General Relativistic Hydrodynamics (or) Towards a Realistic Neutron Star Binary Inspiral

KITP
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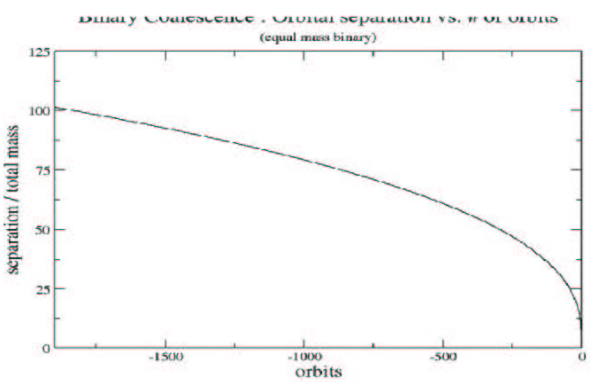


length scale for compact object: ~ *10 M*

discretization length scale: ~ *M/10*

timescale of last orbit: ~ *400 M*

characteristic GR wavelength: ~ *200 M*

Numerical Relativity needed for last stage of coalescence

Problem # 1: astrophysically realistic initial data

Problem # 2: stable, accurate evolution codes

• Circular Orbit Approximation

- ➔ post–Newtonian instantaneously circular orbit configurations
- ➔ evolution using initially circular orbit configurations vs. quasicircular solutions

• Evolution Code and Initial Data for Binary Neutron Stars / Analysis of Initial Data

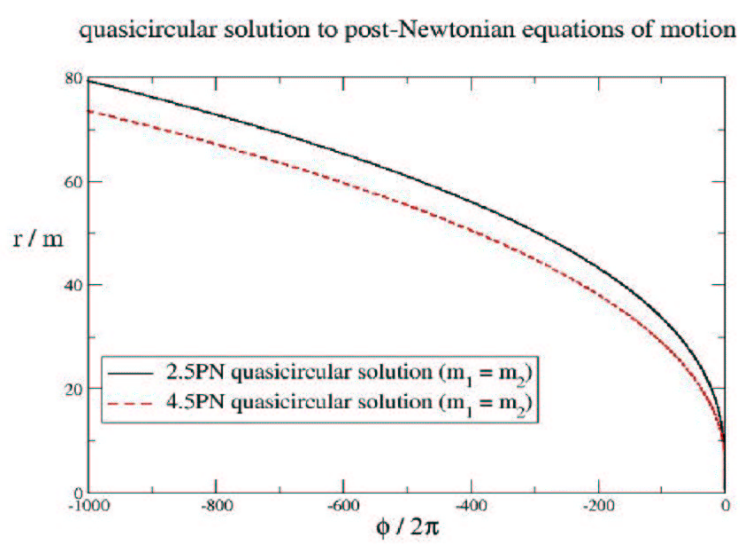
- ➔ Evolution code
- ➔ Conformally Flat, QuasiEquilibrium (CFQE) initial data
- ➔ Error analysis methods
- ➔ CFQE approximation: use evolution code to analyze the approximations used in building initial data

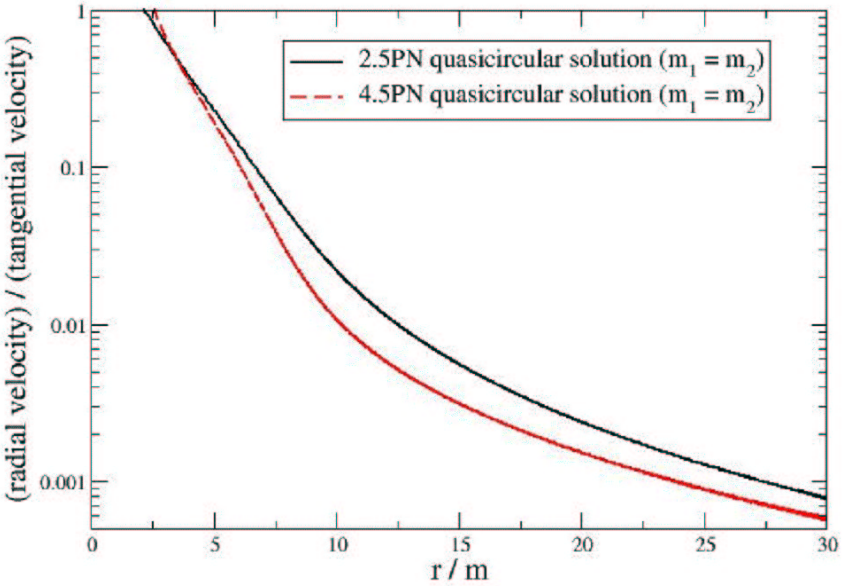
• Multiple Orbit Evolutions

- ➔ stable, multiple orbit evolutions (with error analysis)

Quasicircular solutions to PN equations of motion (fully general: no circular orbit assumption)

(MM: gr–qc/0305024)

