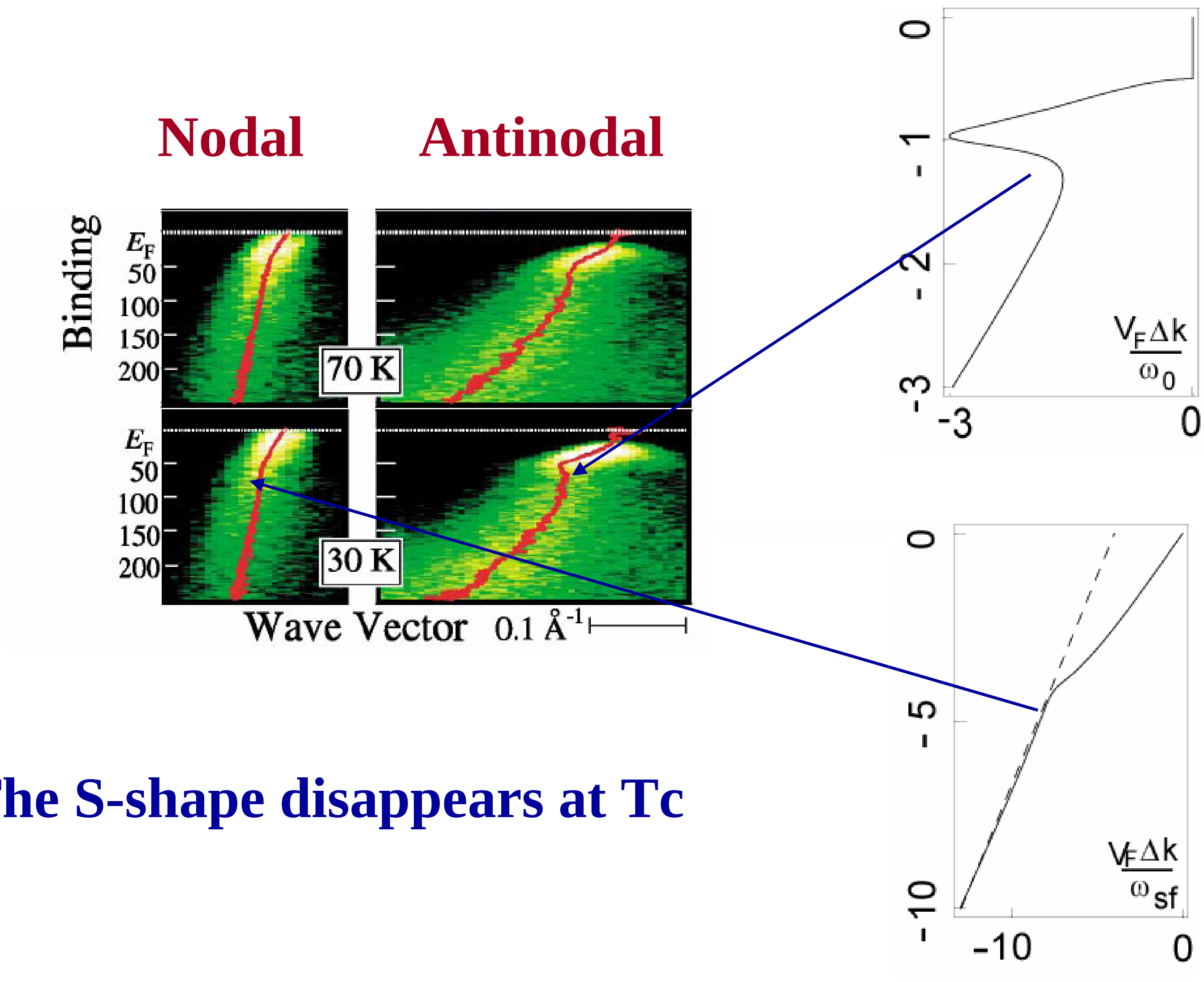
$$\omega_0 = \Delta + \Omega_{res}$$

Nodal direction





The kink



The S-shape disappears at Tc

Another role of the resonance mode: at T=0 we have a superconductor with a low-energy mode,

