An update on Cauchy-Characteristic Matching in General Relativity

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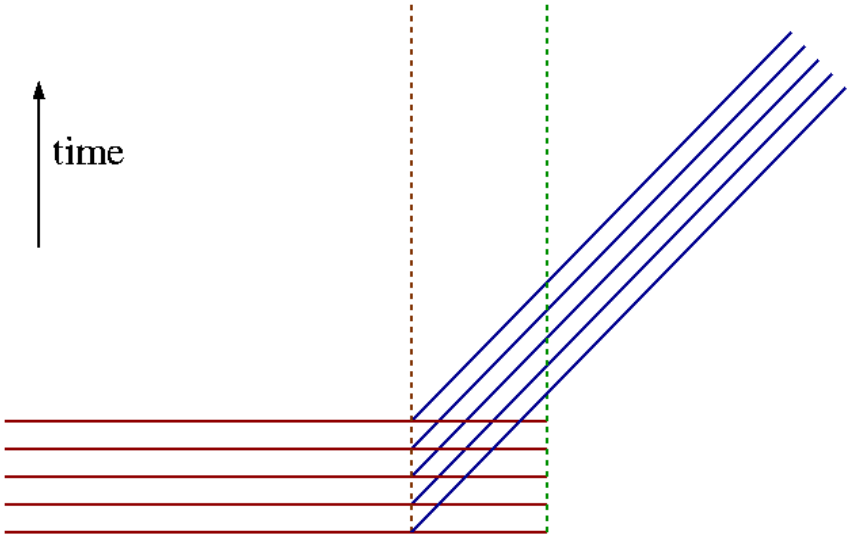
Cauchy Characteristic Matching (CCM)

What is it?

In this approach one covers space-time by a Cauchy and a Characteristic coordinate-patch.



Well-posedness of CCM

Does it make sense?

Hopefully.

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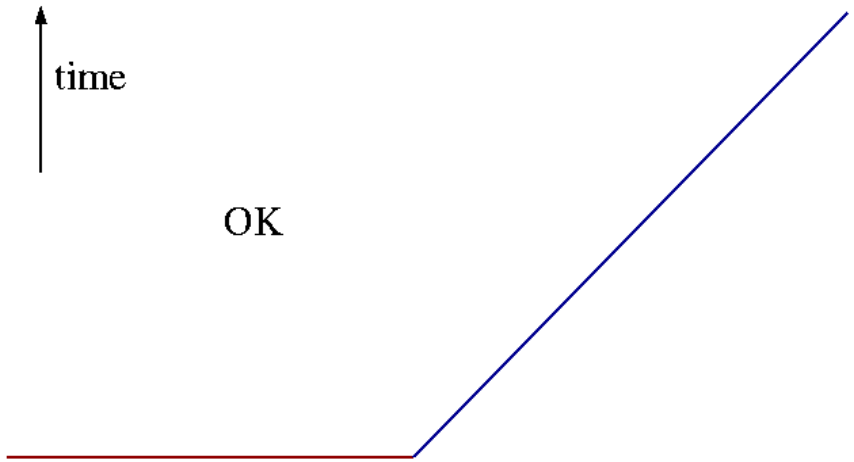

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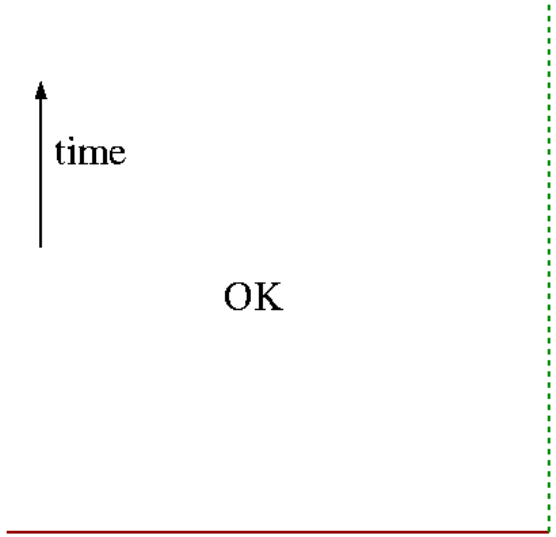

