Gravitational Waves from Oscillation Modes in neutron Stars

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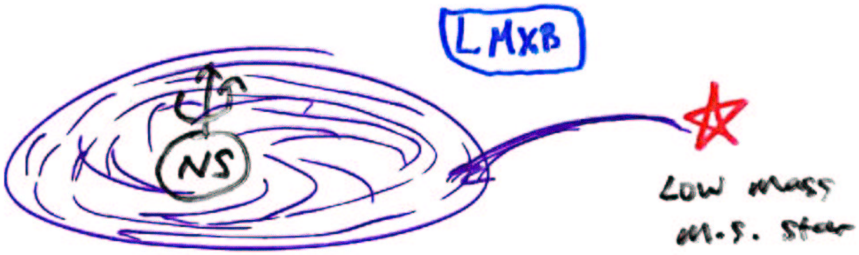
Outline

1) Observations of neutron star spins.

2) Role of grav. waves
↑possible

3) R-mode instability.
Basics ⊕ new developments.

Spin Frequency from X-ray Bursts & Accreting MSP



NS can rotate at $\nu_{spin} \gtrsim 1\,KHZ$.

How much mass does it need to accrete?

$$\frac{1}{3} M R^2 \cdot 2\pi \nu_{spin} \approx \Delta M \sqrt{\frac{GM}{R}}\, R$$

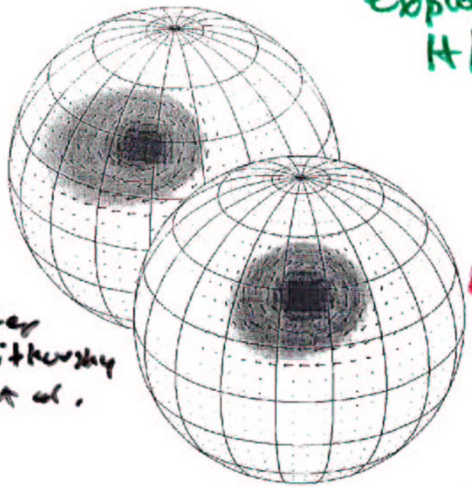
$$\Rightarrow \Delta M \simeq 0.1 \frac{\nu_{spin}}{1\,KHZ} M_\odot$$

How long does this take?

$$t_{spin\,up} = \frac{\Delta M}{\dot M} \simeq 10^8 yr \cdot \frac{10^{-9} \frac{M_\odot}{yr}}{\dot M} \frac{\nu_{spin}}{1\,kHz}$$

$\ll$ Age

Type 1 x-ray burst: thermonuclear explosion of accreted H/He fuel

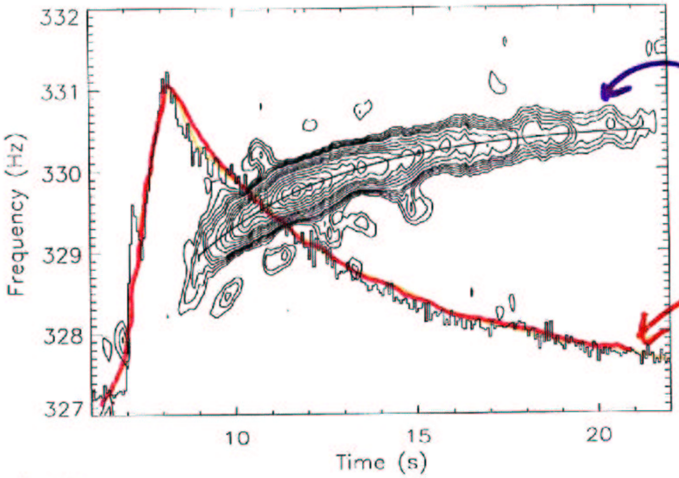


after Spitkovsky et al.

$L_x \sim 10^{38}$ erg/sec

$\Delta t \sim 10-100$ sec

"hot spots" modulated by rotation $\Longrightarrow$ spin frequency



ν_{spin} in tail

Lightcurve of Burst.

