Modeling the Inspiral of Compact Objects into Supermassive Black Holes

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Outline

- Extreme mass-ratio inspirals as sources for LISA: Capture mechanism, estimates of S/N and detection rates
- Theoretical challenge: high accuracy computation of strong-field orbital evolution & waveforms (generic orbits, Kerr)
- Energy-momentum balance approach
- Self-force approach:
  - issues of principle (regularization, gauge,….) ✓
  - development of practical calculation methods ✓
  - implementation of calculation methods

Massive black holes are abundant

- Growing evidence that Supermassive BHs are present in the nuclei of most galaxies (Kormendy & Richstone 1995)
  - theoretical (models of AGN engines)
  - circumstantial (time variability, superluminal jets)
  - observational (stellar and gas dymanics in galaxy cusps)
- A few dozens listed already, with masses 10^6-10^9 $M_\odot$ (table)
- Evidence that in many cases SMBHs are rapidly rotating (Elvis, Risaliti & Zamorani 2002)
- Recently: observed examples of mid-size holes (~1000 $M_\odot$) (Miller; Strohmayer 2003)

Composition of stellar cusps (after 10^{10} yr)

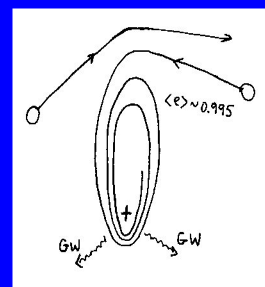
	Main seq.	WD	NS	BH
<*m*>	0.3 $M_\odot$	0.7 $M_\odot$	1.4 $M_\odot$	9 $M_\odot$
% by mass	76%	18%	2%	3%
% by no.	90%	8%	2%	0.1%
	But since detection volume $\propto m^3$ …			
LISA sees		1 :	2 :	26

(S. Phinney 2001)

- Further bias in favor of BHs (capture cross section higher, dynamical friction more effective).

Capture of a CO by a SMBH: mechanism

- CO kicked into "loss cone" through multibody scattering
- Orbit still highly eccentric (e=0.4-0.6) when enters LISA band.
- 10^5-10^6 orbits during last year, inside LISA band. (Finn & Thorne 2000)
- Gradual circularization, but still substantial ecc. at isco. (Cutler, Kennefick, & Poisson 1994)

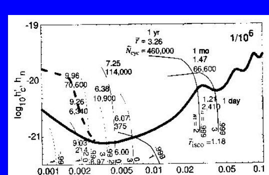


- Rate: ~10^{-8}/yr/galaxy

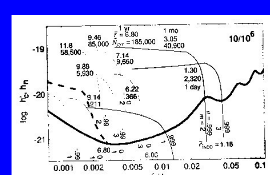
⇒ ~1/year out to 1 Gpc (w/ very conservative mass function)

If only one $50M_{\odot}$ CO at each galaxy, then a few/year at z>1, all detectable by LISA! (Hils & Bender 1995, Sigurdson & Rees 1997)

LISA S/N and detection rates



$1/10^6$ at 1 Gpc, circular orbit



$10/10^6$ at 1 Gpc, circular orbit

- During last year: S/N~10(m/$M_{\odot}$)(1 Gpc/r) (Finn & Thorne 2000)
- Similar values for eccentric orbits (LB & Cutler 2003)

⇒ LISA sees m>$50M_{\odot}$ inspirals from anywhere in the universe, but m~$M_{\odot}$ inspirals only out to a few hundreds Mpc.

- But detection rate highly uncertain, mainly due to unknown ease of data analysis (how much S/N do we need?)

Need high-accuracy theoretical modeling

- Data analysis difficult (17d para. Space/ 8-10 para. important)
  - Will loose much S/N due do template discretization -- crucial to minimize S/N loss due to inaccurate templates!
- To assure $(\Delta\Phi)_{\text{total}}\ \delta\ 1$, require

$$\Delta\Phi / \Phi\ \delta\ 1/N_{\text{cycles}} \sim m/M$$

- So need $\boxed{\Delta\Phi / \Phi\ \delta\ 10^{-6}}$

The theoretical challenge:

- Compute orbital evolution and subsequent waveform to fractional accuracy of 10^{-6}, for
  - 1-100 $M_\odot$ inspiralling onto 10^6-10^8 $M_\odot$ ($\Rightarrow m/M=10^{-4}$-10^{-8})
  - Generic orbits (inclined, eccentric)
  - Kerr black hole*
- Over the last 6 years ~70 papers, 6 dedicated conferences ("Capra Ranch"). Much progress.

