Looking beyond horizons via AdS/CFT

Veronika Hubeny

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1 Motivation

2 Probing Schwarzschild-AdS singularity via CFT correlators
   - Review of FHKS
   - Collapsed vs. eternal BH

3 Inflation in AdS/CFT
   - Constructions of deSitter in Schw-AdS
   - Entropy paradox

4 Further geometrical puzzles/complications

5 Summary
Key question of quantum gravity:
What is the fundamental nature of spacetime?

Invaluable tool in recent years: AdS/CFT correspondence

- string theory in $AdS \times S$
- $\leftrightarrow$ gauge theory on boundary

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Schwarzschild-AdS black hole

$\leftrightarrow$ (approximately) thermal state
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Invaluable tool in recent years: AdS/CFT correspondence

string theory in $AdS \times S$ ↔ gauge theory on boundary

Schwarzschild-AdS black hole ↔ (approximately) thermal state

Need to probe AdS/CFT dictionary further:
- what is the nature of (spacelike) singularity?
- what is the CFT description of causal structure?
- (how) does the CFT describe physics behind an event horizon?
Key questions in cosmology:

Physics in the early universe?
What is the fate of our universe?

Need to understand inflation
∃ difficulties with string theory in de Sitter background

Can we cast this into AdS/CFT framework?

We will

- construct asymp. AdS spacetimes with region of eternally inflating dS
- argue that CFT correlators are sensitive to dS region
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**BTZ and CFT correlators**

**BTZ** = Schw-AdS black hole in 3-d much studied system

Balasubramanian, Louko, Marolf, Ross

Maldacena; KOS

using thermoﬁeld formalism

**scalar field w/ mass** $m \to \infty$

$$\langle \Phi(x_1) \Phi(x_2) \rangle \sim e^{-m \mathcal{L}}$$

w/ $\mathcal{L} =$ regularized proper length along geodesic betw. $x_1$ and $x_2$

**main point**

correlators between operators in CFT$_1$ and CFT$_2$

$\sim$ access to full spacetime

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precursor gedanken experiment
Schwarzschild-AdS black hole

\[ ds^2 = -f(r) \, dt^2 + \frac{dr^2}{f(r)} + r^2 \, d\Omega^2 \]

in \( d \geq 3 \) dimensions, size of AdS = \( R \), size of BH = \( r_+ \)

\[ f(r) \equiv \frac{r^2}{R^2} + 1 - \left( \frac{r_+^2 + R^2}{R^2} \right) \left( \frac{r_+}{r} \right)^{d-3} \]

Symmetric spacelike geodesics (for \( d = 5 \)):
\[ \exists \text{ critical time } t = t_c \]

after which \( \nexists \) sym. sp-like geods.

\[ t_c = -\frac{1}{T_H} \sqrt{r_+^2 + R^2} \]
Schwarzschild-AdS black hole

\[ ds^2 = -f(r) \, dt^2 + \frac{dr^2}{f(r)} + r^2 \, d\Omega^2 \]

in \( d \geq 3 \) dimensions, large BH limit \( f(r) \equiv r^2 - \frac{1}{r^{d-3}} \)

Symmetric spacelike geodesics (for \( d = 5 \)):

\[ t_c = -\frac{1}{T_H} \frac{\sqrt{r_+^2 + R^2}}{r_+} \]

\( \exists \) critical time \( t = t_c \)

after which \( \notin \) sym. sp-like geods.

[cf. 3-D]
Consider correlator $\langle \Phi \Phi \rangle(t)$

$$
\langle \Phi \Phi \rangle(t) \sim e^{-m\mathcal{L}(t)} \sim \frac{1}{(t - t_c)^{2m}}
$$

$\Rightarrow$ singularity in CFT correl. fn. as $t \rightarrow t_c$

We can view correl. $\langle \Phi \Phi \rangle$ between CFT\(_1\) and CFT\(_2\) as

$$
\langle \Phi \Phi \rangle(t) \equiv \langle \Phi(-t + \frac{i\beta}{2}) \Phi(t) \rangle_\beta
$$

corr. in single CFT in thermal state at inverse temperature $\beta$
Main points of FHKS

most importantly:

\exists \text{ distinct (albeit subtle) signals of BH singularity in CFT correlators}