$$\Omega_{\rm res} \sim g/\lambda \ll \Delta \sim g$$

which is not a fluctuation of the sc order parameter

$$\chi(\Omega) \sim \frac{1}{\Omega^2 - \Omega_{res}^2}$$

attractive at $\Omega < \Omega_{\rm res} < \Delta$

repulsive at $\Omega > \Omega_{\rm res}$

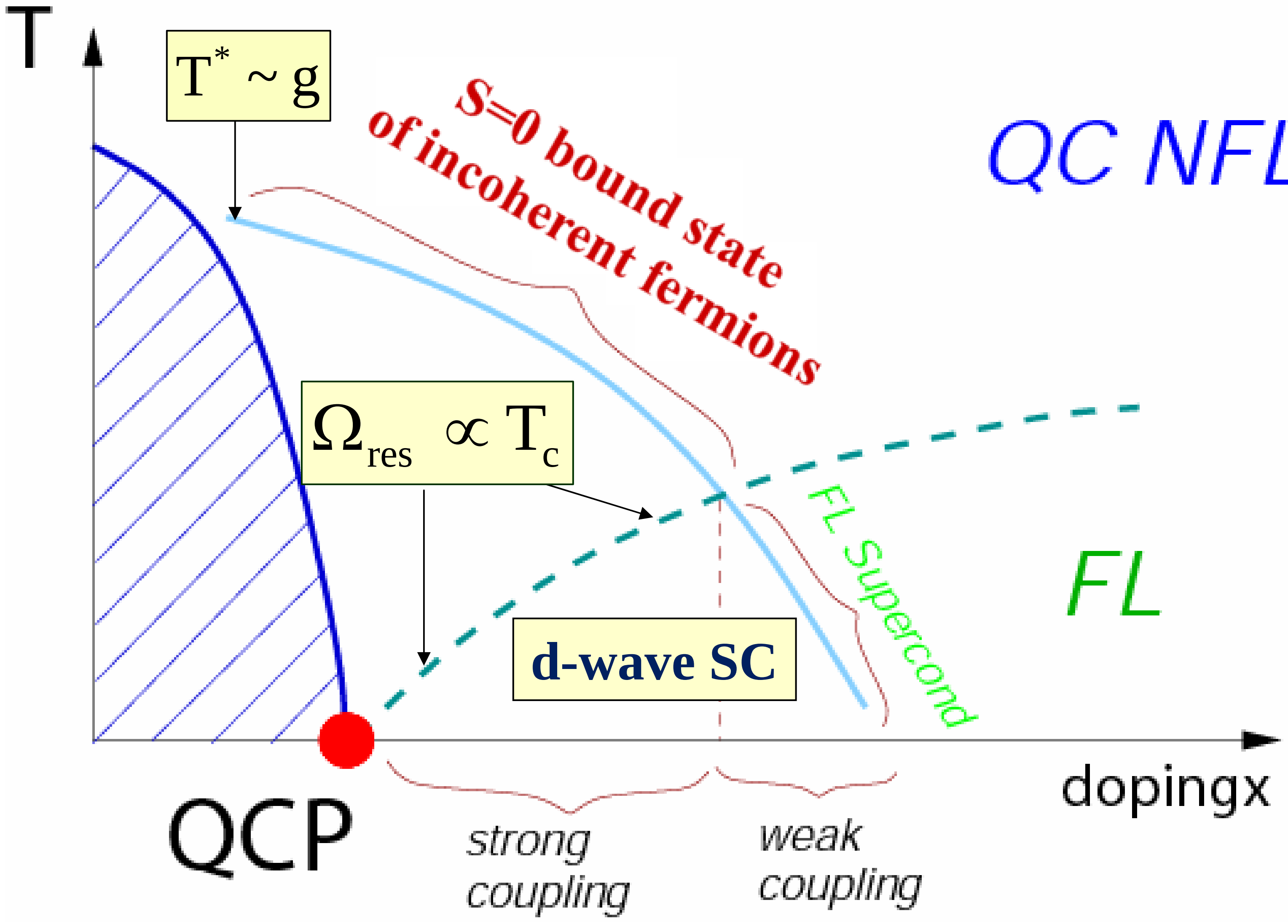
The pairing gap

$$\Delta\,(T=0) \sim T^* \sim g \quad (2\,\Delta(0)/T^* \approx 4)$$

The superconducting stiffness (estimates)

$$\rho_s \sim \Omega_{\rm res} \sim g/\lambda$$

Abanov, AC


T
T* ~ g
S=0 bound state
of incoherent fermions
QC NFL
Ω_res ∝ T_c
FL Supercond
FL
d-wave SC
dopingx
QCP
strong
coupling
weak
coupling

