Mode Coupling in the Nonlinear Response of Black Holes

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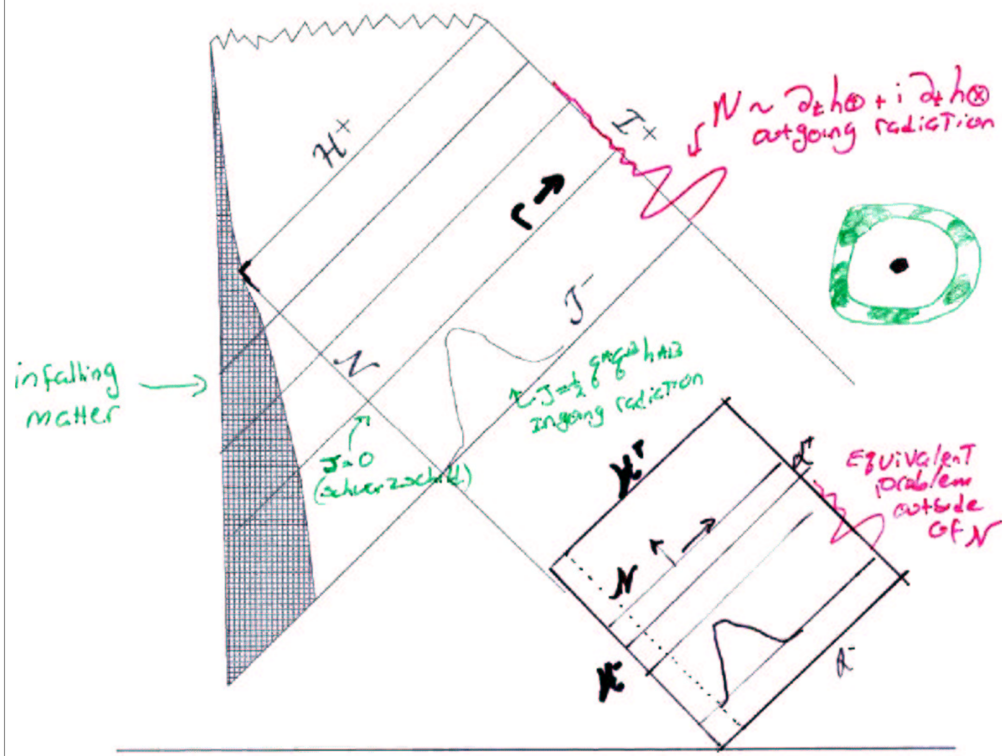
Overview

Analyze the mode-coupling in the scattering off of a Schwarzschild Black hole in full GR using characteristic techniques.



Past Results

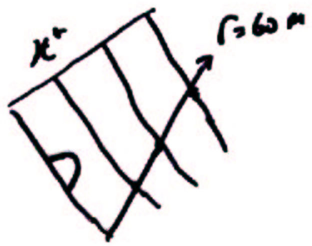
N.T. Bishop et al. Phys. Rev. D **56** 6298 (1997)

Used ' Pitt Null Code' to Analyze Bondi News from nonlinear scattering off of a Schwarzschild black hole (did not analyze mode coupling)

G. Allen et al. gr-qc/9806014

Analyzed mode coupling from a Cauchy approach. Performed axisymmetric non-linear runs.

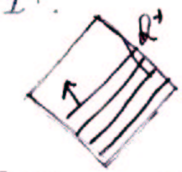
P. Papadopoulos, Phys. Rev. D **65**, 084016 (2002) Analyzed mode coupling with an axisymmetric characteristic code for an outgoing pulse. (evolution on ingoing null hypersurfaces with outer boundary at $r = 60M$)



The Setup

Characteristic Evolution with the 'Pitt Null Code'

'Pitt Null Code' is a characteristic code with evolution along a family of outgoing null hypersurfaces that terminate at $\mathcal{I}^+$.



$$ds^2 = -\left[e^{2\beta}(1 + r\tilde{W}) - r^2 h_{AB}U^A U^B\right] du^2$$
$$- 2e^{2\beta}dudr - 2r^2 h_{AB}U^B du dx^A + r^2 h_{AB} dx^A dx^B$$

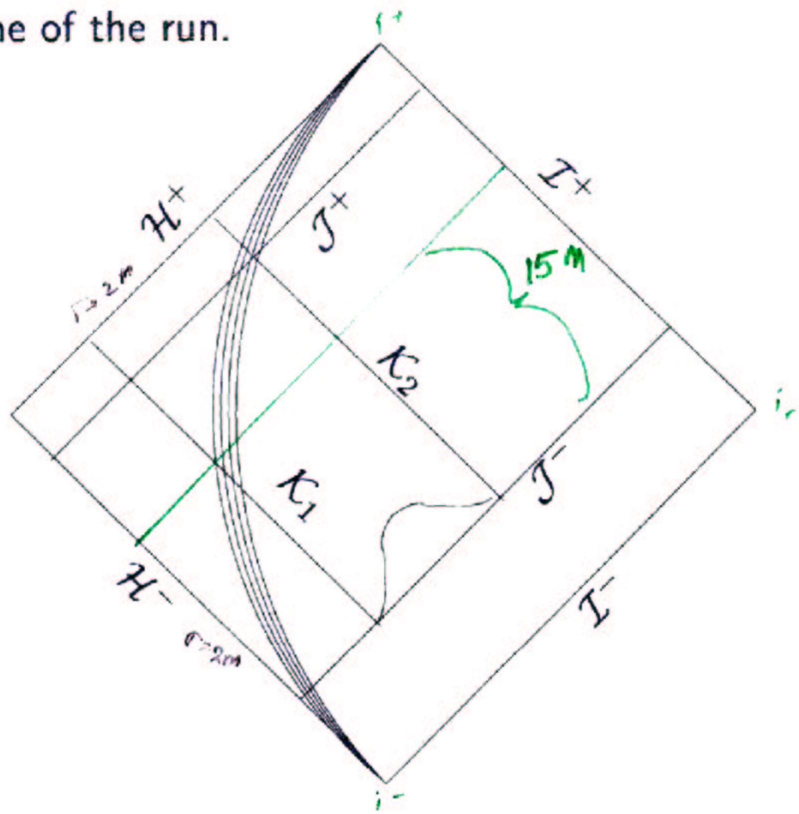
$det(h_{AB}) = det(q_{AB})$

$J = \frac{1}{2} h_{AB} q^A q^B$

On the initial slice J gives the ingoing radiation (constraint free). On $\mathcal{I}^+$ $N = 1/2 J_{lu} + \cdots$