Strohmayer et al 1999

$$\boxed{300\,Hz \le \nu_{spin} \lesssim 600\,Hz}$$

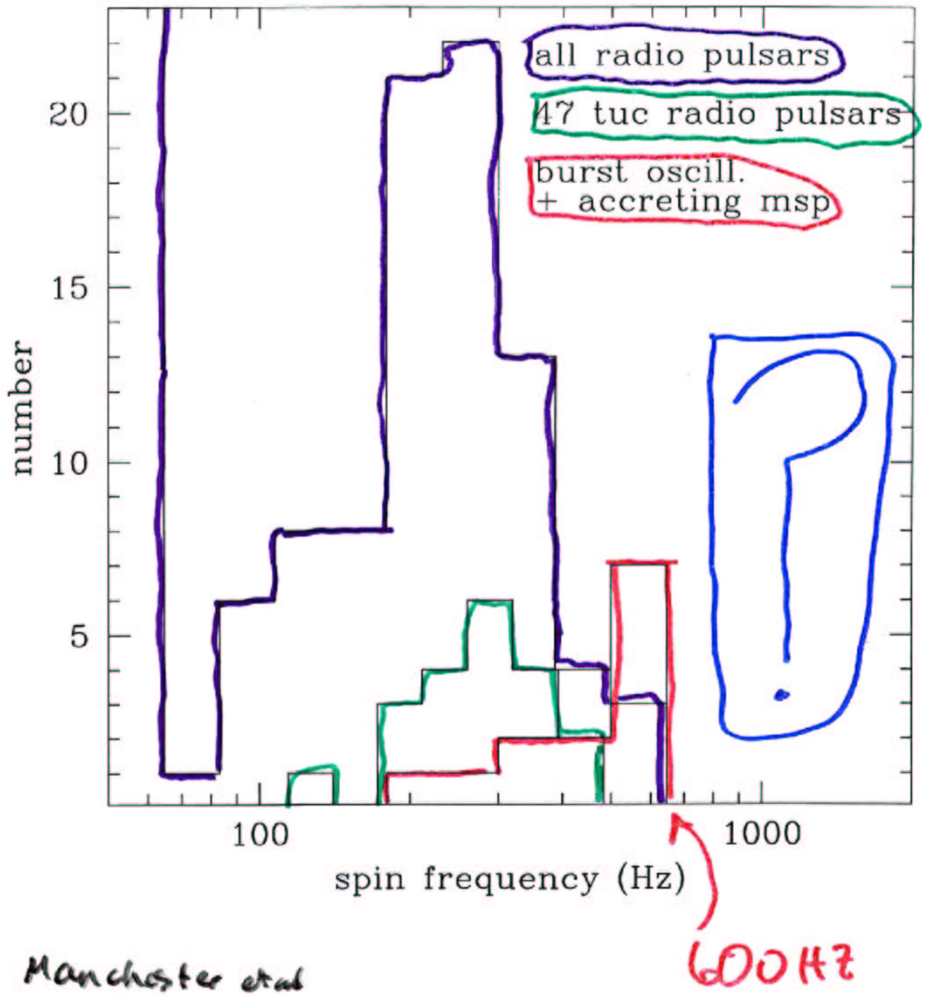
Most LMXB have $B \lesssim 10^{8-9}\,G$ since no persistent pulsation seen

except...

3 accreting millisecond pulsars.

$$\boxed{\nu_{spin} = 185, 401, 435 \text{ Hz}}$$

Sax J1808 has persistent pulse & burst oscillation. They agree.



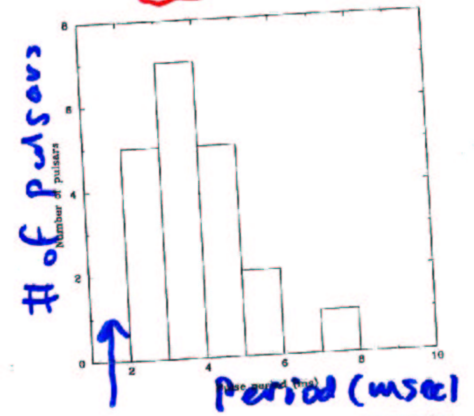
Manchester et al

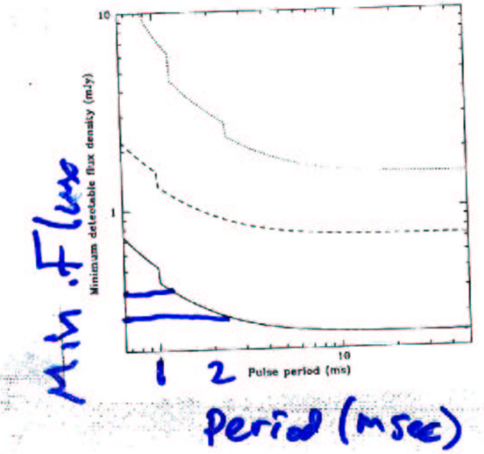
Camilo et al 2000

Strohmayer & Bildsten 2003

Can we detect them?

