The Proton Radius Puzzle

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Introduction: The proton radius puzzle

Form Factors

- Matrix element of EM current between nucleon states give rise to two form factors ($q = p_f - p_i$)

$$\langle N(p_f)| \sum_q e_q \, \bar{q}\gamma^\mu q | N(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2(q^2) q^\nu\right] u(p_i)$$

- Sachs electric and magnetic form factors

$$G_E(q^2) = F_1(q^2) + \frac{q^2}{4m_p^2} F_2(q^2) \qquad G_M(q^2) = F_1(q^2) + F_2(q^2)$$

$$G_E^p(0) = 1 \qquad G_M^p(0) = \mu_p \approx 2.793$$

- The slope of G_E^p

$$\boxed{\langle r^2\rangle_E^p = 6 \left.\frac{dG_E^p}{dq^2}\right|_{q^2=0}}$$

determines the charge radius $r_E^p \equiv \sqrt{\langle r^2\rangle_E^p}$

- The proton *magnetic* radius

$$\langle r^2\rangle_M^p = \frac{6}{G_M^p(0)} \left.\frac{dG_M^p(q^2)}{dq^2}\right|_{q^2=0}$$

Charge radius from atomic physics

$$\langle p(p_f)| \sum_q e_q \, \bar{q}\gamma^\mu q |p(p_i)\rangle = \bar{u}(p_f)\left[\gamma^\mu F_1^p(q^2) + \frac{i\sigma_{\mu\nu}}{2m} F_2^p(q^2) q^\nu\right] u(p_i)$$

- For a point particle amplitude for $p + \ell \to p + \ell$

$$\mathcal{M} \propto \frac{1}{q^2} \quad \Rightarrow \quad U(r) = -\frac{Z\alpha}{r}$$

- Including q^2 corrections from proton structure

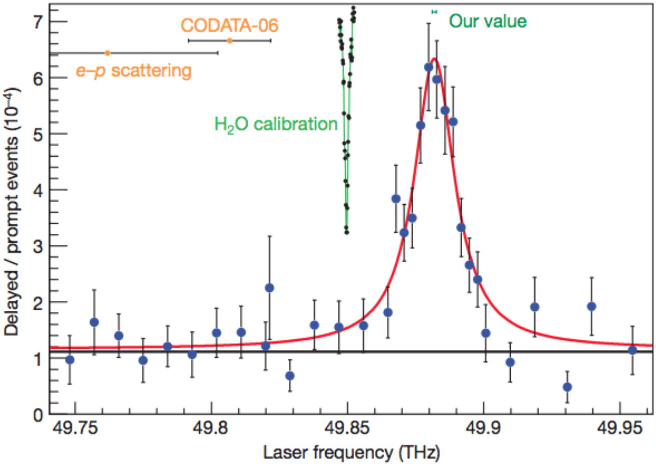
$$\mathcal{M} \propto \frac{1}{q^2} q^2 = 1 \quad \Rightarrow \quad U(r) = \frac{4\pi Z\alpha}{6}\delta^3(r)(r_E^p)^2$$

- Proton structure corrections $\left(m_r = m_\ell m_p/(m_\ell + m_p) \approx m_\ell\right)$

$$\boxed{\Delta E_{r_E^p} = \frac{2(Z\alpha)^4}{3n^3} m_r^3 (r_E^p)^2 \delta_{\ell 0}}$$

- **Muonic hydrogen can give the best measurement of r_E^p!**

Charge radius from Muonic Hydrogen



- CREMA Collaboration measured for the **first time** $2S_{1/2}^{F=1} - 2P_{3/2}^{F=2}$ transition in Muonic Hydrogen [Pohl et al. Nature **466**, 213 (2010)]

Charge radius from atomic physics

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
  $r_E^p = 0.84184(67)$ fm
  more recently $r_E^p = 0.84087(39)$ fm [Antognini et al. Science **339**, 417 (2013)]
- CODATA value [Mohr et al. RMP **80**, 633 (2008)]
  $r_E^p = 0.87680(690)$ fm
  more recently $r_E^p = 0.87510(610)$ fm [Mohr et al. RMP **88**, 035009 (2016)]
  extracted mainly from (electronic) hydrogen
- **5σ discrepancy!**

- This is the proton radius puzzle

What could be the reason for the discrepancy?

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2) Hadronic Uncertainty? (Part 2 of this talk)

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1) Problem with the electronic extraction? (Part 1 of this talk)

2) Hadronic Uncertainty? (Part 2 of this talk)

3) New Physics?

- Declaimer: I will focus on published work I am involved in

Outline

- Introduction: The proton radius puzzle
- Part 1: Proton radii from scattering
- Part 2: Hadronic Uncertainty?
- Part 3: Connecting muon-proton scattering and muonic hydrogen
- Conclusions and outlook

Part 1: Proton radii from scattering

What did PDG 2010 say?

K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010)

p CHARGE RADIUS

This is the rms charge radius, $\sqrt{\langle r^2 \rangle}$.