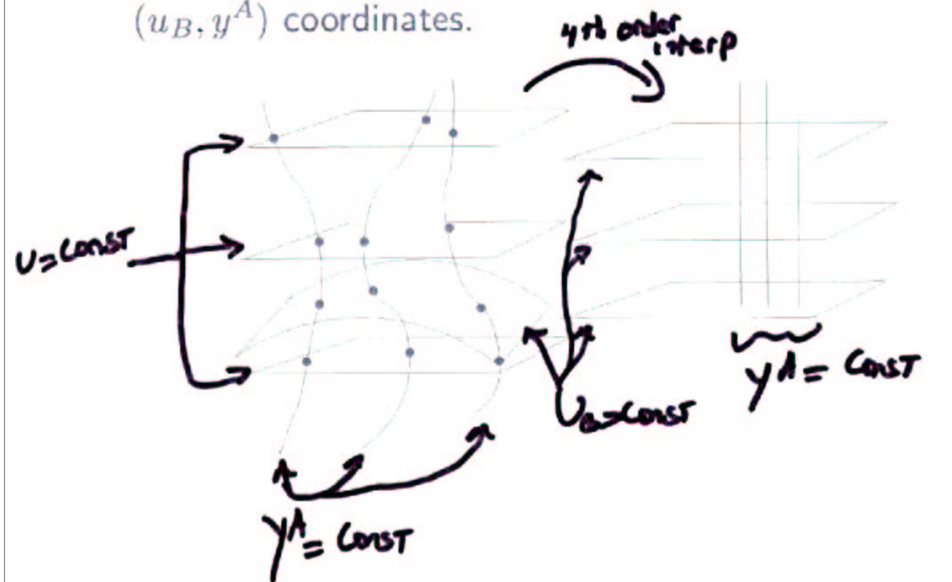
coordinate limitations

Our choice of uniform radial coordinate limits the lifetime of the run.



Transforming The News Into a Bondi Frame

One need to decompose the News in an inertial Bondi coordinate system. This requires interpolations (4th order) from the (u, x^A) coordinates of the code to (u_B, y^A) coordinates.

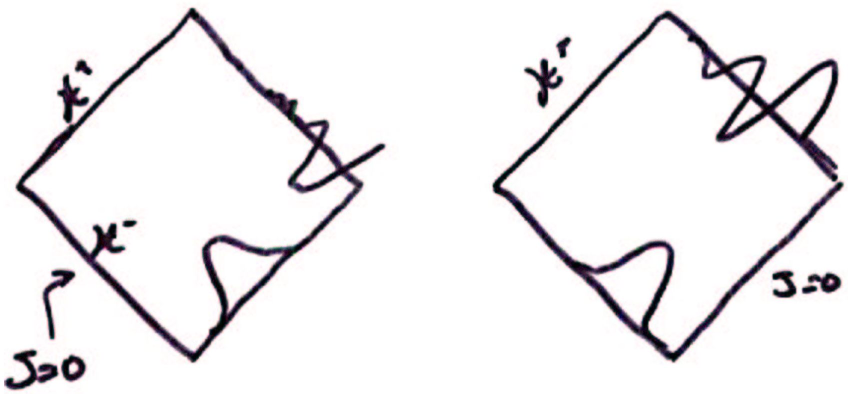


Linearized Calibration Tests

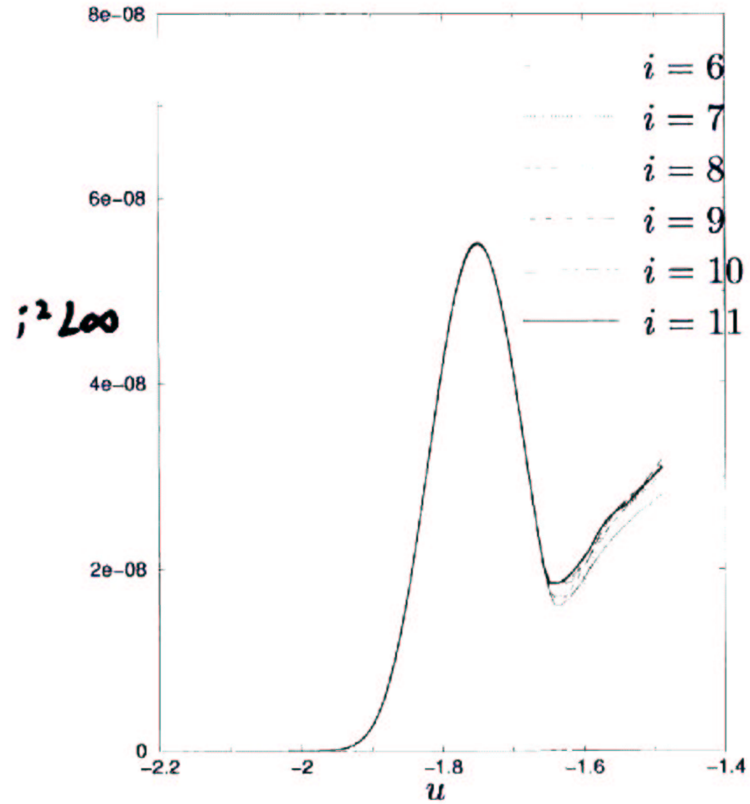
We place linearize data ($\ell = 2, m = 0$) on $\mathcal{J}^-$ or $\mathcal{H}^-$. Evolve using Pitt Null Code and a previously developed Teukolsky code (high accuracy). We ran the Pitt Null Code with grids of $(12*i+5)\times(2i+5)$ angular gridpoints $\times 30i$ radial gridpoints ($i = 6, 7, 8, 9, 10, 11$).

Confirmed $2nd$ order convergence of the Bondi News.

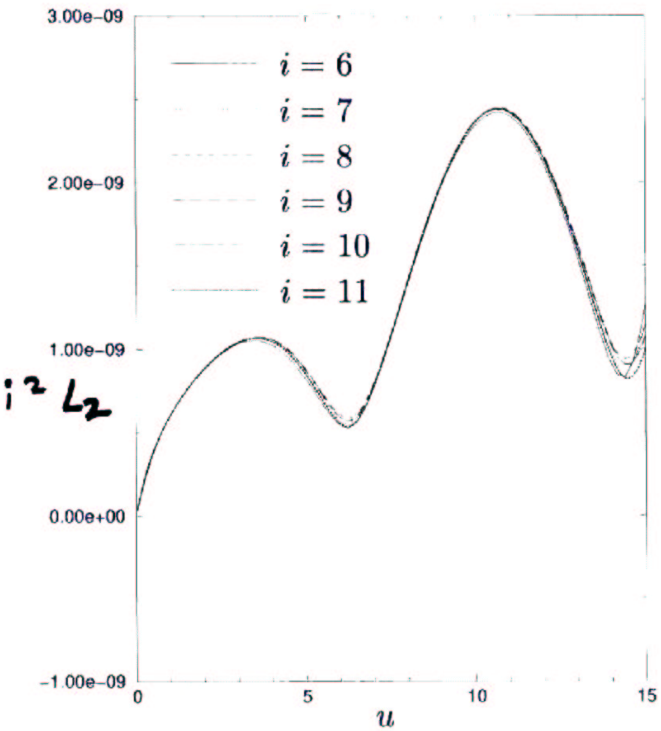
Data were compact pulses



Outgoing pulse (L_∞ norm)