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* Spin of CO may be neglected; no sign of chaotic behavior in extreme-mass-ratio inspirals (Hartl 2003)

Scientific payoffs

- Accurate mapping of the hole's strong field, test of "no hair" theorem [Kerr has only 2 independent mass multipoles; LISA could accurately measure at least 3-5 (Ryan 1997)]
- Test of alternative theories of Gravity (scalar, YM,…)
- Precise measurement of SMBHs masses and spins: $\delta M/M \sim 10^{-4}$-10^{-5}, $\delta S/S \sim 0.01$ (Ryan 1997)
- Decide on SMBH growth mechanism (Hughes & Blandford 2002)

➢ December 2001: LIST decides that LISA noise floor is to be determined by the requirement of detecting CO inspirals.

Possible approaches

- PN methods well developed, but, in our case, not useful!
(most S/N comes from strong-field region, v/c~1)
- Full Numerical Relativity: Recent 1st attempt (Bishop, Gómez, Husa, Lehner, & Winicour 2003). Computed one orbit in Schwarzschild.
- Expansion in m/M: assuming point particle on a fixed background and using black hole perturbation theory (Teukolsky-Sasaki-Nakamura formalism). Two variants:
  - ➢ conservation law
  - ➢ local force ("self force")

Conservation law method

Tanaka, Shibata, Sasaki, Tagoshi, & Nakamura (1993)
Cutler, Kennefick, & Poisson (1994)
Mino, Tanaka, Shibata, Sasaki, Tagoshi, Nakamura (1998)
} *a=0*

Kennefick (1998)
Finn & Thorne (2000)
} ⇨ circular, equatorial

Hughes (2000, 2001) ⇨ circular, inclined

Glampedakis & Kennefick (2002) ⇨ eccentric, equatorial

Conservation law method - basic idea

- Evolution is adiabatic: $(\tau_{rad\text{-}reaction}/\tau_{orbit}) \propto M/m \gg 1$; orbit is approximately geodesic along a few cycles
- Solve for $\psi_4^{lm\omega}$ with geodesic orbit $\{E, L_z, Q\}$ as a source (use Teukolsky/Sasaki-Nakamura to reduce PDEs to ODEs)
- From fluxes at infinity and through the horizon infer $\{\dot{E}, \dot{L}_z, \dot{Q}\}$
- Generate "radiation reaction grid" in phase space of orbits (e.g.)
- Evolve trajectories across the grid (e.g.)

- Failure of method: cannot calculate $\dot{Q}$ for generic orbits in Kerr (also: does not account for conservative piece of force)

Local (self-)force approach

- Perturbation $h_{\alpha\beta}$ ($\propto m$) exerts a "self force" F^α ($\propto m^2$):

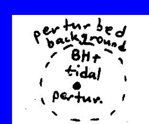
$$m(du^\alpha/d\tau) = 0 + F^\alpha$$

- Analogous to Abraham-Lorentz-Dirac EOM for electric charge in flat space: $m\vec{a} = \vec{F}_{ext}(\propto q) + \frac{2}{3}(q^2/c^2)\dot{\vec{a}}$
- Given F_α, can
  - ➢ directly integrate EOM, or
  - ➢ obtain local $\dot{C} = m^{-1}(\partial C/\partial u_\alpha)F_\alpha$ $\quad (C \equiv \{E, L_z, Q\})$
- Generalization to curved space by DeWitt & Brehme (1960) (feature)
- Generalization to gravitational SF (in arbitrary vacuum spacetime) by Mino , Sasaki, & Tanaka (1997), Quinn & Wald (1997)

Gravitational self-force: regularization methods

- Mino, Sasaki, & Tanaka (1997):
  - (1) Hadamard expansion+ world-tube integration+ local conservation

  - (2) Matched asymptotic expansions



- Quinn & Wald (1997): (3) "Comparison axiom"

