Looking beyond horizons via AdS/CFT

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1. Motivation
2. Probing Schwarzschild-AdS singularity via CFT correlators
   - Review of FHKS — hep-th/0306170, rev. in hep-th/0401138
   - Collapsed vs. eternal BH
3. Inflation in AdS/CFT — hep-th/0510046
   - Constructions of deSitter in Schw-AdS
   - Entropy paradox
4. Further geometrical puzzles/complications
5. Summary

Motivation

Key question of quantum gravity:

What is the fundamental nature of spacetime?

Invaluable tool in recent years: AdS/CFT correspondence

string theory in $AdS \times S$

$\leftrightarrow$ gauge theory on boundary

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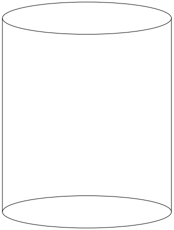

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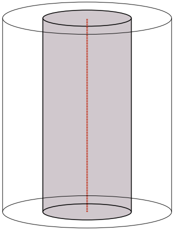

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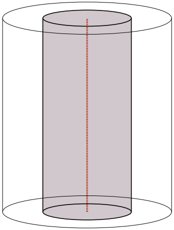

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Need to probe AdS/CFT dictionary further:

- what is the nature of (spacelike) singularity?
- what is the CFT description of causal structure?
- (how) does the CFT describe physics behind an event horizon?

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Key questions in cosmology:

Physics in the early universe?
What is the fate of our universe?

Need to understand inflation
∃ difficulties with string theory in de Sitter background

Can we cast this into AdS/CFT framework?
We will

- construct asymp. AdS spacetimes with region of eternally inflating dS
- argue that CFT correlators are sensitive to dS region

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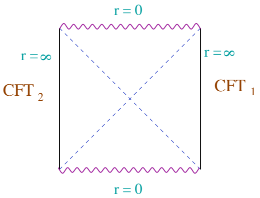
BTZ and CFT correlators



BTZ = Schw-AdS black hole in 3-d
much studied system

Balasubramanian, Louko, Marolf, Ross

Maldacena; KOS

using ▸ thermofield formalism

scalar field w/ mass $m \to \infty$

$$\langle \Phi(x_1)\, \Phi(x_2) \rangle \sim e^{-m\,\mathcal{L}}$$

w/ $\mathcal{L}$ = regularized proper length along geodesic betw. x_1 and x_2

main point KOS

correlators between operators in CFT_1 and CFT_2
⇝ access to full spacetime

▸ precursor gedanken experiment

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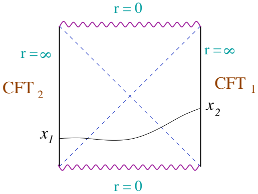

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Schwarzschild-AdS black hole

$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2$$

in $d \geq 3$ dimensions, size of AdS $= R$, size of BH $= r_+$

$$f(r) \equiv \frac{r^2}{R^2} + 1 - (\frac{r_+^2 + R^2}{R^2})\,(\frac{r_+}{r})^{d-3}$$

Symmetric spacelike geodesics (for $d = 5$):

$\exists$ critical time $t = t_c$
after which $\nexists$ sym. sp-like geods.

$$t_c = -\frac{1}{T_H}\,\frac{\sqrt{r_+^2 + R^2}}{r_+}$$

cf. 3-D

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$$ds^2 = -f(r)\,dt^2 + \frac{dr^2}{f(r)} + r^2\,d\Omega^2$$

in $d \geq 3$ dimensions, large BH limit $f(r) \equiv r^2 - \frac{1}{r^{d-3}}$

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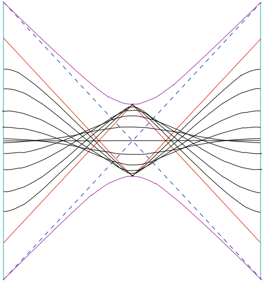

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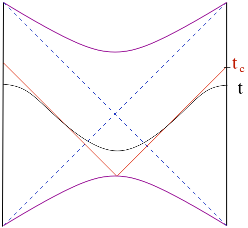
Expectations for CFT correlators

Consider correlator $\langle \Phi\Phi \rangle(t)$

$$\langle \Phi\Phi \rangle(t) \sim e^{-m\mathcal{L}(t)} \sim \frac{1}{(t - t_c)^{2m}}$$