Ingoing pulse (L_2 norm)



Spin Weighted Spherical Harmonics

$$\eth F = q^A \nabla_A F \leftarrow \text{scalar}$$

$$\eth f = P^{1-s}\partial_{\bar z}(f P^s)$$

$P = 1 + z\bar z$, z stereographic coord

$$_sR_{\ell m} = \sqrt{\frac{(\ell-s)!}{(\ell+s)!}} \begin{cases} \frac{1}{\sqrt{2}}\eth^s \left(Y_{\ell m} + (-1)^m Y_{\ell\, -m}\right), & m > 0 \\ \eth^s Y_{\ell 0}, & m = 0 \\ \frac{i}{\sqrt{2}}\eth^s \left((-1)^m Y_{\ell m} - Y_{\ell\, -m}\right), & m < 0 \end{cases}$$

for $s>0$

$$\oint {}_sR_{\ell m}\, {}_s\bar R_{\ell' m'} d\Omega = \delta_{\ell\ell'}\delta_{mm'}$$

$$N = \sum (N_{\ell m}\, {}_2R_{\ell m}) \text{ where } N_{\ell m} = \oint (N\, {}_2\bar R_{\ell m}) d\Omega$$

$$J = \sum J_{\ell m}(u, l)\, {}_2R_{\ell m}$$

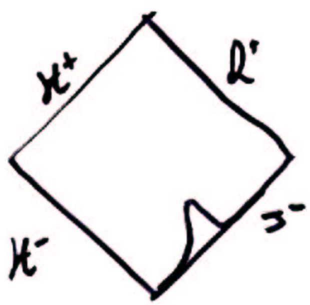
In linear theory

$$N_{\ell m}(u) = \frac{1}{2} J_{\ell m, lu}(u, 0) - \frac{\ell(\ell+1)}{4} \Re J_{\ell m}(u, 0)$$

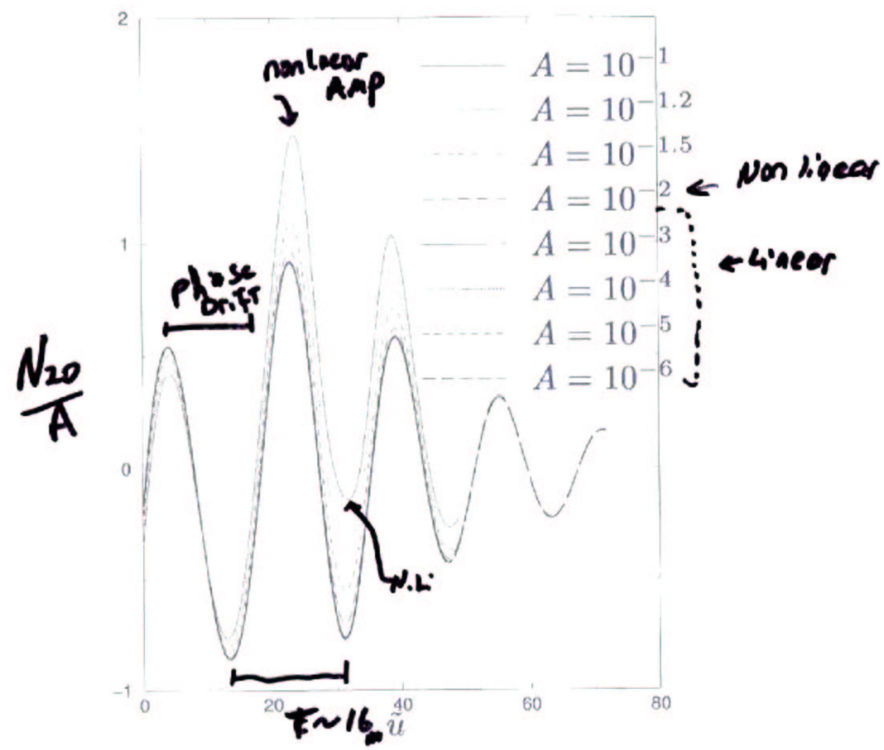
No mode-coupling in Linear Theory

Axisymmetric Mode Coupling

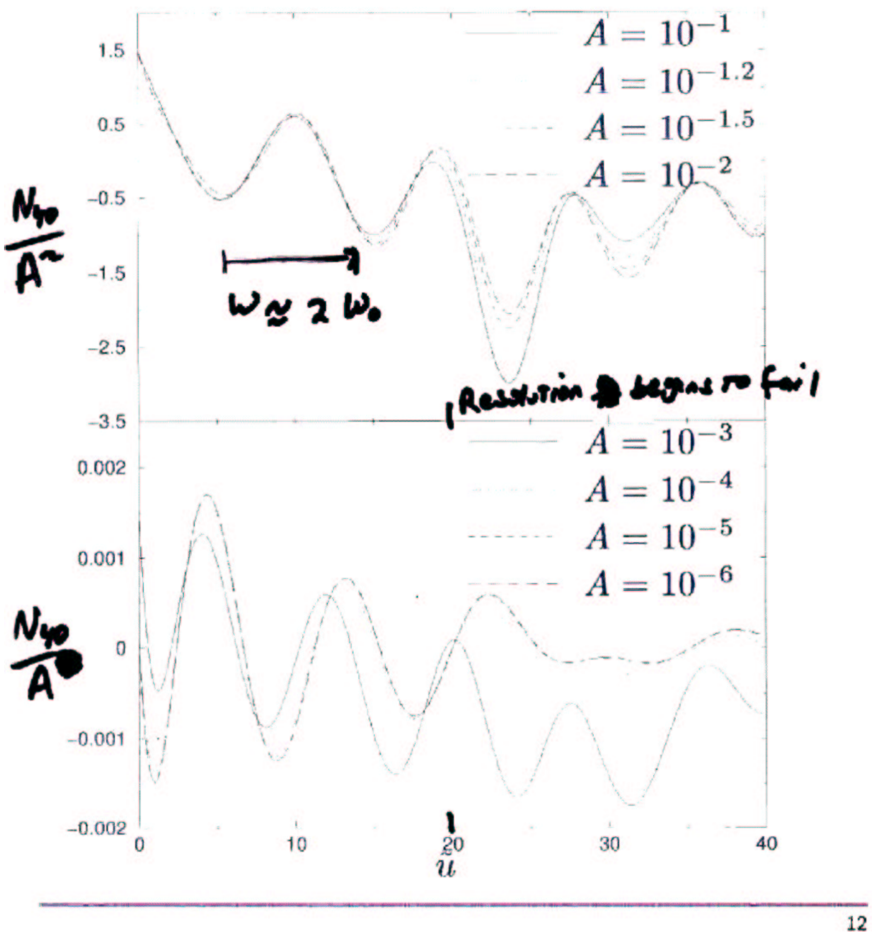
- Compact radial profile (C^2) $2.54M < r < 8M$
- data contains $\ell = 2$ $m = 0$ angular mode
- Expect only $m = 0$ modes (axisymmetry) in the News
- Expect only even l modes (parity) in the News



