

# Eccentricity: What, where, and why?

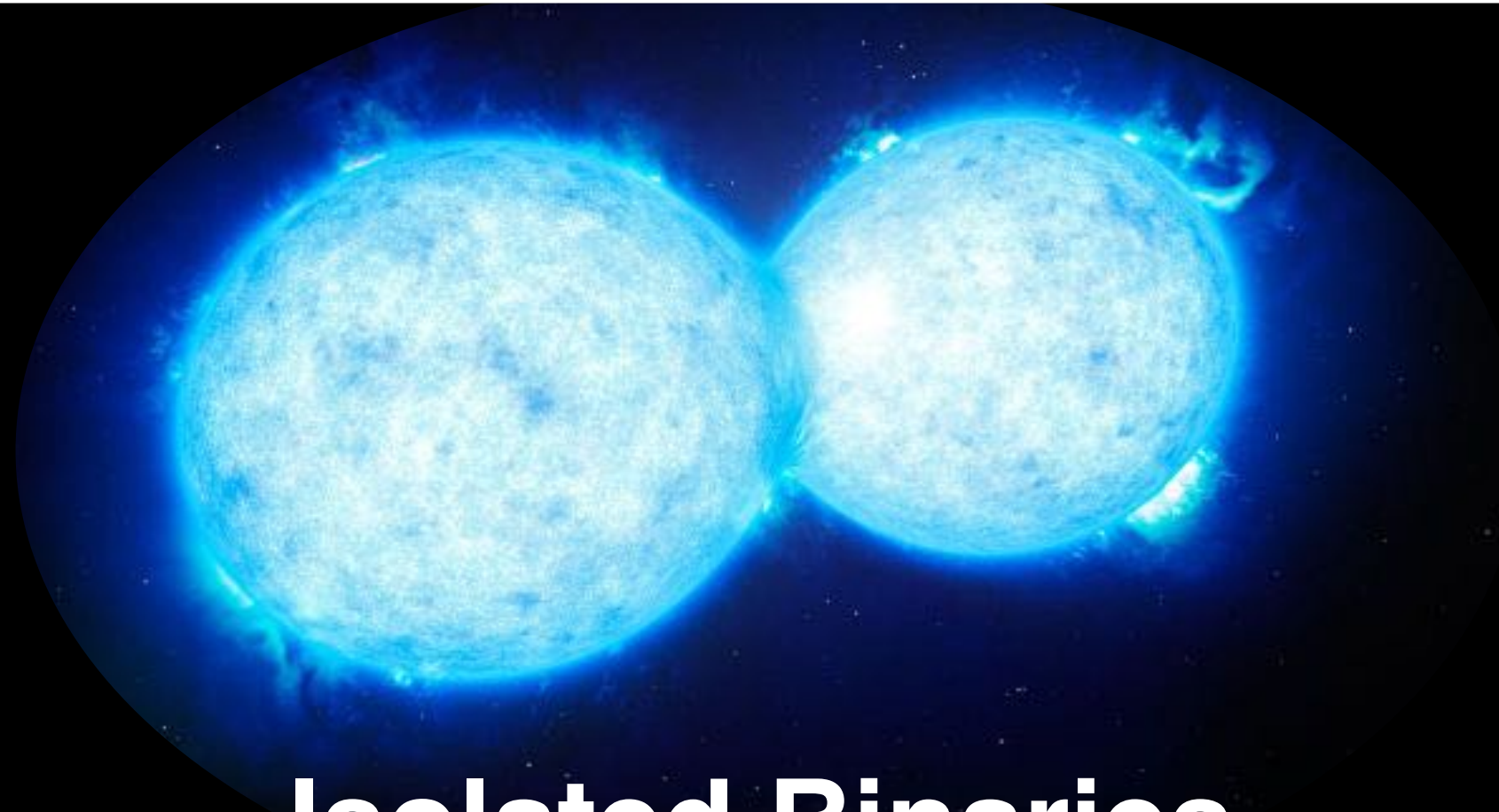
Carl Rodriguez  
MIT

Michael Zevin, Pau Amaro-Seoane, Sourav Chatterjee,  
Kyle Kremer, Fred Rasio, Claire Ye, Johan Samsing, Avi Loeb