$\Rightarrow$ singularity in CFT correl. fn. as $t \to t_c$



detour into sublety

We can view correl. $\langle \Phi\Phi \rangle$ between CFT_1 and CFT_2 as

$$\langle \Phi\Phi \rangle(t) \equiv \langle \Phi(-t + \frac{i\beta}{2})\, \Phi(t) \rangle_\beta$$

correl. in single CFT in thermal state at inverse temperature β

Main points of FHKS

technical results of FHKS

most importantly:

$\exists$ distinct (albeit subtle) signals of BH singularity in CFT correlators

- Properties of singularity are computationally accessible, given the CFT data $\langle \Phi\Phi \rangle$
- t_c singularity persists to all orders in $\frac{1}{m}$, g_s, and α'
  $\rightsquigarrow$ sensitive to stringy & quantum behaviour near BH singularity
- All this rests on analyticity...

Collapsed vs. eternal black hole

At late times, collapsed BH looks like eternal BH.

$\leftrightarrow$ CFT is in (approx.) thermal state

$\Rightarrow$ similar $\langle \Phi\Phi \rangle (t \to \infty)$

Puzzle: would we see a singularity in $\langle \Phi\Phi \rangle$?

$\exists$ only 1 boundary

$\Rightarrow$ no t_c geodesic (between 2 bdys)

$\Rightarrow$ no t_c singularity in $\langle \Phi\Phi \rangle$

$\langle \Phi(-t + \frac{i\beta}{2})\, \Phi(t) \rangle =$ finite

instead, signal of singularity in

$\langle \Phi(t, \Omega)\, \Phi(t', -\Omega) \rangle$

Hence CFT correls $\langle \Phi\Phi \rangle$ would see difference between collapsed BH and eternal BH.

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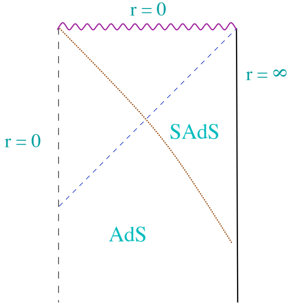

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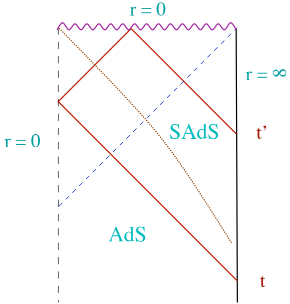

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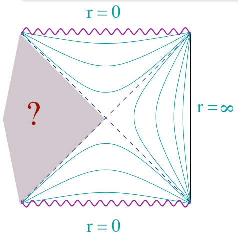
Other geometries?

How far can we push this?

Since the CFT correlators are sensitive to differences in geometry "far away," could we probe differences between spacetimes which are (almost) identical everywhere outside the horizon?

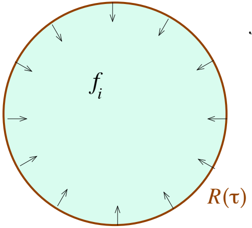


What sorts of spacetimes can we have in the "?" region?

- FRW ("Wheeler bag of gold")
- flat
- de Sitter

Thin domain wall construction

Consider two metrics patched across a spherical shell:



$$ds_\alpha^2 = -f_\alpha(r)\, dt_\alpha^2 + \frac{dr^2}{f_\alpha(r)} + r^2\, d\Omega^2$$

where $\alpha = i$ (inside), or o (outside)
Induced metric on shell's world-volume:

$$ds_{shell}^2 = -d\tau^2 + R(\tau)^2\, d\Omega^2$$

Junction conditions $\rightsquigarrow$ EOM of shell:
radial motion in effective potential V_{eff}:

$$\dot{R}^2 + \underbrace{f_o(R) - \frac{(f_i(R) - f_o(R) - \kappa^2 R^2)^2}{4\,\kappa^2 R^2}}_{V_{eff}(R)} = 0$$

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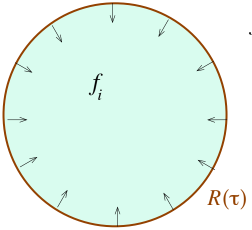

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$$\sqrt{\dot{R}^2 + f_i(R)} - \sqrt{\dot{R}^2 + f_o(R)} = \kappa\,R$$

with $\dot{R} \equiv \frac{dR(\tau)}{d\tau}$
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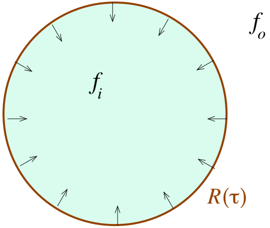

