# Simulation of NS-NS binaries

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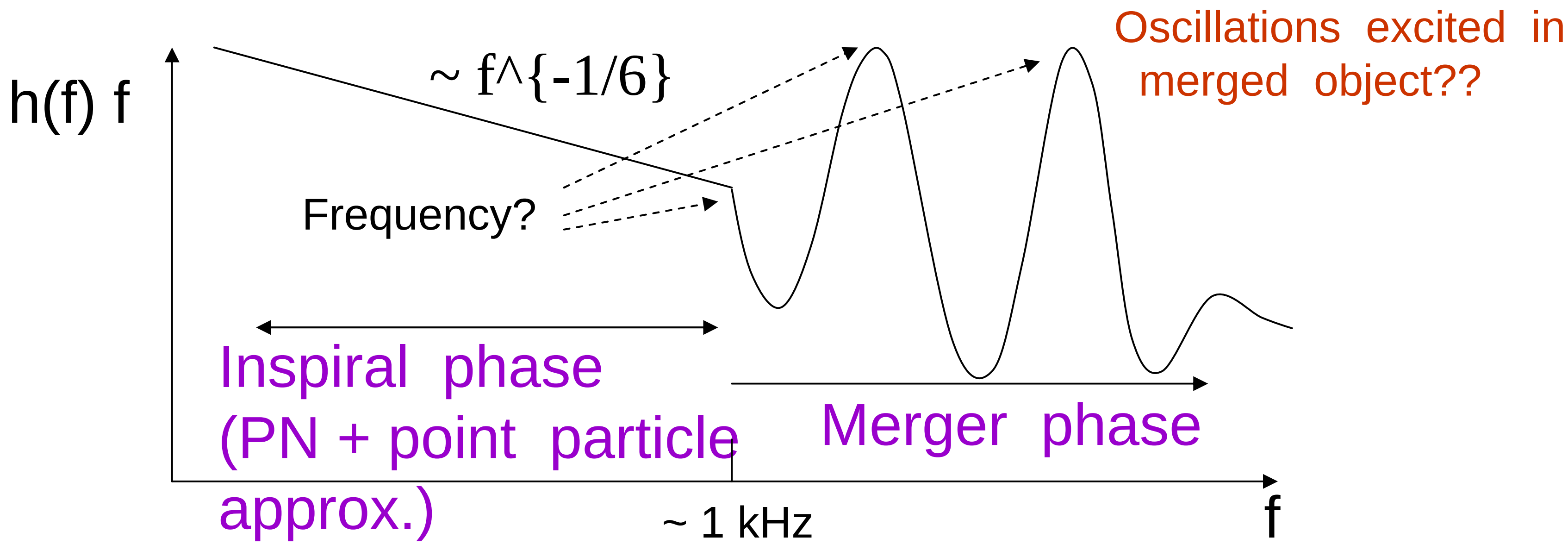
## Contents

# 0. Introduction: Why we perform simulation for merger of 2NSs

## (A) To clarify gravitational waveforms

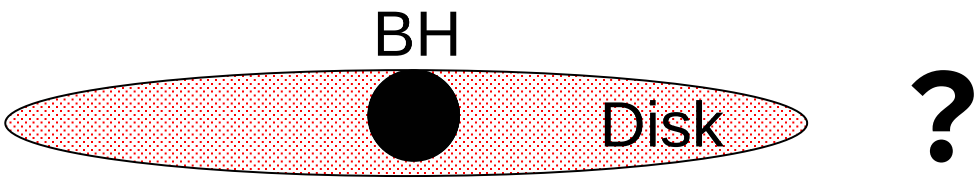



f > 1 kHz at merger: too high for LIGO I to be detected.
But may be a good source for future Interferometers
or resonant – mass detectors

(B) To compute fraction of disk mass in the formation of black hole

Astrophysical

Is it large enough to power GRBs (short-duration)?




(C) Which is the final product ? Black hole or Neutron star?

Academic

$1.4M_{\odot} + 1.4M_{\odot} = 2.8M_{\odot} > 2M_{\odot}$, NS+NS=BH?

| Maximum mass | Soft EOS | Stiff EOS |
|---|---|---|
| Spherical | $\sim 1.5M_{\odot}$ | $\sim 2.0M_{\odot}$ |
| Rigid Rotation +20% | $\sim 1.8M_{\odot}$ | $\sim 2.4M_{\odot}$ |
| Differential Rotation +>50% | $>2.3M_{\odot}$ | $>3.0M_{\odot}$ |

# Active groups in GR simulations

- M. Miller, Suen… (WashU)
- Illinois (Shapiro, Baumgarte, Duez et al.)
- Euro Network (Potsdam, Valencia …???)
- (Oohara-Nakamura)
- Shibata (with Uryu, Taniguchi)

Use similar formulations & implementations

↓

I will review the status based on ours.

# 1. Necessary implementations for GR simulations

- Einstein evolution equations solver
- Gauge conditions (coordinate conditions)
- GR Hydrodynamic equations solver
- Realistic initial conditions in GR
- Gravitational wave extraction techniques (Radiation reaction)
- Powerful supercomputer
- Special techniques for handling BHs.

# Summary of current implementations I

- Einstein's evolution equation :

  BSSN (Nakamura-Shibata) formalism

Choose variables:

$$\phi \equiv \frac{1}{12}\ln\left(\det(\gamma)\right)$$

$$\tilde{\gamma}_{ij} \equiv e^{-4\phi}\gamma_{ij}$$

$$K \equiv K_k^k$$

$$\tilde{A}_{ij} \equiv e^{-4\phi}\left(K_{ij} - \frac{1}{3}\gamma_{ij}K\right)$$

$$F_i \equiv \delta^{jk}\partial_j\tilde{\gamma}_{ik}$$

17 components

→

Rewrite equations using

$$\left\{\begin{array}{l} constraint\ \ equations \\ \det\left(\tilde{\gamma}_{ij}\right) = 1 \end{array}\right\}$$

↓

Stable numerical simulation (So far no problem in the absence of black holes)

# Summary of current implementations II

- Spatial gauge condition :

Previous belief: Minimal distortion type gauge

$$\text{MD gauge : } \Delta\beta^k + \frac{1}{3} D^k D_j \beta^j = S^k$$

Time consuming

New: Dynamical gauge (Alcubierre et al, Lindblom & Scheel)

Dynamical gauge (I use):

$$\dot{\beta}^k = \tilde{\gamma}^{kl}(F_l + \Delta t\ \dot{F}_l), \quad F_l \equiv \delta^{ij}\tilde{\gamma}_{il,j}$$

$$\dot{F}_l = \Delta\beta_l + \frac{1}{3} D_l D_j \beta^j - S'_l$$

Works very well.
Much smaller
CPU TIME!

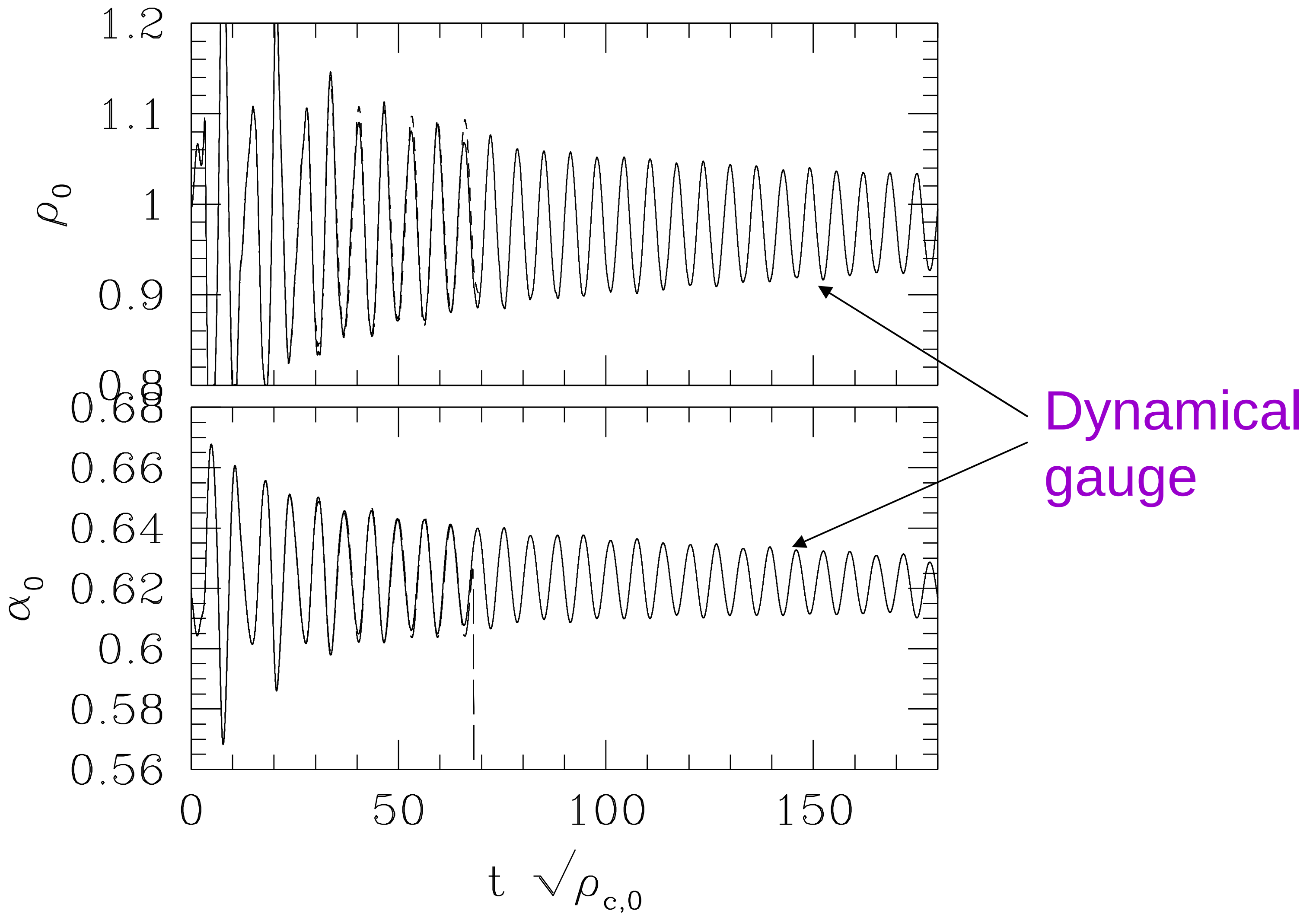
- Slicing condition : Maximal slicing or