# *Forming Black Hole Binaries*

Carl • Rodriguez



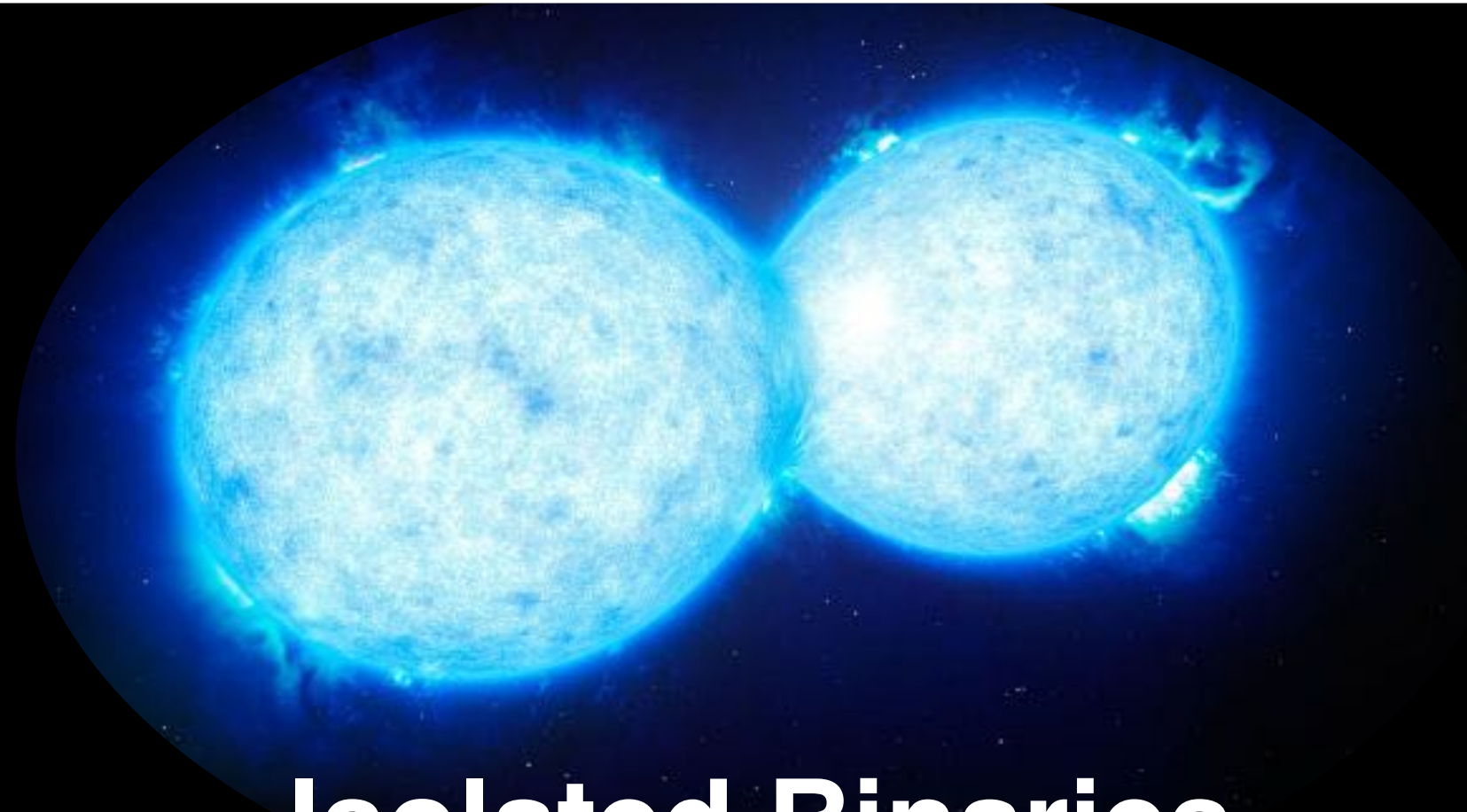
**Isolated Binaries**



**Dynamical Formation**



# Forming Black Hole Binaries



**Isolated Binaries**

- Chemically Homogeneous Evolution
- Triples (Lidov-Kozai)
  - Stellar triples (field or dynamical)
  - Stellar BBH/SMBH
- Primordial black holes
- Highly-eccentric captures in scattering encounters
- Formation in AGN disks
- And many more...