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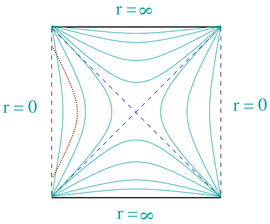
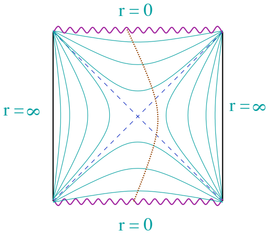
Example construction

On each spacetime (given by f_i and f_o),
shell's trajectory is $r = R(\tau)$.
Excise spacetime f_i outside (to right of) the shell
and spacetime f_o inside (to left of) the shell,
and patch together across shell.

Example: bubble of de Sitter inside Schwarzschild-AdS:





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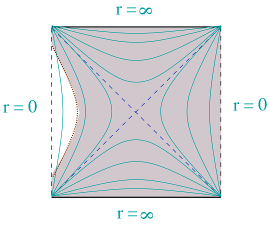

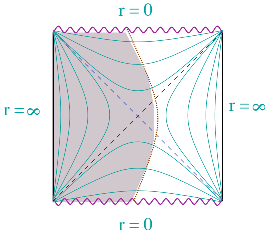

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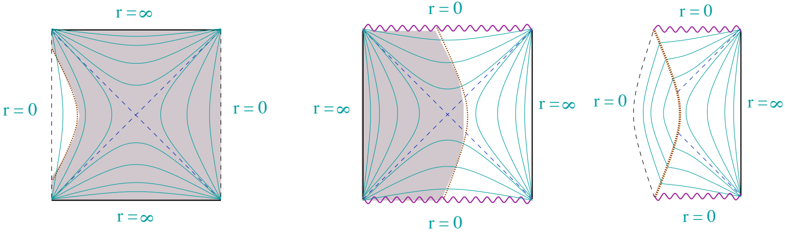


Possibilities for de Sitter / Schw-AdS junction

For de Sitter, $f_i(r) = 1 - \lambda\, r^2$
for Schw-AdS, $f_o(r) = r^2 + 1 - \frac{\mu}{r}$

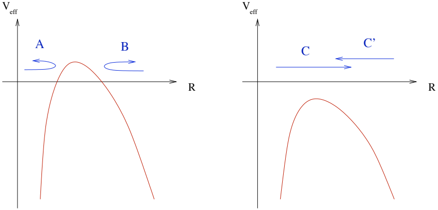
$$V_{eff}(r) = -\frac{(\lambda + \kappa^2 - 1)^2 + 4\,\lambda}{4\,\kappa^2}\, r^2 + 1 + \mu\, \frac{(1 + \lambda - \kappa^2)}{2\,\kappa^2}\, \frac{1}{r} - \frac{\mu^2}{4\,\kappa^2}\, \frac{1}{r^4}$$

where $\lambda \sim$ cosmol. const.
$\mu \sim$ BH mass
$\kappa \sim$ bubble wall tension

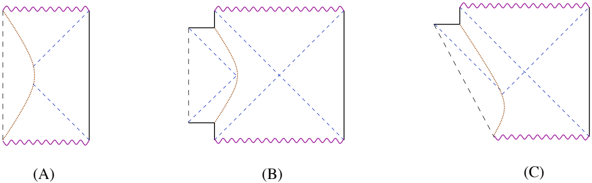
Possibilities for de Sitter / Schw-AdS junction

Possible effective potentials:

▸ actual plots



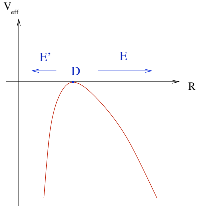
Corresponding Penrose diagrams:



Possibilities for de Sitter / Schw-AdS junction

Possible effective potentials: (fine-tuned V_{eff})

actual plots



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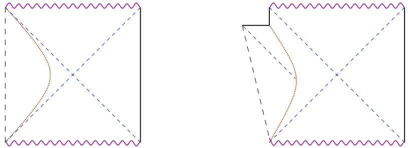
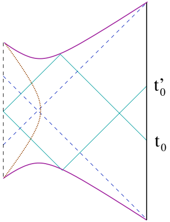


(D) (E)

CFT signatures of de Sitter



- geodesics bounce off singularity
- geodesics pass through origin $\Omega \to -\Omega$
- geodesics bounce off dS scri

$\Rightarrow$ geodesics return to same boundary . . .