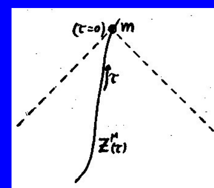
Alternative methods:

- ➢ Zeta function (Lousto 2000)
- ➢ Extended object (Ori & Rosenthal 2002)
- ➢ power expansion (Mino, Nakano, & Sasaki 2002)
- ➢ Radiative Green's function (Detweiler & Whiting 2002)
- ➢ Kerr symmetry (Mino 2003)

} All consistent with Mino-Quinn-Wald

The gravitational self-force

- Has only "tail" contribution, no "light-cone" contribution:



$$F^{\alpha} = F^{\alpha}_{\text{tail}} = \lim_{\varepsilon\to 0^+} m^2 \int_{-\infty}^{-\varepsilon} d\tau\, \nabla^{\alpha\beta\gamma} G_{\beta\gamma\beta'\gamma'}[z(0), z(\tau)]\, u^{\beta'}(\tau)\, u^{\gamma'}(\tau)$$

- Alternative, useful, formulation:

$$F^{\alpha} = \lim_{x\to z(0)} F^{\alpha}_{\text{tail}}(x), \qquad F^{\alpha}_{\text{tail}}(x) = \underbrace{\left(F^{\alpha}_{\text{full}}(x) = m\nabla^{\alpha\beta\gamma} h_{\beta\gamma}(x)\right)}_{\text{from full mertic pert.}} - \underbrace{\left(F^{\alpha}_{\text{direct}}(x)\right)}_{\text{only light-cone cont.}}$$

- In practice: know how to get multipoles of h_{full} and F_{full}. But: each mode carries a mixed imprint of both "tail" and "direct".

Practical calculation scheme: the mode-sum method

- LB & Ori 2000, 2003 (scalar,EM)
- LB 2001 (gravitational)

$$F_{\text{tail}} = \lim_{x\to z(0)} \sum_l \left(F^l_{\text{full}}(x) - F^l_{\text{direct}}(x)\right)$$

Multipole contributions finite at the particle, $F^l_{\text{full}}, F^l_{\text{direct}}(l\to\infty) \propto l$

$$\Downarrow$$

$$F_{\text{tail}} = \sum_l \left(F^l_{\text{full}}(0) - A\cdot l - B - C/l\right) - \underbrace{\sum_l \left(F^l_{\text{direct}}(0) - A\cdot l - B - C/l\right)}_{D}$$

- "Regularization parameters" A_α, B_α, C_α, D_α derived analytically by local analysis:
  - LB, Mino, Nakano, Ori, & Sasaki (PRL 2002) - equ. orbit in Schwarz.
  - LB & Ori (PRL 2003) - generic orbit in Kerr

Summary of prescription for computing the local self-force

- Solve (numerically) Tuekolsky-Sasaki-Nakamura equations, with a geodesic source → get $\psi_0^{lm\omega}, \psi_4^{lm\omega}$
- Construct metric perturbation $h_{\alpha\beta}^{lm\omega}$ using Wald-Chrzanowski-Ori (In Schwarz. can solve directly for $h_{\alpha\beta}$ using RW-Zerilli-Moncrief)
- Construct the "full force" modes $F_{\rm full}^{\alpha l} = \sum_{m\omega} \nabla^{\alpha\beta\gamma} h_{\beta\gamma}^{lm\omega}$
- Apply mode-sum formula:

$$F^{\alpha} = \sum_{l} \left(F_{\rm full}^{\alpha l} - A^{\alpha} \cdot l - B^{\alpha} - C^{\alpha}/l \right) - D^{\alpha}$$

Implementation

Scalar force:

- Static particle in Schwarz. (Burko 2000)
- Circular orbit in Schwarz. (Burko 2000)
- Radial trajectories in Schwarz. (LB & Burko 2000)
- Static particle in Kerr (Burko & Liu 2001)

Gravitational force:

- Radial trajectories in Schwarz. (LB & Lousto 2002)
- Circular orbit in Schwarz. (LB & Lousto, in progress)