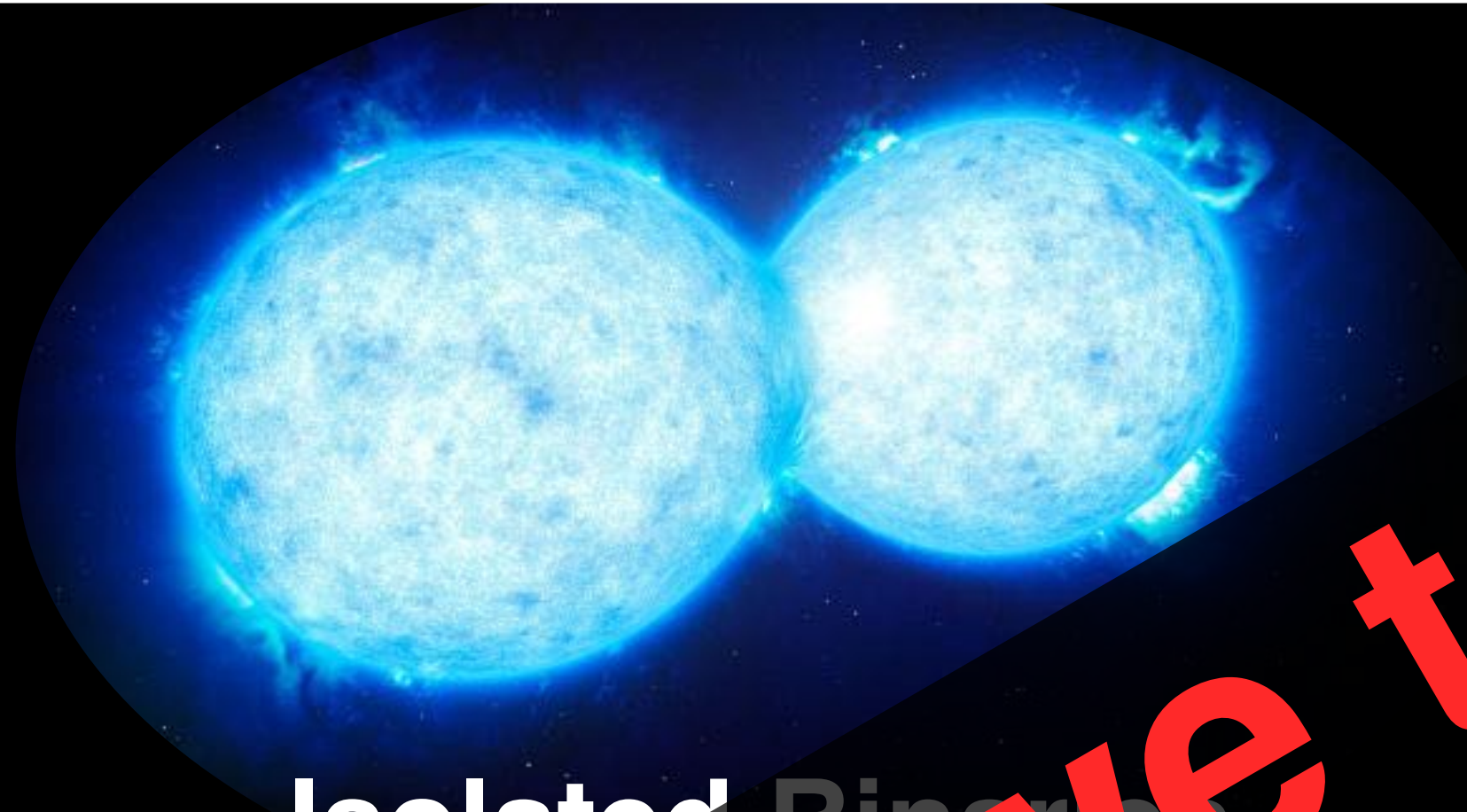


**Dynamical Formation**



# Forming Black Hole Binaries

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Isolated Binaries