$\Rightarrow$ expected singularity in (analytically continued) correl. function

$$\langle \Phi(t,\Omega)\Phi(s,-\Omega)\rangle \sim \frac{1}{(s-t'(t))^{2m}}$$

conclusion:

CFT correlators allow us to see bulk ST geometry

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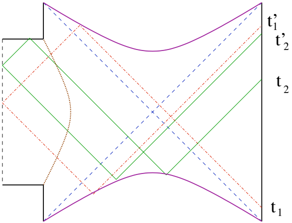

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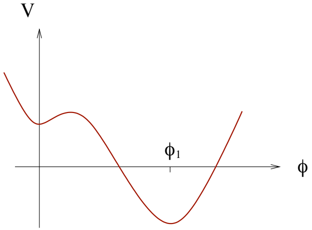
$$\langle \Phi(t,\Omega)\Phi(s,-\Omega)\rangle \sim \frac{1}{(s-t'(t))^{2m}}$$

conclusion:

CFT correlators allow us to see bulk ST geometry (including the late-time eternal inflation region)

de Sitter via scalar field in potential

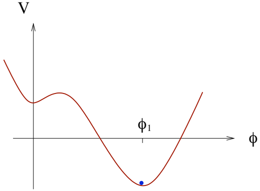
Physically more realistic case: instead of domain wall, consider scalar field Φ in a given potential $V(\Phi)$.
(motivated via string landscape...)

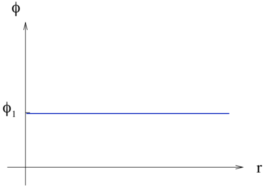


$\Phi = \Phi_1 \quad \rightsquigarrow$ AdS minimum
(defines a CFT...)

$\Phi = 0 \quad \rightsquigarrow$ dS minimum

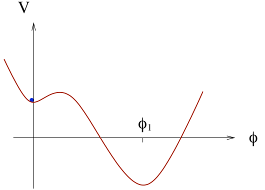
de Sitter via scalar field in potential



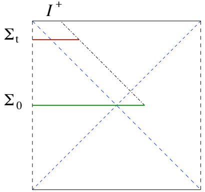


Field in AdS minimum
$\rightsquigarrow$ AdS

de Sitter via scalar field in potential



Field in dS minimum ⇝ dS (in domain of dependence of initial slice Σ)



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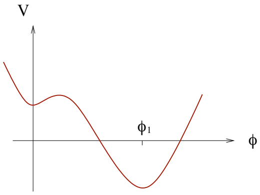
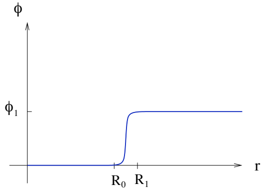
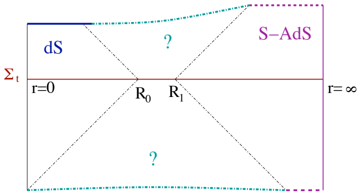




Interpolating field
$\rightsquigarrow$ asymp. AdS spacetime
with dS region

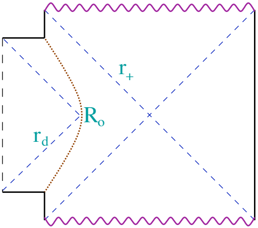


Note: dS boundary must be causally disconnected from AdS boundary

argument

Entropy paradox

For all time-symmetric configurations, the shell reaches extremal size at $\tau = 0$; denote $R(\tau = 0) \equiv R_0$.



$$R_0 \leq r_d$$

$$R_0 \geq r_+$$

$$\Rightarrow \qquad r_d \geq r_+$$

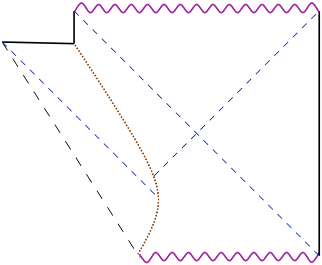
$\Rightarrow$ dS horizon entropy $>$ (can be $\gg$) black hole entropy ($\sim$ active DOFs in CFT)

Puzzle:

How can CFT state w/ $e^{S_{BH}}$ active degrees of freedom describe large inflating region w/ $e^{S_{dS}} \gg e^{S_{BH}}$ degrees of freedom?