◆ Local analysis can be pushed to higher orders in $1/l$, providing analytic approximation for the self force. For example:

$$F^{rl} = -\frac{15}{16} m^2 \frac{E^2}{r^2} \left(E^2 + 4M/r - 1\right)\left(l + 1/2\right)^{-2} + O(l^{-4})$$

Radial infall in Schwarz. (LB & Lousto 2002)

The gauge problem

- Local self-force is gauge dependent:

  $x^\mu \to x^\mu - \xi^\mu$:

  $h_{\alpha\beta} \to h_{\alpha\beta} + \xi_{\alpha;\beta} + \xi_{\beta;\alpha}$

  $F^\alpha \to F^\alpha - m\left[\left(g^{\alpha\lambda} + u^\alpha u^\lambda\right)\ddot{\xi}_\lambda + R^\alpha{}_{\mu\lambda\nu} u^\mu \xi^\lambda u^\nu\right]$ (LB & Ori 2001)

  Gauge invariants (e.g., orbit integrated $\dot{C}$) can be derived from *F* in whatever regular gauge.
- Mino/Quinn/Wald's result (and, consequently, mode-sum scheme) formulated in the "harmonic gauge".
- Problem: to implement mode-sum scheme, need *h* in harmonic gauge; but standard pert. tools give *h* in "radiation" (or RW) gauge.
- Mode-sum formula "gauge invariant" if gauge regular (LB & Ori 2001):

$$F^\alpha(\text{any gauge}) = \sum_l \left[F^{\alpha l}_{\text{full}}(\text{any gauge}) - A^\alpha \cdot l - B^\alpha - C^\alpha / l\right] - D^\alpha$$

- Problem: radiation gauge becomes irregular in the presence of a particle [*h* is singular along a null ray - (LB & Ori 2001)]

Solution to the gauge problem

(LB & Ori 2003)

- We devised an "intermediate" gauge, which is
  - regular (*h* singular only at the particle)
  - simply related to the radiation gauge
- Then:

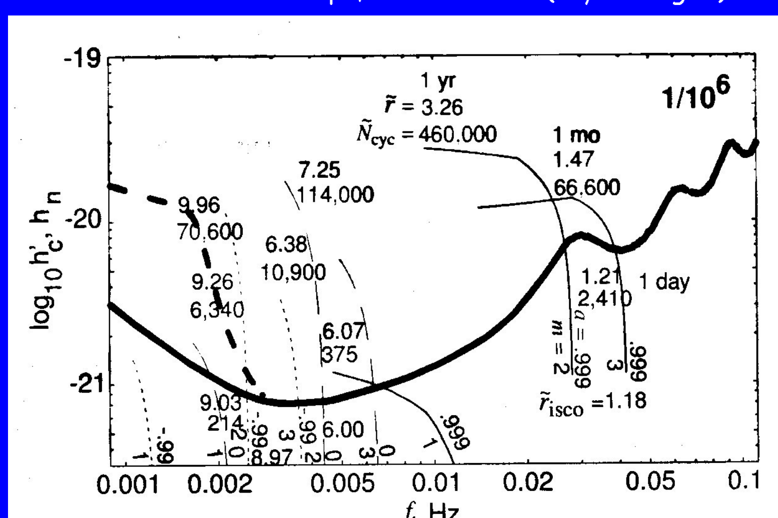
$$F^\alpha(\text{intermediate}) = \sum_l \left[F^{\alpha l}_{\text{full}}(\text{radiation}) - A^\alpha \cdot l - B^\alpha - C^\alpha / l\right] - D^\alpha - \delta F^\alpha$$

- "Gauge correction term" δF^α recently calculated explicitly for generic orbits in Kerr.
- F(intermediate) can now be used to calculate the long-term, gauge-invariant, orbital evolution