$\ell = 2$

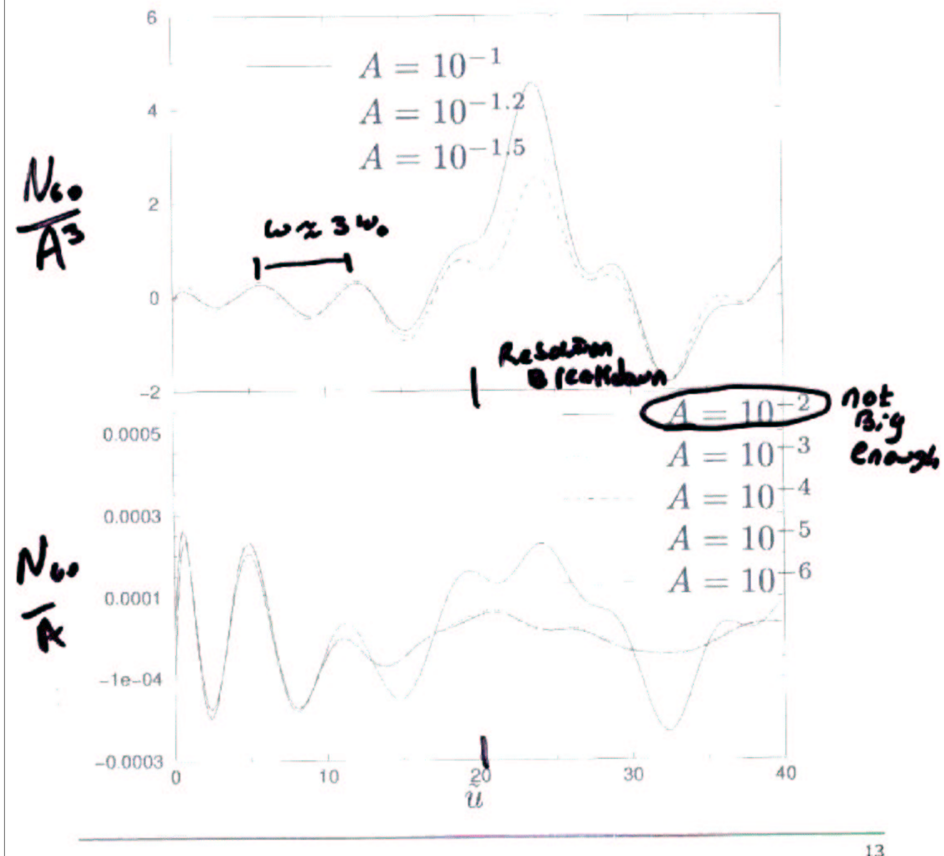


$\ell = 4$



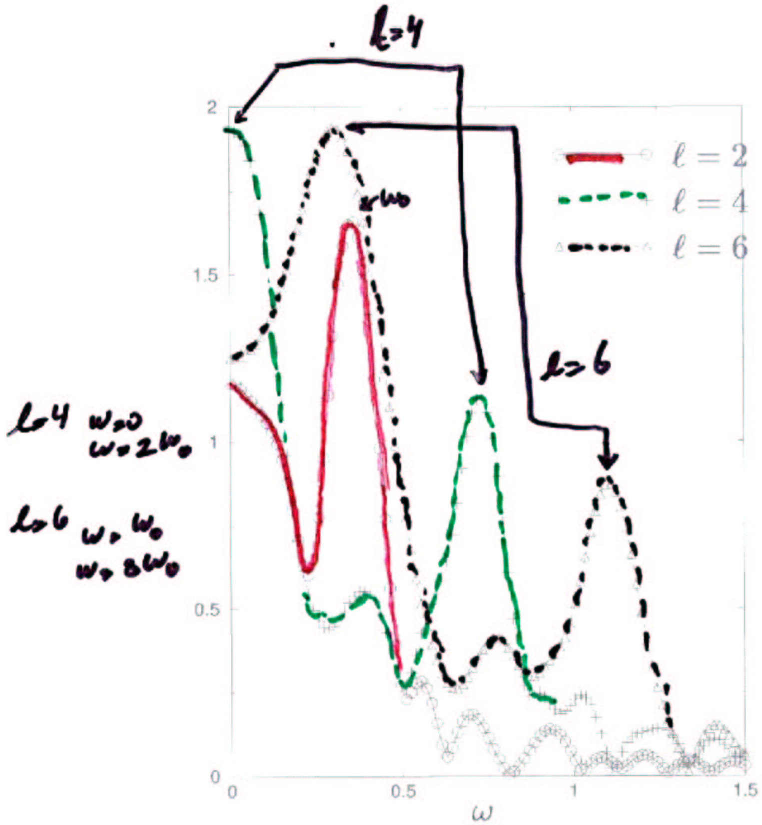
A = 10^-1
A = 10^-1.2
A = 10^-1.5
A = 10^-2
A = 10^-3
A = 10^-4
A = 10^-5
A = 10^-6
N40 / A^2
N40 / A
ω ≈ 2 ω0
Resolution begins to fail
u
12