Possible resolutions

- ∃ (time-asymmetric) geometries with $S_{dS} < S_{BH}$



- de Sitter entropy has different interpretation (cosmol. horizon is observer-dependent)
- CFT states assoc. w/ inflating regions in AdS are mixed:

  active DOF in CFT are entangled w/ DOF in inflating region
  S_{BH} is a measure of this entanglement

$$S_{BH} = Tr(\rho \log \rho)$$

(with $\rho =$ boundary density matrix)

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argument for mixed state

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- CFT states assoc. w/ inflating regions in AdS are mixed:

  active DOF in CFT are entangled w/ DOF in inflating region S_{BH} is a measure of this entanglement

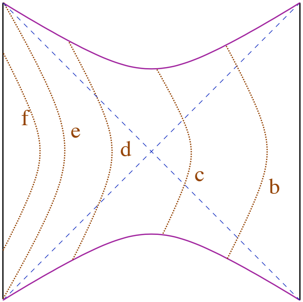
$$S_{BH} = Tr(\rho \log \rho)$$

  (with ρ = boundary density matrix)

argument for mixed state

Progression of spacetimes

Consider a family of solutions obtained by letting the wall pass progressively further left on Schw-AdS Penrose diagram



▸ causal features in progression

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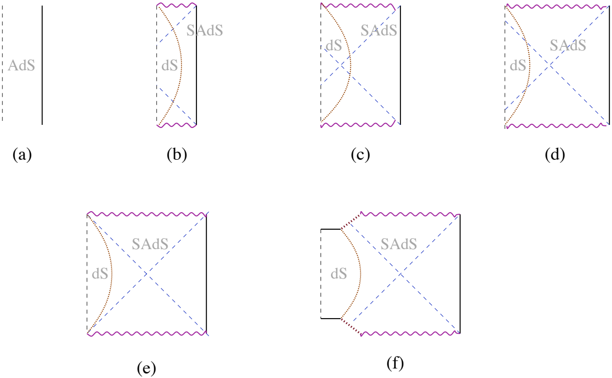


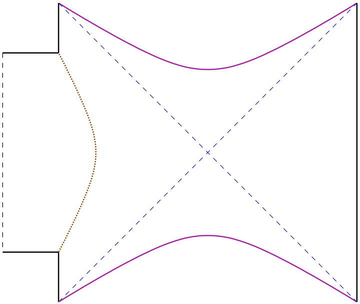
▸ causal features in progression

When is CFT dual described by a mixed state?

Geometrical complications and open questions

How realistic is the thin wall geometry?

- shell
- left AdS boundaries
- black hole singularity
- de Sitter scri



thin shell Penrose diagram

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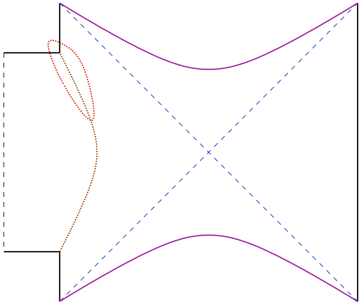

shell could be subject to instabilities

- dynamical breaking of spherical symmetry

  Cf. Aguirre & Johnson

- spreading (shell becomes thick)

Geometrical complications and open questions

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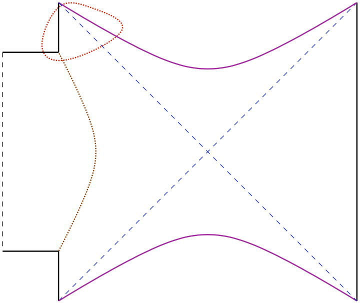

no left AdS bdy
– probably replaced by (almost null) FRW-like singularity

Geometrical complications and open questions

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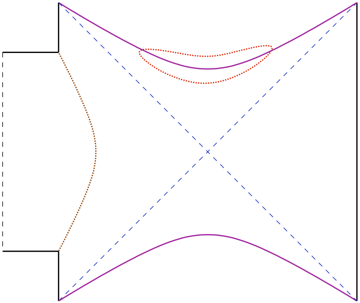

black hole singularity $\rightsquigarrow$
Mixmaster-like behaviour
(no longer clear that geodesics bounce...)

Geometrical complications and open questions

How realistic is the thin wall geometry?