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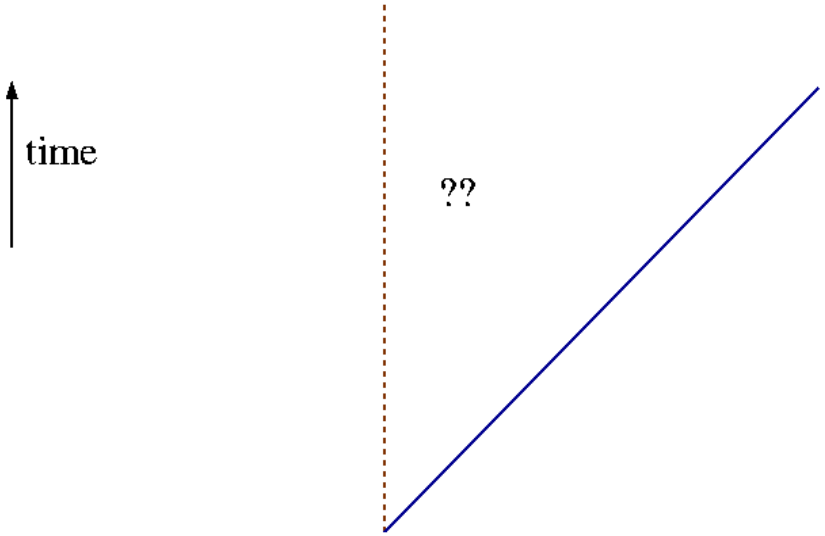
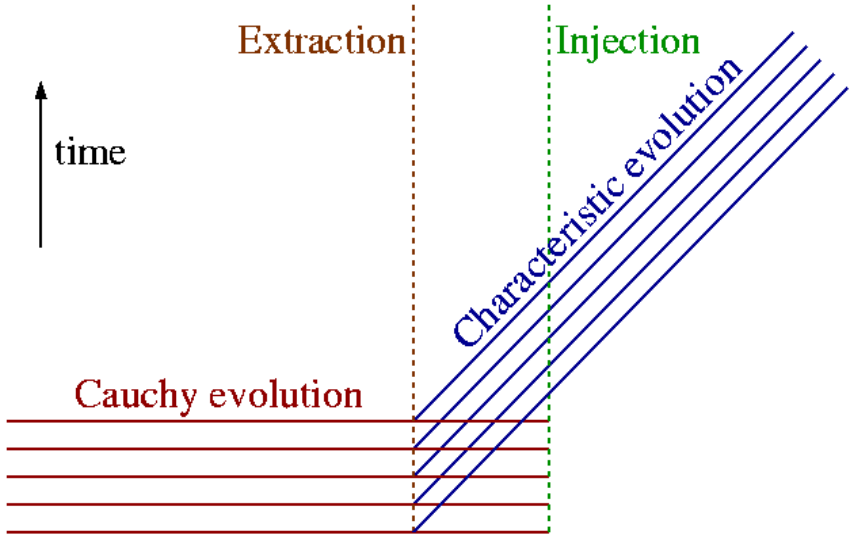


Well-posedness of CCM

Does it make sense?

Hopefully.

One of the issues is the definition of the matching boundaries.



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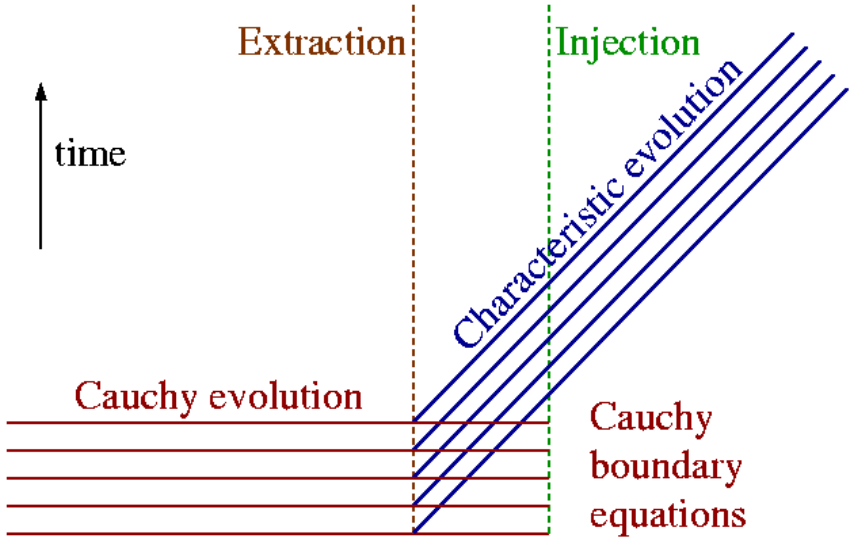
Does it make sense?