Dynamical Formation

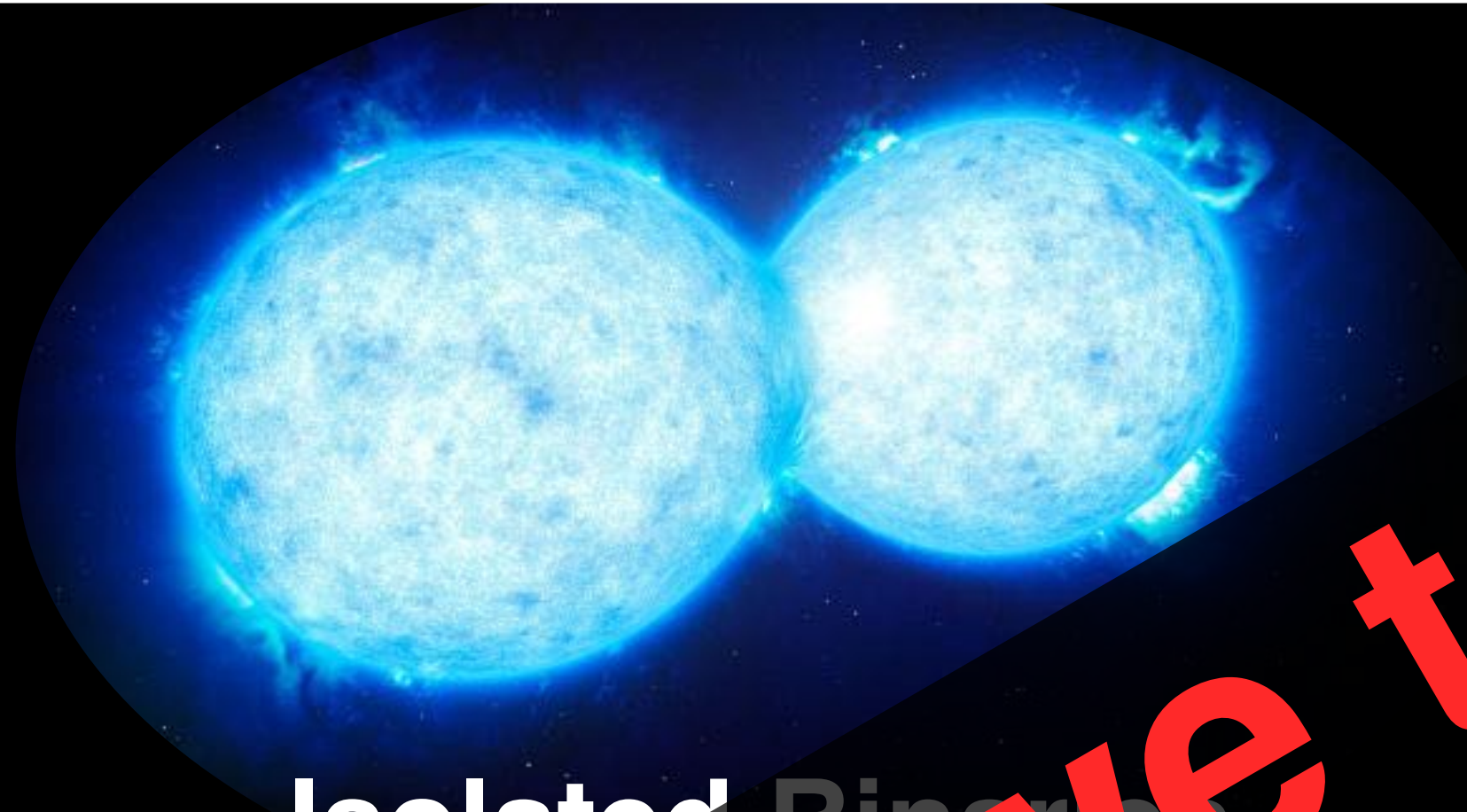
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Can we tell the difference?