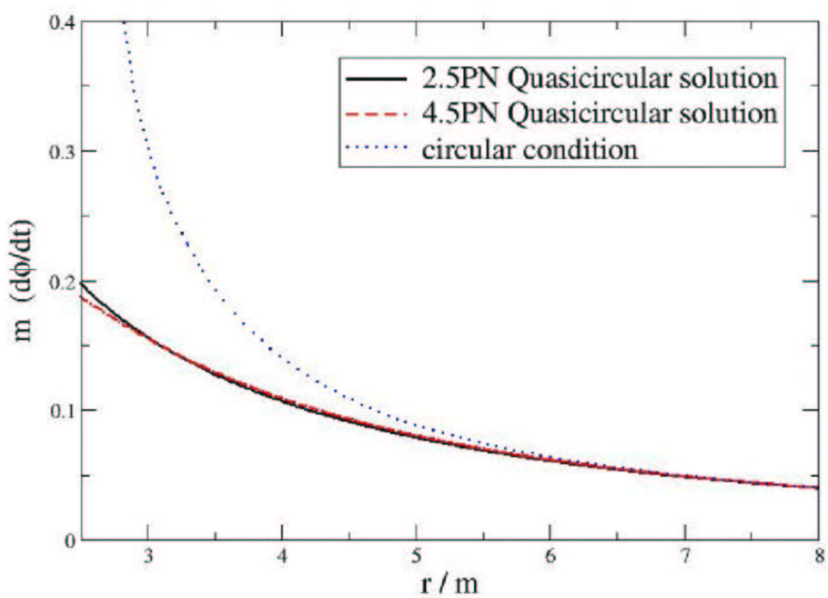


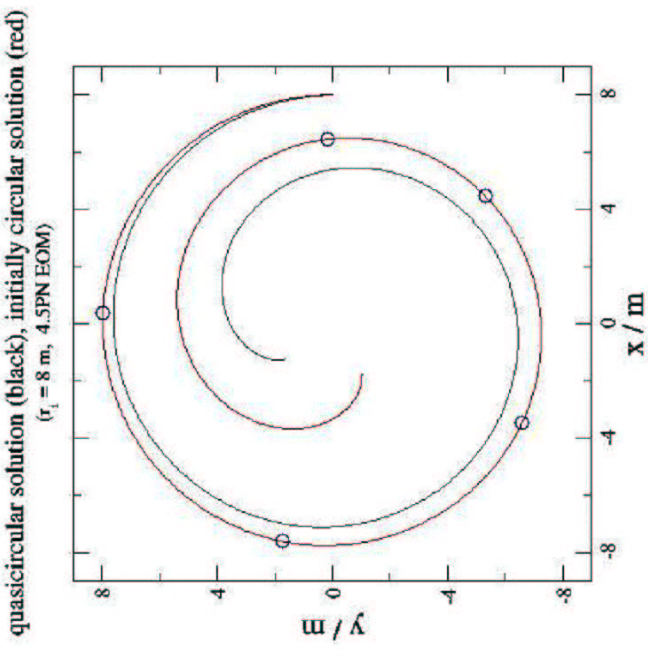


Circular initial data for the PN equations of motion

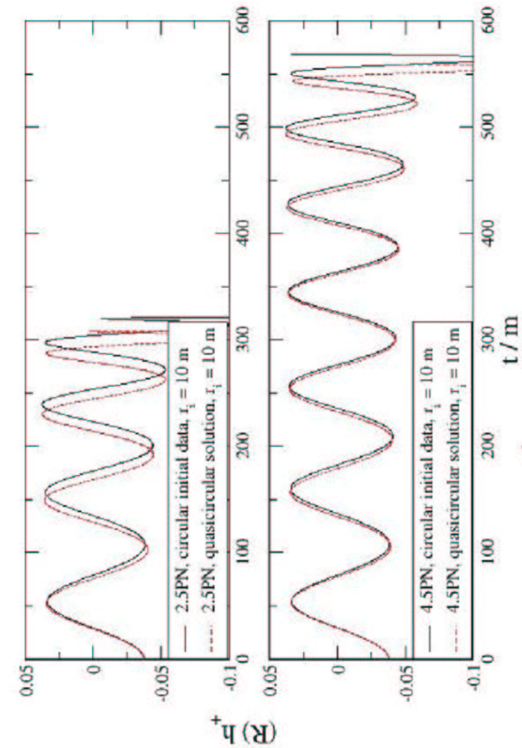
$$\dot{r}_i = \frac{dr}{dt}(t=0) = 0 \qquad \ddot{r}_i = \frac{d^2 r}{dt^2}(t=0) = 0$$

angular velocity for quasicircular solution and circular initial data



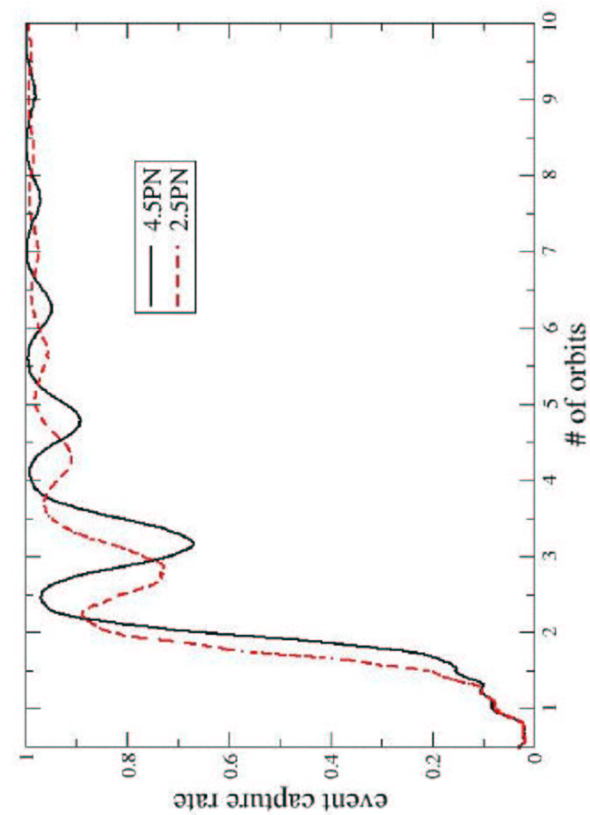


Gravitational Waveforms:

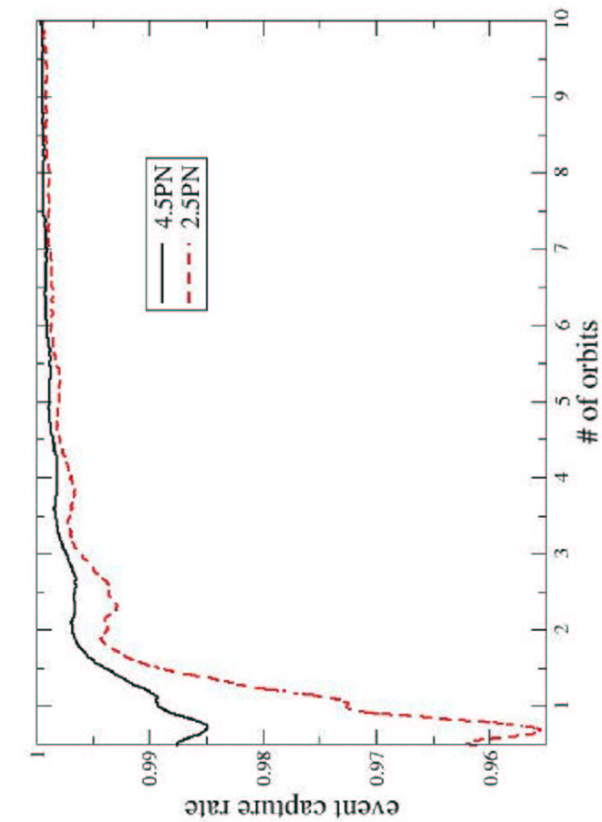


$$C[h_c(t), h_{qc}(t)] \equiv \max_{\tau} \left\{ \frac{\int_0^{t_f} h_c(t)\, h_{qc}(t-\tau)\, dt}{\sqrt{\left(\int_0^{t_f} h_c(t)^2\, dt\right)\left(\int_0^{t_f} h_{qc}(t-\tau)^2\, dt\right)}} \right\}$$

Equal mass binary black hole waveform error due to the circular orbit approximation used in initial data



Equal mass binary neutron star waveform error due to the circular orbit approximation used in initial data



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General Relativistic Hydrodynamics: MAHC

$$T^{ab}=(\rho+\rho\epsilon+P)u^a u^b+P g^{ab} \qquad P=P(\rho,\epsilon)$$

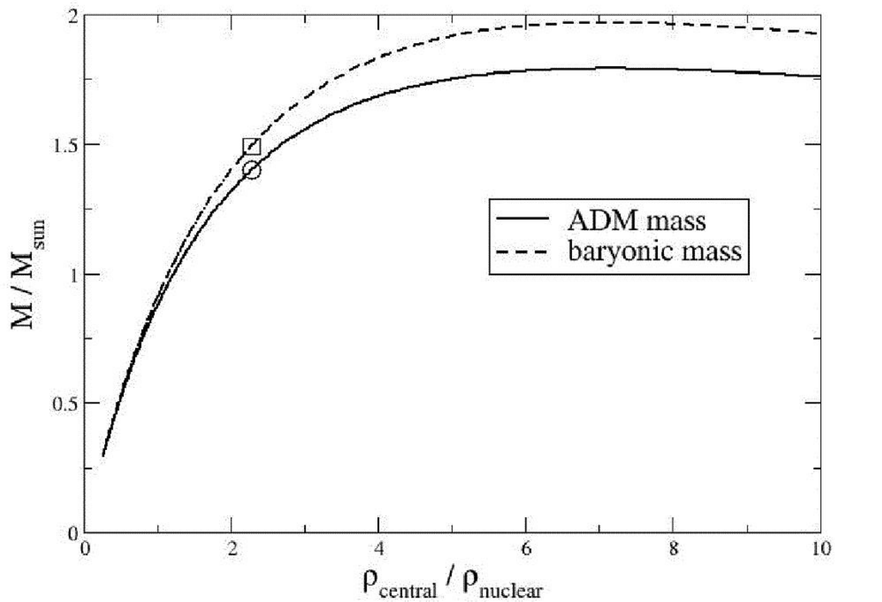
$$G_{ab}=8\pi T_{ab} \qquad \nabla_a T^{ab}=0 \qquad \nabla_a(\rho u^a)=0$$

- High Resolution Shock Capturing Methods (PPM)
- BSSN (standard and symmetric hyperbolic versions) formulation for spacetime
- 2nd order accurate (both space and time)
- Rigorous consistency code tests: (shock tubes, OS collapse, static stars, rapidly rotating stars, boosted stars, head–on NS collisions, independent residual evaluators for all 10 EEs)
- stable boundary conditions
- robust gauge conditions: lapse: maximal or "1+log"

  shift: $\partial_t \beta^i = c_1 \tilde{\Gamma}^i - c_2 \beta^i$

$$P = k\,\rho^{\Gamma} \qquad \Gamma = 2 \qquad k = 0.0445\,c^2/\rho_{nuclear}$$

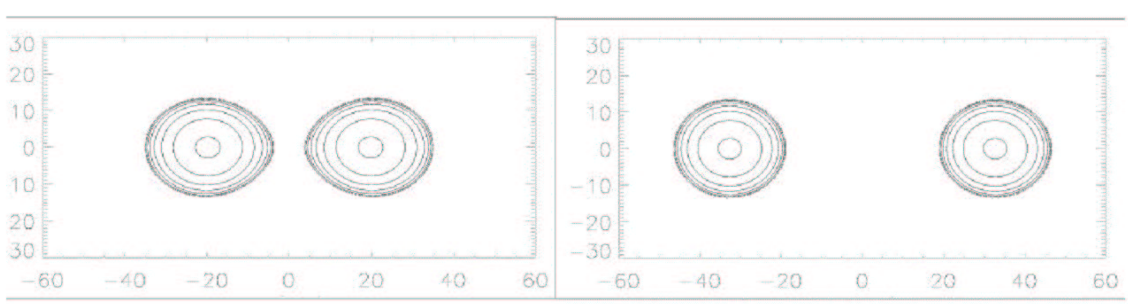
static, isolated, polytropic neutron stars