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0.8768±0.0069	MOHR	08	RVUE	2006 CODATA value
• • • We do not use the following data for averages, fits, limits, etc. • • •				
0.897 ±0.018	BLUNDEN	05		SICK 03 + 2γ correction
0.8750±0.0068	MOHR	05	RVUE	2002 CODATA value
0.895 ±0.010 ±0.013	SICK	03		$e p \to e p$ reanalysis
0.830 ±0.040 ±0.040	[24] ESCHRICH	01		$e p \to e p$
0.883 ±0.014	MELNIKOV	00		1S Lamb Shift in H
0.880 ±0.015	ROSENFELDR.	00		$e p$ + Coul. corrections
0.847 ±0.008	MERGELL	96		$e p$ + disp. relations
0.877 ±0.024	WONG	94		reanalysis of Mainz $e p$ data
0.865 ±0.020	MCCORD	91		$e p \to e p$
0.862 ±0.012	SIMON	80		$e p \to e p$
0.880 ±0.030	BORKOWSKI	74		$e p \to e p$
0.810 ±0.020	AKIMOV	72		$e p \to e p$
0.800 ±0.025	FREREJACQ...	66		$e p \to e p$ (CH_2 tgt.)
0.805 ±0.011	HAND	63		$e p \to e p$

Citation: K. Nakamura *et al.* (Particle Data Group), JPG **37**, 075021 (2010) (URL: http://pdg.lbl.gov)

[24] ESCHRICH 01 actually gives $\langle r^2 \rangle = (0.69 \pm 0.06 \pm 0.06)$ fm^2.

Proton charge radius from scattering

- The PDG 2010 lists proton radii starting from 1963
- This is about 50 years of radii extraction.
- You can find almost any value between 0.8-0.9 fm....
- Data sets have changed over the last 50 years
  but even using the same data sets different people get different values
- What is the problem?

Form Factors: What we don't know

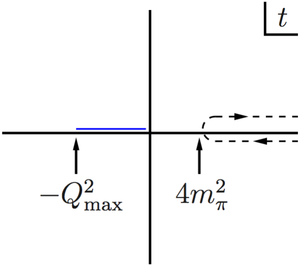
- The form factors are non-perturbative objects.
- **Nobody** knows the *exact* functional form of G_E^p and G_M^p
- They don't have to have a dipole/polynomial/spline
  or any other functional form
- Including such models can bias your extraction of r_E^p and r_M^p

Form Factors: What we do know

- Notation: $q^2 = t = -Q^2$
- Analytic properties of $G_E^p(t)$ and $G_M^p(t)$ are known
- They are analytic outside a cut $t \in [4m_\pi^2, \infty]$

  [Federbush, Goldberger, Treiman, Phys. Rev. **112**, 642 (1958)]

- $e - p$ scattering data is in $t < 0$ region

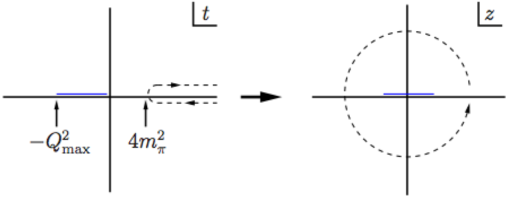


z expansion

- z expansion: map domain of analyticity onto unit circle

$$z(t, t_{\rm cut}, t_0) = \frac{\sqrt{t_{\rm cut} - t} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - t} + \sqrt{t_{\rm cut} - t_0}}$$

where $t_{\rm cut} = 4m_\pi^2$, $z(t_0, t_{\rm cut}, t_0) = 0$



- Expand $G^p_{E,M}$ in a Taylor series in z: $G^p_{E,M}(q^2) = \sum_{k=0}^{\infty} a_k \, z(q^2)^k$

- **The** method for **meson** form factors

  [Flavor Lattice Averaging Group, EPJ C **74**, 2890 (2014)]

z expansion

- [Zachary Epstein, GP, Joydeep Roy PRD **90**, 074027 (2014)]

z expansion

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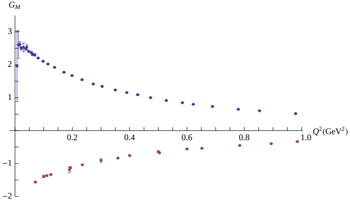
  $G_M(Q^2)$ for proton (blue, above axis) and neutron (red, below axis)



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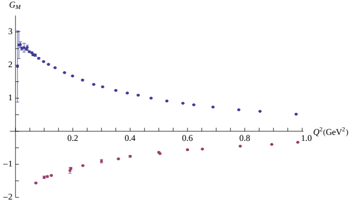

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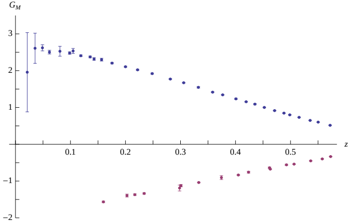

- See also R.J. Hill talk at FPCP 2006 [hep-ph/0606023]

Extracting r_E^p using the z expansion

- First use of the z expansion to extract r_E^p
  [Richard J. Hill, GP PRD **82** 113005 (2010)]

- Proton: $Q^2 < 0.5$ GeV^2

$$r_E^p = 0.870 \pm 0.023 \pm 0.012\,\mathrm{fm}$$

- Proton and neutron data

$$r_E^p = 0.880^{+0.017}_{-0.020} \pm 0.007\,\mathrm{fm}$$

- Proton, neutron and $\pi\,\pi$ data

$$r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$$

- Lamb shift in muonic hydrogen [Pohl et al. Nature **466**, 213 (2010)]
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PDG 2016

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, **40**, 100001 (2016)

p CHARGE RADIUS

This is the rms electric charge radius, $\sqrt{\langle r_E^2 \rangle}$.