# Forming Black Hole Binaries

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Isolated Binaries



Dynamical Formation

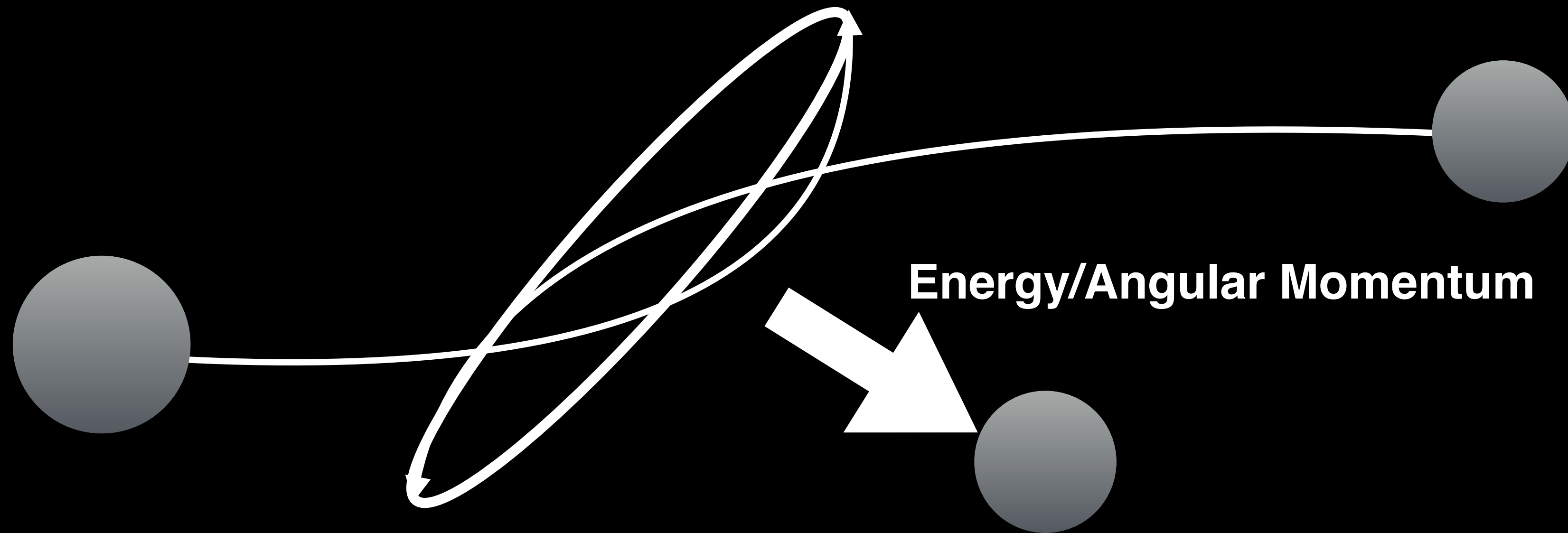
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Can we tell the difference?