- Properties of singularity are computationally accessible, given the CFT data \( \langle \Phi \Phi \rangle \)
- \( t_c \) singularity persists to all orders in \( \frac{1}{m'}, g_s, \) and \( \alpha' \)
  \( \leadsto \) sensitive to stringy & quantum behaviour near BH singularity
- All this rests on analyticity...
Collapsed vs. eternal black hole

At late times, collapsed BH looks like eternal BH.

\[ \leftrightarrow \text{ CFT is in (approx.) thermal state} \]

\[ \Rightarrow \text{ similar } \langle \Phi \Phi \rangle (t \to \infty) \]

**Puzzle:** would we see a singularity in \( \langle \Phi \Phi \rangle \)?

\[ \exists \text{ only 1 boundary} \]

\[ \Rightarrow \text{ no } t_c \text{ geodesic (between 2 bdys)} \]

\[ \Rightarrow \text{ no } t_c \text{ singularity in } \langle \Phi \Phi \rangle \]

\[ \langle \Phi(-t + \frac{i\beta}{2}) \Phi(t) \rangle = \text{ finite} \]

instead, signal of singularity in

\[ \langle \Phi(t, \Omega) \Phi(t', -\Omega) \rangle \]

Hence CFT correlates \( \langle \Phi \Phi \rangle \) would see difference between collapsed BH and eternal BH.
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Other geometries?

How far can we push this?

Since the CFT correlators are sensitive to differences in geometry “far away,” could we probe differences between spacetimes which are (almost) identical everywhere outside the horizon?

What sorts of spacetimes can we have in the “?” region?

- FRW ("Wheeler bag of gold")
- flat
- de Sitter
Thin domain wall construction

Consider two metrics patched across a spherical shell:

\[ ds^2_{\alpha} = -f_{\alpha}(r) \, dt_{\alpha}^2 + \frac{dr^2}{f_{\alpha}(r)} + r^2 \, d\Omega^2 \]

where \( \alpha = i \) (inside), or \( o \) (outside)

Induced metric on shell’s world-volume:

\[ ds^2_{\text{shell}} = -d\tau^2 + R(\tau)^2 \, d\Omega^2 \]

Junction conditions \( \rightsquigarrow \) EOM of shell:
radial motion in effective potential \( V_{\text{eff}} \):

\[ \dot{R}^2 + f_o(R) - \frac{(f_i(R) - f_o(R) - \kappa^2 R^2)^2}{4 \kappa^2 R^2} = 0 \]

\[ \text{Veronika Hubeny} \quad \text{Looking beyond horizons via AdS/CFT} \]
Consider two metrics patched across a spherical shell:

\[
\begin{align*}
    ds_\alpha^2 &= -f_\alpha(r) \, dt_\alpha^2 + \frac{dr^2}{f_\alpha(r)} + r^2 \, d\Omega^2 \\
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Induced metric on shell's world-volume:

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ds_{\text{shell}}^2 = -d\tau^2 + R(\tau)^2 \, d\Omega^2
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Junction conditions \( \Rightarrow \) EOM of shell:

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\sqrt{\dot{R}^2 + f_i(R)} - \sqrt{\dot{R}^2 + f_o(R)} = \kappa R
\]

with \( \dot{R} \equiv \frac{dR(\tau)}{d\tau} \)

radial motion in effective potential \( V_{\text{eff}} \):

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Example construction

On each spacetime (given by $f_i$ and $f_o$), shell’s trajectory is $r = R(\tau)$.

Excise spacetime $f_i$ outside (to right of) the shell and spacetime $f_o$ inside (to left of) the shell, and patch together across shell.

Example: bubble of de Sitter inside Schwarzschild-AdS:
Example construction

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Example: bubble of de Sitter inside Schwarzschild-AdS:
For de Sitter, \( f_i(r) = 1 - \lambda r^2 \)
for Schw-AdS, \( f_o(r) = r^2 + 1 - \frac{\mu}{r} \)

\[
V_{\text{eff}}(r) = -\frac{(\lambda + \kappa^2 - 1)^2 + 4 \lambda}{4 \kappa^2} r^2 + 1 + \mu \left(1 + \lambda - \kappa^2\right) \frac{1}{r} - \frac{\mu^2}{4 \kappa^2} \frac{1}{r^4}
\]

where \( \lambda \sim \text{cosmol. const.} \)
\( \mu \sim \text{BH mass} \)
\( \kappa \sim \text{bubble wall tension} \)
Possibilities for de Sitter / Schw-AdS junction

Possible effective potentials:

\[ V_{\text{eff}} \]

\[ R \]

A \quad B

\[ V_{\text{eff}} \]

\[ R \]

C \quad C'

Corresponding Penrose diagrams:

(A) \quad (B) \quad (C)
Possibilities for de Sitter / Schw-AdS junction

Possible effective potentials: \( (\text{fine-tuned } V_{\text{eff}}) \)

Corresponding Penrose diagrams:
CFT signatures of de Sitter

- geodesics bounce off singularity
- geodesics pass through origin $\Omega \rightarrow -\Omega$
- geodesics bounce off dS scri

$\Rightarrow$ geodesics return to same boundary . . .