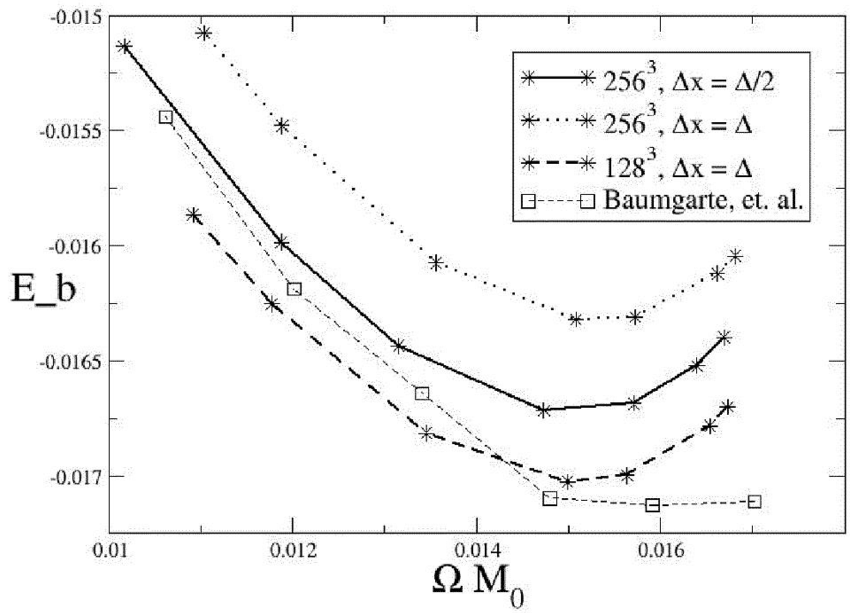
Conformally Flat QuasiEquilibrium (CFQE) Approximation

Major Assumptions:

1) conformally flat: $g_{ij} = \phi^4 \delta_{ij}$
2) $K = L_t K = 0$
3) $L_t \tilde{g}_{ij} = 0$
4) approximate Killing vector field: $k^a = t^a + \Omega \left(\frac{\partial}{\partial \phi} \right)^a$
5) co–rotating matter: $u^a \sim k^a$



$$E_b = \frac{M_{ADM} - 2M_{NS}}{M_0}$$



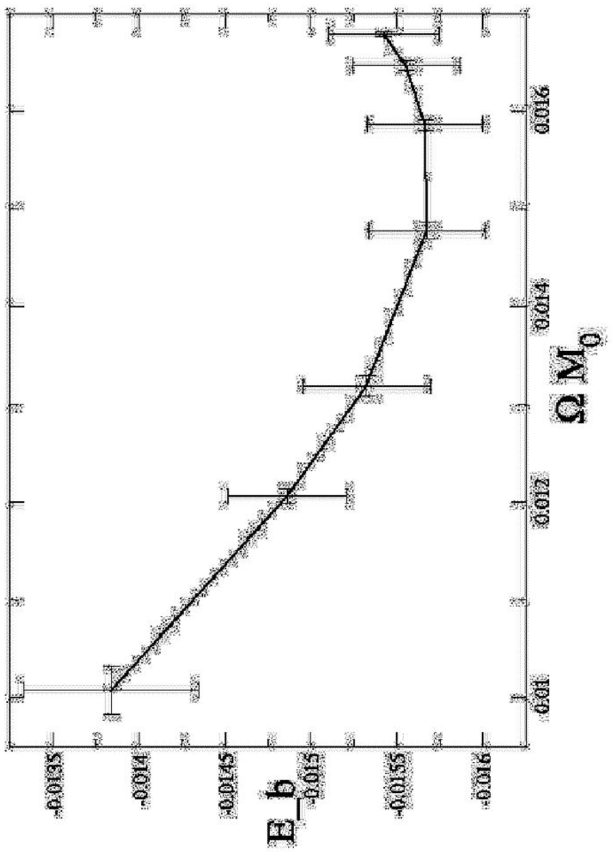
Convergence: used to extract physics from the code!

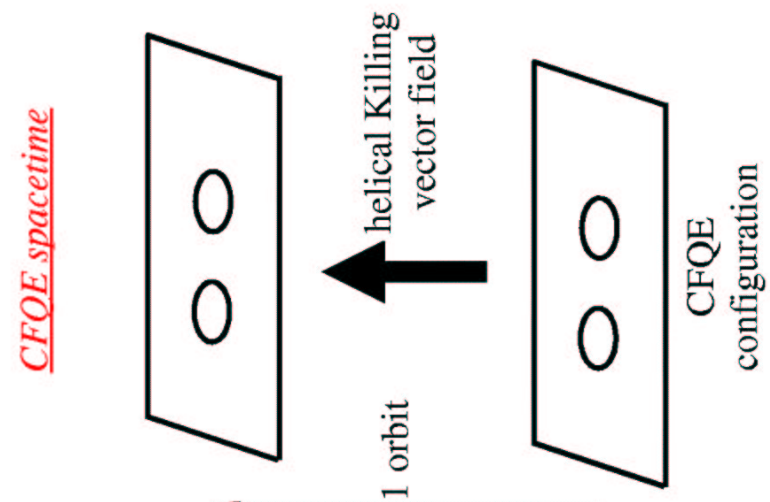
$$Q_{numerical} = Q_{exact} + O((\Delta x)^n)$$