VALUE (fm)			*DOCUMENT ID*		*TECN*	*COMMENT*
0.8751	**±0.0061**		MOHR	16	RVUE	2014 CODATA value
0.84087	**±0.00026**	**±0.00029**	ANTOGNINI	13	LASR	μp-atom Lamb shift
• • • We do not use the following data for averages, fits, limits, etc. • • •						
0.895	±0.014	±0.014	[1] LEE	15	SPEC	Just 2010 Mainz data
0.916	±0.024		LEE	15	SPEC	World data, no Mainz
0.8775	±0.0051		MOHR	12	RVUE	2010 CODATA, $e p$ data
0.875	±0.008	±0.006	ZHAN	11	SPEC	Recoil polarimetry
0.879	±0.005	±0.006	BERNAUER	10	SPEC	$e p \to e p$ form factor
0.912	±0.009	±0.007	BORISYUK	10		reanalyzes old $e p$ data
0.871	±0.009	±0.003	HILL	10		z-expansion reanalysis
0.84184	±0.00036	±0.00056	POHL	10	LASR	See ANTOGNINI 13
0.8768	±0.0069		MOHR	08	RVUE	2006 CODATA value
0.844	$^{+0.008}_{-0.004}$		BELUSHKIN	07		Dispersion analysis
0.897	±0.018		BLUNDEN	05		SICK 03 + 2γ correction
0.8750	±0.0068		MOHR	05	RVUE	2002 CODATA value
0.895	±0.010	±0.013	SICK	03		$e p \to e p$ reanalysis

[Hill, GP PRD **82** 113005 (2010)]

Extracting r_M^p using the z expansion

- z expansion study

  [Zachary Epstein, GP, Joydeep Roy PRD **90**, 074027 (2014)]

- Proton data :$r_M^p = 0.91^{+0.03}_{-0.06} \pm 0.02$ fm
- Proton and neutron data: $r_M^p = 0.87^{+0.04}_{-0.05} \pm 0.01$ fm
- Proton, neutron and $\pi\,\pi$ data: $r_M^p = 0.87 \pm 0.02$ fm

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[1] Authors also provide values for a combination of all available data.

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

Neutron magnetic radius: z expansion

- The neutron has zero charge but non-zero magnetic moment
- Can extract the **neutron** magnetic radius from the same data
- Using proton, neutron and $\pi\pi$ data:

$$r_M^n = 0.89 \pm 0.03 \text{ fm}$$

PDG 2016

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$0.864^{+0.009}_{-0.008}$ OUR AVERAGE			
0.89 ± 0.03	EPSTEIN	14	Using $e\,p$, $e\,n$, $\pi\pi$ data
$0.862^{+0.009}_{-0.008}$	BELUSHKIN	07	Dispersion analysis

[Epstein, GP, Roy PRD **90**, 074027 (2014)]

Latest z expansion fit

- Most recent study using the z expansion
  [Gabriel Lee, J. R. Arrington, and R. J. Hill, PRD **92**, 013013 (2015)]
  Analyze the "Mainz" data set
  [J. C. Bernauer et al. PRL **105**, 242001 (2010)]
  and world data (excluding Mainz)

- World data
  [Lee, Arrington, Hill '15] $r_E^p = 0.918 \pm 0.024$ fm
  [Hill , GP '10] $r_E^p = 0.870 \pm 0.023 \pm 0.012$ fm

  [Lee, Arrington, Hill '15] $r_M^p = 0.913 \pm 0.037$ fm
  [Epstein, GP, Roy '14] $r_M^p = 0.910^{+0.030}_{-0.060} \pm 0.020$ fm

- Mainz data
  $r_E^p = 0.895 \pm 0.020$ fm $\quad r_M^p = 0.773 \pm 0.038$ fm

PDG 2016: r_E^p

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[Lee, Arrington, Hill, PRD **92**, 013013 (2015)]

Part 2: Hadronic Uncertainty?

[Hill, GP PRD **95**, 094017 (2017), arXiv:1611.09917]

The bottom line

- Scattering:
- World $e-p$ data [Lee, Arrington, Hill '15]
  $r_E^p = 0.918 \pm 0.024$ fm
- Mainz $e-p$ data [Lee, Arrington, Hill '15]
  $r_E^p = 0.895 \pm 0.020$ fm
- Proton, neutron and π data [Hill , GP '10]
  $r_E^p = 0.871 \pm 0.009 \pm 0.002 \pm 0.002\,\mathrm{fm}$

- Muonic hydrogen
- [Pohl et al. Nature **466**, 213 (2010)]
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- The bottom line:
  using z expansion scattering disfavors muonic hydrogen

- Is there a problem with muonic hydrogen *theory*?

Muonic hydrogen theory

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- Potentially yes!
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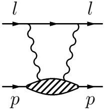
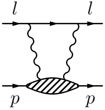
- [Antognini et al. Science **339**, 417 (2013), Ann. of Phy. **331**, 127]
  $\Delta E = 206.0336(15) - 5.2275(10)(r_E^p)^2 + 0.0332(20)$ meV

- Apart from r_E^p need two-photon exchange



Two photon exchange



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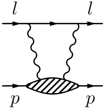
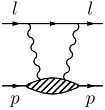


$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle k,s|\, T\{J^{\mu}_{\mathrm{e.m.}}(x)J^{\nu}_{\mathrm{e.m.}}(0)\}|k,s\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^{\mu}q^{\nu}}{q^2}\right) W_1(\nu,q^2) + \left(k^{\mu} - \frac{k\cdot q\, q^{\mu}}{q^2}\right)\left(k^{\nu} - \frac{k\cdot q\, q^{\nu}}{q^2}\right) W_2(\nu,q^2)$$