$\Rightarrow$ expected singularity in (analytically continued) correl. function

$$\langle \Phi(t, \Omega)\Phi(s, -\Omega) \rangle \sim \frac{1}{(s - t'(t))^{2m}}$$

**Conclusion:**

CFT correlators allow us to see bulk ST geometry
CFT signatures of de Sitter

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correlation function

**Conclusion:**

CFT correlators allow us to see bulk ST geometry (including the late-time eternal inflation region)
Physically more realistic case: instead of domain wall, consider scalar field $\Phi$ in a given potential $V(\Phi)$.
(motivated via string landscape...)

$\Phi = \Phi_1 \leadsto \text{AdS minimum}$
(defines a CFT...)

$\Phi = 0 \leadsto \text{dS minimum}$
de Sitter via scalar field in potential

Field in AdS minimum
⇝ AdS
de Sitter via scalar field in potential

Field in dS minimum $\leadsto$ dS (in domain of dependence of initial slice $\Sigma$)
de Sitter via scalar field in potential

Interpolating field
\[ \sim \] asymp. AdS spacetime
with dS region

Note: dS boundary must be causally disconnected from AdS boundary
Entropy paradox

For all time-symmetric configurations, the shell reaches extremal size at $\tau = 0$; denote $R(\tau = 0) \equiv R_0$.

\[ R_0 \leq r_d \]
\[ R_0 \geq r_+ \]
\[ \Rightarrow \quad r_d \geq r_+ \]

\[ \Rightarrow \text{dS horizon entropy} \geq \text{(can be } \gg\text{) black hole entropy} \]

\((\sim \text{active DOFs in CFT})\)

Puzzle:

How can CFT state w/ $e^{S_{BH}}$ active degrees of freedom describe large inflating region w/ $e^{S_{dS}} \gg e^{S_{BH}}$ degrees of freedom?
Possible resolutions

\[ \exists \text{ (time-asymmetric) geometries with } S_{dS} < S_{BH} \]

- de Sitter entropy has different interpretation (cosmol. horizon is observer-dependent)
- CFT states assoc. w/ inflating regions in AdS are mixed:
  active DOF in CFT are entangled w/ DOF in inflating region
  \( S_{BH} \) is a measure of this entanglement

\[
S_{BH} = Tr(\rho \log \rho)
\]
(with \( \rho = \text{boundary density matrix} \))
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argument for mixed state
Progression of spacetimes

Consider a family of solutions obtained by letting the wall pass progressively further left on Schw-AdS Penrose diagram.
Progression of spacetimes

Consider a family of solutions obtained by letting the wall pass progressively further left on Schw-AdS Penrose diagram.

(a) AdS
(b) SAdS
c (c) SAdS
(d) dS
(e) dS
dS
(f) SAdS
SAdS

When is CFT dual described by a mixed state?
How realistic is the thin wall geometry?

- shell
- left AdS boundaries
- black hole singularity
- de Sitter scri

thin shell Penrose diagram
Geometrical complications and open questions

How realistic is the thin wall geometry?

- shell
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Shell could be subject to instabilities

- dynamical breaking of spherical symmetry
- spreading (shell becomes thick)

Cf. Aguirre & Johnson
Geometrical complications and open questions

How realistic is the thin wall geometry?

- shell
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no left AdS bdy
– probably replaced by (almost null) FRW-like singularity
Geometrical complications and open questions

How realistic is the thin wall geometry?