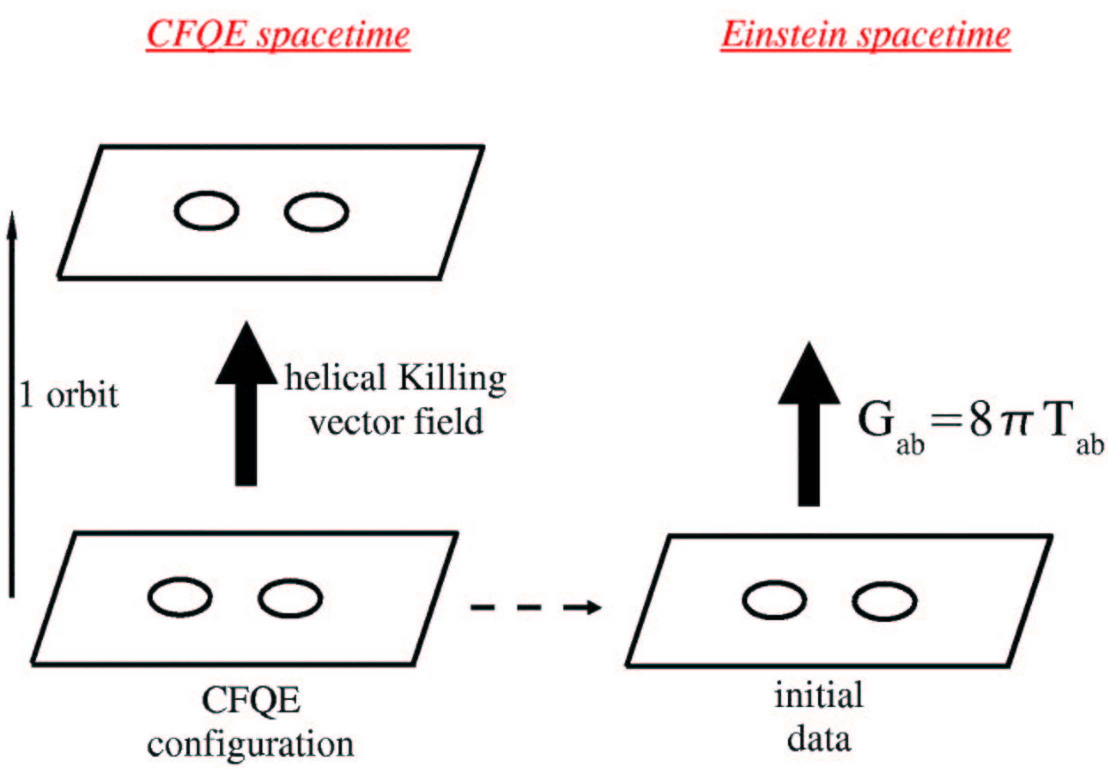
Typical case: 2 sources of error (truncation, boundary)

$$Q_n = Q_e + \frac{c_1}{r_{bound}} + c_2(\Delta x) + c_3(\Delta x)^2 + \ldots$$

$$Error = max\{|Q_n - Q_e| \ , \ \frac{c_1}{r_{bestbound}} \ , \ c_2(\Delta x_{best}) + c_3(\Delta x_{best})^2\}$$