What's next

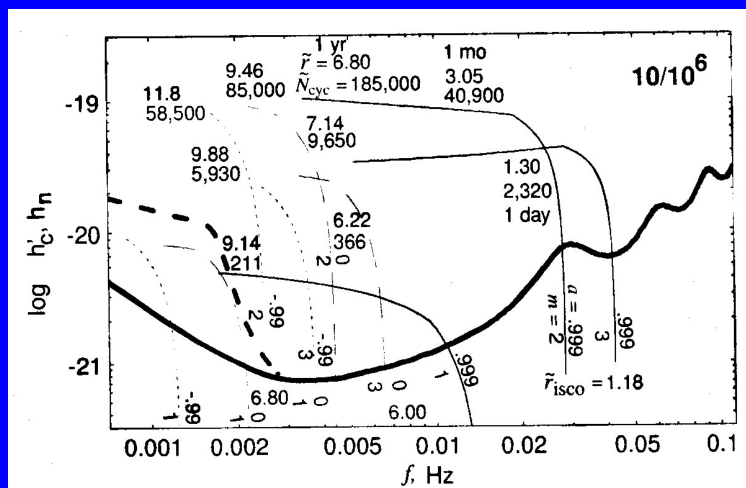
- Compare outcome from self force conservation law full numerical calculations in cases where possible. Ongoing Work on circular orbits in Schwarzschild will provide a first opportunity.
- Implement calculation scheme for generic orbits, Kerr.
- In the longer term: 2nd-order perturbation theory and 2nd-order self-force needed for improving template accuracy.

1 M⊙ onto 10^6 M⊙ at 1 Gpc, circular orbit (sky averaged)



(Finn & Thorne 2000)

10 M⊙ onto 10^6 M⊙ at 1 Gpc, circular orbit (sky averaged)



(Finn & Thorne 2000)

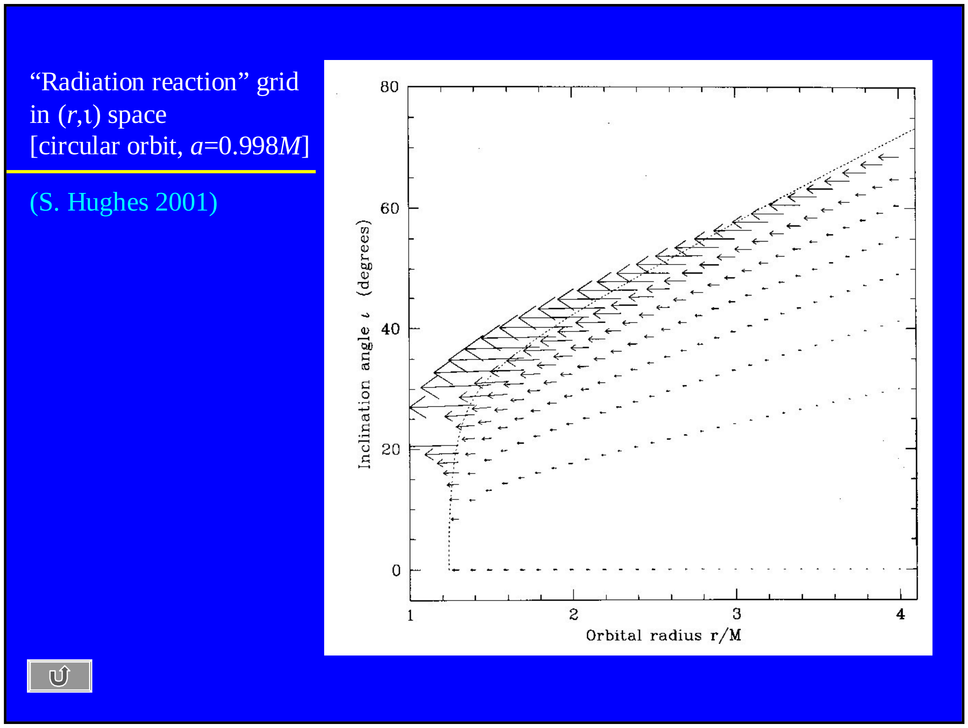
Census of Supermassive black holes as of March 2001