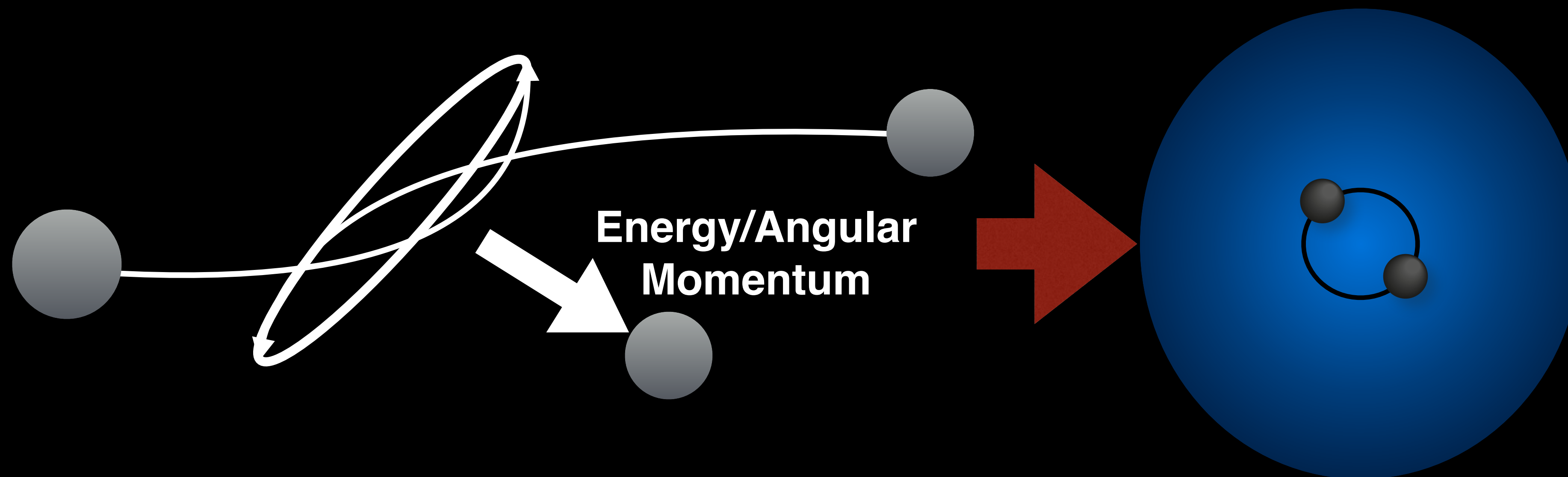
# Forming Black Hole Binaries

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# Chaotic Interactions



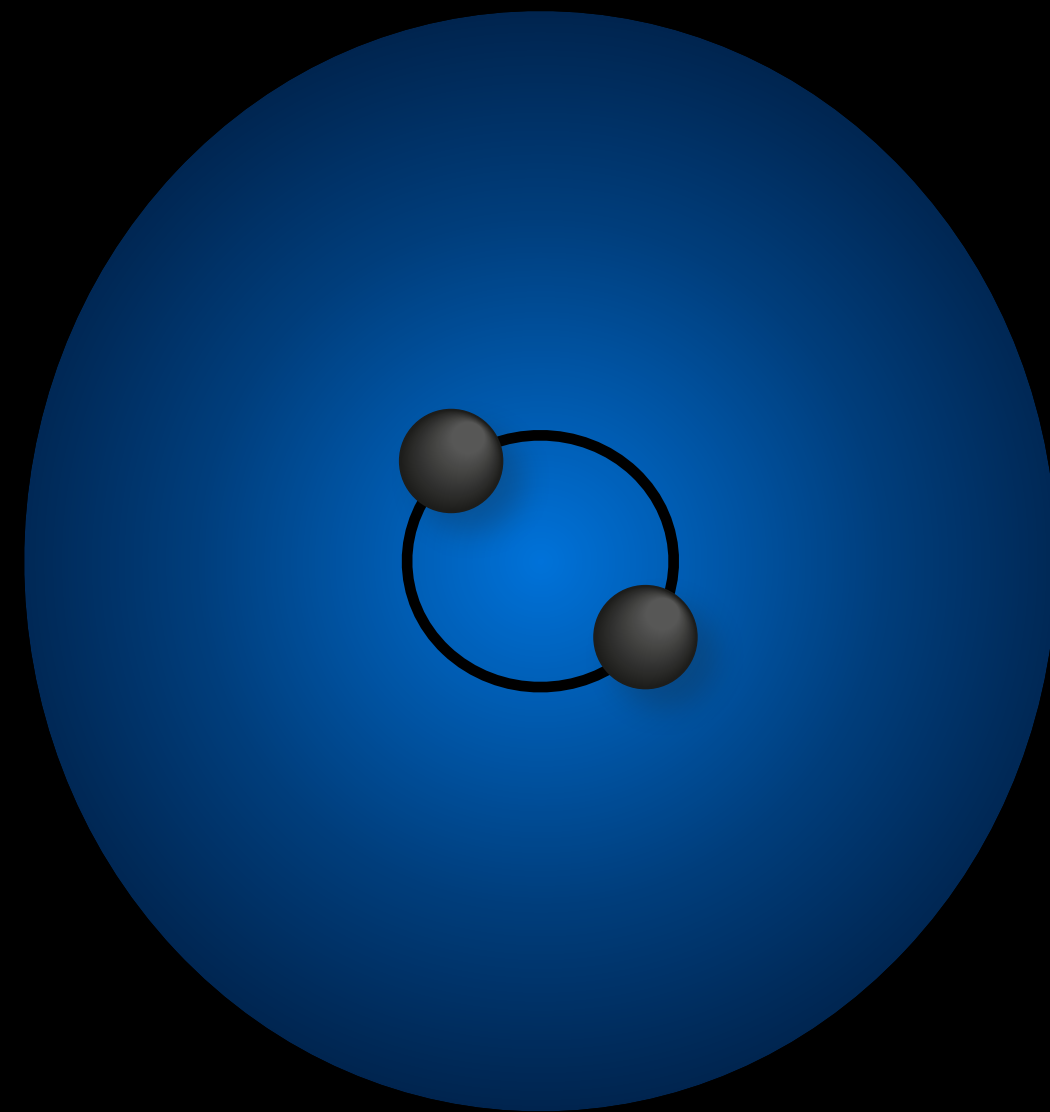