Camilo et al 2000





20 MSP in glob. 47 Tuc

only ~30% less sensitive from 2-1 msec.

Magnetic Accretion Model

White Zhang '97

Star spins up until it corotates with inner edge of disk at boundary of magnetosphere. Predicts

$$\nu^{\rm spin}_{\rm equil.} \propto B^{-6/7} \dot{M}^{3/7}.$$

Since $\nu_{\rm spin}$ only varies by factor of 2 over a factor of 10^3 in $\dot{M}$

$\Rightarrow$ B varies by a factor of $\underline{\underline{\sim 30}}$

<u>and</u>

You must hide the persistent pulsation! (Bildsten)

Gravitational Radiation Torques

Gravitational waves emitted by lumpy NS exert torque to balance accretion

$$N_{acc} \simeq \dot{M}\sqrt{GMR}$$

$$N_{GW} \simeq \frac{32}{5}\frac{GQ^2\omega_{spin}^5}{c^5}$$

$$\Longrightarrow$$

only $10^{-7} = \frac{Q}{MR^2}$ needed to balance accretion at typical ν_{spin} & $\dot{M}$.

Also, $h \simeq \frac{G\omega^2 Q}{d c^4}$, $F = \frac{L}{4\pi d^2} = \frac{GM\dot{M}/R}{4\pi d^2}$

$$\Longrightarrow$$

$$h \simeq 5\times10^{-27}\left(\frac{F}{10^{-8}\,\mathrm{ergs/cm^2/sec}}\right)^{\frac{1}{2}}\left(\frac{300\,\mathrm{Hz}}{\nu_{spin}}\right)^{\frac{1}{2}}$$

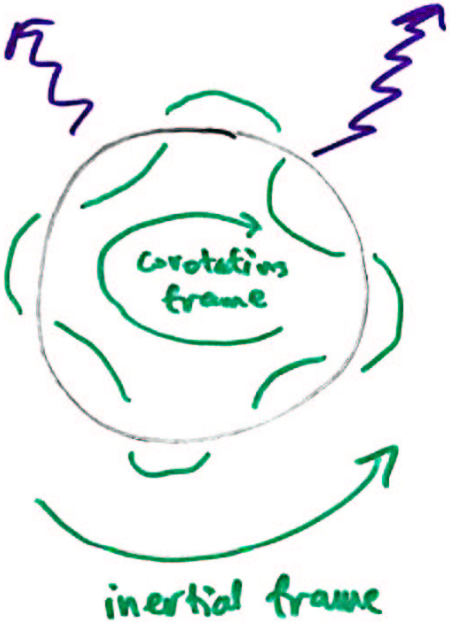
Estimate for h from observed Flux!

Bildsten '98, Wagoner '84

R mode Instability

All rotating, inviscid stars are unstable to a GR instability which causes the star to spin down.

(Chandrasekhar, Friedman & Schutz, Andersson, Friedman & Morsink)



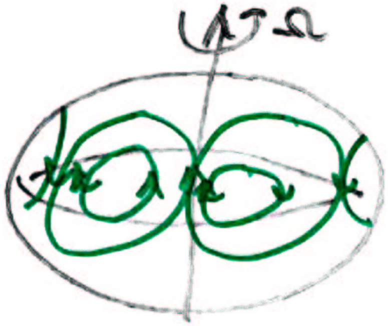
$$\Delta J_{GW} > 0$$

$$\Rightarrow$$

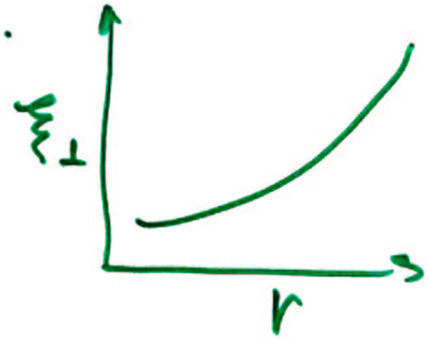
$$\Delta J_{rmode} < 0$$

$\Rightarrow$ amplitude increase

· What do r-modes look like?