Hopefully.

Another issue is that of the constraints of the two formulations.

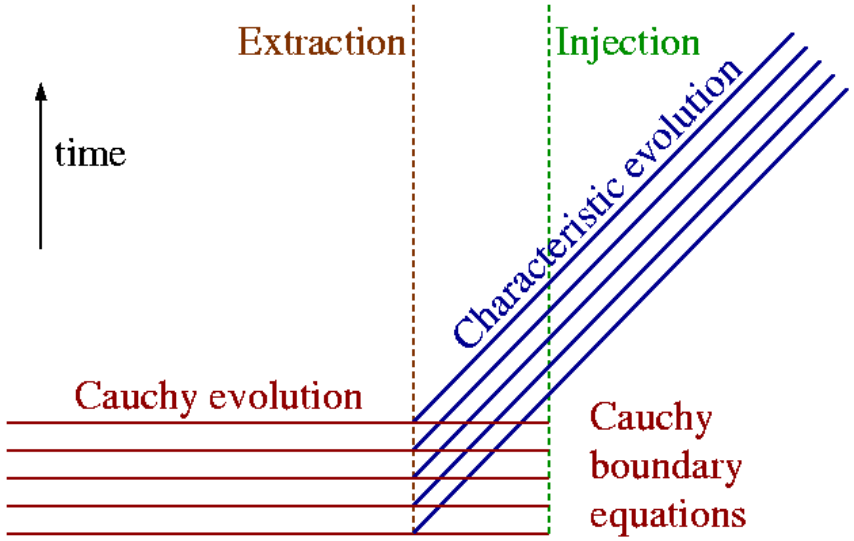


Well-posedness of CCM

Does it make sense?

Hopefully.

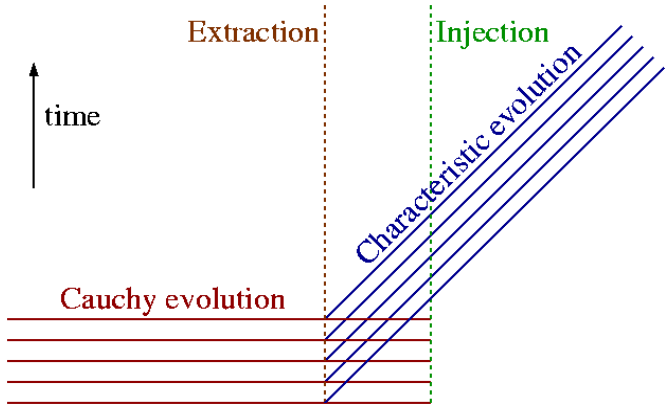
Yet another issue is that of consistent initialization.



Successful CCM schemes

Can it work?

Apparently.



Examples:

- 3D non-linear scalar evolution on a fixed Euclidean background

  Bishop et al, J. Comp. Phys. **136**, 140 (1997)

- 1D (non-linear) GR evolution, with Black Hole excision

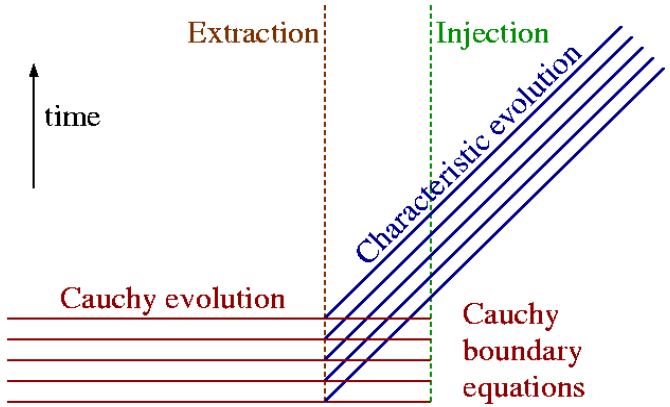
  Gomez et al, PRD **56**, 6310 (1997)

- *[2D (non-linear) GR evolution – Southampton group]*

CCM in linearized GR

Can it work?