Two photon exchange



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$$= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1(\nu,q^2) + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2(\nu,q^2)$$

$\nu = 2k\cdot q =$ virtual photon energy, $Q^2 = -q^2$, virtual photon 4-momentum

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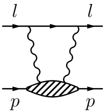

$$W^{\mu\nu} = \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle k,s|\,T\{J^\mu_{\rm e.m.}(x)J^\nu_{\rm e.m.}(0)\}|k,s\rangle$$

$$= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right)W_1(\nu,q^2) + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2(\nu,q^2)$$

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- Dispersion relations

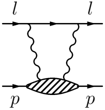
$$W_1(\nu,Q^2) = W_1(0,Q^2) + \frac{\nu^2}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\,\frac{{\rm Im}\,W_1(\nu',Q^2)}{\nu'^2(\nu'^2-\nu^2)}$$

$$W_2(\nu,Q^2) = \frac{1}{\pi}\int_{\nu_{\rm cut}(Q^2)^2}^{\infty} d\nu'^2\,\frac{{\rm Im}\,W_2(\nu',Q^2)}{\nu'^2-\nu^2}$$

- W_1 requires subtraction...

Two photon exchange

- In both cases apart from r_E^p we have two-photon exchange



- Imaginary part of TPE related to data:
  form factors, structure functions

- Cannot reproduce it from its imaginary part:
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- Need poorly constrained non-perturbative function $W_1(0, Q^2)$

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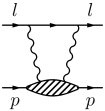

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- Need poorly constrained non-perturbative function $W_1(0, Q^2)$

- $W_1(0, Q^2)$ is calculable in small Q^2 limit using NRQED
  [Hill, GP, PRL **107** 160402 (2011)]

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- Organize operators in $1/m_p$, Lagrangian form:

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \left\{ iD_t + \frac{\boldsymbol{D}^2}{2m_p} + \frac{\boldsymbol{D}^4}{8m_p^3} + c_F e \frac{\boldsymbol{\sigma} \cdot \boldsymbol{B}}{2m_p} + c_D e \frac{[\boldsymbol{\partial} \cdot \boldsymbol{E}]}{8m_p^2} + \cdots \right\} \psi$$

NRQED (NRQCD) Lagrangian

- The $1/m_p^2$ were given in Caswell, Lepage '86

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \Big\{ iD_t + \frac{\boldsymbol{D}^2}{2m_p} + \frac{\boldsymbol{D}^4}{8m_p^3} + c_F g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}}{2m_p} + c_D g \frac{[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{8m_p^2}$$
$$+ i c_S g \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D})}{8m_p^2} + c_{W1} g \frac{\{\boldsymbol{D}^2, \boldsymbol{\sigma}\cdot\boldsymbol{B}\}}{8m_p^3} \Big\} \psi$$

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- The $1/m_p^3$ were given in Manohar '97

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \Big\{ \ldots - c_{W2} g \frac{D^i \boldsymbol{\sigma}\cdot\boldsymbol{B} D^i}{4m_p^3} + c_{p'p} g \frac{\boldsymbol{\sigma}\cdot\boldsymbol{D}\boldsymbol{B}\cdot\boldsymbol{D} + \boldsymbol{D}\cdot\boldsymbol{B}\boldsymbol{\sigma}\cdot\boldsymbol{D}}{8m_p^3} + ic_M g \frac{\{\boldsymbol{D}^i,[\boldsymbol{\partial}\times\boldsymbol{B}]^i\}}{8m_p^3} + c_{A1} g^2 \frac{\left(\boldsymbol{B}_a^i\boldsymbol{B}_b^i - \boldsymbol{E}_a^i\boldsymbol{E}_b^i\right)\ T^a T^b}{8m_p^3} - c_{A2} g^2 \frac{\boldsymbol{E}_a^i\boldsymbol{E}_b^i\ T^a T^b}{16m_p^3} + c_{A3} g^2 \frac{\left(\boldsymbol{B}_a^i\boldsymbol{B}_b^i - \boldsymbol{E}_a^i\boldsymbol{E}_b^i\right)\delta^{ab}}{8m_p^3} - c_{A4} g^2 \frac{\boldsymbol{E}_a^i\boldsymbol{E}_b^i\,\delta^{ab}}{16m_p^3} + - c_{B1} g^2 \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{B}_a\times\boldsymbol{B}_b - \boldsymbol{E}_a\times\boldsymbol{E}_b) f^{abc} T^c}{16m_p^3} + c_{B2} g^2 \frac{\boldsymbol{\sigma}\cdot(\boldsymbol{E}_a\times\boldsymbol{E}_b) f^{abc} T^c}{16m_p^3} \Big\}\psi$$

- Last line is non-zero only for Non-Relativistic Quantum Chromo Dynamics (NRQCD)