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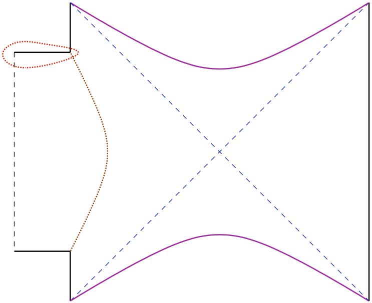
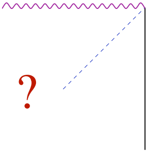


de Sitter fragmentation

Summary (probing bulk spacetime)

- (Spatial dependence of) CFT correlators contains lots of information about the bulk geometry
- Despite very similar bulk (outside horizon, late time) behaviour, CFT correlators can have quite different properties



- Horizon is not an obstacle to probing geometry via CFT
  - can probe BH singularity
  - can probe dS scri
  - but can not probe cosmol. (FRW) singularity in this way

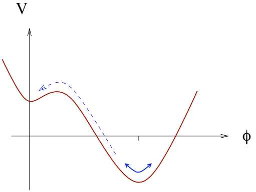
reason: BH vs. FRW

- All this rests on analyticity

Summary (inflation)

Potential framework to study inflation in string theory (AdS/CFT)

- string landscape with dS and AdS vacua
  $\rightsquigarrow$ stable AdS min. described by bdy CFT
- inflating regions involve excitations around AdS
  $\rightsquigarrow$ excitations of CFT



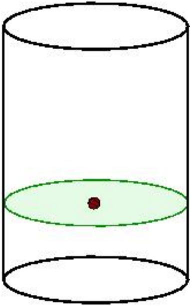
$\Rightarrow$ CFT contains some information about dS

- bulk containing inflating regions (dS) described by mixed state (density matrix) in CFT

consequences of mixed state

Gauge theory can see inside black holes

- Start with pure AdS
  measure event by means of a precursor
- After measurement, collapse a shell
  with sufficiently large energy ...
- ... that event horizon of resulting black hole
  encompasses the measured event



Conclusion (V.H.)

global nature of event horizon
and gauge theory acting as a nonlocal observer
$\Rightarrow$ AdS/CFT probes physics inside horizon

back

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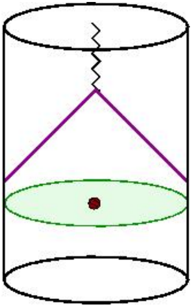

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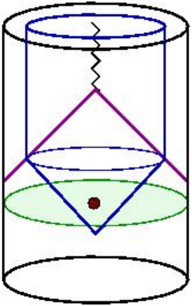

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back

Thermofield formalism

Cons. tensor product of 2 copies of the original CFT:

$$\mathcal{H} = \mathcal{H}_1 \times \mathcal{H}_2$$

Construct a pure, entangled state ($\rightsquigarrow$ Hartle-Hawking WF)

$$|\psi\rangle \sim \sum_n e^{-\frac{\beta E_n}{2}} \, |E_n\rangle_1 \otimes |E_n\rangle_2$$

Reproduces thermal correlators:

$$\langle\psi|\mathcal{O}_1|\psi\rangle \sim Tr[\rho_\beta \, \mathcal{O}_1]$$

but because of entanglement,

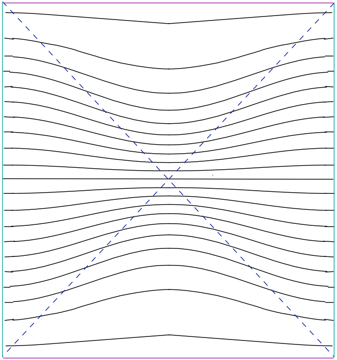
$$\langle\psi|\mathcal{O}_1 \, \mathcal{O}_2|\psi\rangle \neq 0$$

(Thermal state counting entropy arises from entanglement of the pure entangled Hartle-Hawking state.)

back

BTZ geometry

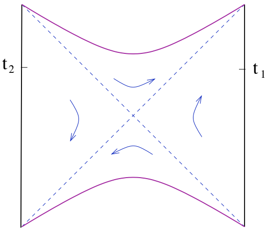
cf. 3-D:



nothing special happens:
symmetric geods $\exists \; \forall \; t$
insensitive to singularity

back to 5-D

Puzzle: CFT correl's can't diverge



$$t_1 = t$$

$$t_2 = -t + \frac{i\,\beta}{2}$$

$$\langle \Phi\Phi \rangle(t) \equiv \langle \Phi(-t+\frac{i\beta}{2})\Phi(t)\rangle_\beta = \sum_{n,m} e^{-\frac{\beta}{2}(E_n+E_m)}\, e^{2\,i\,t\,(E_n-E_m)}\ |\Phi_{nm}|^2$$

$$|\ \langle \Phi\Phi \rangle(t)\ | \leq \langle \Phi\Phi \rangle(0) < \infty$$

$\Rightarrow$ There can't be the expected singularity in the correlator $\langle \Phi\Phi \rangle$!