It also works for 3D linearized GR.



CCM in linearized GR

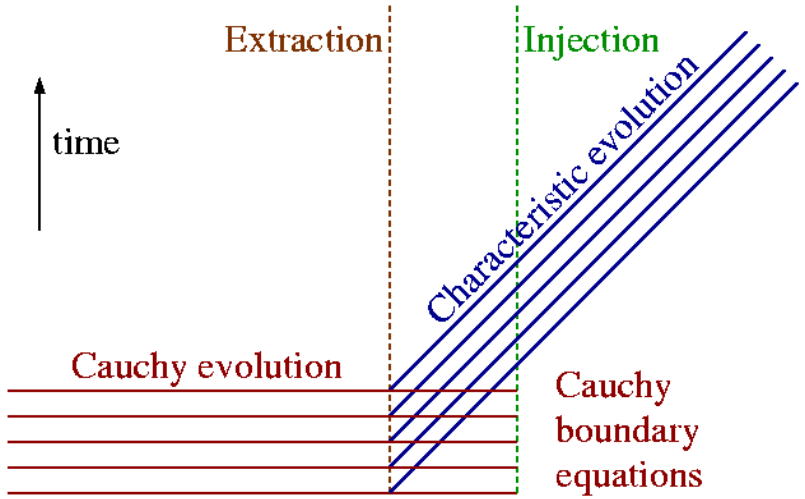
Details:

Cauchy evol: linearized harmonic

Cauchy bdry: enforce 4 constraints

Characteristic evol:

- linearized Bondi
- Gauge evolution – this allows knowledge of the Bondi -> Cartesian Jacobian