Dynamical slicing which is also likely to work

# Evolution of compact rotating stars in a dynamical gauge

Central density

Lapse Function At r=0



# Summary of current implementations III

- Hydro code: Current trend

High-resolution shock-capturing scheme

(Approximate Riemann solver with PPM interpolation)

Developed by Valencia & Munchen groups

Now used by many groups (including myself)

⇒ Shocks & oscillations are computed accurately

⇒ Current best choice in

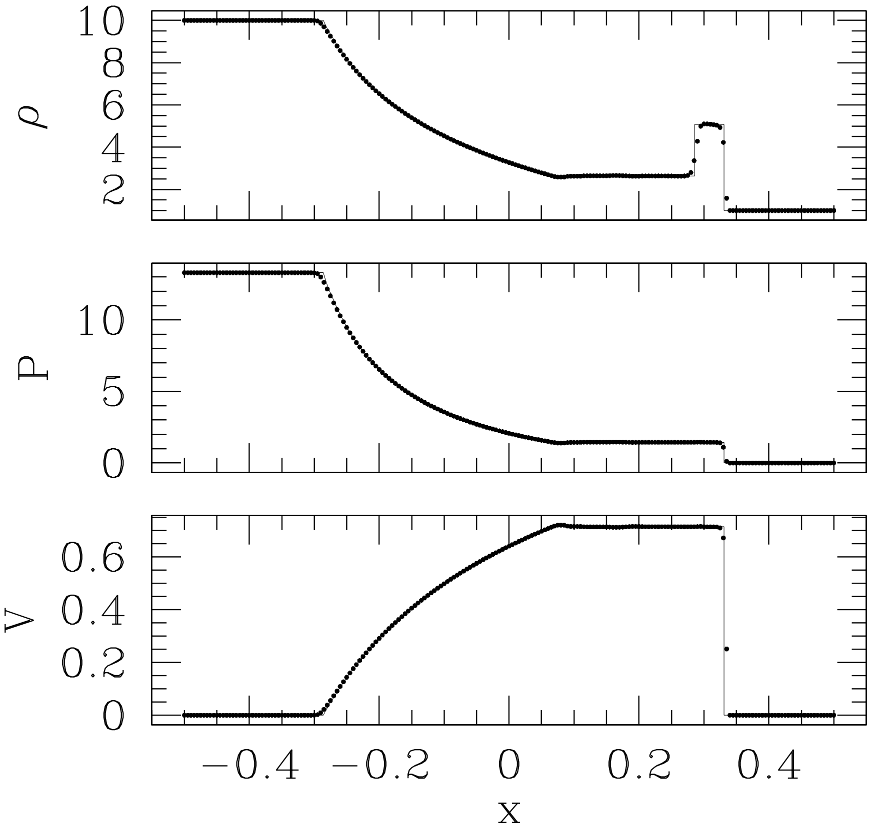
*stellar collapse

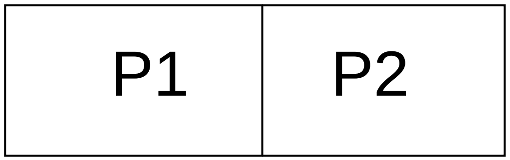
*detailed study of NS-NS merger

# Standard tests for hydro code in special relativity

Riemann Shock Tube

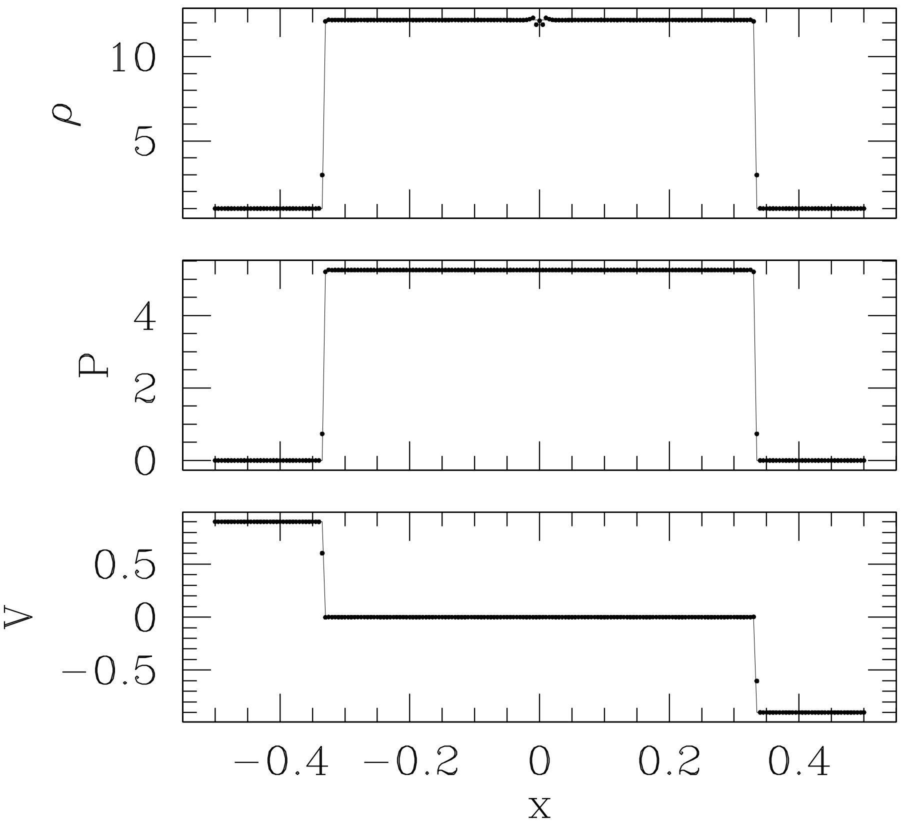
N = 400, Γ = 5/3

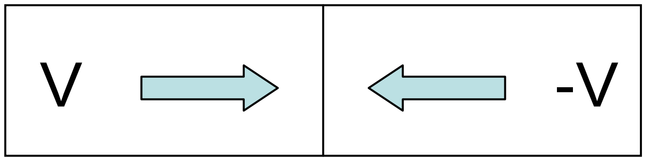



V = 0.9c. Wall Shock

N = 400, Γ = 4/3



# Summary of current implementations IV

## Initial condition for BNS (in quasiequilibrium)

So far : Conformal flatness approximation

$$ds^2 = -\left(\alpha^2 - \beta^k \beta_k\right) dt^2 + 2\beta_k dx^k dt + \psi^4 \delta_{ij} dx^i dx^j$$

$$\begin{cases} \Delta(\alpha\psi) = \ldots. \\ \Delta\psi = \ldots. \\ \Delta\beta^k + \dfrac{1}{3}\nabla^k \nabla_j \beta^j = \ldots. \end{cases}$$

5 elliptic PDEs

$$K = 0, \quad \tilde{A}_{ij} \equiv \psi^{-4}\left(K_{ij} - \frac{1}{3}\gamma_{ij} K\right) = \tilde{\gamma}_{jk}\tilde{\nabla}_i \beta^k + \tilde{\gamma}_{ik}\tilde{\nabla}_j \beta^k - \frac{2}{3}\tilde{\gamma}_{ij}\tilde{\nabla}_k \beta^k$$

$$\tilde{\gamma}_{ij} \equiv \psi^{-4}\gamma_{ij} = \delta_{ij}$$

Adequate for qualitative study of merger (probably), but not for quantitative study since

$\gamma_{ij} - \delta_{ij} \sim v^4 \sim O(0.1)$ ; Not small → Need better one

# What is a good formulation?

1. Binary evolves as a result of gravitational radiation

$\Rightarrow$ $dE/dt = \Omega\, dJ/dt$ holds for gravitational waves

$\Rightarrow$ The first law $\delta E = \Omega \delta J$ should be satisfied

2. Binary is in approximately stationary state.

Virial relation should be satisfied (at least approximately).