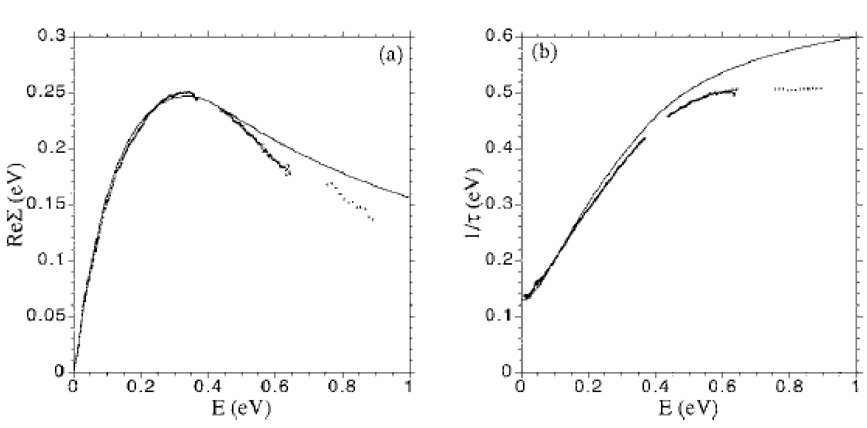
3. If your theory ignores phonons, can you really get away with that? Do you think phonons are even relevant?

4. What is the spectroscopic signatures predicted for your theory? Is a McMillan-Rowell inversion or related procedure possible for your theory? Is this question meaningful?

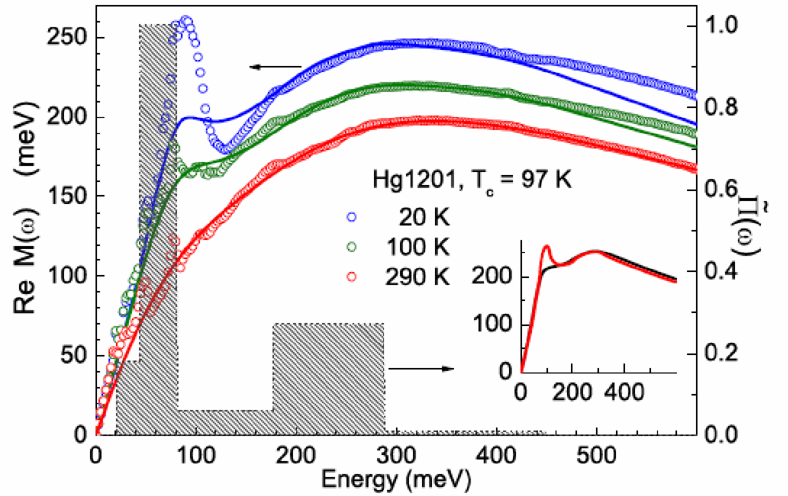
The effective

$$\alpha^2 F(\omega) = \oint_{FS} \mathrm{Im}\,\chi(q,\omega)\, d^2 q$$

Above Tc : "spin" interaction is with the continuum.