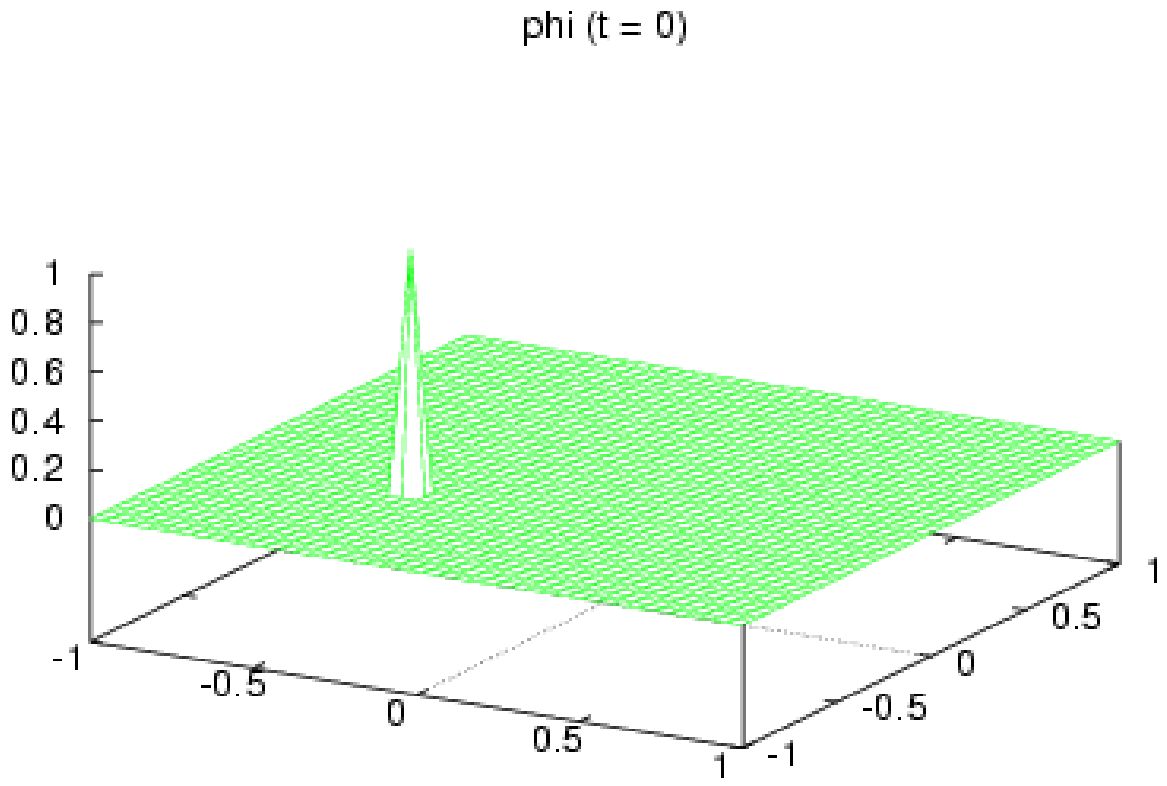


Extraction: polynomial interp & lots of algebra

Injection: “Sommerfeld”-style, normal to faces of the cube

CCM in linearized GR

Result:



CCM in linearized GR

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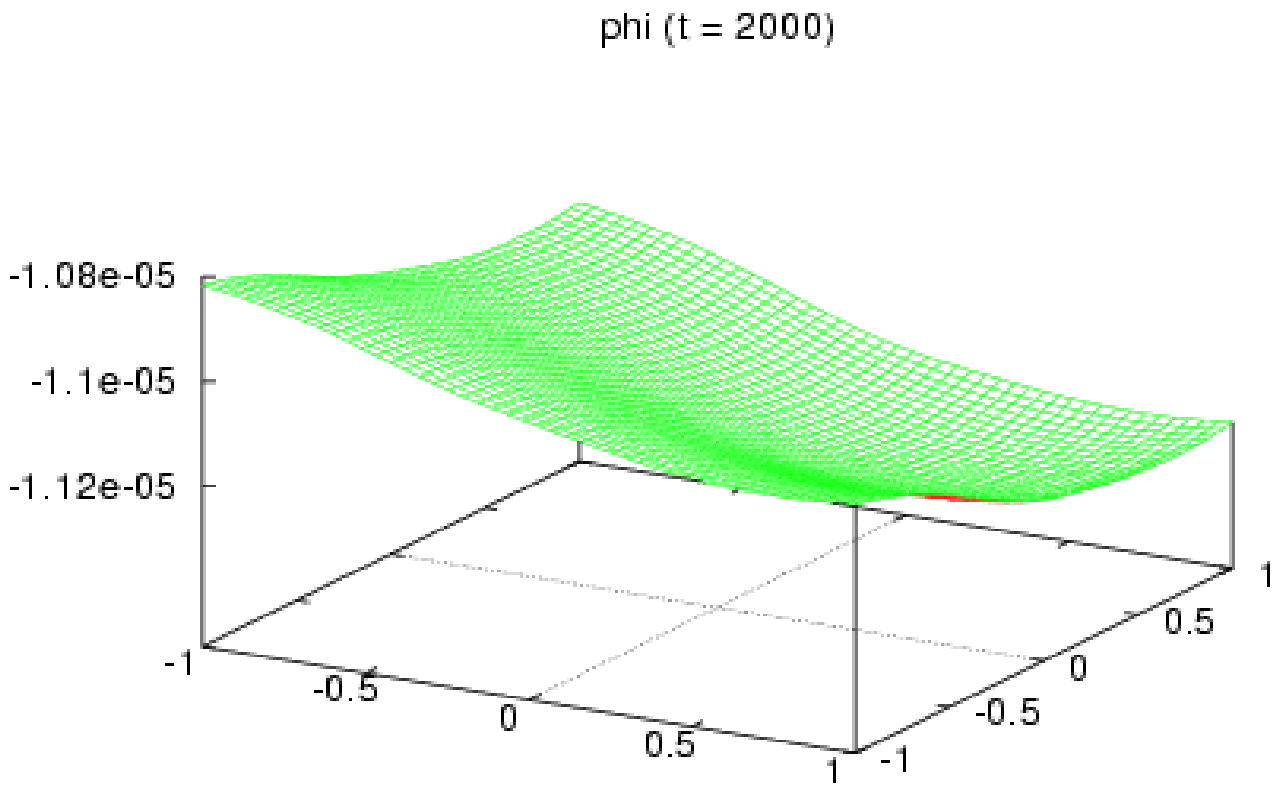
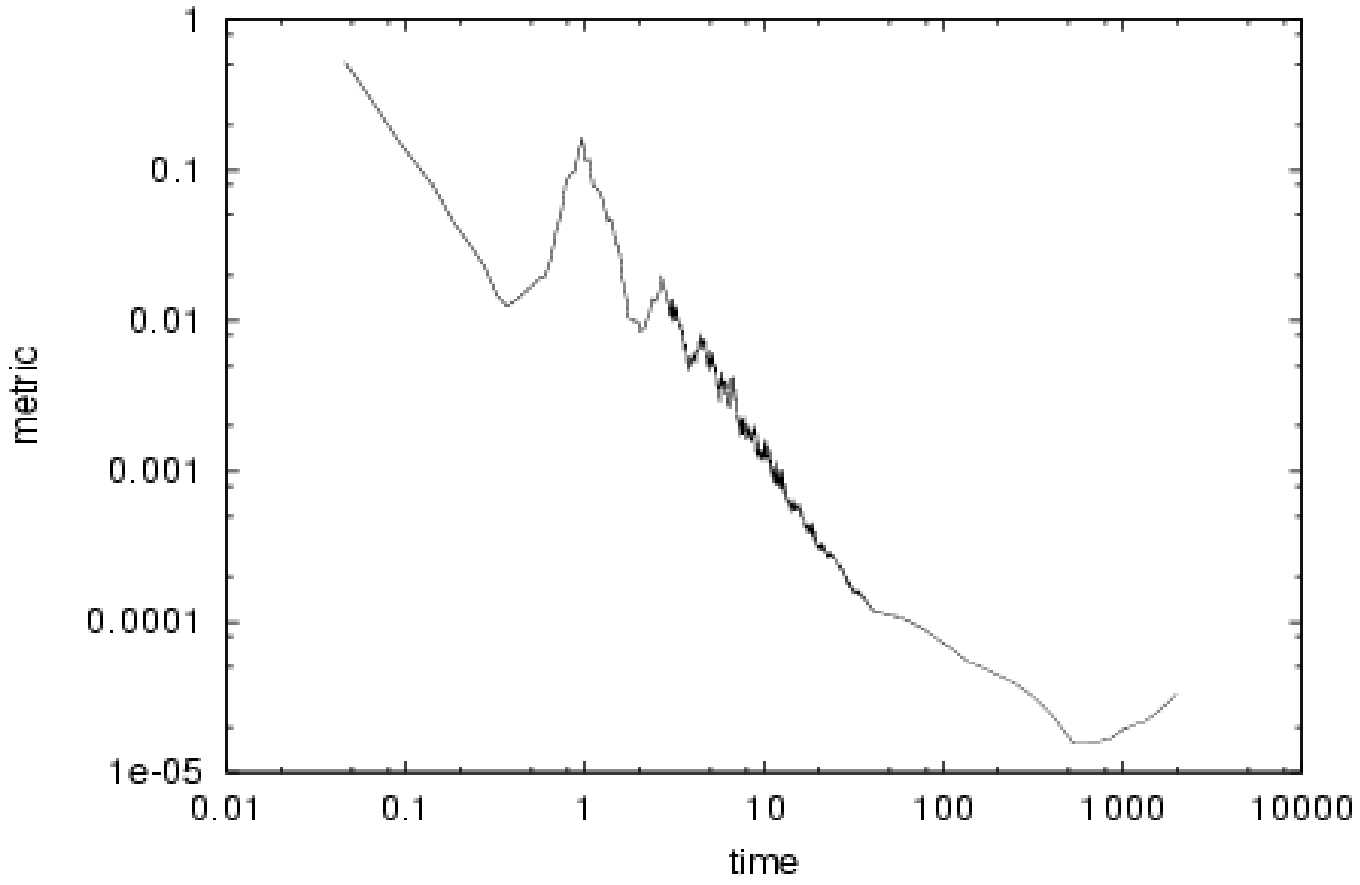


CCM in linearized GR

Result:



CCM in linearized GR

Lessons learned so far:

- Need working Cauchy (and Characteristic) initial-boundary code(s)
- Need constraint preserving boundary conditions (Cauchy boundary equations)
- Need finite distance between Injection and Extraction (?)
- Need Sommerfeld style Injection
- Need parallelization (Yosef is getting there)

Status of the non-linear Cauchy code

Numerical evolution scheme:

- 1^{st} differential order in time, 2^{nd} differential order in space
  (Does anybody know how to keep it fully 2^{nd} differential order?)