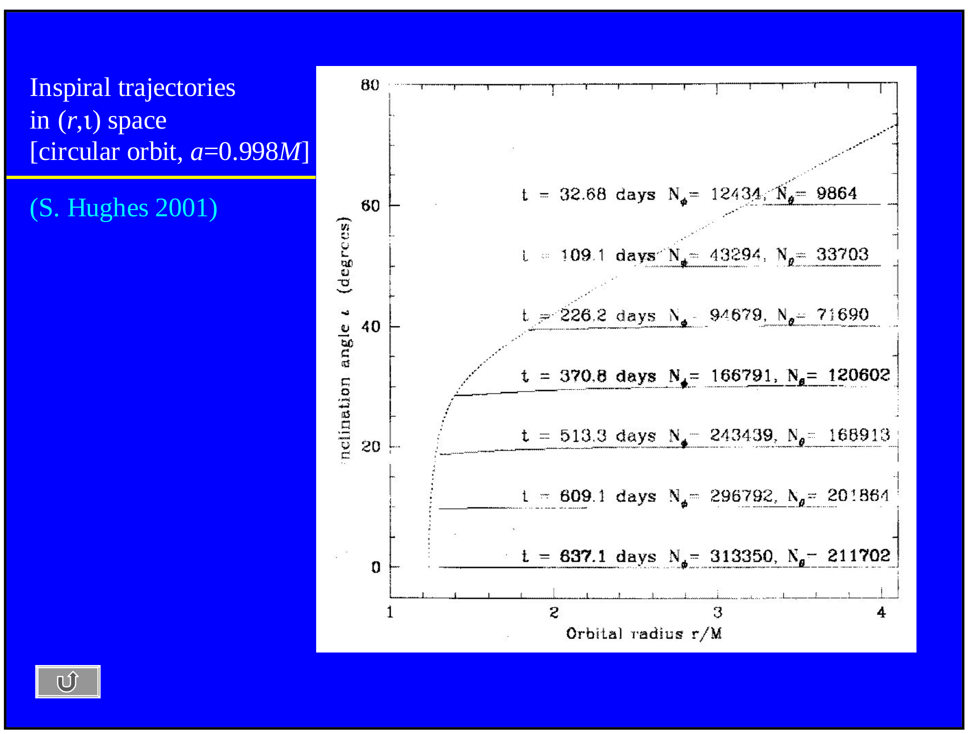
(Kormendy & Gebhardt 2001)

TABLE 1

Census of Supermassive Black Holes (2001 March)