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  [Hill, Lee, GP, Solon, PRD **87** 053017 (2013)]

$$\mathcal{L}_{\text{NRQED}} = \psi^\dagger \Big\{ \ldots + c_{X1} g \frac{[\boldsymbol{D}^2, \boldsymbol{D}\cdot\boldsymbol{E} + \boldsymbol{E}\cdot\boldsymbol{D}]}{m_p^4} + c_{X2} g \frac{\{\boldsymbol{D}^2, [\boldsymbol{\partial}\cdot\boldsymbol{E}]\}}{m_p^4}$$
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$$+ i c_{X5} g \frac{D^i \boldsymbol{\sigma}\cdot(\boldsymbol{D}\times\boldsymbol{E} - \boldsymbol{E}\times\boldsymbol{D}) D^i}{m_p^4} + i c_{X6} g \frac{\epsilon^{ijk}\sigma^i D^j [\boldsymbol{\partial}\cdot\boldsymbol{E}] D^k}{m_p^4}$$
$$+ c_{X7} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{B}[\boldsymbol{\partial}\cdot\boldsymbol{E}]}{m_p^4} + c_{X8} g^2 \frac{[\boldsymbol{E}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{B}]}{m_p^4} + c_{X9} g^2 \frac{[\boldsymbol{B}\cdot\boldsymbol{\partial}\boldsymbol{\sigma}\cdot\boldsymbol{E}]}{m_p^4}$$
$$+ c_{X10} g^2 \frac{[E^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} B^i]}{m_p^4} + c_{X11} g^2 \frac{[B^i \boldsymbol{\sigma}\cdot\boldsymbol{\partial} E^i]}{m_p^4}$$
$$+ c_{X12} g^2 \frac{\boldsymbol{\sigma}\cdot\boldsymbol{E}\times[\partial_t \boldsymbol{E} - \boldsymbol{\partial}\times\boldsymbol{B}]}{m_p^4} \Big\} \psi$$

- Method explained in [GP MPLA **30**, 1550128 (2015)]

NRQED (NRQCD) Lagrangian beyond $1/m_p^4$

- This direct construction is very tedious:
- Not clear how many operators there are
- Are two given operators linearly independent?

- Not easy to generalize to $1/m_p^5$
  $1/m_p^5$ can be used to control $W_1(0, Q^2)$

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- Can we do better?

- Yes!

- The trick is to map NRQED (NRQCD) to
  HQET (Heavy Quark Effective Theory) B-meson matrix elements
  [Gunawardna, GP JHEP **1707** 137 (2017)]

- Allows to *construct* terms to arbitrary higher power in $1/m_p$

Hilbert Series method

- Kobach and Pal have used the Hilbert series method to find the NRQED and NRQCD/HQET operators up to $1/m_p^4$.
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- Still, a very useful check

NRQCD Lagrangian $1/m_p^4$

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$$+ic_{X4a}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b \{T^a, T^b\}\}}{2M^4} + ic_{X4b}\, g^2 \frac{\{\boldsymbol{D}^i, \epsilon^{ijk} \boldsymbol{E}^j_a \boldsymbol{B}^k_b\, \delta^{ab}\}}{m_p^4} + ic_{X5} g \frac{\boldsymbol{D}^i \boldsymbol{\sigma} \cdot (\boldsymbol{D} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{D}) \boldsymbol{D}^i}{m_p^4}$$

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$$+c_{X8a}\, g^2 \frac{\{\boldsymbol{E}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_b T^b\}}{2M^4} + c_{X8b}\, g^2 \frac{\boldsymbol{E}^i_a [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{B}]_a}{m_p^4} + c_{X9a}\, g^2 \frac{\{\boldsymbol{B}^i_a T^a, [\boldsymbol{D}^i, \boldsymbol{\sigma} \cdot \boldsymbol{E}]_b T^b\}}{2M^4}$$

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c_{Xib} start at $\mathcal{O}(\alpha_s)$

NRQCD Lagrangian $1/m_p^4$

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$$
\begin{aligned}
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$$

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We now have methods that allow to find arbitrary higher powers

Two Photon exchange: small Q^2 limit

- *Small* Q^2 limit using NRQED [Hill, GP, PRL **107** 160402 (2011)]
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$$W_1(0,Q^2) = 2a_p(2+a_p) + \frac{Q^2}{m_p^2}\left\{\frac{2m_p^3\bar{\beta}}{\alpha} - a_p - \frac{2}{3}\left[(1+a_p)^2 m_p^2 (r_M^p)^2 - m_p^2 (r_E^p)^2\right]\right\} + \mathcal{O}\left(Q^4\right)$$

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- $a_p=1.793$, $\bar{\beta}=2.5(4)\times10^{-4}$ fm^3
- $r_M=0.776(34)(17)$ fm,
- $r_E^H=0.8751(61)$ fm or $r_E^{\mu H}=0.84087(26)(29)$ fm

$$W_1(0,Q^2)=13.6+\frac{Q^2}{m_p^2}(-54\pm7)+\mathcal{O}\left(Q^4\right)$$

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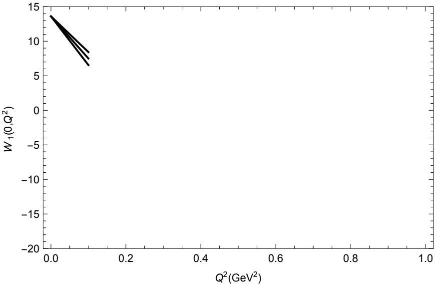
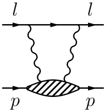


Two Photon Exchange: large Q^2 limit



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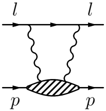

- Need poorly constrained non-perturbative function $W_1(0, Q^2)$
- Calculable in *large* Q^2 limit using Operator Product Expansion (OPE) [J. C. Collins, NPB **149**, 90 (1979)]
  The photon "sees" the quarks and gluons inside the proton

$$W_1(0, Q^2) = c/Q^2 + \mathcal{O}\left(1/Q^4\right)$$

- Result was used to estimate two photon exchange effects
- c calculated in [J. C. Collins, NPB **149**, 90 (1979)]

RENORMALIZATION OF THE COTTINGHAM FORMULA

John C. COLLINS *
Joseph Henry Laboratories, Princeton University, Princeton, New Jersey 08540, USA

Received 23 October 1978

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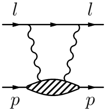

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$$W_1(0, Q^2) = c/Q^2 + \mathcal{O}\left(1/Q^4\right)$$

- Result was used to estimate two photon exchange effects
- c calculated in [J. C. Collins, NPB **149**, 90 (1979)]

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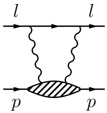
- Was it?