What went wrong?

Subtlety: branch structure of $\mathcal{L}(t)$

captured by

$$t = \frac{1}{2}\ln\left(\frac{\frac{1}{2}E^2 - E + 1}{\sqrt{1+\frac{1}{4}E^4}}\right) - \frac{1}{2}i\ln\left(\frac{-\frac{1}{2}E^2 + iE + 1}{\sqrt{1+\frac{1}{4}E^4}}\right)$$

$$\mathcal{L} = \ln\left(\frac{2}{\sqrt{1+\frac{1}{4}E^4}}\right)$$

at $E \to 0$, $\quad \begin{array}{l} t(E) \sim E^3 \\ \mathcal{L}(E) \sim -E^4 \end{array} \quad \Rightarrow \quad \mathcal{L}(t) \sim -t^{4/3}$

$\Rightarrow \quad \exists$ a branch cut

geometrically corresponds to 3 coincident geodesics at $t = 0$.

Can see this explicitly by resolving the branch cut

geometrical resolution

$\mathcal{L}(t) \rightsquigarrow$ 3-sheeted Riemann surface;

on one sheet appears t_c singularity;

but CFT correls $\langle \Phi\Phi \rangle$ given by other 2 sheets.

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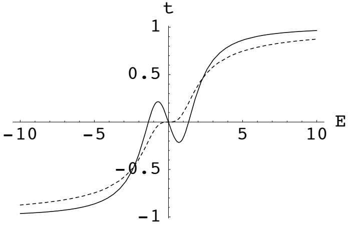
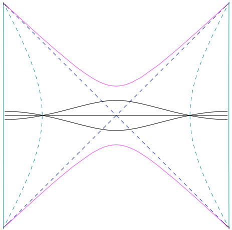
resolutions of branch cut

Geometrically, branch cut corresponds to 3 coincident geodesics resolved at:

- finite cut-off





- finite BH size (Euclidean set-up)
  - complex geods contribute equally
  - real geod. is subdominant

back

resolutions of branch cut

Geometrically, branch cut corresponds to 3 coincident geodesics resolved at:

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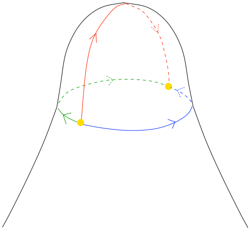


- complex geods contribute equally
- real geod. is subdominant

back

Results of FHKS (including subtlety)

- although t_c geod $=$ dominant saddle, it doesn't contribute to path integral (since not on path of steepest descent)
- unique prescription from Euclidean set-up: sum over contributions from 2 complex solns $\mathcal{L}(t)$
- by analytic cont., t_c singularity visible on secondary sheet

most importantly:

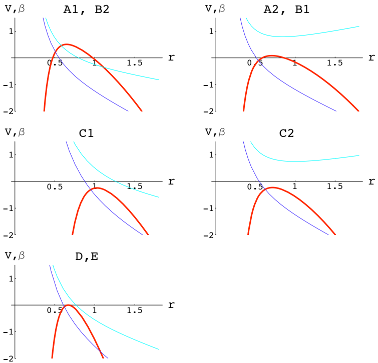
$\exists$ distinct (albeit subtle) signals of BH singularity in CFT correlators

back to main points

Example plots of V_{eff} for dS/SAdS junction

Effective potential V_{eff} and extrinsic curvature β_α, with

$$\beta_\alpha(r) = \frac{f_i(r) - f_o(r) - \kappa^2\, r^2}{2\,\kappa\, r} = \pm\sqrt{\dot{R}^2 + f_\alpha(R)}$$