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black hole singularity $\rightsquigarrow$ Mixmaster-like behaviour (no longer clear that geodesics bounce...)
Geometrical complications and open questions

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![de Sitter fragmentation diagram]
Summary (probing bulk spacetime)

- (Spatial dependence of) CFT correlators contains lots of information about the bulk geometry.
- Despite very similar bulk (outside horizon, late time) behaviour, CFT correlators can have quite different properties.
- Horizon is not an obstacle to probing geometry via CFT:
  - can probe BH singularity
  - can probe dS scri
  - but can not probe cosm. (FRW) singularity in this way
  - reason: BH vs. FRW
- All this rests on analyticity.
Potential framework to study inflation in string theory (AdS/CFT)

- string landscape with dS and AdS vacua
  - $\leadsto$ stable AdS min. described by bdy CFT
- inflating regions involve excitations around AdS
  - $\leadsto$ excitations of CFT

$V \rightarrow$ CFT contains some information about dS

- bulk containing inflating regions (dS) described by mixed state (density matrix) in CFT
Gauge theory can see inside black holes

- Start with pure AdS
  measure event by means of a precursor
- After measurement, collapse a shell with sufficiently large energy ...
- ... that event horizon of resulting black hole encompasses the measured event

Conclusion

(V.H.)

Global nature of event horizon and gauge theory acting as a nonlocal observer

⇒ AdS/CFT probes physics inside horizon
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**Conclusion**
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global nature of event horizon
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Thermofield formalism

Cons. tensor product of 2 copies of the original CFT:

$$\mathcal{H} = \mathcal{H}_1 \times \mathcal{H}_2$$

Construct a pure, entangled state (\(\leadsto\) Hartle-Hawking WF)

$$|\psi\rangle \sim \sum_n e^{-\frac{\beta E_n}{2}} |E_n\rangle_1 \otimes |E_n\rangle_2$$

Reproduces thermal correlators:

$$\langle \psi | \mathcal{O}_1 | \psi \rangle \sim Tr[\rho_\beta \mathcal{O}_1]$$

but because of entanglement,

$$\langle \psi | \mathcal{O}_1 \mathcal{O}_2 | \psi \rangle \neq 0$$

(Thermal state counting entropy arises from entanglement of the pure entangled Hartle-Hawking state.)
cf. 3-D:

nothing special happens: symmetric geods $\exists \forall t$ insensitive to singularity
Puzzle: CFT correl’s can’t diverge

\[ t_1 = t \]
\[ t_2 = -t + \frac{i \beta}{2} \]

\[ \langle \Phi \Phi \rangle(t) \equiv \langle \Phi(-t + \frac{i \beta}{2})\Phi(t) \rangle_{\beta} = \sum_{n,m} e^{-\frac{\beta}{2} (E_n + E_m)} e^{2 i t (E_n - E_m)} |\Phi_{nm}|^2 \]

\[ |\langle \Phi \Phi \rangle(t)| \leq \langle \Phi \Phi \rangle(0) < \infty \]

⇒ There can’t be the expected singularity in the correlator \( \langle \Phi \Phi \rangle \)!

What went wrong?
Subtlety: branch structure of $\mathcal{L}(t)$

captured by

$$ t = \frac{1}{2} \ln\left( \frac{\frac{1}{2} E^2 - E + 1}{\sqrt{1 + \frac{1}{4} E^4}} \right) - \frac{1}{2} i \ln\left( \frac{-\frac{1}{2} E^2 + iE + 1}{\sqrt{1 + \frac{1}{4} E^4}} \right) $$

$$ \mathcal{L} = \ln\left( \frac{2}{\sqrt{1 + \frac{1}{4} E^4}} \right) $$

at $E \to 0$, $t(E) \sim E^3$, $\mathcal{L}(E) \sim -E^4$  \Rightarrow  $\mathcal{L}(t) \sim -t^{4/3}$

$\Rightarrow$  $\exists$ a branch cut

geo\textit{metrically corresponds to 3 coincident geodesics at } t = 0.
Can see this explicitly by resolving the branch cut

$\mathcal{L}(t) \rightsquigarrow$ 3-sheeted Riemann surface;
on one sheet appears $t_c$ singularity;
but CFT correls $\langle \Phi \Phi \rangle$ given by other 2 sheets.
Subtlety: branch structure of $\mathcal{L}(t)$

at $E \to 0$, 
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\begin{align*}
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$\mathcal{L}(t) \rightsquigarrow$ 3-sheeted Riemann surface;
on one sheet appears $t_c$ singularity;
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Nevertheless, we can analytically continue through the branch cut
at $t = 0$ to extract the $t_c$ singularity.
Subtlety: branch structure of $\mathcal{L}(t)$

\[ t(E) \sim E^3 \]
\[ \mathcal{L}(E) \sim -E^4 \]

\[ \Rightarrow \exists \text{ a branch cut} \]

does not geometrically correspond to 3 coincident geodesics at $t = 0$.
Can see this explicitly by resolving the branch cut