# Chaotic Interactions

Carl Rodriguez



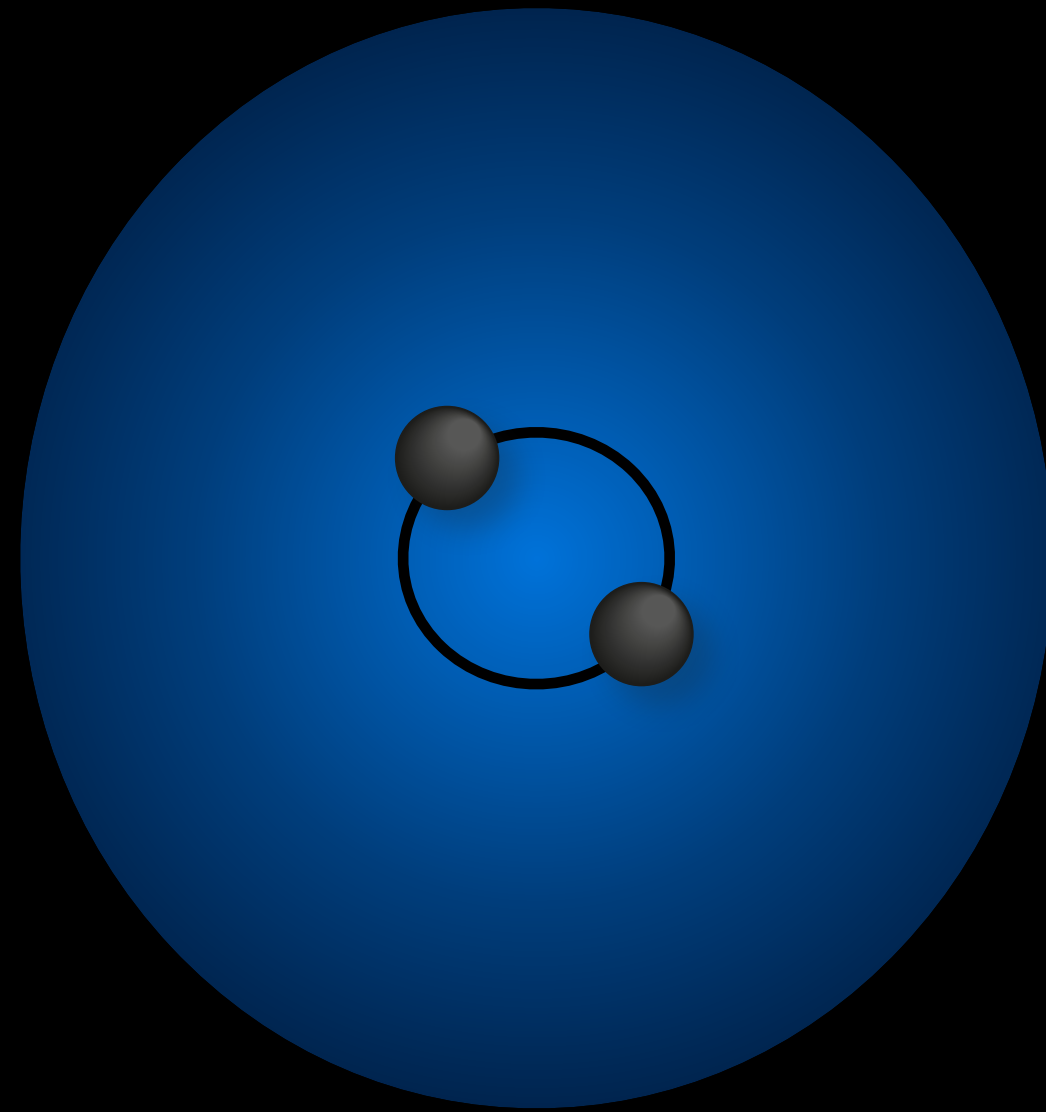
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# *Chaotic Interactions*