$\lambda = 0.5$
$\mu = 0.75$
$\kappa = 2$

$\lambda = 1$
$\mu = 0.5$
$\kappa = 1$

$\lambda = 1$
$\mu = 2$
$\kappa = 1$

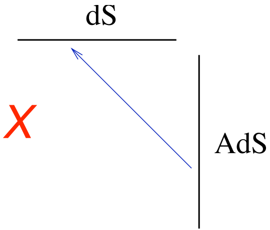
$\lambda = 1$
$\mu = 1$
$\kappa = 2$

$\lambda = 2$
$\mu = 0.89$
$\kappa = 1$

back

dS bdy is causally disconnected from AdS bdy

Simple argument based on Raychaudhuri's equation:
(gravity is attractive)



converging congruence of null geodesics cannot begin diverging (without self-crossing)

Hence ∃ future & past singularities
⇒ de Sitter region must be behind a horizon.

back

Argument for mixed state

- Inflating region (de Sitter with scri) must appear beyond the black hole horizon.
- Hence large region of spacetime looks similar to the eternal Schw-AdS black hole.
- Boundary CFT should therefore be similar to the thermal field theory dual to the eternal BH, which is in mixed state, with thermal density matrix

$$\rho_\beta = e^{-\beta H}$$

(for sufficiently low cut-off scale, in thin-wall approx., the two are identical...)

- Hence states on left and right of BH horizon are entangled.

back to resolutions

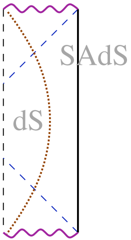
Detailed progression of spacetimes

(a)

causally trivial
(pure AdS; no bubble)

back

Detailed progression of spacetimes

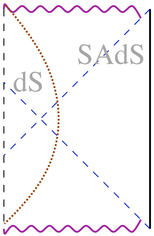


(b)

$\exists$ event horizon
(Schw-AdS + small bubble of dS)

back

Detailed progression of spacetimes

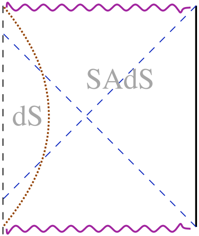


(c)

∃ region causally disconnected from AdS bdy

back

Detailed progression of spacetimes

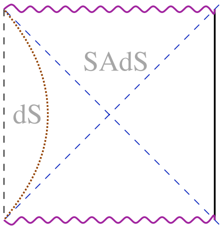


(d)

bubble passes through
causally disconnected region

back

Detailed progression of spacetimes

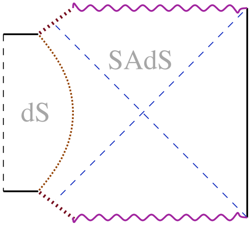


(e)

entire bubble trajectory is causally disconnected from AdS bdy (static de Sitter)

back

Detailed progression of spacetimes

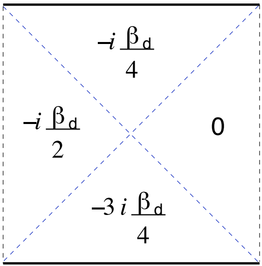


(f)

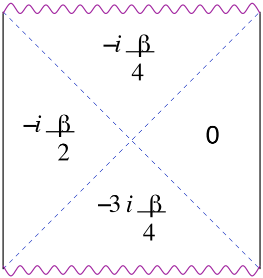
∃ additional asymptotic regions (inflating de Sitter)

back

dS & S-AdS complex time coordinate conventions



(a)



(b)

Cosmological vs. black hole singularities

Can we study Big Crunch (FRW) singularity by the same method?
(i.e. via almost null bouncing geodesics)

No: because geodesics do not bounce off big crunch singularities!
FRW metric:

$$ds^2 = -dt^2 + a(t)^2 \left(dr^2 + \cdots\right)$$

$\rightsquigarrow$ geodesics:

$$\ddot{t} + a(t)\, a'(t)\, \dot{r}^2 = 0$$

$\Rightarrow$ for big crunch, $a'(t) < 0$, so that $\ddot{t} > 0$

Whereas for bounce, we require $\ddot{t} < 0$ when $\dot{t} = 0$.

Lesson:

Cosmological singularities differ from black hole singularities
(cannot probe them via the same methods...)

back to summary

Consequences of mixed state description

- resolves entropy puzzle $S_{dS} \gg S_{BH}$
  ($\rightsquigarrow$ large # of inflating DOFs need not be explicitly represented in CFT)
- mixed state despite only a single asymp. (AdS) region
  ($\rightsquigarrow$ study non-boundary description of non-pert. QG)
- Re: Can inflaction begin by tunneling? Farhi, Guth, Guven
  inflating regions can't be produced in a scattering process (even by QM tunneling)
  since a pure state can't evolve into a mixed state...

back to summary