E_b
-0.0135
-0.014
-0.0145
-0.015
-0.0155
-0.016
Ω M0
0.01
0.012
0.014
0.016


CFQE spacetime
helical Killing
vector field
CFQE
configuration
1 orbit

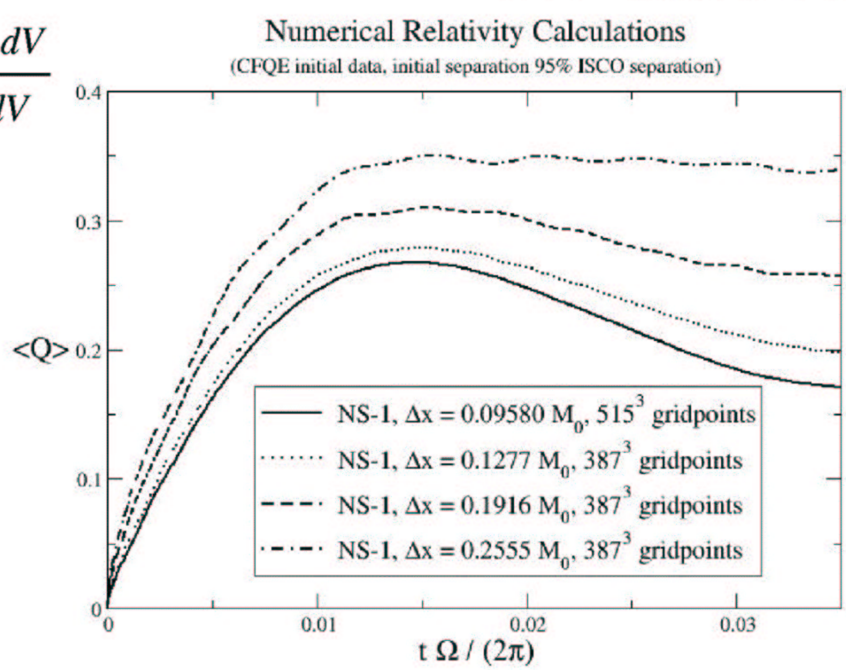


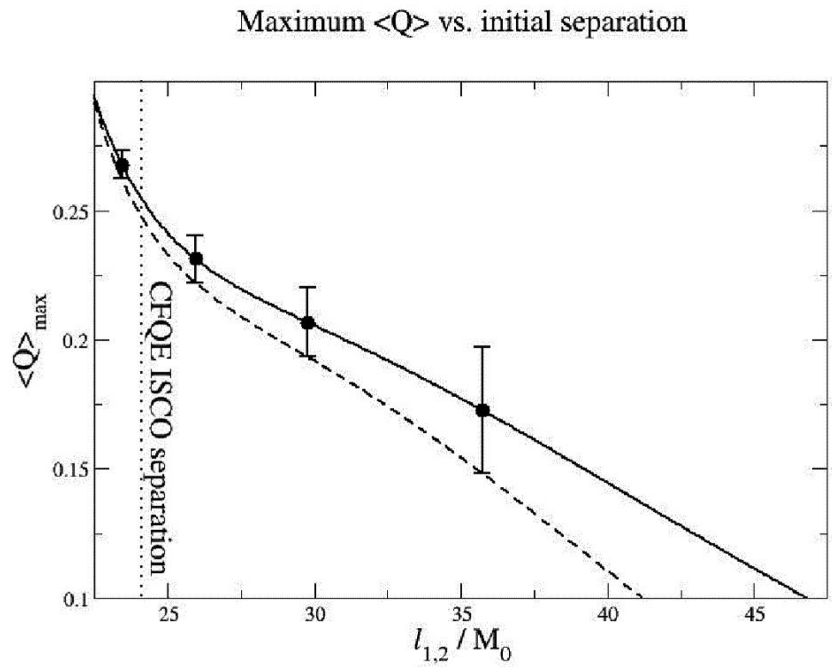
Quasi–Equilibrium Assumption

$$Q_{ab} = \nabla_a u_b + \nabla_b u_a + u_a a_b + a_a u_b$$

$$Q = \frac{|Q_{ab}|}{max\{|\nabla_a u_b + \nabla_b u_a|, |u_a a_b + a_a u_b|\}}$$

$$\langle Q \rangle = \frac{\int \rho Q \, dV}{\int \rho \, dV}$$

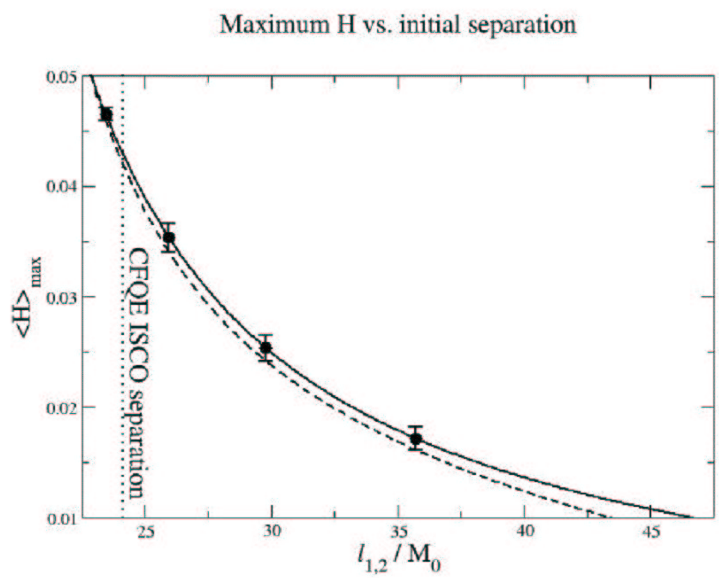




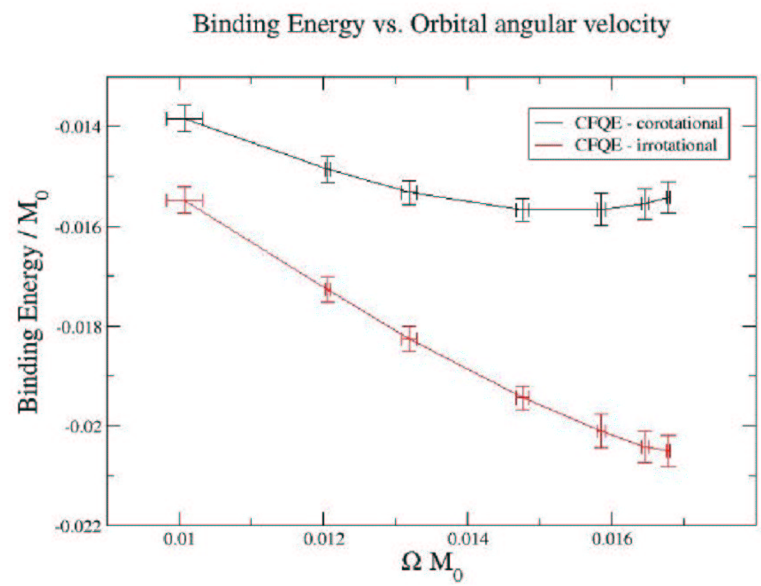
Conformal Flatness Assumption

$$H_{ij} = \epsilon^{kl}{}_i D_k \left({}^{(3)}R_{lj} - \frac{1}{4} g_{lj} {}^{(3)}R \right) \qquad H = \frac{|H_{ij}|}{\sqrt{(D_i R_{jk})(D^i R^{jk})}}$$