$\mathcal{L}(t) \rightsquigarrow$ 3-sheeted Riemann surface;
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resolutions of branch cut

Geometrically, branch cut corresponds to 3 coincident geodesics resolved at:

- finite cut-off

- finite BH size (Euclidean set-up)

  - complex geods contribute equally

  - real geod. is subdominant
resolutions of branch cut

Geometrically, branch cut corresponds to 3 coincident geodesics resolved at:

- finite cut-off
- finite BH size (Euclidean set-up)

- complex geods contribute equally
- real geod. is subdominant
Results of FHKS (including subtlety)

- although $t_c$ geod = dominant saddle, it doesn’t contribute to path integral (since not on path of steepest descent)
- unique prescription from Euclidean set-up: sum over contributions from 2 complex solns $\mathcal{L}(t)$
- by analytic cont., $t_c$ singularity visible on secondary sheet

most importantly:

$\exists$ distinct (albeit subtle) signals of BH singularity in CFT correlators
Example plots of $V_{\text{eff}}$ for dS/SAdS junction

Effective potential $V_{\text{eff}}$ and extrinsic curvature $\beta_\alpha$, with

$$\beta_\alpha(r) = \frac{f_i(r) - f_o(r) - \kappa^2 r^2}{2\kappa r} = \pm \sqrt{\dot{R}^2 + f_\alpha(R)}$$

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<td>1</td>
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dS bdy is causally disconnected from AdS bdy

Simple argument based on Raychaudhuri’s equation: (gravity is attractive)

\[
\begin{array}{c}
\text{dS} \\
\text{X} \\
\text{AdS}
\end{array}
\]

converging congruence of null geodesics cannot begin diverging (without self-crossing)

Hence ∃ future & past singularities
⇒ de Sitter region must be behind a horizon.
Argument for mixed state

- Inflating region (de Sitter with scri) must appear beyond the black hole horizon.
- Hence large region of spacetime looks similar to the eternal Schw-AdS black hole.
- Boundary CFT should therefore be similar to the thermal field theory dual to the eternal BH, which is in mixed state, with thermal density matrix

$$\rho_\beta = e^{-\beta H}$$

(for sufficiently low cut-off scale, in thin-wall approx., the two are identical...)

- Hence states on left and right of BH horizon are entangled.
Detailed progression of spacetimes

(a) causally trivial
(pure AdS; no bubble)
Detailed progression of spacetimes

∃ event horizon
(Schw-AdS + small bubble of dS)
Detailed progression of spacetimes

∃ region causally disconnected from AdS bdy

(c)
Detailed progression of spacetimes

bubble passes through causally disconnected region

(d)
Detailed progression of spacetimes

entire bubble trajectory is causally disconnected from AdS bdy
(static de Sitter)

(e)
Detailed progression of spacetimes

∃ additional asymptotic regions (inflating de Sitter)
dS & S-AdS complex time coordinate conventions

(a)

(b)

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Looking beyond horizons via AdS/CFT
Cosmological vs. black hole singularities

Can we study Big Crunch (FRW) singularity by the same method? (i.e. via almost null bouncing geodesics)

No: because geodesics do not bounce off big crunch singularities!
FRW metric:

\[ ds^2 = -dt^2 + a(t)^2 (dr^2 + \cdots) \]

\[ \Rightarrow \] geodesics:

\[ \ddot{t} + a(t) a'(t) \dot{r}^2 = 0 \]

\[ \Rightarrow \] for big crunch, \( a'(t) < 0 \), so that \( \ddot{t} > 0 \)

Whereas for bounce, we require \( \ddot{t} < 0 \) when \( \dot{t} = 0 \).

Lesson:

Cosmological singularities differ from black hole singularities (cannot probe them via the same methods...)

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Consequences of mixed state description

- resolves entropy puzzle $S_{dS} \gg S_{BH}$
  ($\sim$ large # of inflating DOFs need not be explicitly represented in CFT)
- mixed state despite only a single asympt. (AdS) region
  ($\sim$ study non-boundary description of non-pert. QG)
- Re: Can inflection begin by tunneling? 
  inflating regions can’t be produced in a scattering process
  (even by QM tunneling)
  since a pure state can’t evolve into a mixed state...

Farhi, Guth, Guven

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