$\Rightarrow$ ADM mass = 'Komar-like' mass

$$\psi = 1 + \frac{M}{2r} + O\left(r^{-2}\right)$$

$$\alpha = 1 - \frac{M_K}{r} + O\left(r^{-2}\right)$$

$$\Rightarrow M = M_K$$

We require 2 conditions

(By the way, conformal flatness approx. satisfies these conditions.)

# First possibility: Assume helical symmetry

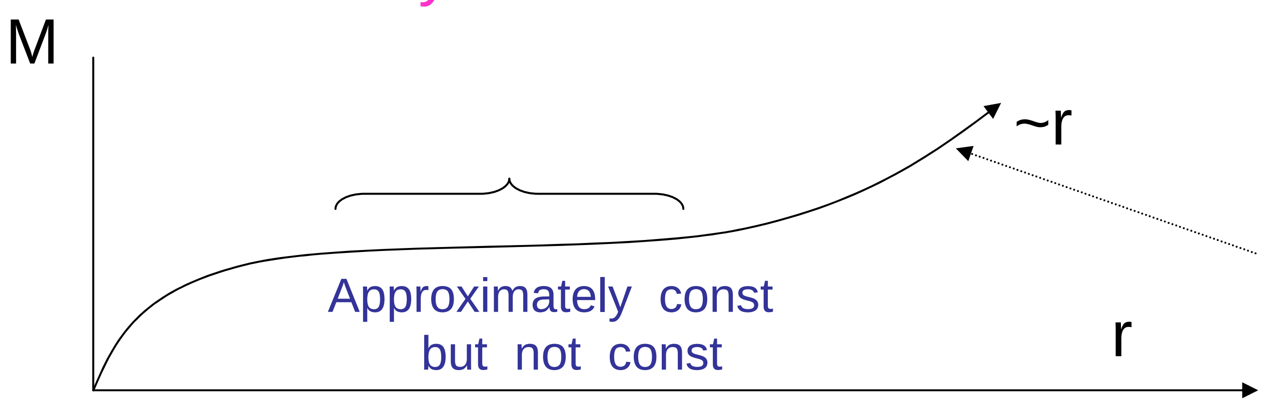
$$L_\xi \tilde{\gamma}_{ij} = 0 = L_\xi \tilde{A}_{ij} \;; \quad \xi^\mu = \left(\frac{\partial}{\partial t}\right)^\mu + \Omega\left(\frac{\partial}{\partial \varphi}\right)^\mu$$

$$\left\{\begin{aligned} &\Delta(\alpha\psi) = \ldots. \\ &\Delta\psi = \ldots. \\ &\Delta\beta^k + \frac{1}{3}\nabla^k\nabla_j\beta^j = \ldots. \\ &(-\partial_{tt} + \Delta)\tilde{\gamma}_{ij} = \ldots. \end{aligned}\right\}$$

RHS=Not compact [O(r^-2)] for r → infinity

$$(e.g., \quad K = 0, \quad \tilde{\gamma}_{ij,j} = 0)$$

It is not very clear how M & J are defined




First law??
Virial ??

Due to standing waves

# A prescription: Asymptotically waveless

$$L_\xi \tilde{\gamma}_{ij} = 0 = L_\xi \tilde{A}_{ij} \quad \text{for} \quad r \ll r_{\text{light cylinder}}$$

OR

$$L_t \tilde{\gamma}_{ij} = 0 = L_t \tilde{A}_{ij} \quad \text{for} \quad r \gg r_{\text{light cylinder}}$$

$$\xi^{\mu} = \left(\frac{\partial}{\partial t}\right)^{\mu} + F(r)\Omega\left(\frac{\partial}{\partial \varphi}\right)^{\mu}$$

$$F = \begin{cases} 1 \quad \text{for} \quad r < r_{\text{light cylinder}} \\ 0 \quad \text{for} \quad r \gg r_{\text{light cylinder}} \end{cases}$$

$$\Delta(\alpha\psi) = \ldots.$$

$$\Delta\psi = \ldots.$$

$$\Delta\beta^{k} + \frac{1}{3}\nabla^{k}\nabla_{j}\beta^{j} = \ldots.$$

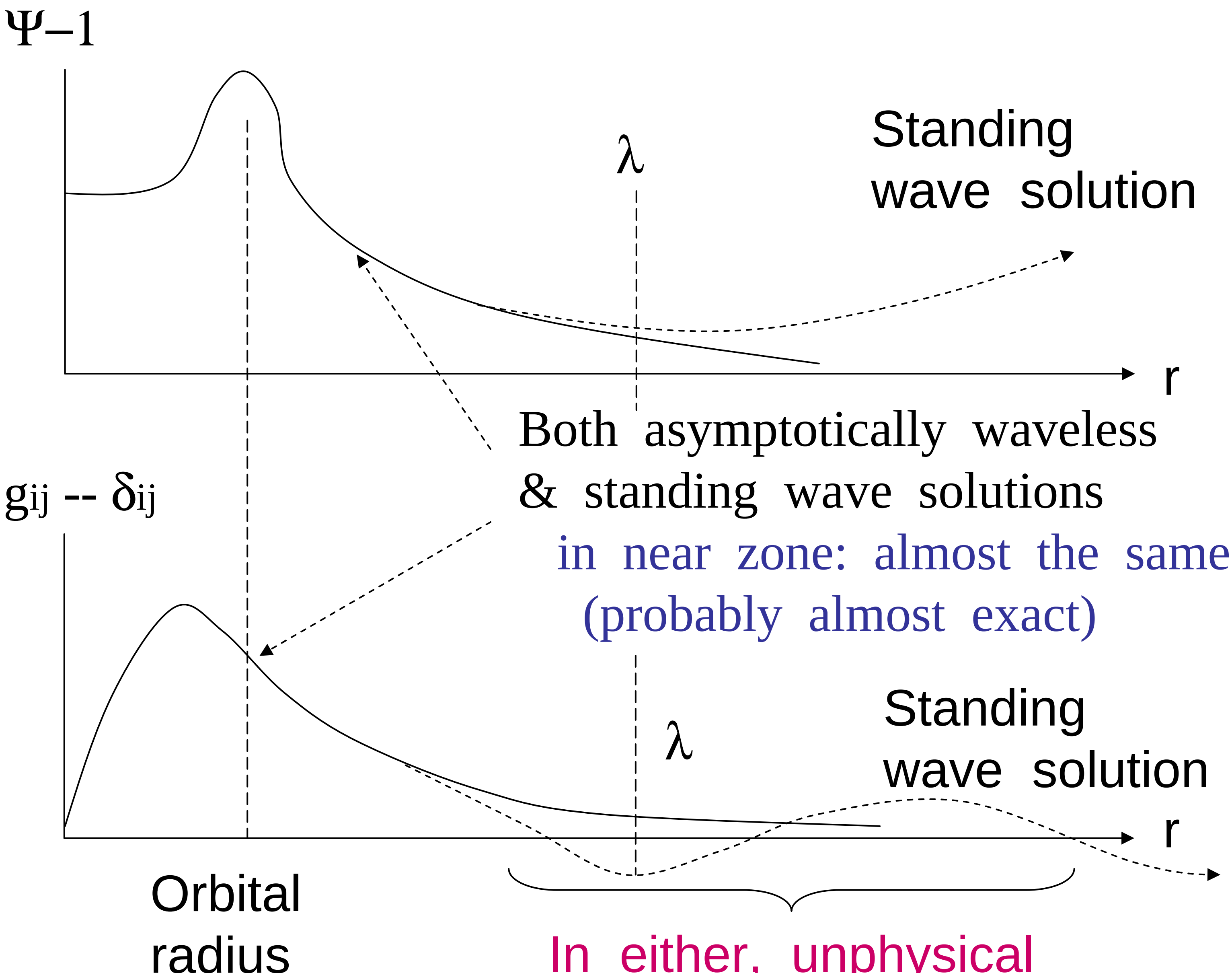
$$\Delta\tilde{\gamma}_{ij} = \ldots. \quad \text{for} \quad r \to \infty$$