- Discretization: finite differencing (2^{nd} or 4^{th} order accurate)

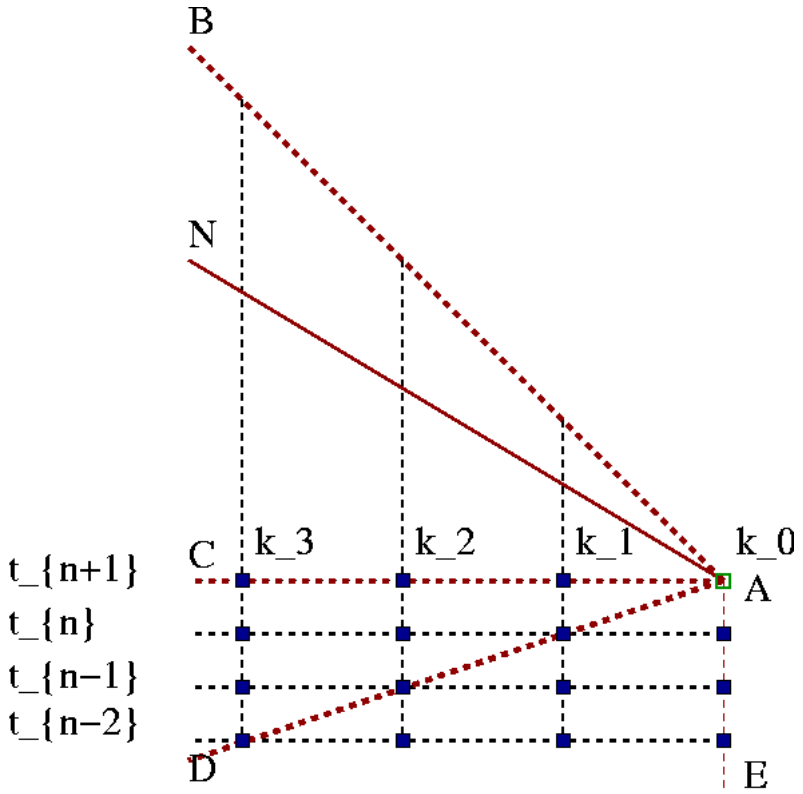
- Time-integration:
  - Iterative Crank-Nicholson
  - 3^{rd} and 4^{th} order accurate predictor-corrector schemes
  - 4^{th} order Runge Kutta

Status of the non-linear Cauchy code

Numerical boundary scheme:

- A first major issue is the need of a good Neumann algorithm

1) Sideways algorithm:

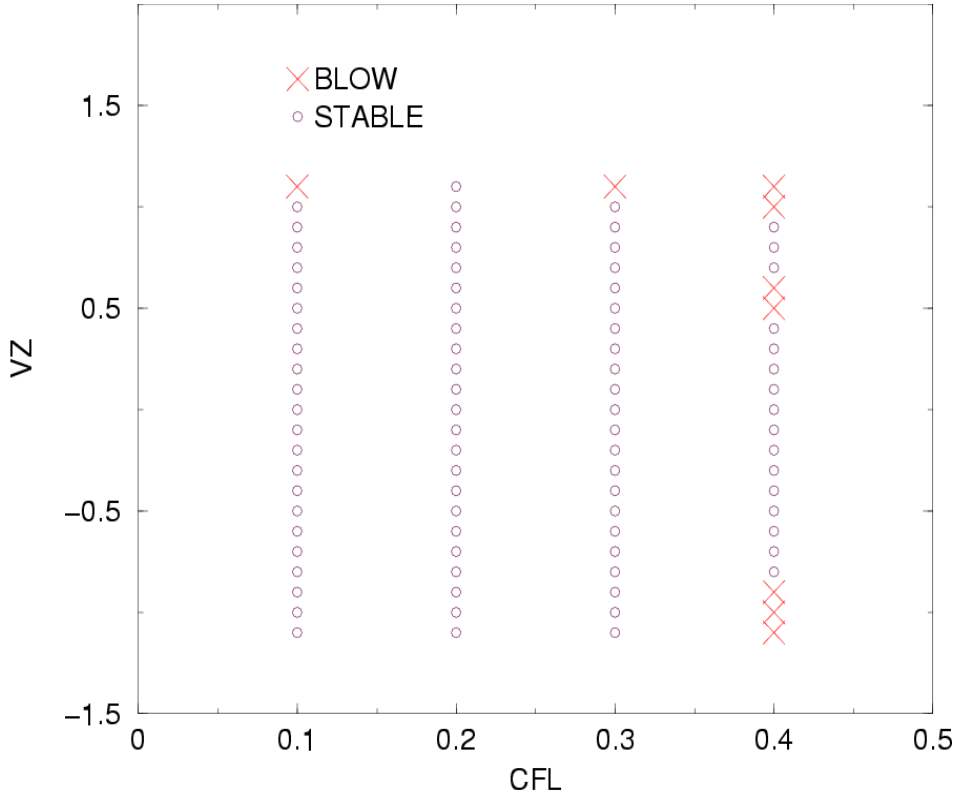


Status of the non-linear Cauchy code

Numerical boundary scheme:

- A first major issue is the need of a good Neumann algorithm

1) Sideways algorithm:
   - works (too) well in 1D
   - uses future time-levels
   - no obvious (smooth) extension to edges



Status of the non-linear Cauchy code

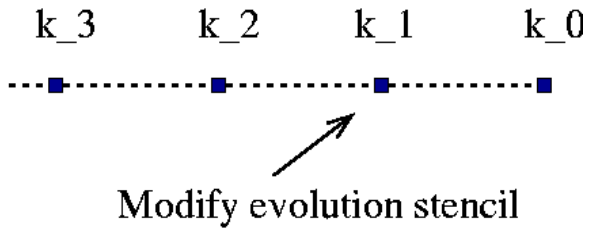
Numerical boundary scheme:

- A first major issue is the need of a good Neumann algorithm

  1) Sideways algorithm

  2) "Evolution-based" algorithm:

  - simple stencil
  - good for edges
  - provides no boundary value for evolution variables

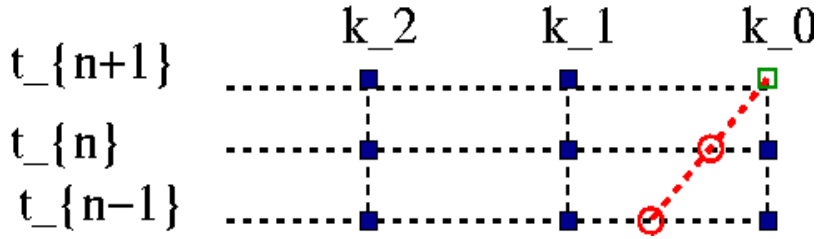


Status of the non-linear Cauchy code

Numerical boundary scheme:

- A first major issue is the need of a good Neumann algorithm

1) Sideways algorithm
2) "Evolution-based" algorithm
3) Characteristic algorithm:
   - no future levels
   - Generalization of 1st diff. order schemes
   - no good edge algorithm



Status of the non-linear Cauchy code

Numerical boundary scheme:

- A first major issue is the need of a good Neumann algorithm

  1) Sideways algorithm

  2) "Evolution-based" algorithm

  3) Characteristic algorithm

- The boundary equations ask for both the metric components and their Neumann derivatives at the boundary point. As a result, we plan to use a combination of the 2nd and 3rd algorithms.

Conclusion

- CCM works in linearized (harmonic) GR
- Implementation of a non-linear (harmonic) Cauchy evolution-boundary algorithm is well under way
- We already have:
  - A non-linear characteristic code
  - A non-linear extraction module (on 1 CPU)
- Still need to work out a non-linear injection module (in it's current setup)