$\ell = 6$

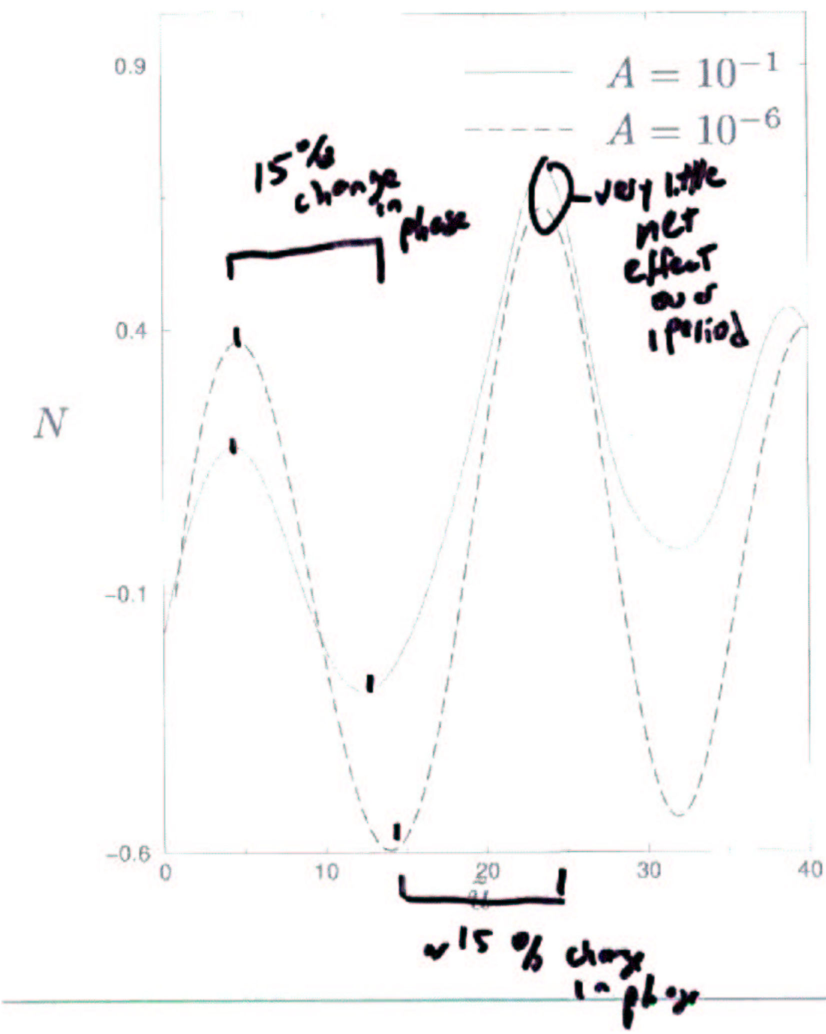


A = 10^-1
A = 10^-1.2
A = 10^-1.5
A = 10^-2
A = 10^-3
A = 10^-4
A = 10^-5
A = 10^-6
N60 / A^3
N60 / A
ω ≈ 3 ω0
Resolution Breakdown
not Big enough
u
13

Fourier Transform



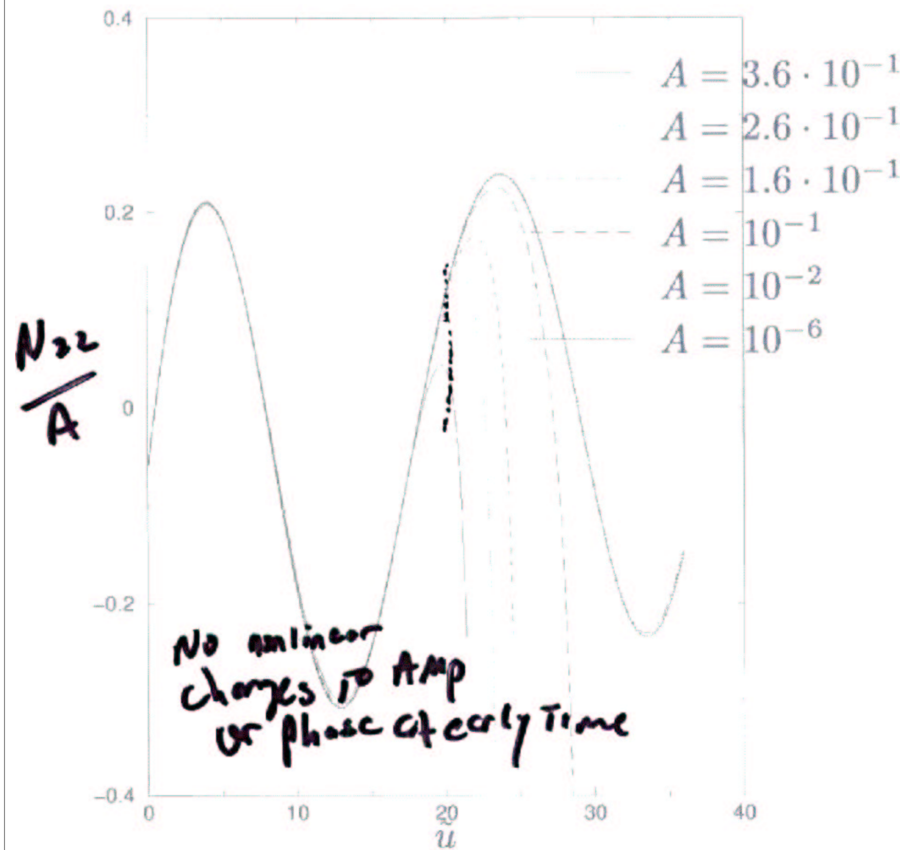
News on Equator



Non-Axisymmetric Coupling

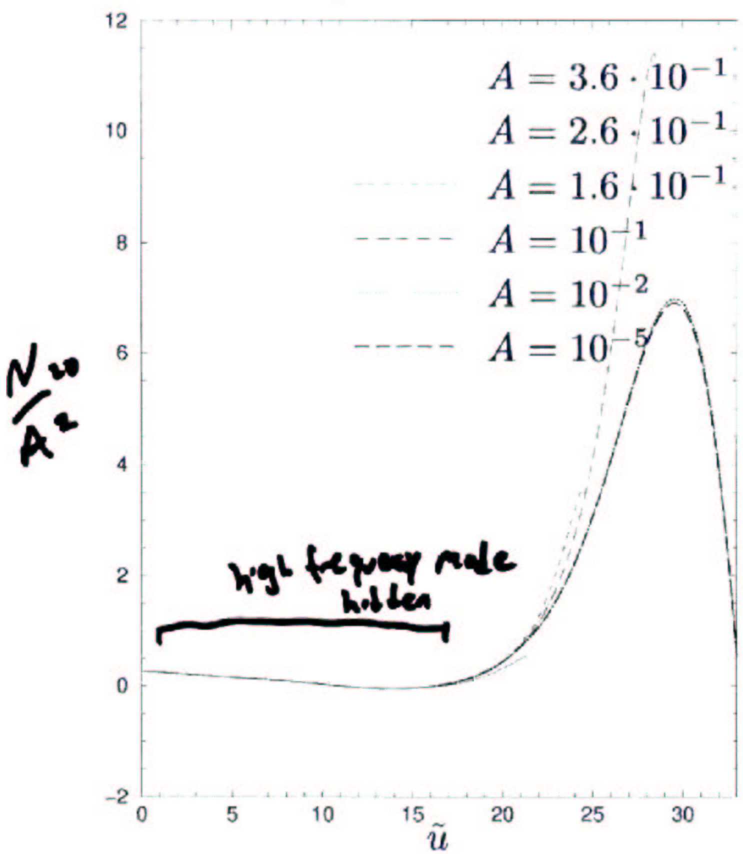
- Compact radial profile ($2.54M < r < 8M$), $\ell = 2$ $m = 2$ angular profile.
- Expect only even ℓ (parity)
- Expect even m (parity)
- Expect $m > 0$ (reflection symmetry)