Norman, AC

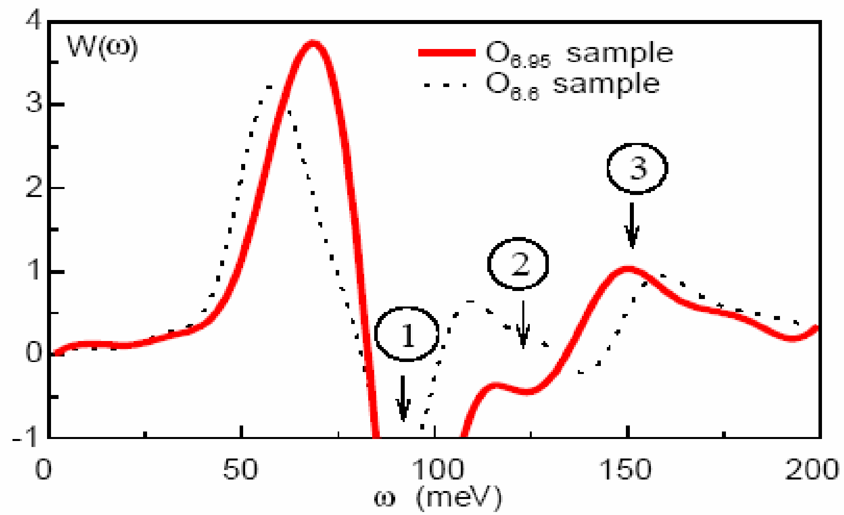


Van Heumen et al

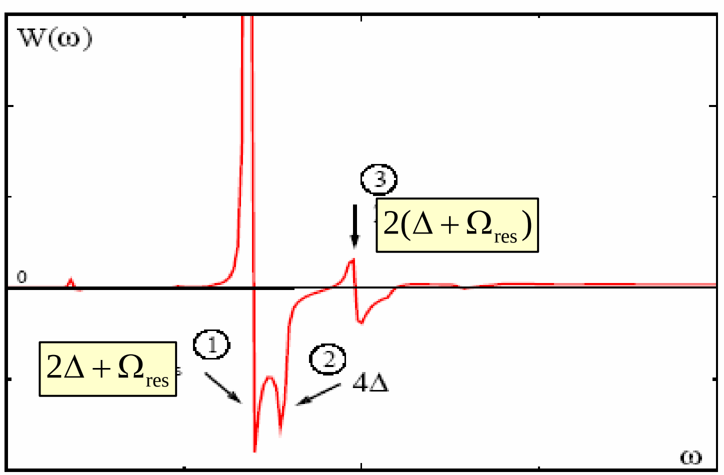
Below Tc --mode surely affects optical conductivity

$$W(\omega) = \frac{d^2}{d\omega^2}\left[\omega \,\mathrm{Re}\, \frac{1}{\sigma(\omega)}\right]$$

YBCO6.95



Theory



$$\Delta \approx 30\,\mathrm{meV}, \quad \Omega_{res} \approx 40\,\mathrm{meV}$$

Basov et al,
Timusk et al,
J. Tu et al…..

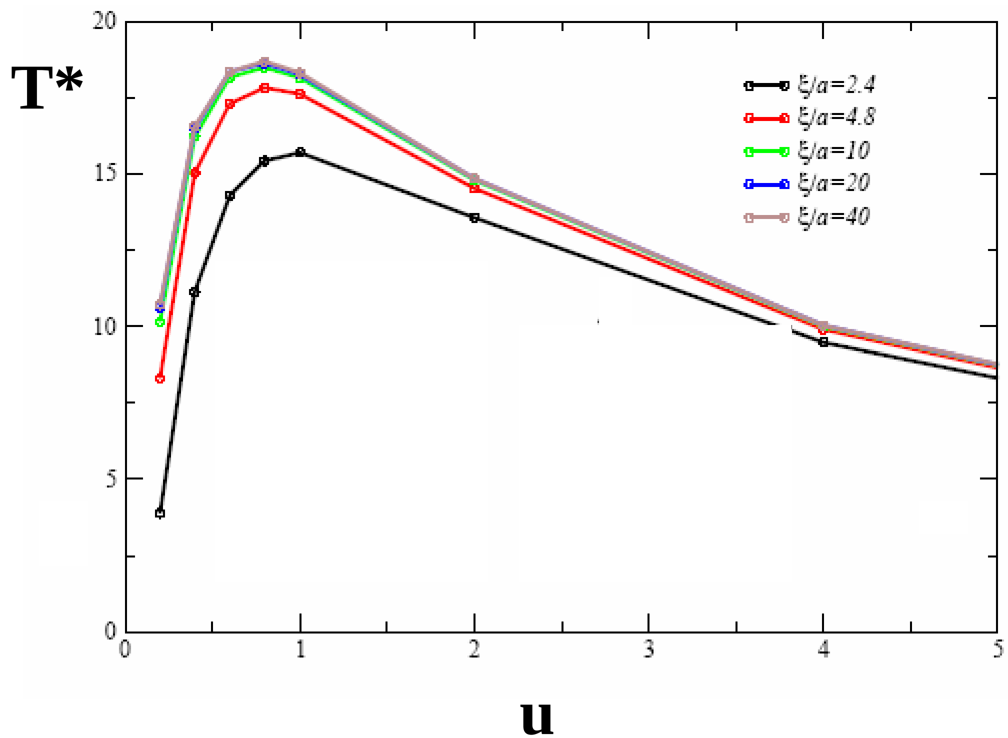
Abanov et al
Carbotte et al

Conclusions

Effective interaction between low-energy fermions, mediated by a collective degree of freedom

Some phenomenology is unavoidable (or RPA)

Once we set the model, to get $\Sigma(\omega)$ and the pairing is a legitimate theoretical issue (and not only for the cuprates)