$$H\rangle = \frac{\int \rho H \, dV}{\int \rho \, dV}$$



$$(E_b)_{irrot} = \frac{M_{ADM} - 2M_{NS}(\Omega)}{M_0}$$



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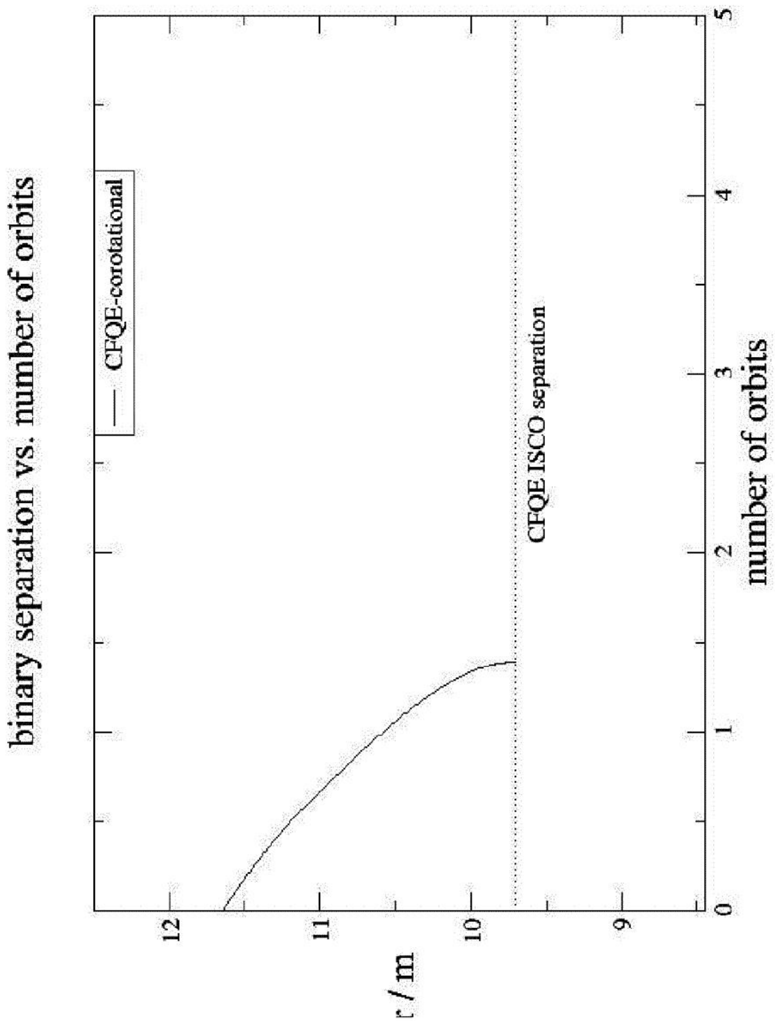
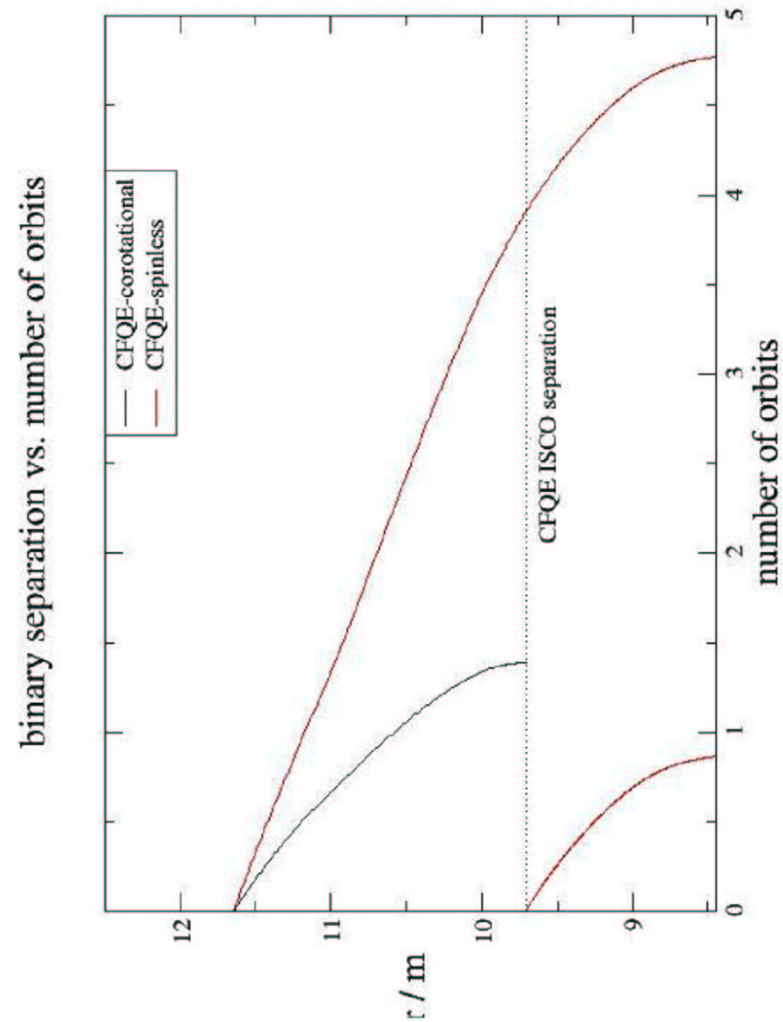
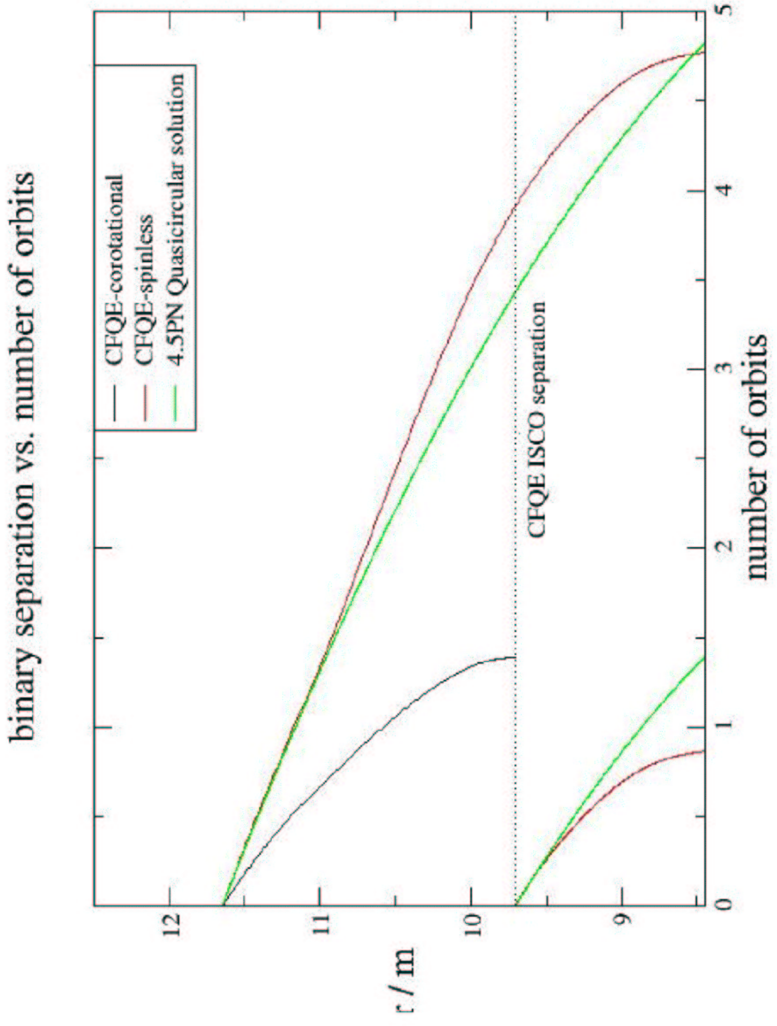
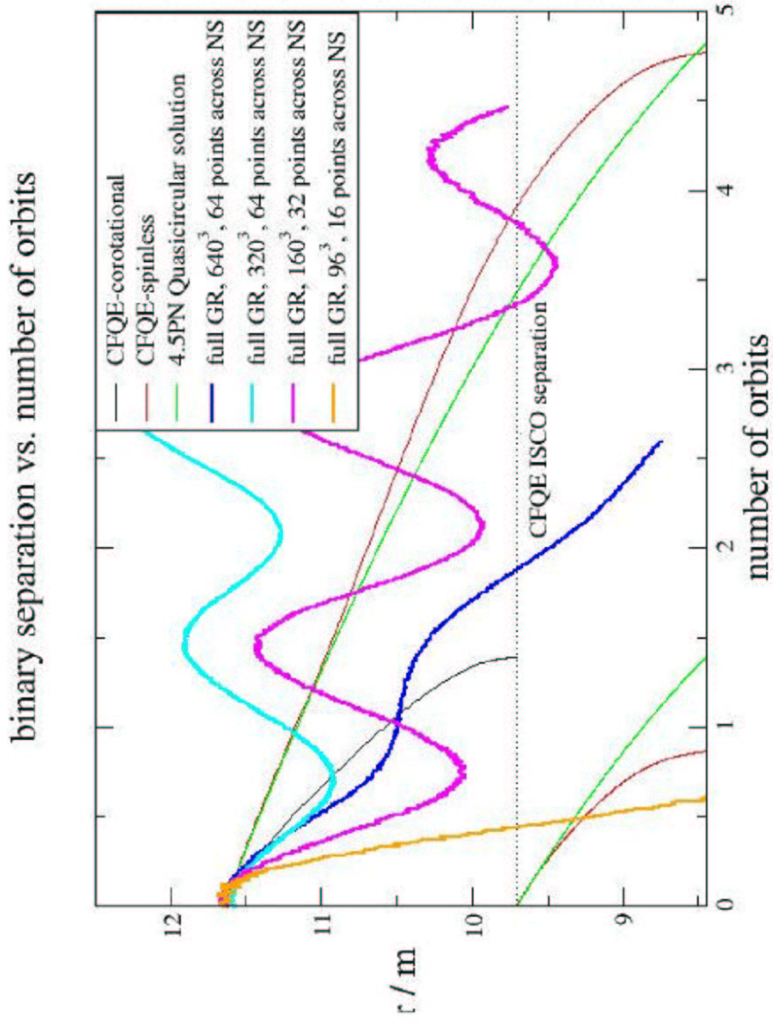
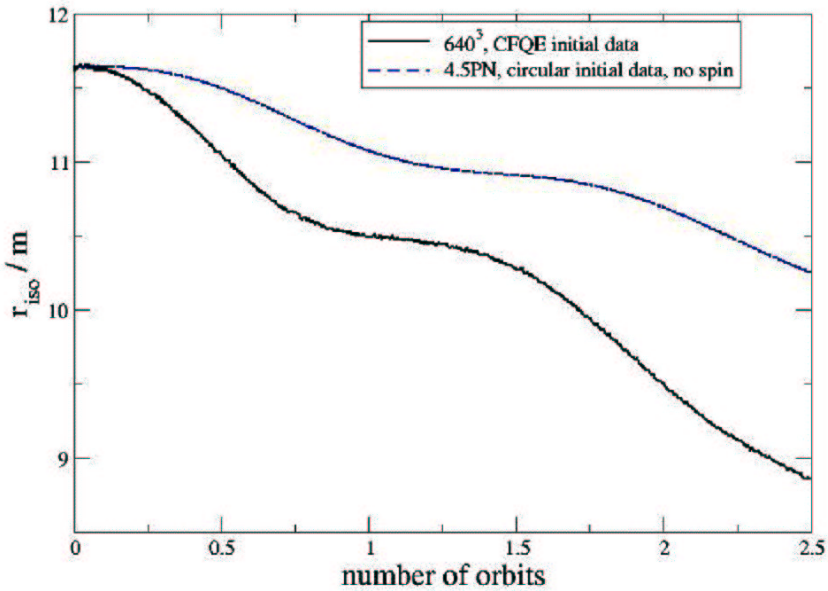

binary separation vs. number of orbits
CFQE-corotational
CFQE ISCO separation
number of orbits
r / m


binary separation vs. number of orbits
CFQE-corotational
CFQE-spinless
CFQE ISCO separation
number of orbits
r / m


binary separation vs. number of orbits
CFQE-corotational
CFQE-spinless
4.5PN Quasicircular solution
CFQE ISCO separation
r / m
number of orbits
12
11
10
9
0
1
2
3
4
5


binary separation vs. number of orbits
CFQE-corotational
CFQE-spinless
4.5PN Quasicircular solution
full GR, 640³, 64 points across NS
full GR, 320³, 64 points across NS
full GR, 160³, 32 points across NS
full GR, 96³, 16 points across NS
CFQE ISCO separation
r / m
number of orbits
12
11
10
9
0
1
2
3
4
5



Conclusions:

- stable code for NS/NS evolutions
- evaluation of CFQE initial data, also testing post–Newtonian initial data (noncircular initial data)
- FARM (Fixed, yet Adaptively Refined Meshes) needed to accurately simulate all relevant lengthscales of the problem
- symmetric–hyperbolic formulation of BSSN equations: better boundary conditions.