RHS=compact [O(r^-4)] for r → infinity

M & J are well-defined

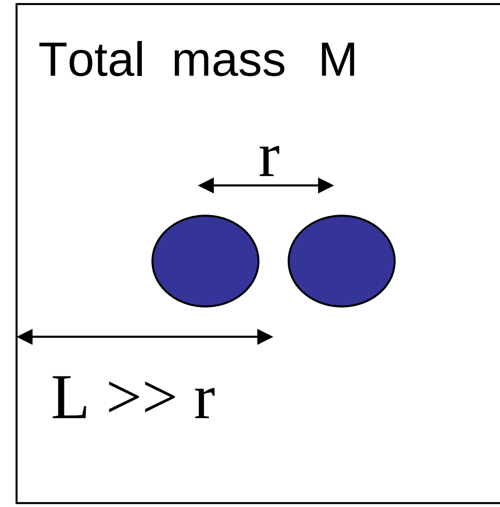

First law & Virial are guaranteed to be satisfied approx. in this formalism (Shibata et al. 2003)

# Expected relation between two



# Summary of computational resources

Required grid number for extraction of accurate gravitational waveforms




$$\lambda_{GW} \le \lambda_{ISCO} \approx 58\left(\frac{GM}{c^2}\right)\left(\frac{rc^2}{7GM}\right)^{3/2}$$

$$\text{Require} \quad \mathrm{L} \ge \lambda_{GW} \quad \& \quad \Delta \mathrm{x} \le 0.2\left(\frac{\mathrm{GM}}{\mathrm{c}^2}\right)$$

$$\Rightarrow \quad \frac{\mathrm{L}}{\Delta \mathrm{x}} \ge 290\left(\frac{rc^2}{7GM}\right)^{3/2} \quad \& \quad \mathrm{N} \ge 580\left(\frac{rc^2}{7GM}\right)^{3/2}$$

Minimum grid required (in uniform grid):
600 * 600 *300 (equatorial symmetry is assumed)
$\Rightarrow$ Memory ~ 200 GBytes (~200 variables)

# An example of current supercomputer

FACOM VPP5000 at NAOJ

- Vector-Parallel type ~ max: 48PEs
- Maximum memory ~ 0.7TBytes
- Our typical run

633*633*317 grid points = 240Gbyte memory (in my code)

About 20000 timesteps ~ 100 CPU hours

Minimum grid numbers can be taken

Hopefully we would like to use more powerful one.
(e.g. As Earth simulator)
Or need to develop mesh refinement techniques

# Summary of Current Status

- Einstein evolution equations solver — OK
- Gauge conditions (coordinate conditions) — OK
- GR Hydrodynamic equations solvers — OK
- Realistic initial conditions in GR — ~OK
- Gravitational wave extraction techniques
- Powerful supercomputer
- Special techniques for handling BHs.

But to be developed

~OK, but need Mesh-refinement or hypercomputers

To be developed

# 2. Numerical results : example

## My current implementation

1. GR : Nakamura-Shibata (modified gradually)

   Improve transport term = crucial for M & J conservation

   Solve: $\partial_t \sqrt{\gamma} - \partial_i \left( \sqrt{\gamma}\, \beta^i \right) = -\alpha K \sqrt{\gamma}$ & use the hydro scheme

   Not: $\partial_t \sqrt{\gamma} - \beta^i \partial_i \sqrt{\gamma} = -\alpha K \sqrt{\gamma} + \partial_i \beta^i \sqrt{\gamma}$ (previously used)

2. Gauge : ~ Maximal Slicing + dynamical gauge
3. Hydro : High-resolution shock-capturing scheme
   (Roe-type method with PPM interpolation)
4. Initial conditions : Still conformal flatness approx.
   (computation with new formulation in progress)
5. Wave extraction : Extract gauge-invariant variable
6. Typical grid size : 633 * 633 * 317

# Setting

- Equation of state t = 0 : $P = K \rho^\Gamma$
  t > 0 : $P = (\Gamma - 1)\rho\varepsilon$ with $\Gamma = 2$

| Compactness $(M/R)_\infty$ | Total rest mass $M_{*Tot} / M_{*Max}$ (J=0) | Spin $J/M^2$ | $m_2/m_1$ | Model | Fate |
|---|---|---|---|---|---|
| 0.14 | 1.62 | 0.951 | 1 | M1414 | NS |
| 0.16 | 1.78 | 0.914 | 1 | M1616 | BH |
| 0.13 vs 0.15 | 1.62 | 0.961 | 0.90 | M1315 | NS |
| 0.15 vs 0.17 | 1.77 | 0.923 | 0.925 | M1517 | BH |
| 0.14 vs 0.18 | 1.76 | 0.933 | 0.855 | M1418 | BH |

$M_{*Max}$: Maximum rest-mass of spherical star in isolation

Unequal mass (new)