Universal pairing scale

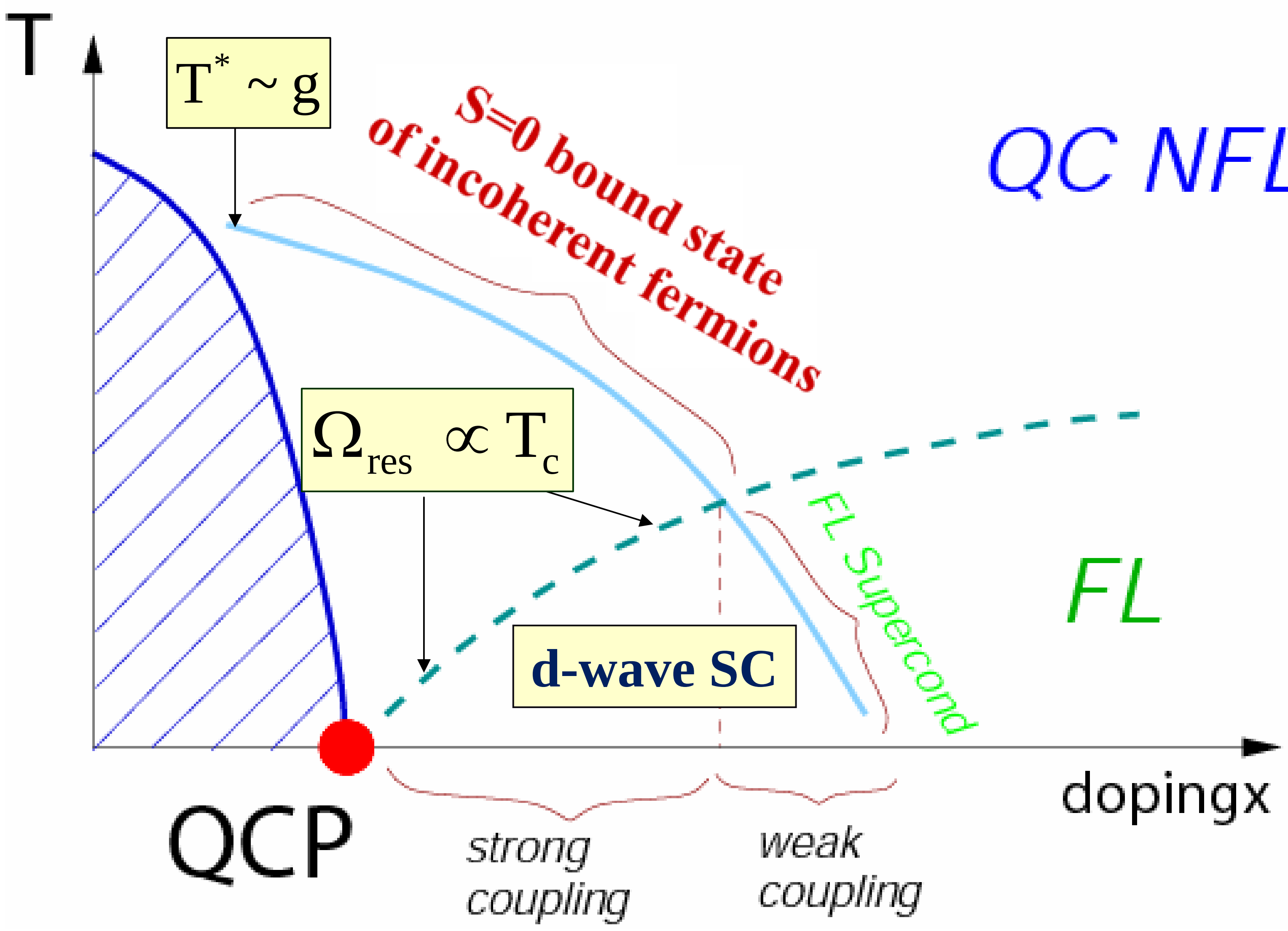
$$T^*_{max} \sim 0.02 \frac{v_F}{a}$$

The gap

$$\Delta(k) \approx \Delta\,(\cos k_x - \cos k_y)$$

Low-energy collective mode

$$\Omega_{res} \sim \Delta / \lambda < \Delta$$


T
$T^* \sim g$
S=0 bound state
of incoherent fermions
QC NFL
$\Omega_{res} \propto T_c$
FL Supercond
FL
d-wave SC
QCP
dopingx
strong
coupling
weak
coupling

THANK YOU