Two Photon Exchange: large Q^2 limit



$$
\begin{aligned}
W^{\mu\nu} &= \frac{1}{2}\sum_s i\int d^4x\, e^{iq\cdot x}\langle \boldsymbol{k}, s|\, T\{J^{\mu}_{\rm e.m.}(x)J^{\nu}_{\rm e.m.}(0)\}|\boldsymbol{k}, s\rangle \\
&= \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)W_1 + \left(k^\mu - \frac{k\cdot q\, q^\mu}{q^2}\right)\left(k^\nu - \frac{k\cdot q\, q^\nu}{q^2}\right)W_2
\end{aligned}
$$

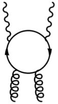
- $W_1(0, Q^2)$ is dimensionless

$$
W_1 \sim \frac{\langle \text{Proton}|O|\text{Proton}\rangle}{Q^2} + \mathcal{O}\left(\frac{1}{Q^4}\right)
$$

- O is a dimension 4 operator:

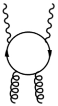
- Quarks: Spin 0: $m_q\bar{q}q$ Spin 2: $\bar{q}\left(iD^\mu\gamma^\nu + iD^\nu\gamma^\mu - \frac{1}{4}i\not{D}\, g^{\mu\nu}\right)q$
- Gluons: must be color singlet: $G_a^{\alpha\beta}G_a^{\rho\sigma}$
- What gluon operators can we have?

Gluon operators



- Gluons: must be color singlet $\quad G_a^{\alpha\beta} G_a^{\rho\sigma}$
  A product of (E^i, B^i) and (E^j, B^j) has $7 \times 6/2 = 21$ components:

Gluon operators



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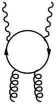
Gluon operators



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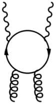
Gluon operators



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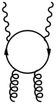
- 9 components of traceless symmetric tensor: $G^{\mu\alpha} G_{\alpha}^{\ \nu} - \dfrac{1}{4} G^{\alpha\beta} G_{\alpha\beta} g^{\mu\nu}$
  chromomagnetic stress-energy tensor
- What else? 10 components of

Gluon operators



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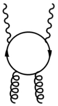
$$O^{\mu\alpha\nu\beta} = -\frac{1}{4}\left(\epsilon^{\mu\alpha\rho\sigma}\epsilon^{\nu\beta\kappa\lambda} + \epsilon^{\mu\beta\rho\sigma}\epsilon^{\nu\alpha\kappa\lambda}\right) G_{\rho\kappa} G_{\sigma\lambda} - \text{all possible traces}$$

For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$

Gluon operators



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  For example $O^{0123} = G^{01} G^{23} + G^{03} G^{21} = E^1 B^1 - E^3 B^3$
- For protons: $\langle \text{Proton} | O^{\mu\alpha\nu\beta} | \text{Proton} \rangle = 0$

  What about $\langle \text{Medium} | O^{\mu\alpha\nu\beta} | \text{Medium} \rangle$?

  Solution looking for a problem...

Summary: Possible operators

- In total we have four operators with non-zero proton matrix elements.

- Quarks:
- Spin 0: $m_q \bar{q} q$
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Large Q^2 behavior

- In 1978 Collins calculated EM corrections to the nucleon mass with an emphasis on $m_n - m_p$
  The mass only depends on spin-0 operators (q quark, $G^{\mu\nu}$ gluon)

$$\langle P|m_q\bar{q}q|P\rangle, \qquad \langle P|G^{\mu\nu}G_{\mu\nu}|P\rangle$$

	Quark	Gluon
Spin-0	Collins '78	Collins '78

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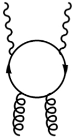
- Need to calculate the spin-2 contribution [Hill, GP arXiv:1611.09917]

	Quark	Gluon
Spin-0	Collins '78	Collins '78
Spin-2	Hill, GP '16	Hill, GP '16

- Collins's result is not enough for muonic hydrogen!

Large Q^2 behavior

- Requires 1-loop calculation



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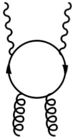

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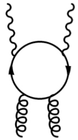

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- Collins didn't calculate the spin-0 gluon contribution directly
  He extracted it from another calculation

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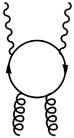

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- For three light quark u, d, s
  Correct result: $\sum_q e_q^2 = (\frac{2}{3})^2 + (\frac{1}{3})^2 + (\frac{1}{3})^2 = \frac{2}{3}$
  Collins: $\sum_q = 3$
  Too large by a factor of 4.5...

- 1978:

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- 2016:

Corrigendum to "Renormalization of the Cottingham formula" [Nucl. Phys. B 149 (1979) 90–100]

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I thank Richard Hill and Gil Paz for pointing out the important error about the coefficient of the gluonic operator, as reported in Ref. [2]. This work was supported in part by the U.S. Department of Energy under Grant No. DE-SC0013699.

