Critical gravitational collapse of a massive vector field

(Garfinkle, Mann and Vuille
gr-qc/0305014)

(1) critical gravitational collapse

(2) massive scalar field

(3) massive vector field paradox

(4) numerical simulations

(5) paradox resolved

Critical gravitational collapse

Choptuik's discovery

collapse of a spherically symmetric scalar field ϕ initial data $\phi_p(0,r)$

for $p > p*$ a black hole forms

for $p < p*$ the field disperses

scaling of black hole mass:
for p near $p*$

$$M = c(p - p*)^\gamma$$

$$\gamma \approx .374$$

critical solution

The $p = p*$ solution is discretely self-similar (DSS)

After a certain amount of time the profile of the scalar field repeats itself with the scale of space shrunk.

Massive scalar field

(Brady et. al PRD 56, 6057 (1997))

Choptuik solution is DSS
This is a natural consequence of the scale invariance of the Einstein-scalar system.

What happens when we break scale invariance by treating a massive scalar field?

results of numerical simulations

two critical solutions

one periodic

one identical to that
of the massless case.

How can the same solution
be the critical solution
for both the massless
and massive cases?

terms in the stress-energy
that depend on the mass
depend on the amplitude of
the field, while the other
terms depend on the gradients
of the field.

as the scale of space becomes
small, the mass terms become
negligible

Massive vector field paradox

What is the nature of critical
collapse of a spherically symmetric
massive vector field?

(1) the mass should be negligible
so it should be the same as for
a massless vector field.

(2) it can't be the same as for a
massless vector field (Maxwell field)
because a spherically symmetric
Maxwell field has no degrees of freedom

Numerical simulations

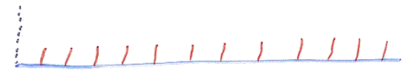
Three numerical methods

Cauchy

Characteristic

Cauchy with AMR

Cauchy codes



use polar-radial coordinates

$$ds^2 = -\alpha^2 dt^2 + a^2 dr^2 + r^2 d\Omega^2$$

Initial data is $X = (a/\alpha)A_t$ and A_r.

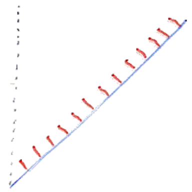
solve constraints for field strength
and metric components.
Use these to evolve X and A_r.

use centered differences
for spatial derivatives
use ICN for time evolution

use Kreiss-Oliger dissipation
for added stability.

AMR Cauchy code is modified
Choptuik EYM code

Characteristic code



use Christodoulou coordinates

$$ds^2 = -e^{2\nu} du^2 - 2e^{\nu+\lambda} du dr + r^2 d\Omega^2$$

Initial data on a null cone (u=const). is $h = \partial_r (r\phi)$ where $\partial_r \phi = \mu A_r$

integrate along the generators to find the other matter variables and the metric components.

evolve h along ingoing light rays. (essentially ODEs at each point)

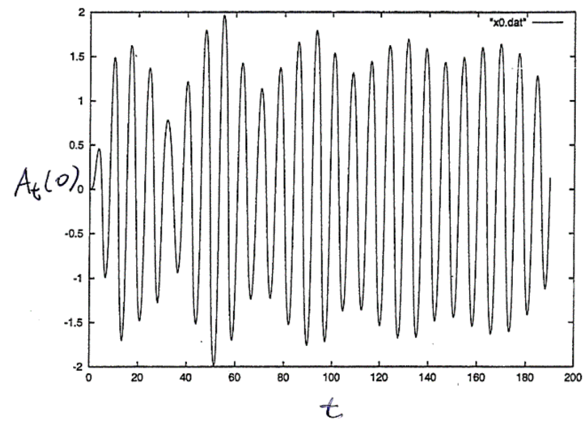
points are lost when the corresponding light ray crosses the origin.
when half the points are lost replace them in between the remaining ones.
In this way resolution is maintained as the relevant distance scale shrinks.

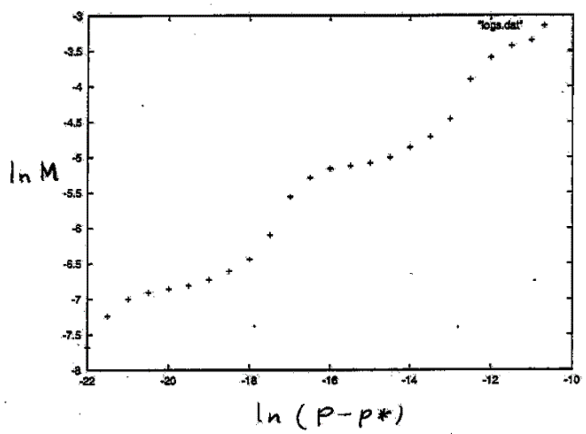
Results

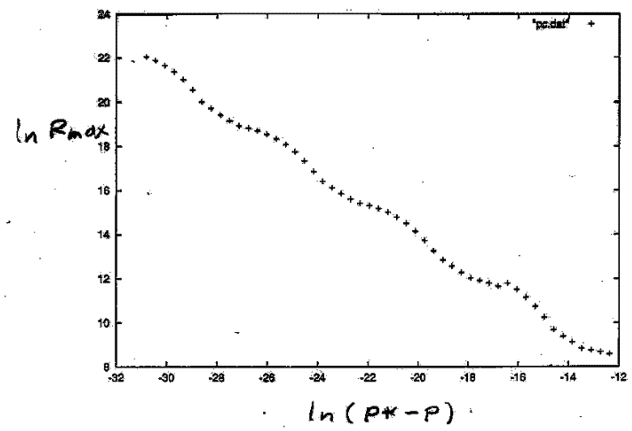
There are two critical solutions one periodic and one DSS

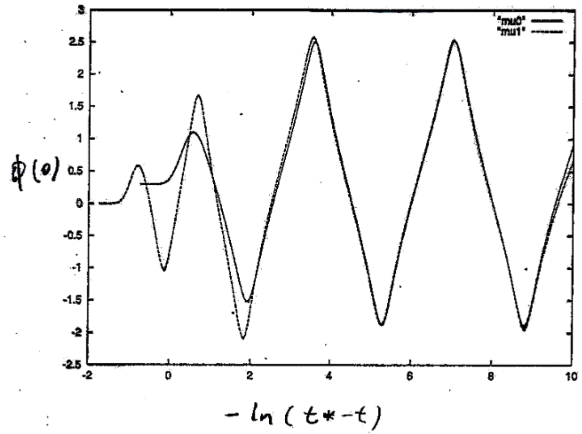
The DSS solution is identical to the Choptuik critical solution for a massless scalar field

(effective scalar field ϕ defined by $\partial_r \phi = \mu A_r$ along outgoing null lines)




"logs.dat"
ln M
ln (P−P*)


"pc.dat"
ln Rmax
ln (P*−P)



Paradox resolved

assume that

$$A_a \to \frac{1}{\mu} P_a + \mu Q_a + \dots$$

in the small μ limit.

Then a (non-singular) limitting stress-energy requires that $\nabla_{[a} P_{b]} = 0$ and therefore that $P_a = \nabla_a \phi$ for some scalar field ϕ.

The stress-energy of A_a then goes over to the stress-energy of ϕ.

The $\mu \to 0$ limit of a massive vector field is a massless scalar field (decoupling of longitudinal modes)