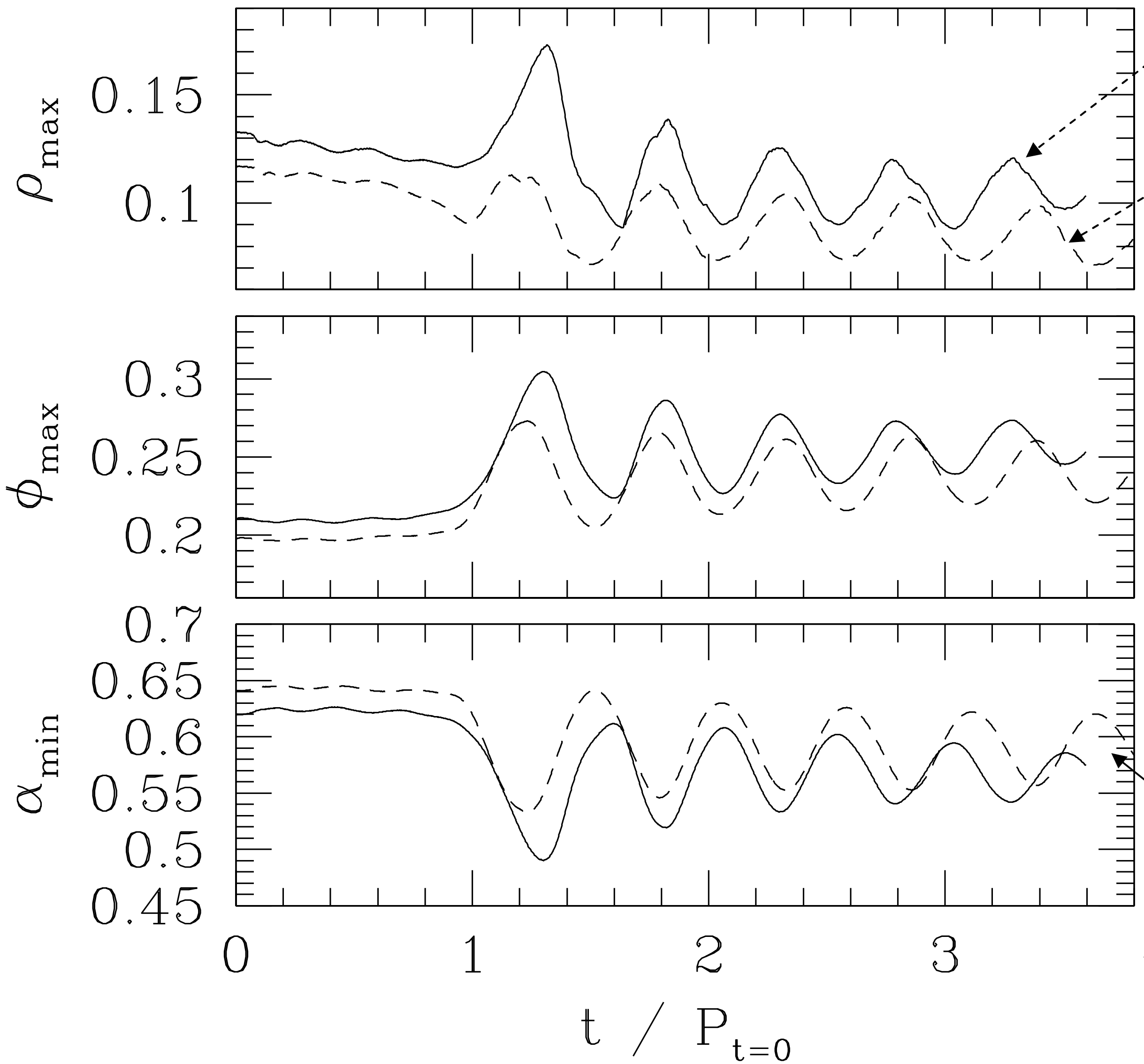
Hypermassive

Note (M/R) = 0.14 & 0.16 mean
R = 15km & 13km if M = 1.4 Solar mass

# Animations

- http://esa.c.u-tokyo.ac.jp/~shibata/anim.html

# Change of maximum density in NS formation




Unequal mass M1315

Equal mass M1414

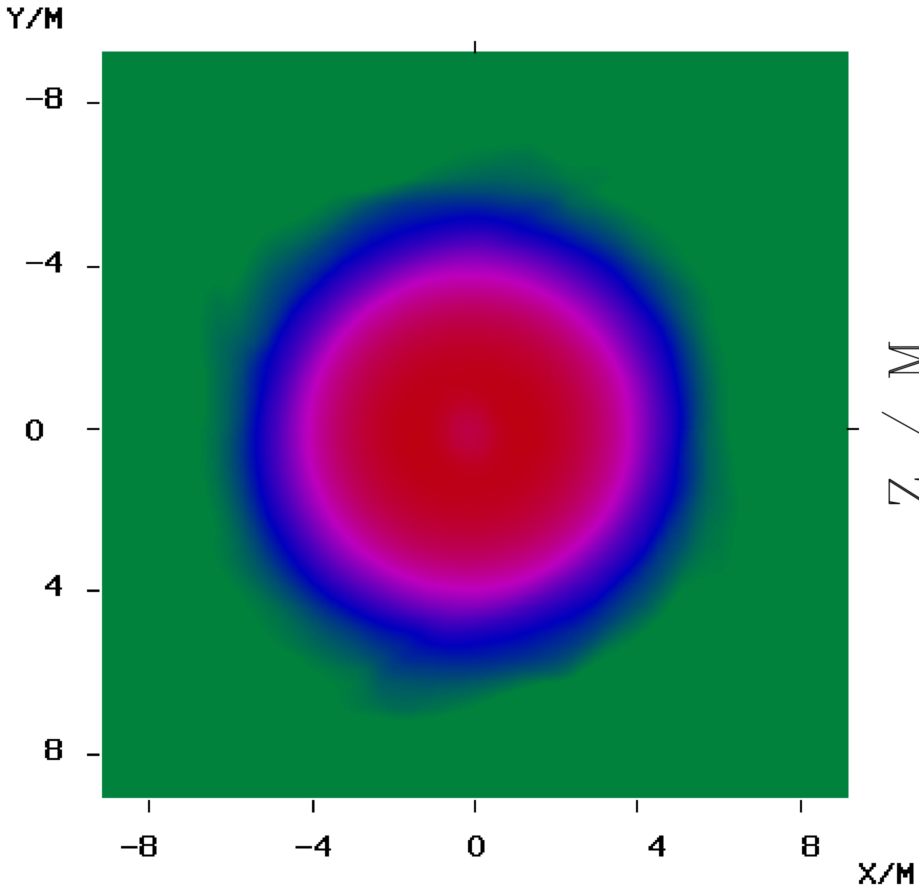
Oscillating hyper massive neutron stars are formed

Not crash.
We stopped simulation.

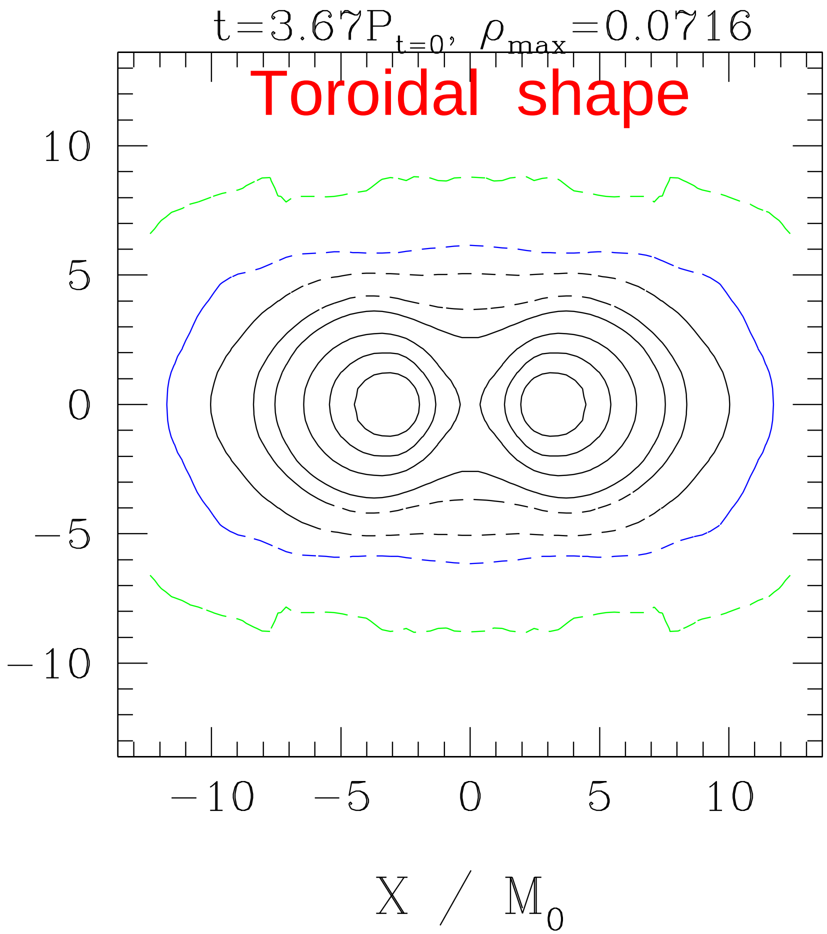
# M/R = 0.14 equal mass case : final snapshot