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[2] R.J. Hill, G. Paz, Nucleon spin-averaged forward virtual Compton tensor at large Q^2, arXiv:1611.09917.

Large Q^2 behavior

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- Even worse, quark spin-0 and gluon spin-0 come with opposite signs
  After correcting the mistake they largely cancel
  $W_1(0, Q^2)$ is **dominated** by spin-2 contribution

- Lesson: It is important to do a full calculation

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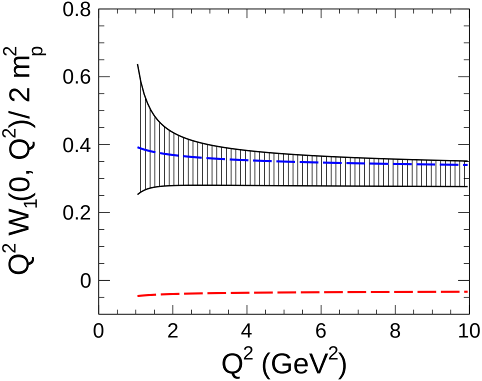
- Flip side: You cannot use $m_n - m_p$ to constrain muonic hydrogen

Large Q^2 behavior: Total contribution

- The total contribution



- Dashed red: spin 0
- Dashed blue: spin 2
- Vertical stripes: total contribution with perturbative and hadronic errors added in quadrature

Two Photon Exchange: Modeling

- Simple modeling: use OPE for $Q^2 \geq 1$ GeV^2
- Model unknown Q^4: add $\Delta_L(Q^2) = \pm Q^2/\Lambda_L^2$ with $\Lambda_L \approx 500$ MeV
- Model unknown $1/Q^4$: add $\Delta_H(Q^2) = \pm \Lambda_H^2/Q^2$ with $\Lambda_H \approx 500$ MeV

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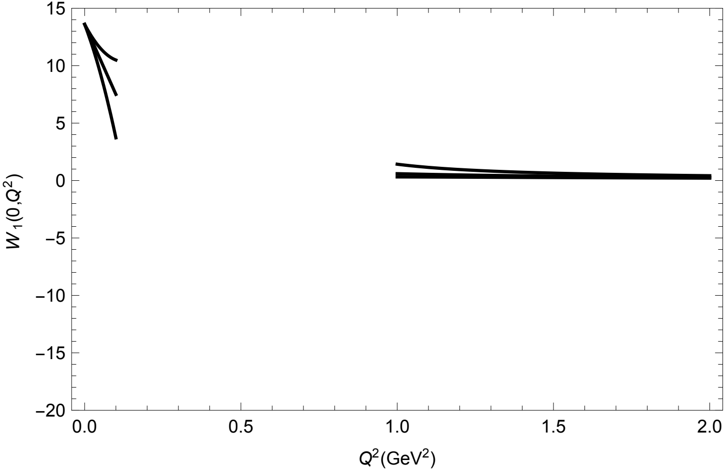
- How to connect the curves?



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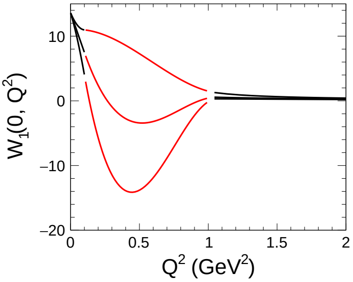

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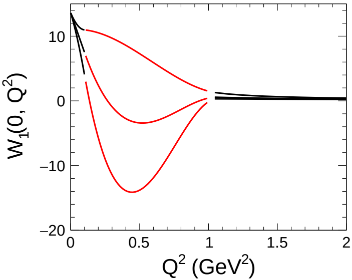
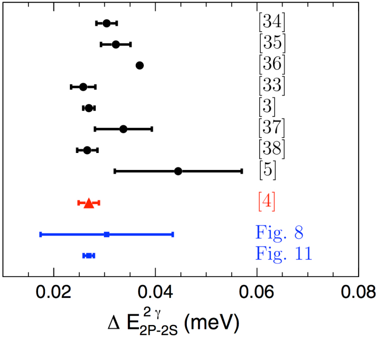


- Energy contribution: $\delta E(2\mathrm{S})^{W_1(0,Q^2)} \in [-0.046\,\mathrm{meV}, -0.021\,\mathrm{meV}]$
  To explain the puzzle need this to be ~ 0.3 meV
- Caveats: OPE might be only valid for larger Q^2
  $W_1(0, Q^2)$ might be different than the interpolated lines

Two Photon Exchange: Other approaches

- Similar results found by other groups



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[Fig. 8] Hill, GP PRD 95, 094017 (2017).