$\ell = 2\, m = 2$



Quadratic terms don't contaminate N_{22}

$\ell = 2\, m = 0$



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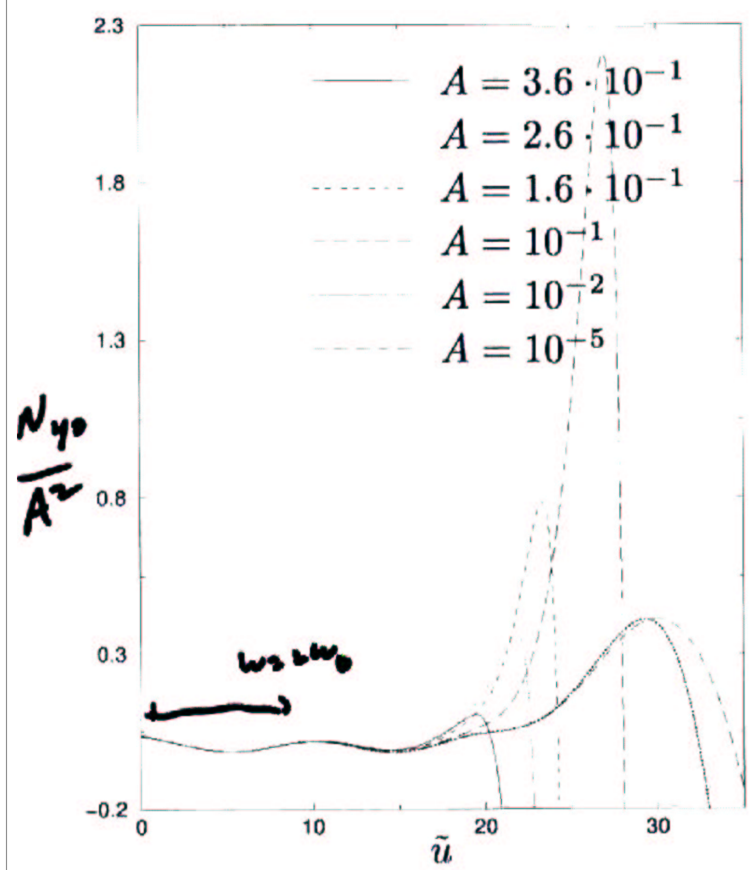
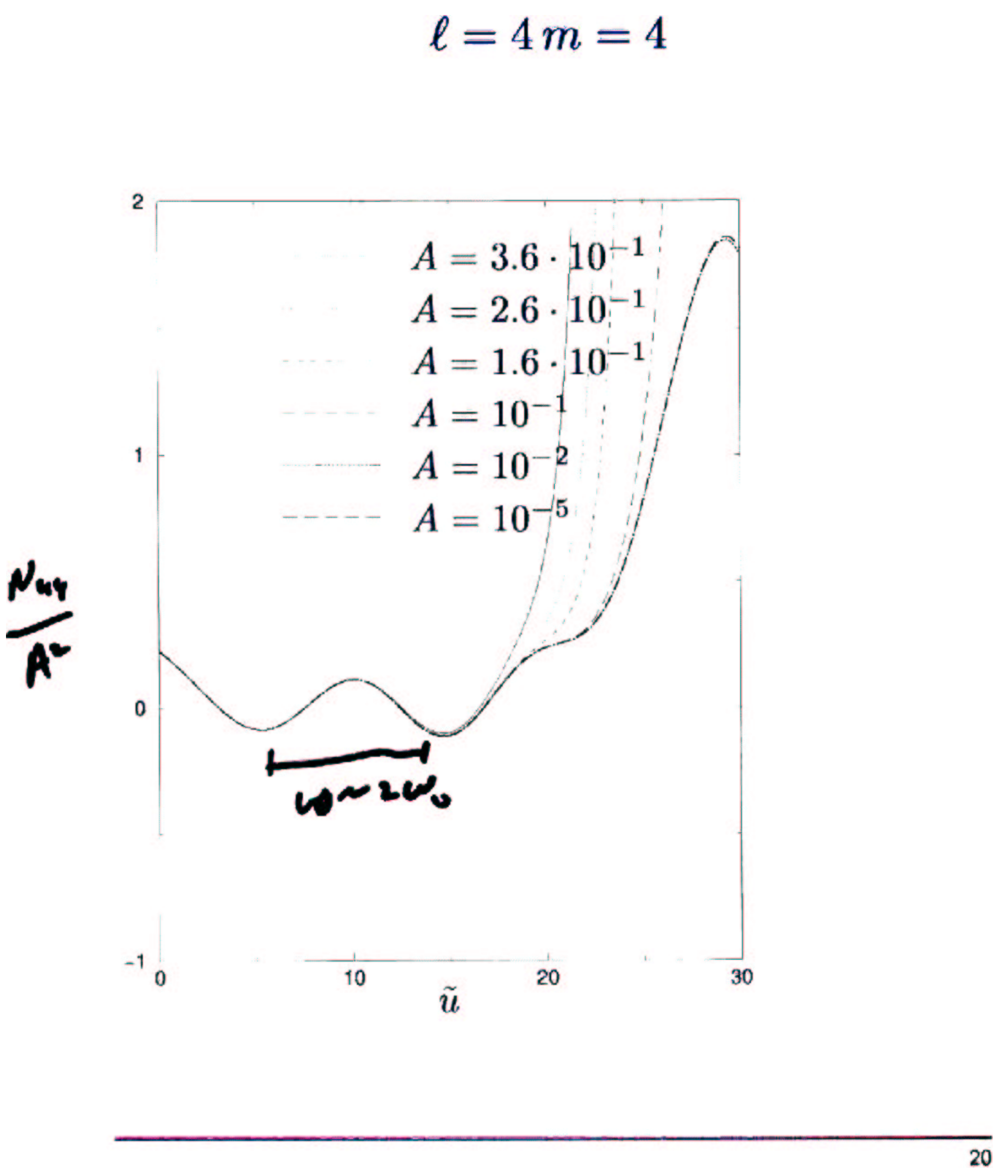
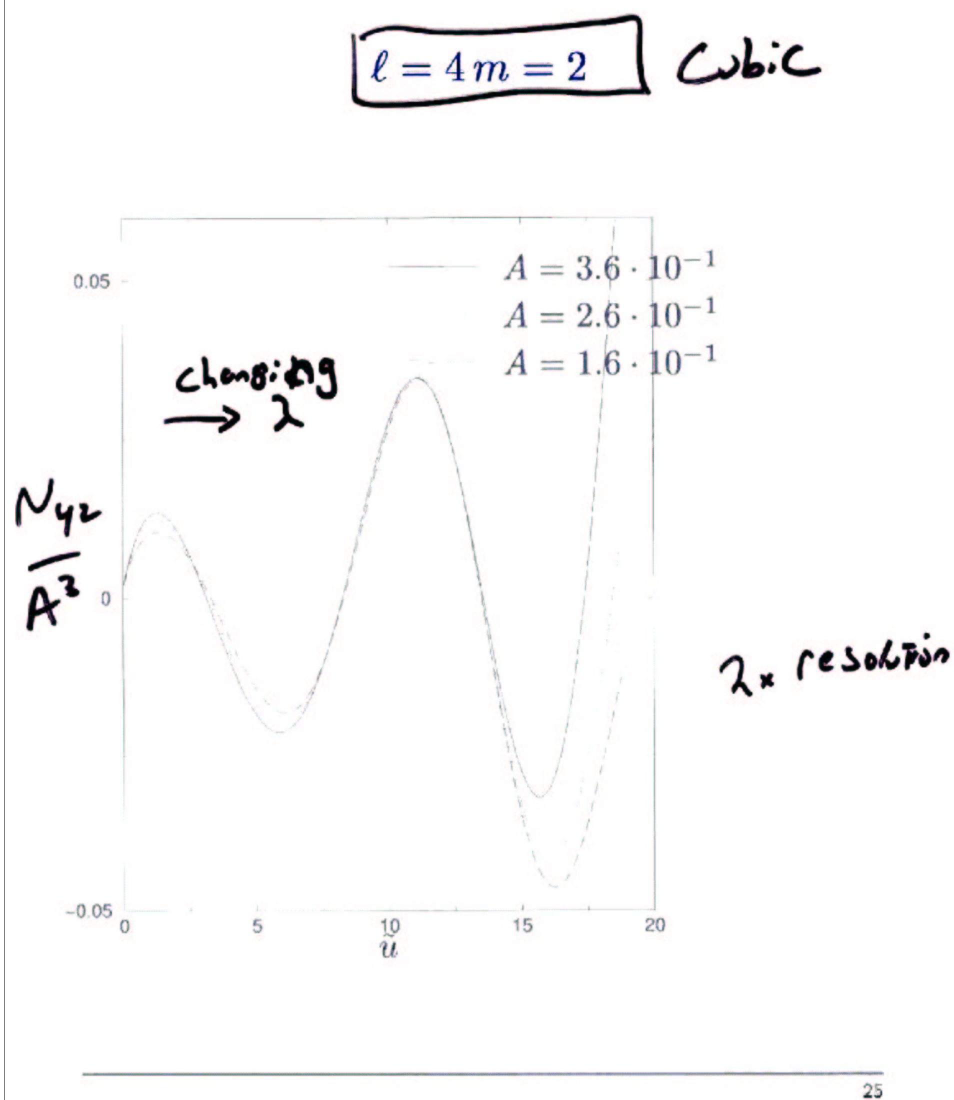
$\ell = 4\, m = 0$