- mainly horizontal motion
- little compression
- rotational pattern moving past fluid elements



amplitude rises toward surface

(for simple star with no buoyancy)

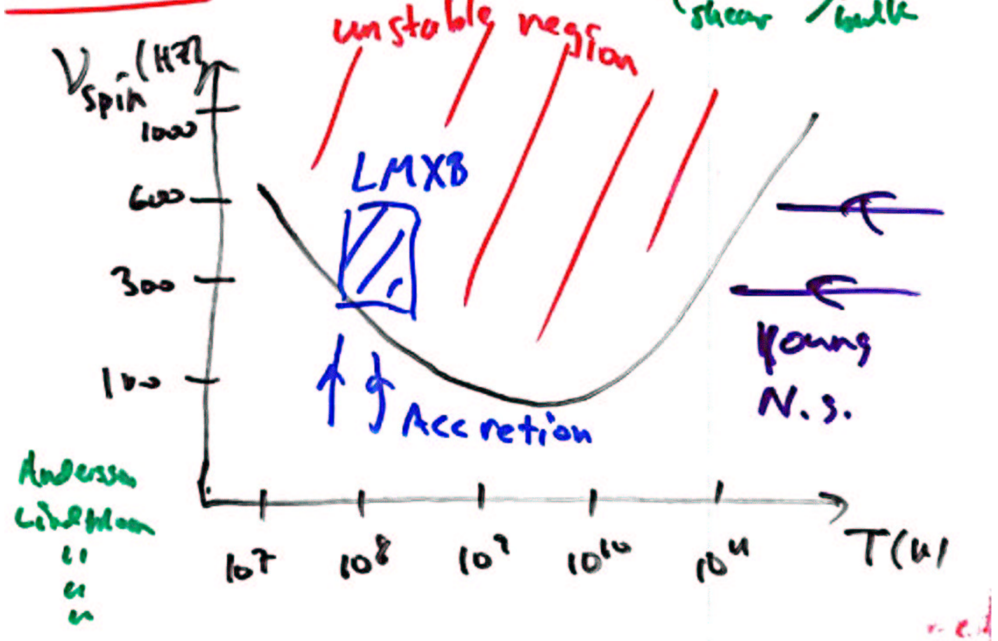
Papaloizou & Pringle

When does the instability operate?

Driving due to GR must be stronger than viscous damping

$$\dot E_{mode} = + \underbrace{\dot E_{GR}}_{\propto \nu_{spin}^6 \times E_{mode}} - \underbrace{\dot E_{visc}}_{\left[\underset{\uparrow \text{shear}}{\frac{(\cdot)}{T^2}} + \underset{\nearrow \text{bulk}}{(\cdot)\nu_{spin}^2 T^6}\right]}$$

"Standard scenario"



G.W. Emission by R-modes

mass current quadrupole ⇒

$$h \sim \frac{GM}{c^2 d} \cdot \left(\frac{\Omega R}{c}\right)^3 \cdot \underbrace{\frac{\xi_\perp}{R}}_{\text{amplitude}}$$

Owen et al.

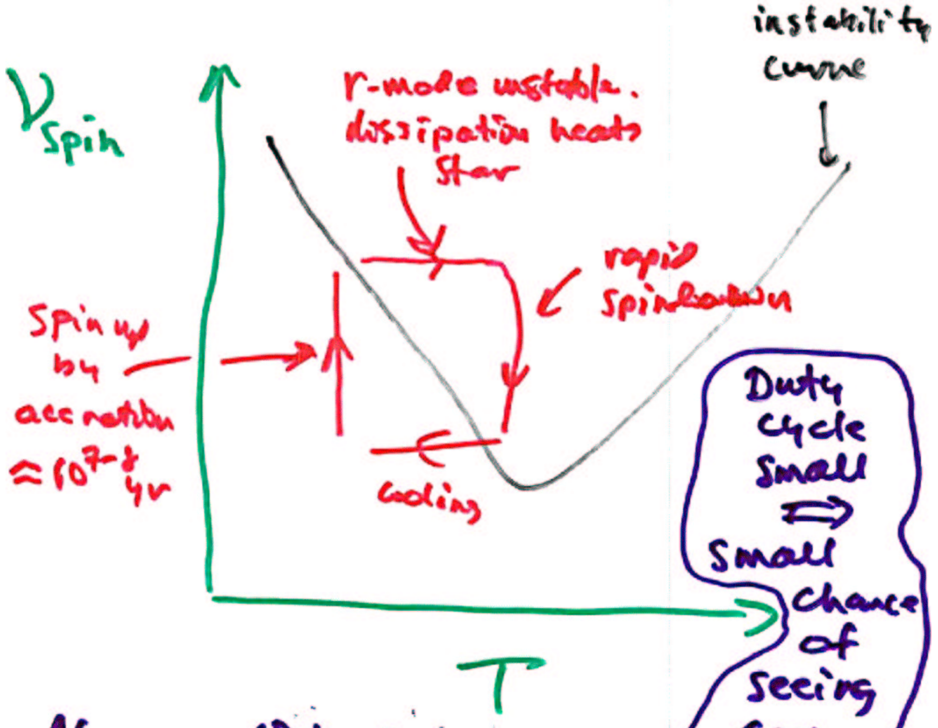
$$\simeq 2\times10^{-24} \cdot \frac{10\,\text{Mpc}}{d} \cdot \left(\frac{\nu_{spin}}{1\,\text{kHz}}\right)^3 \frac{\xi_\perp}{R}.$$