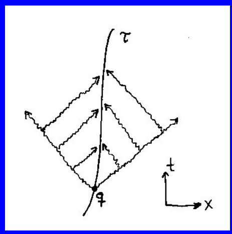
Galaxy	Type	$M_{B,\mathrm{bulge}}$	$M_\bullet$ (M_{low}, M_{high}) ($M_\odot$)	σ_e (km/s)	D (Mpc)	r_{cusp} (arcsec)	Reference
Galaxy	Sbc	−17.65	2.6 (2.4−2.8) e6	75	0.008	51.40	See notes
M 31	Sb	−19.00	4.5 (2.0−8.5) e7	160	0.76	2.06	Dressler + 1988; Kormendy 1988a
M 32	E2	−15.83	3.9 (3.1−4.7) e6	75	0.81	0.76	Tonry 1984, 1987
M 81	Sb	−18.16	6.8 (5.5−7.5) e7	143	3.9	0.76	Bower + 2001b
NGC 821	E4	−20.41	3.9 (2.4−5.6) e7	209	24.1	0.03	Gebhardt + 2001
NGC 1023	S0	−18.40	4.4 (3.8−5.0) e7	205	11.4	0.08	Bower + 2001a
NGC 2778	E2	−18.59	1.3 (0.5−2.9) e7	175	22.9	0.02	Gebhardt + 2001
NGC 3115	S0	−20.21	1.0 (0.4−2.0) e9	230	9.7	1.73	Kormendy + 1992
NGC 3377	E5	−19.05	1.1 (0.6−2.5) e8	145	11.2	0.42	Kormendy + 1998
NGC 3379	E1	−19.94	1.0 (0.5−1.6) e8	206	10.6	0.20	Gebhardt + 2000a
NGC 3384	S0	−18.99	1.4 (1.0−1.9) e7	143	11.6	0.05	Gebhardt + 2001
NGC 3608	E2	−19.86	1.1 (0.8−2.5) e8	182	23.0	0.13	Gebhardt + 2001
NGC 4291	E2	−19.63	1.9 (0.8−3.2) e8	242	26.2	0.11	Gebhardt + 2001
NGC 4342	S0	−17.04	3.0 (2.0−4.7) e8	225	15.3	0.34	Cretton + 1999a
NGC 4473	E5	−19.89	0.8 (0.4−1.8) e8	190	15.7	0.13	Gebhardt + 2001
NGC 4486B	E1	−16.77	5.0 (0.2−9.9) e8	185	16.1	0.81	Kormendy + 1997
NGC 4564	E3	−18.92	5.7 (4.0−7.0) e7	162	15.0	0.13	Gebhardt + 2001
NGC 4594	Sa	−21.35	1.0 (0.3−2.0) e9	240	9.8	1.58	Kormendy + 1988b
NGC 4649	E1	−21.30	2.0 (1.0−2.5) e9	375	16.8	0.75	Gebhardt + 2001
NGC 4697	E4	−20.24	1.7 (1.4−1.9) e8	177	11.7	0.41	Gebhardt + 2001
NGC 4742	E4	−18.94	1.4 (0.9−1.8) e7	90	15.5	0.10	Kaiser + 2001
NGC 5845	E	−18.72	2.9 (0.2−4.6) e8	234	25.9	0.18	Gebhardt + 2001
NGC 7457	S0	−17.69	3.6 (2.5−4.5) e6	67	13.2	0.05	Gebhardt + 2001
NGC 2787	SB0	−17.28	4.1 (3.6−4.5) e7	185	7.5	0.14	Sarzi + 2001
NGC 3245	S0	−19.65	2.1 (1.6−2.6) e8	205	20.9	0.21	Barth + 2001
NGC 4261	E2	−21.09	5.2 (4.1−6.2) e8	315	31.6	0.15	Ferrarese + 1996
NGC 4374	E1	−21.36	4.3 (2.6−7.5) e8	296	18.4	0.24	Bower + 1998
NGC 4459	SA0	−19.15	7.0 (5.7−8.3) e7	167	16.1	0.14	Sarzi + 2001
M 87	E0	−21.53	3.0 (2.0−4.0) e9	375	16.1	1.18	Harms + 1994
NGC 4596	SB0	−19.48	0.8 (0.5−1.2) e8	136	16.8	0.22	Sarzi + 2001
NGC 5128	S0	−20.80	2.4 (0.7−6.0) e8	150	4.2	2.26	Marconi + 2001
NGC 6251	E2	−21.81	6.0 (2.0−8.0) e8	290	106	0.06	Ferrarese + 1999
NGC 7052	E4	−21.31	3.3 (2.0−5.6) e8	266	58.7	0.07	van der Marel + 1998
IC 1459	E3	−21.39	2.0 (1.2−5.7) e8	323	29.2	0.06	Verdoes Kleijn + 2001
NGC 1068	Sb	−18.82	1.7 (1.0−3.0) e7	151	15	0.04	Greenhill + 1996
NGC 4258	Sbc	−17.19	4.0 (3.9−4.1) e7	120	7.2	0.36	Miyoshi + 1995
NGC 4945	Scd	−15.14	1.4 (0.9−2.1) e6		3.7		Greenhill + 1997


"Radiation reaction" grid
in (r,ι) space
[circular orbit, a=0.998M]
(S. Hughes 2001)
Inclination angle ι (degrees)
Orbital radius r/M
0
20
40
60
80
1
2
3
4


Inspiral trajectories
in (r,ι) space
[circular orbit, a=0.998M]
(S. Hughes 2001)
t = 32.68 days N_φ= 12434, N_θ= 9864
t = 109.1 days N_φ= 43294, N_θ= 33703
t = 226.2 days N_φ= 94679, N_θ= 71690
t = 370.8 days N_φ= 166791, N_θ= 120602
t = 513.3 days N_φ= 243439, N_θ= 168913
t = 609.1 days N_φ= 296792, N_θ= 201864
t = 637.1 days N_φ= 313350, N_θ= 211702
Inclination angle ι (degrees)
Orbital radius r/M
0
20
40
60
80
1
2
3
4

Non-local nature of SF in curved space

GW back-scattered off spacetime curvature



Failure of Huygens principle; "breakdown of locality"

E.g.: a static charge inside a massive shell (Burko, Liu, & Soen 2000)