Experimental test

- How to test?
- New experiment: $\mu - p$ scattering
  MUSE (MUon proton Scattering Experiment) at PSI
  [R. Gilman et al. (MUSE Collaboration), arXiv:1303.2160]

- Need to connect muon-proton scattering and muonic hydrogen can use a new effective field theory: QED-NRQED
  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Part 3: Connecting muon-proton scattering and muonic hydrogen

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV $\ll m_\mu, m_p$

  Both proton and muon non-relativistic

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV $\ll m_\mu, m_p$

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- MUSE:

  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

MUSE

- Muonic hydrogen:

  Muon momentum $\sim m_\mu c\alpha \sim 1$ MeV $\ll m_\mu, m_p$

  Both proton and muon non-relativistic

- MUSE:

  Muon momentum $\sim m_\mu \sim 100$ MeV

  Muon is relativistic, proton is still non-relativistic

- QED-NRQED effective theory:

- Use QED for muon
- Use NRQED for proton

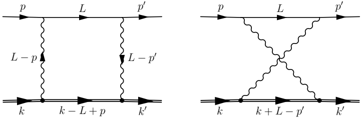
  $m_\mu/m_p \sim 0.1$ as expansion parameter

- A *new* effective field theory suggested in

  [Hill, Lee, GP, Mikhail P. Solon, PRD **87** 053017 (2013)]

QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]



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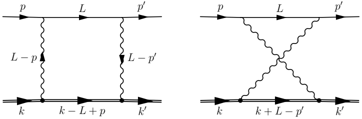

- QED-NRQED result

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1 - v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1 + \frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1 - v^2\sin^2\theta}\right]$$

$Z = 1$, $E =$ muon energy, $v = |\vec{p}|/E$, $q = p' - p, \theta$ scattering angle

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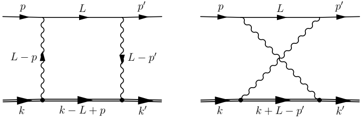

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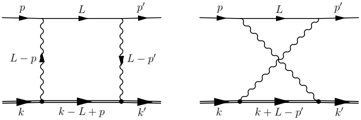
$Z = 1$, $E =$ muon energy, $v = |\vec{p}|/E$, $q = p' - p, \theta$ scattering angle

- *Same result* as scattering relativistic lepton off static $1/r$ potential
  [Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)]
  reproduced in [Itzykson, Zuber, "Quantum Field Theory"]

QED-NRQED Effective Theory

- Example: TPE at the lowest order in $1/m_p$
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]



- QED-NRQED result

$$\frac{d\sigma}{d\Omega} = \frac{Z^2\alpha^2 4E^2\left(1 - v^2\sin^2\frac{\theta}{2}\right)}{\vec{q}^{\,4}}\left[1 + \frac{Z\alpha\pi v\sin\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\right)}{1 - v^2\sin^2\theta}\right]$$

  $Z = 1$, $E =$ muon energy, $v = |\vec{p}|/E$, $q = p' - p, \theta$ scattering angle

- *Same result* as scattering relativistic lepton off static $1/r$ potential
  [Dalitz, Proc. Roy. Soc. Lond. **206**, 509 (1951)]
  reproduced in [Itzykson, Zuber, "Quantum Field Theory"]

- *Same result* as $m_p \to \infty$ of "point particle proton" QED scattering
  (For $m_p \to \infty$ only proton charge is relevant)

QED-NRQED Effective Theory beyond $m_p \to \infty$ limit

- QED-NRQED allows to calculate $1/m_p$ corrections
- Example: one photon exchange $\mu + p \to \mu + p$:
  QED-NRQED $= 1/m_p$ expansion of form factors
  [Steven P. Dye, Matthew Gonderinger, GP, PRD **94** 013006 (2016)]

Connecting muon-proton scattering to muonic hydrogen

- Matching

QED, QCD $\qquad$ $G_{E,M}$, Structure func., $W_1(0, Q^2)$

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- Need to match QED-NRQED contact interaction, e.g. $\bar{\mu}\gamma^0\mu\psi_p^\dagger\psi_p$ to NRQED-NRQED contact interaction, e.g. $\psi_\mu^\dagger\psi_\mu\psi_p^\dagger\psi_p$ [Dye, Gonderinger, GP *in progress*]

Connecting muon-proton scattering to muonic hydrogen

- To do list:

1) Relate QED-NRQED contact interactions
   to NRQED contact interactions and $W_1(0, Q^2)$

2) Calculate $d\sigma(\mu + p \to \mu + p)$ and asymmetry in terms of r_E^p and TPE

3) *Direct* relation between μ–p scattering and muonic H

Proton radius puzzle: Latest developments

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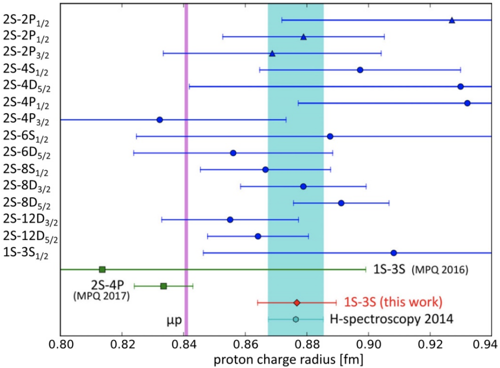

Proton radius puzzle: Future developments

- PRad: New low Q^2 $e-p$ scattering experiment
  Proton radius results in summer 2018

- July 2018: the 4th proton radius puzzle workshop at Mainz Germany
  [Organizers: Richard J. Hill, GP, Randolf Pohl]

- MUSE: new $\mu - p$ scattering experiment
  Data taking starting mid 2018
  [MUSE Collaboration TDR, arXiv:1709.09753 [physics.ins-det]]

Conclusions

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- Proton radius puzzle: $> 5\sigma$ discrepancy between
- r_E^p from muonic hydrogen
- r_E^p from hydrogen and $e - p$ scattering

- After almost 8 years the origin is still not clear

1) Is it a problem with the electronic extraction?
2) Is it a hadronic uncertainty?
3) is it new physics?

- Motivates a reevaluation of our understanding of the proton

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   hydrogen using a new effective field theory: QED-NRQED

- Much more work to do!

- Thank you