## Massive toroidal neutron star is formed

## (slightly elliptical)

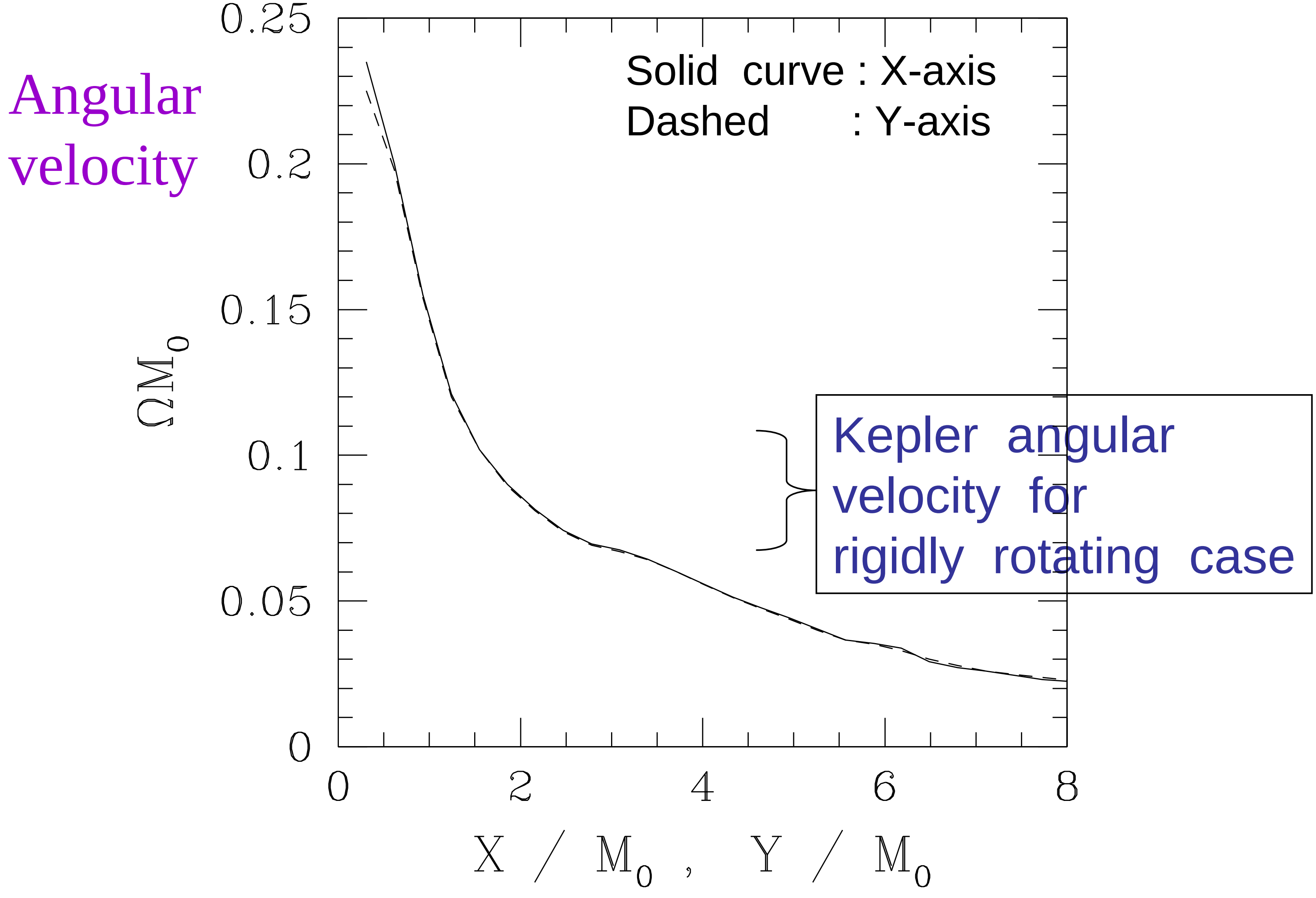



X – Y contour plot




X – Z contour plot

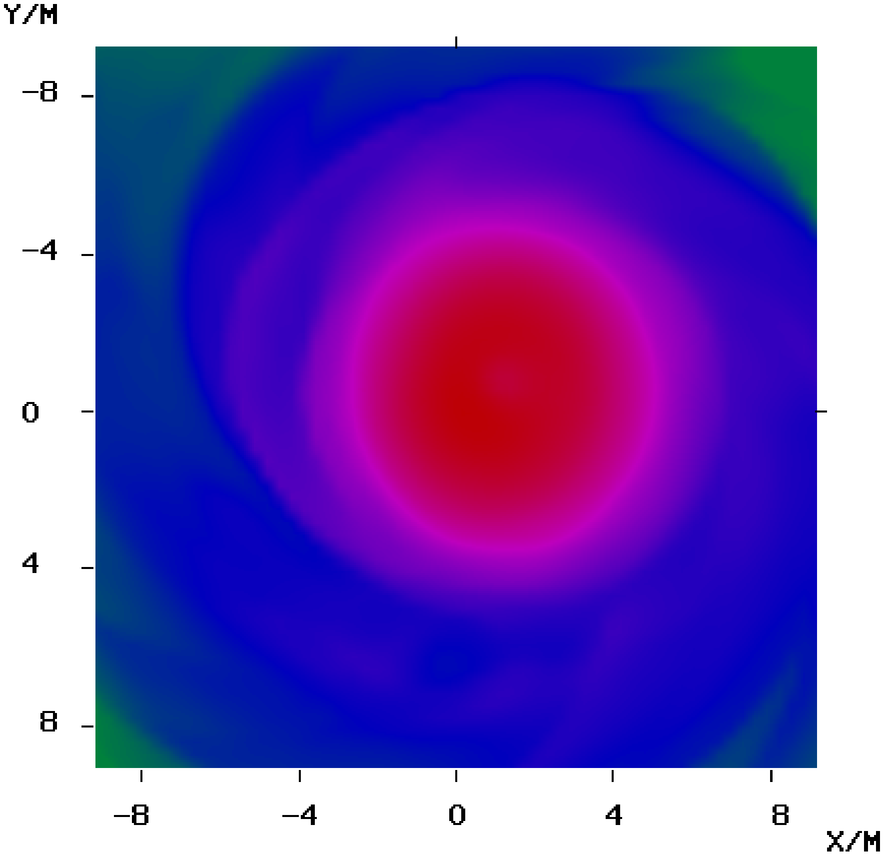
# Formed Massive NS is differentially and rapidly rotating



# Comparison between equal and unequal mass merger

M/R = 0.13 vs 0.15:

Massive NS + disk

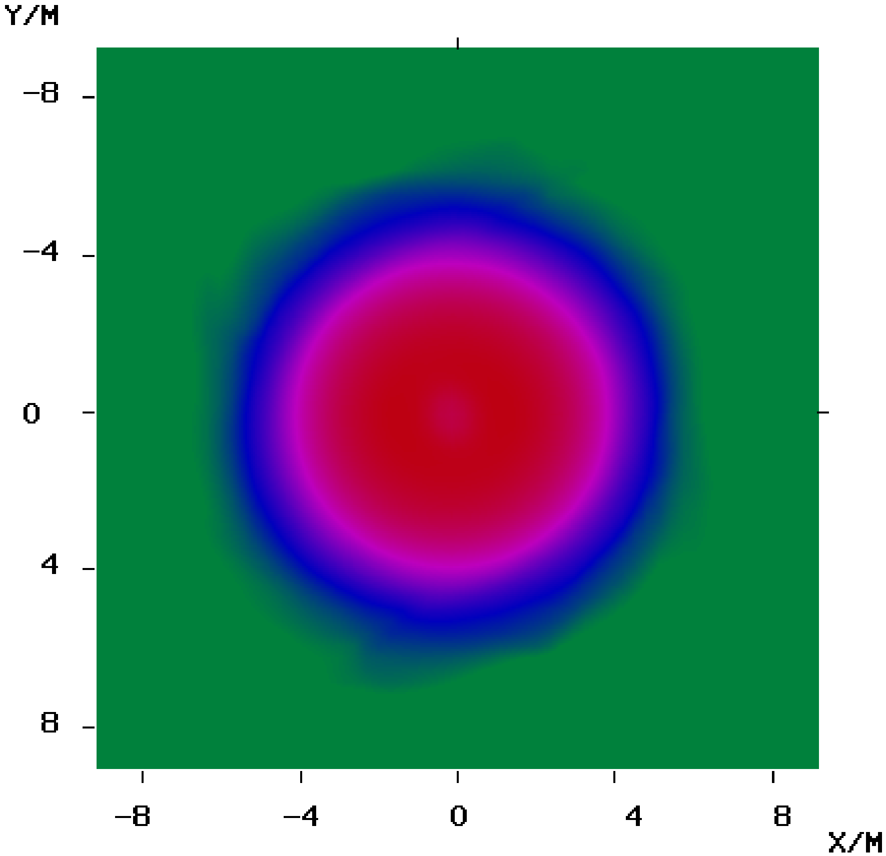



Unequal mass

Mass ratio ~ 0.90

M/R = 0.14 vs 0.14:

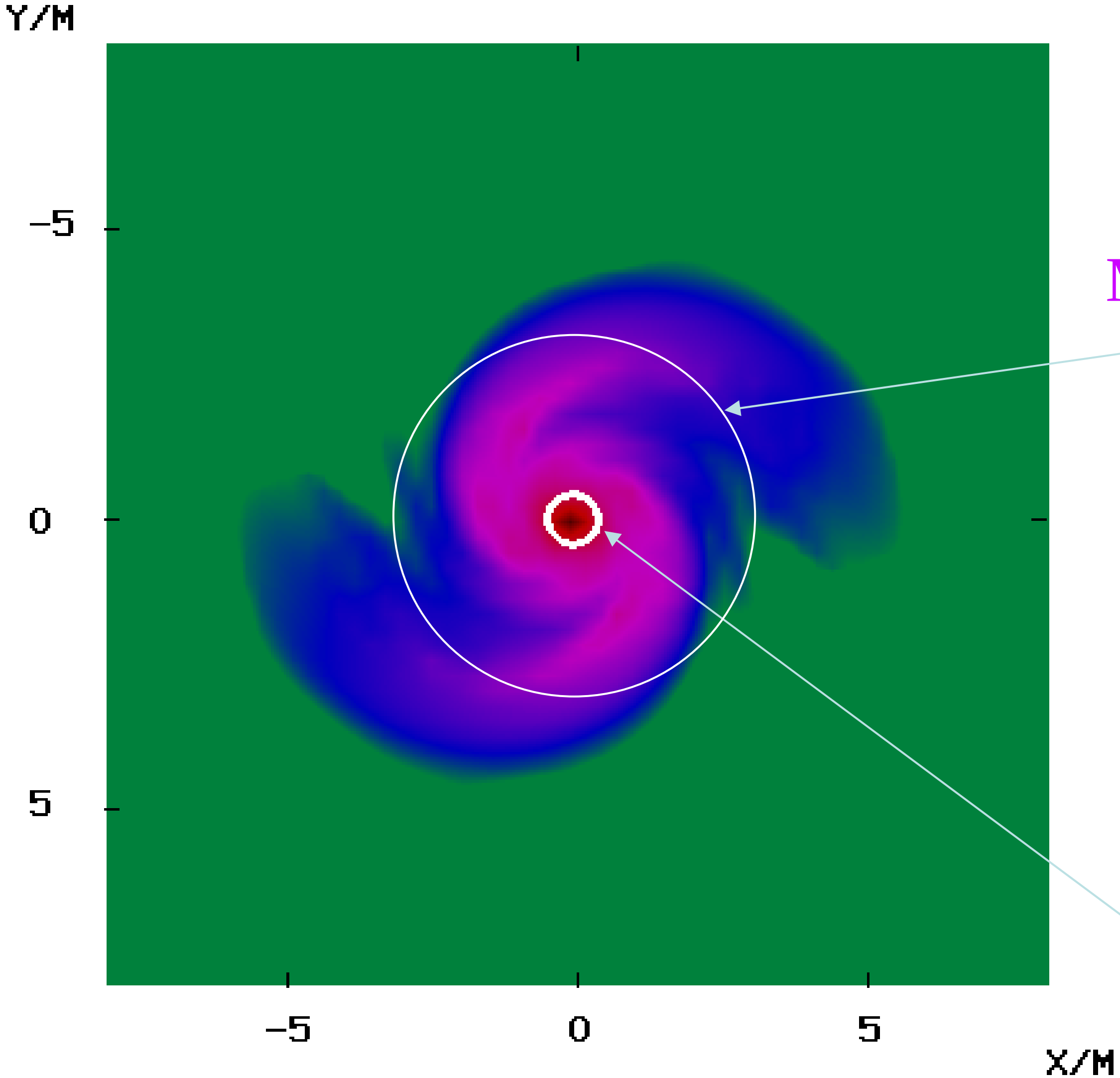
Massive NS




Equal mass

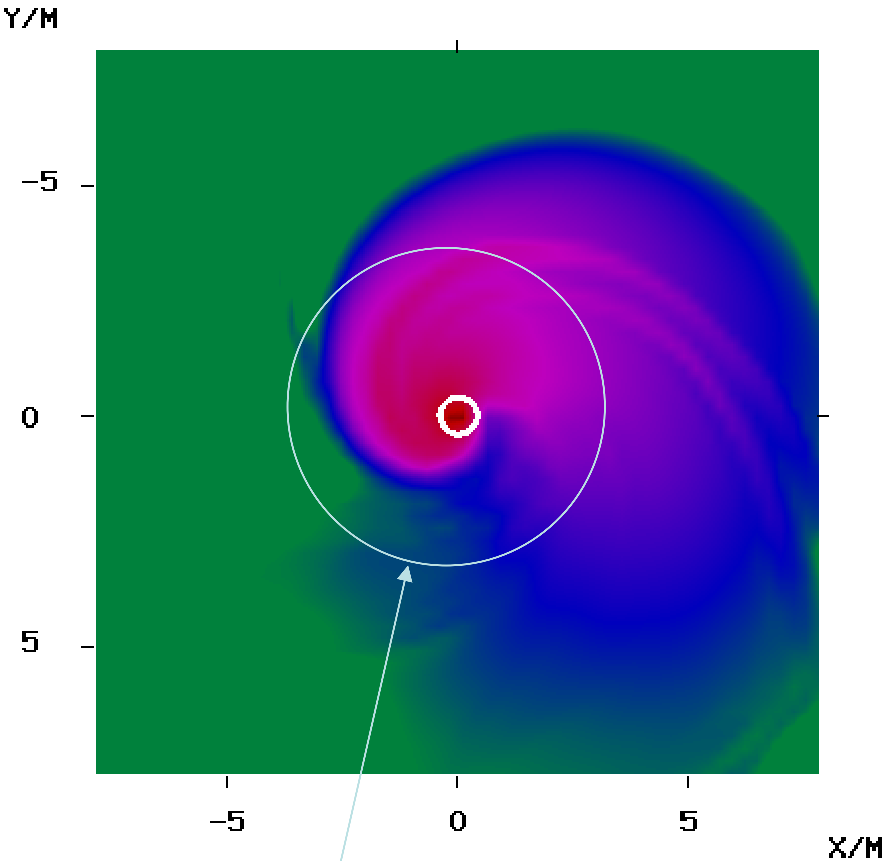
# Black hole formation case: M/R=0.16

## Equal mass

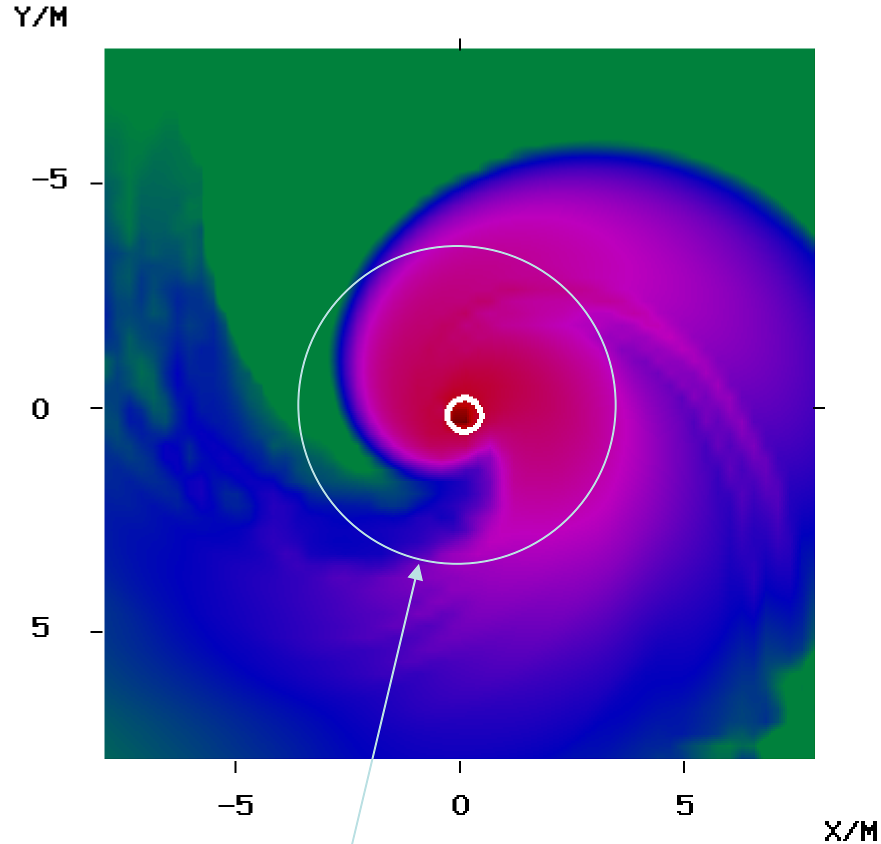


# Disk mass for unequal mass merger

M1517: Mass ratio 0.925




M1418: Mass ratio 0.855




Mass for r > 3M
~ 2%

Mass for r > 3M
~ 5%

# Products of mergers for $\Gamma$=2: Latest results

## Equal – mass cases

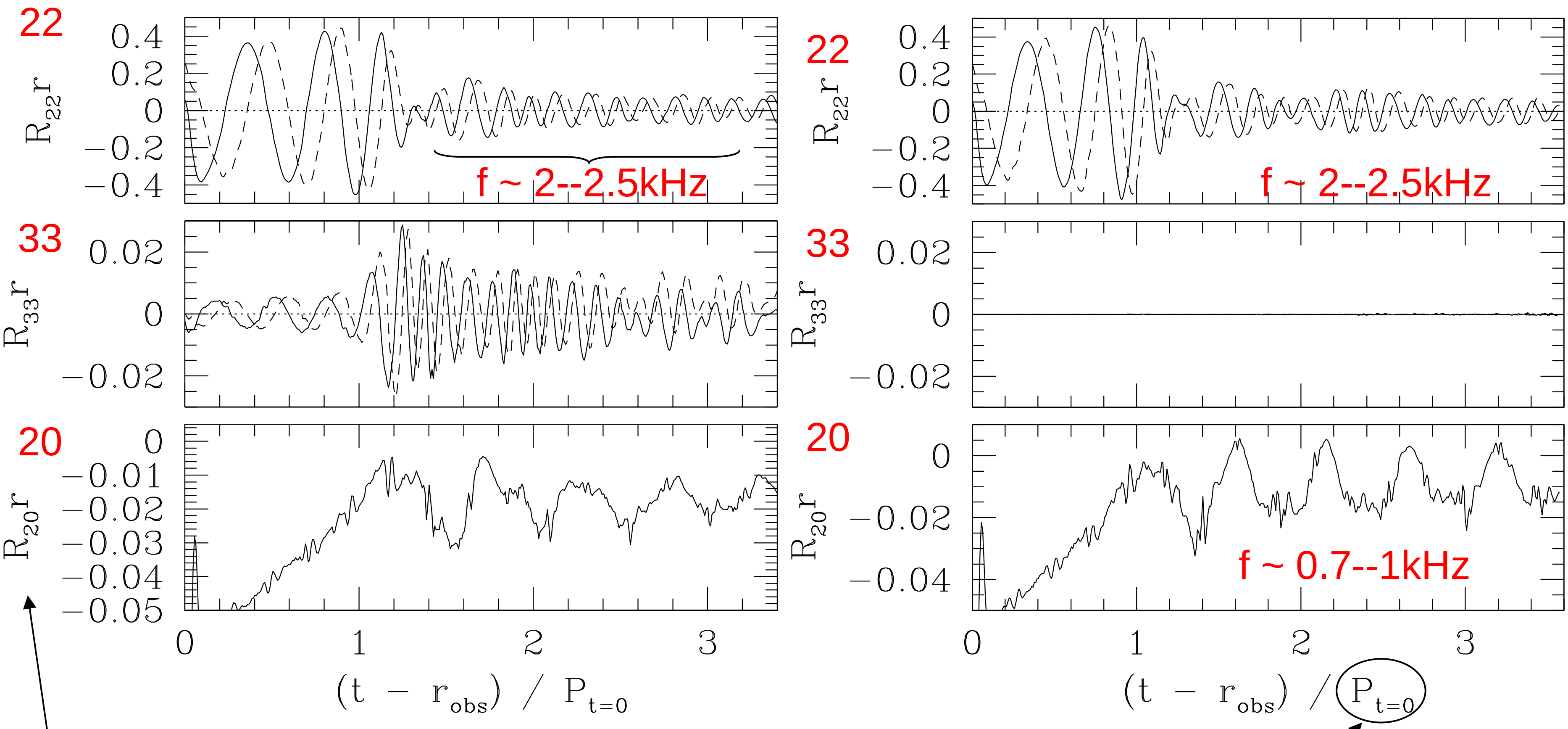
- Low mass cases (r ~ 15 km for 1.4 solar mass)
  Hyper massive neutron stars
  of non-axisymmetric oscillation.
- High mass cases (r < 13 km for 1.4 solar mass)
  Direct formation of Black holes
  with very small disk mass