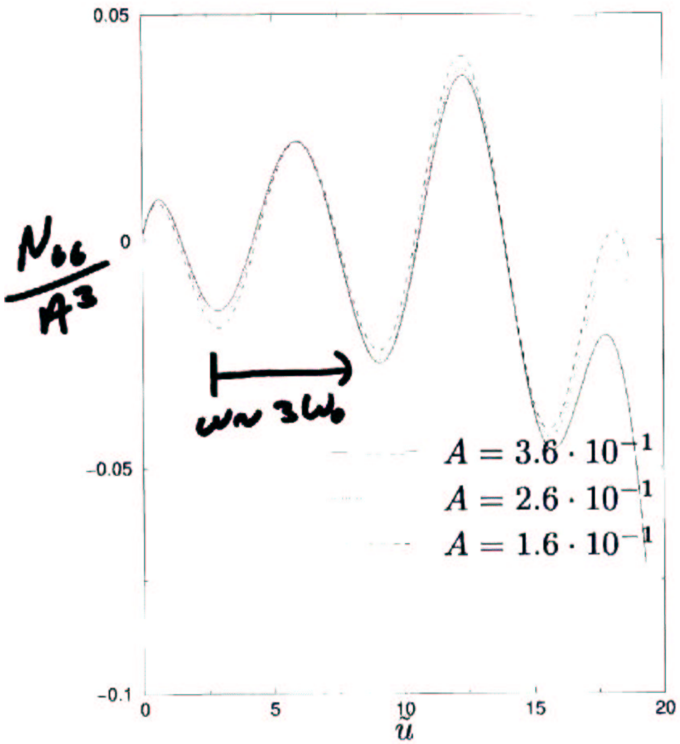


19

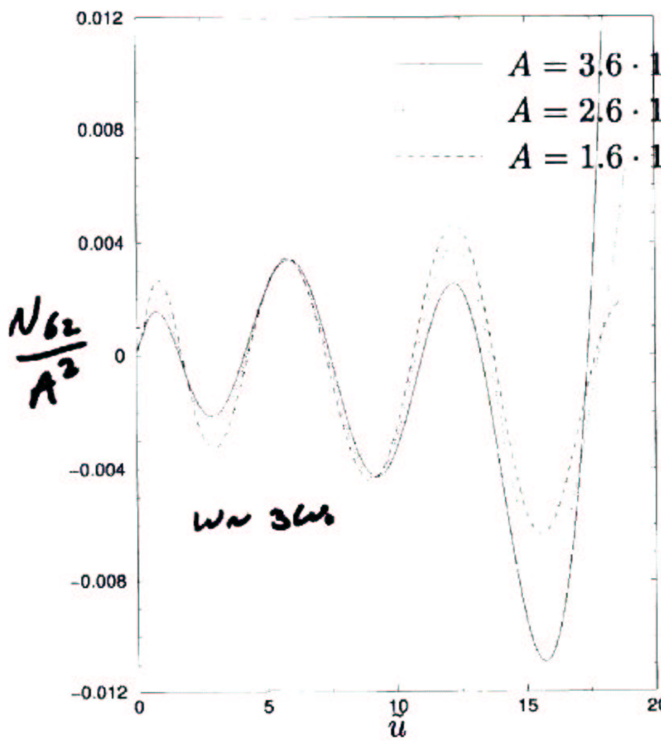

$\ell = 4 m = 4$
$A = 3.6 \cdot 10^{-1}$
$A = 2.6 \cdot 10^{-1}$
$A = 1.6 \cdot 10^{-1}$
$A = 10^{-1}$
$A = 10^{-2}$
$A = 10^{-5}$
$\frac{N_{44}}{A^2}$
$\tilde{u}$
2
1
0
-1
0
10
20
30
$\omega \sim 2\omega_0$
20