$\boxed{\boxed{\text{If}}}$ accretion torque can be stably balanced by r-mode GW torque then you recover simple estimate involving Fx of accreting NS. Amplitude $\xi_\perp$ would adjust to make this happen.

$\boxed{\boxed{\text{If Not}}}$ amplitude set by other process (hydrodynamics?)

Levin's Limit Cycle

If viscosity decreases as temp. increases you get a runaway. Steady balance not possible. Instead you get a limit cycle.



Duty cycle small ⇒ small chance of seeing GW.

$N_{sources} \cdot \left(\begin{smallmatrix}\text{Duty}\\\text{cycle}\end{smallmatrix}\right) > 1$? Heyl

Numerical Evolution of r-modes

Lindblom, Tohline, Vallisneri
2001

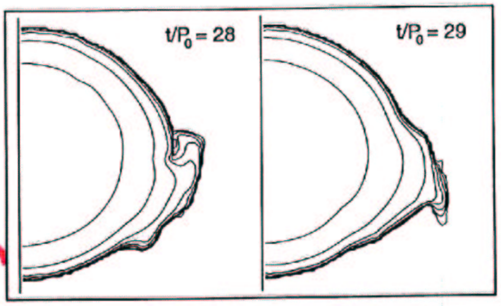


FIG. 7. Density contours (at $10^{-n/2}\rho_{\max}$ with $n = 1, 2, \ldots$) in selected meridional planes at times $t = 28P_0$ and $29P_0$ illustrate the breaking of surface waves. Shocks at the leading edges of these waves appear to be the primary mechanism that suppresses the *r*-mode.

Saturation amplitude large. Shocks near the surface.

$$\underline{\underline{\xi_\perp \sim R}}$$

Saturation by Energy transfer to other waves

$$\dot{E}_{GR} \sim \gamma_{GR} \times \text{Amplitude}$$

$$\dot{E}_{\text{nonlinear}} \sim \Omega \times (\text{Amplitude})^2$$

$$\Rightarrow$$

$$E_{r\text{-mode}} \sim E_{\text{rotation of star}} \cdot \underbrace{\frac{\gamma_{GR}}{\Omega}}_{\simeq \left(\frac{\Omega R}{c}\right)^5 \ll 1}$$

$\Rightarrow$ saturation amplitude small

- Young NS only detectable out to $d \sim 200$ kpc
- Spin down takes 10^{3-4} years
- LMXB detection depends on duty cycle....

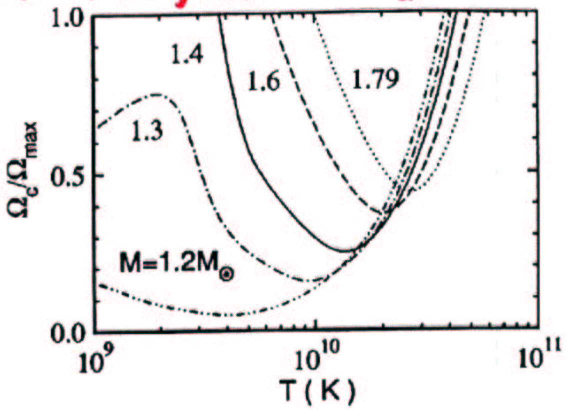
Hyperon Bulk Viscosity

Lindblom
Owen
2002
Jones
2001

$n+n \rightleftharpoons p+\Sigma^-$

$n+p \rightleftharpoons p+\Lambda$

orders of magnitude bigger!



May completely suppress the rmode instability,

but....

Wagoner (2002) notes that high T_c for proton pairing may suppress hyperon viscosity and allow a steady state.

Summary

- R-modes may be responsible for maximum rotation rate of NS.
- Small saturation amplitude $\Rightarrow$ only nearby supernovae produce detectable r-mode GW signal.
- <u>If</u> viscosities allow steady state in LMXB (high T_c superfluids, suppressed hyperon viscosity), they remain promising sources for LIGO.