## Unequal – mass cases (mass ratio ~ 90%)

- Likely to form disks of mass
  ~several percents of total mass
  → BH(NS) + Disk

# Gravitational waves: NS formation

Unequal mass(M1315) | Equal mass(M1414)

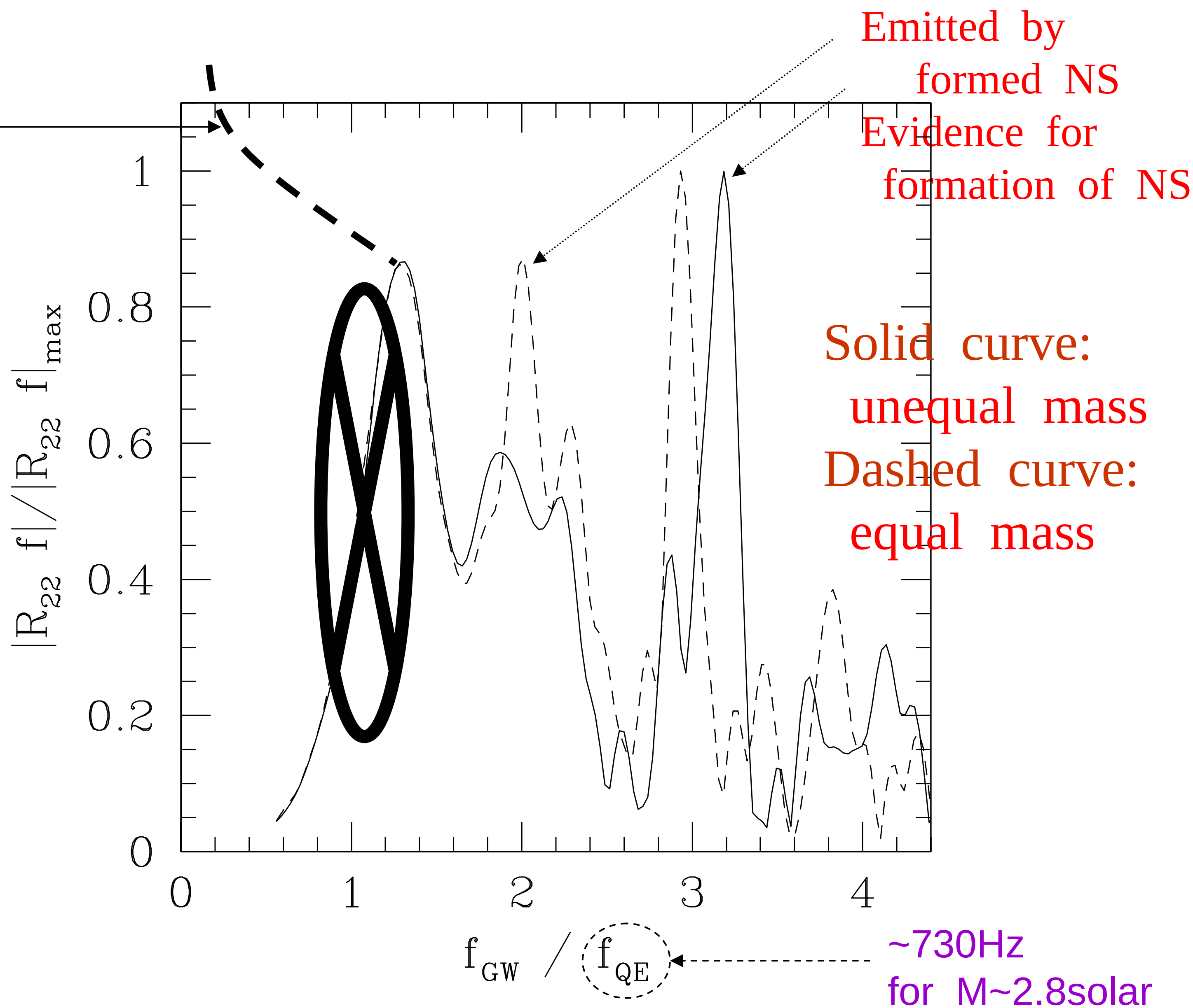



Gauge inv. variables
with (l,m)=(2,2), (3,3) & (2,0)

P ~ 2.7msec (M/2.8solar)

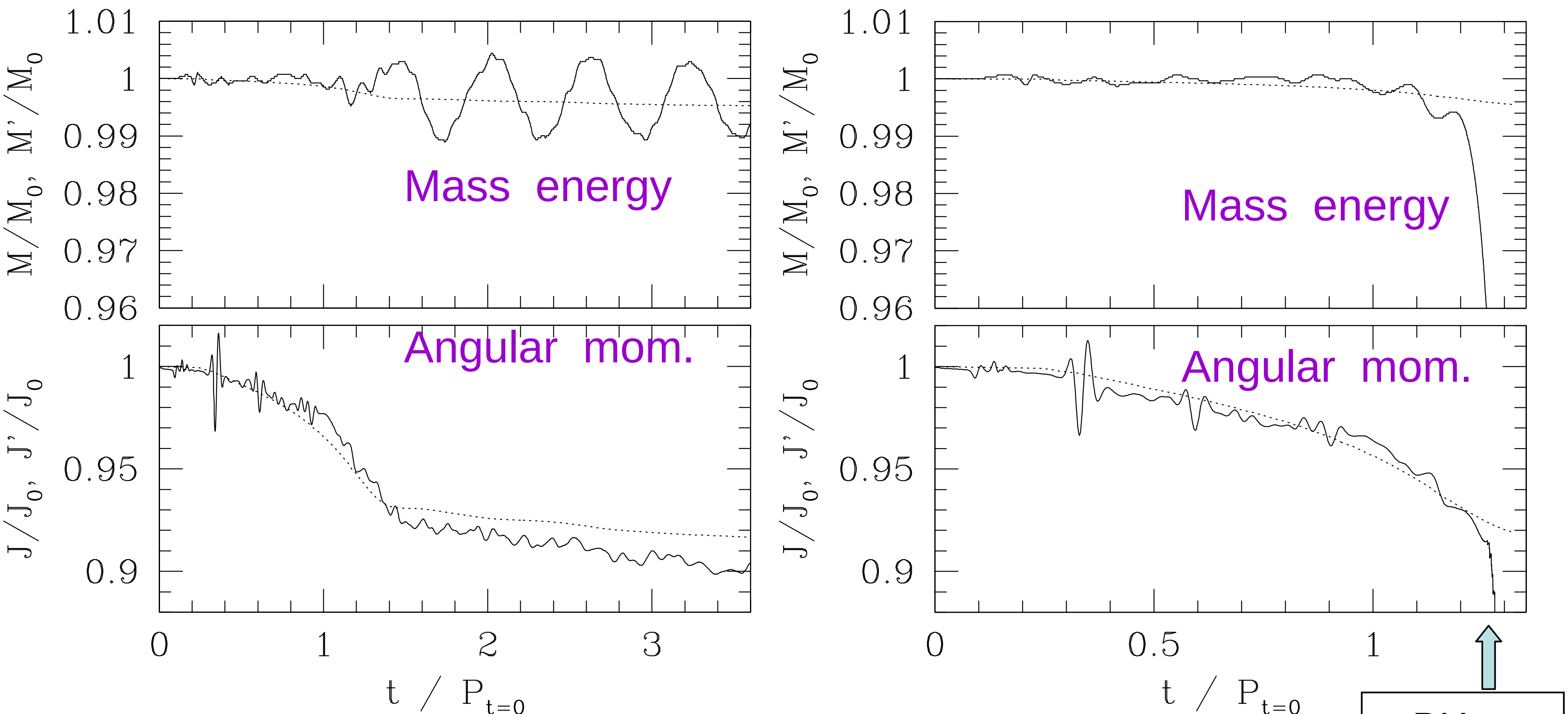
# Fourier spectrum for NS formation

Inspiral Waveform f^{-1/6} is absent



# Radiation reaction : OK within ~ 1%

NS formation: equal mass    BH formation: unequal mass




Solid curves : computed from data sets.

Dotted curves: computed from fluxes of gravitational waves

# 3. Summary

- Simulations are feasible to get scientific results.

  (I think) numerical implementations for fundamental parts have been almost established (for the absence of BHs).

- Still there are technical Issues :
  - Grid numbers are still not large enough

    → We need Mesh-Refinement (AMR/FMR).
  - Computation crashed due to grid stretching

    around BH horizon → We need excision.

    (Dynamical gauges may benefit excising.)
  - Incorporate more physical EOS (probably not very difficult), neutrino cooling, etc.