$\ell = 4 m = 2$
Cubic
$A = 3.6 \cdot 10^{-1}$
$A = 2.6 \cdot 10^{-1}$
$A = 1.6 \cdot 10^{-1}$
changing
$\rightarrow \lambda$
$\frac{N_{42}}{A^3}$
2x resolution
0.05
0
−0.05
0
5
10
15
20
$\tilde{u}$
25

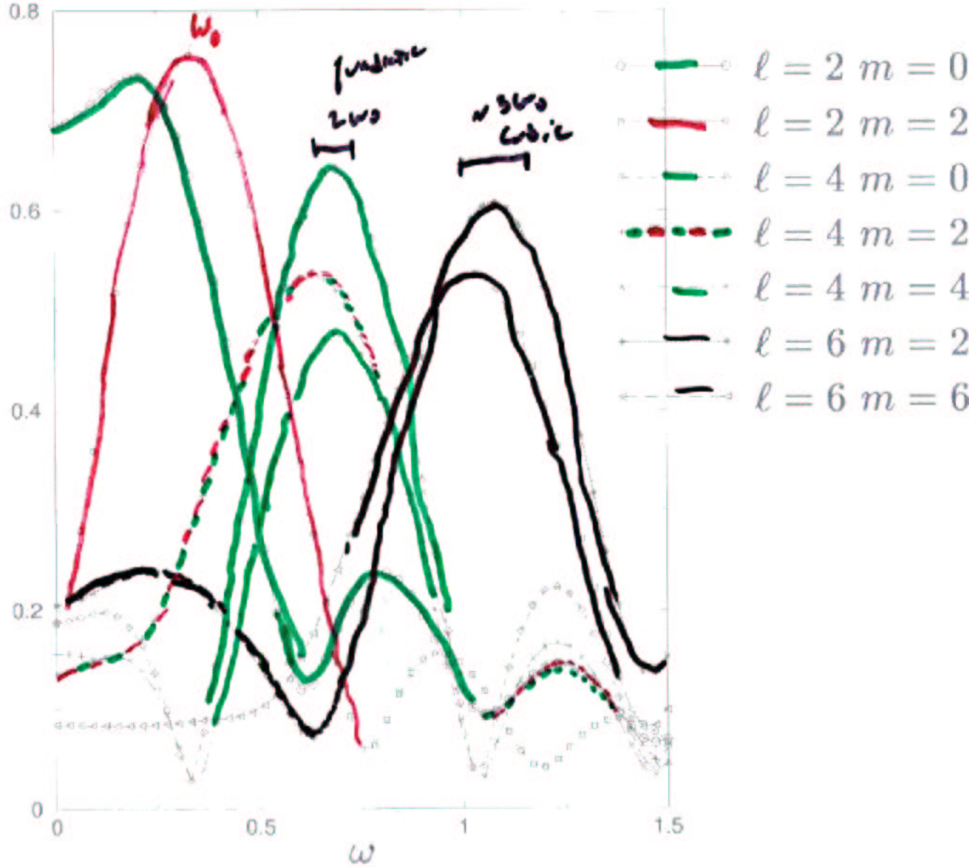
$\ell = 6\, m = 6$



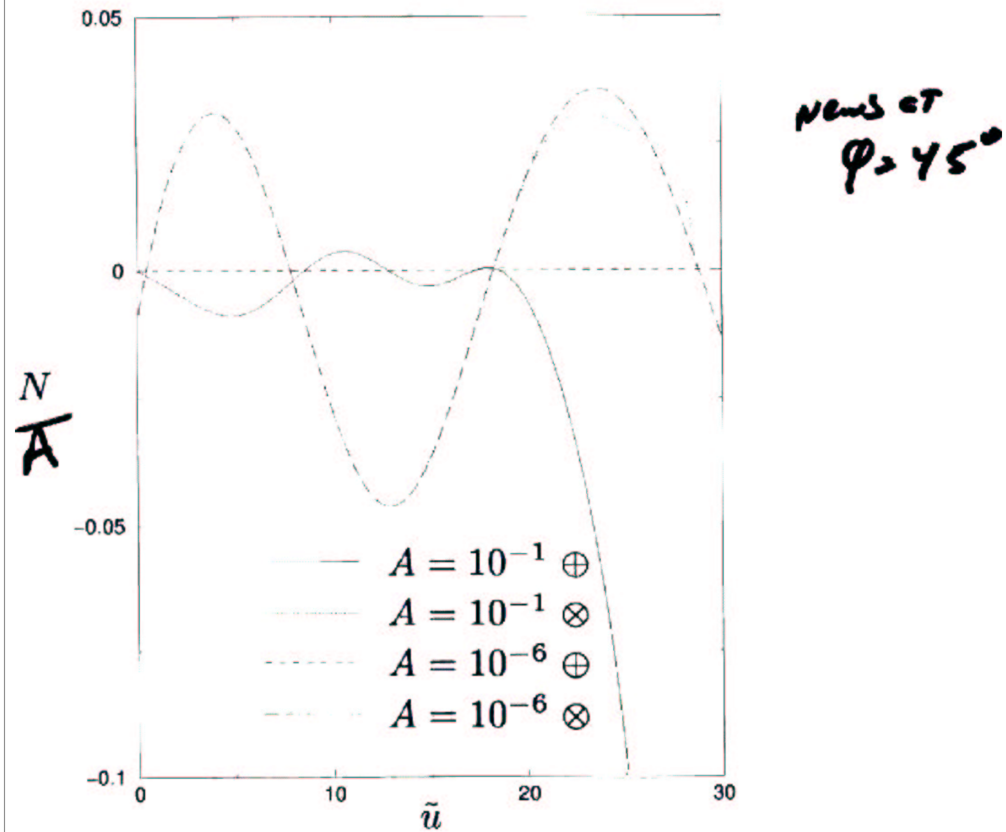
$\ell = 6\, m = 2$



Fourier Transform



⊕ ⊗ coupling



Conclusion

This test was a demonstration. The 'Pitt Null Code' does not require that the input data contain any symmetries. We have shown that this type of analysis is possible with this stable, mature code.

Future Work

- Perform analysis with more general pulses (e.g. compact pulses which 'simulate' ingoing bodies)

  orbiting radiation ($r=3m$)

- Perform analysis using black holes with spin