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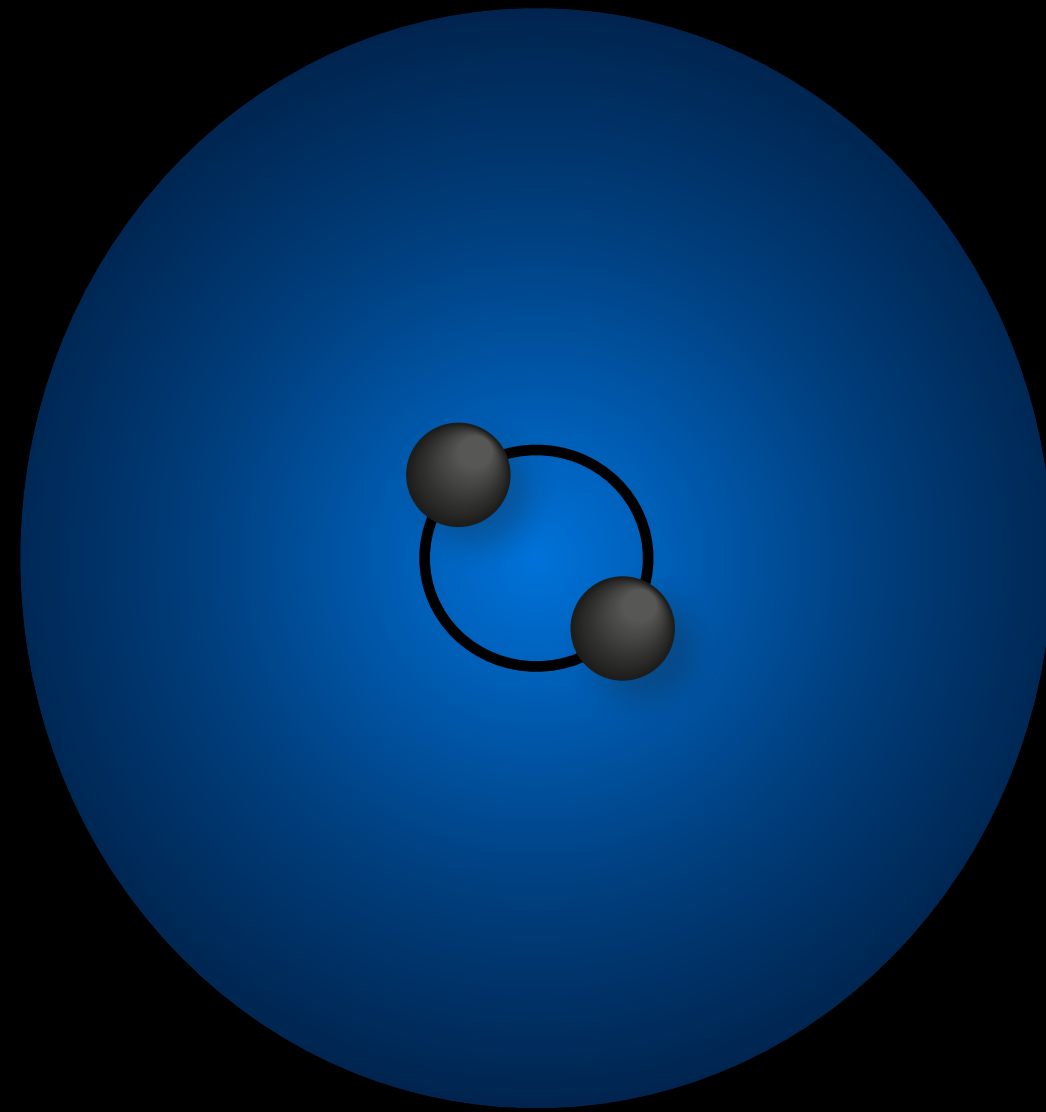


# *Chaotic Interactions*

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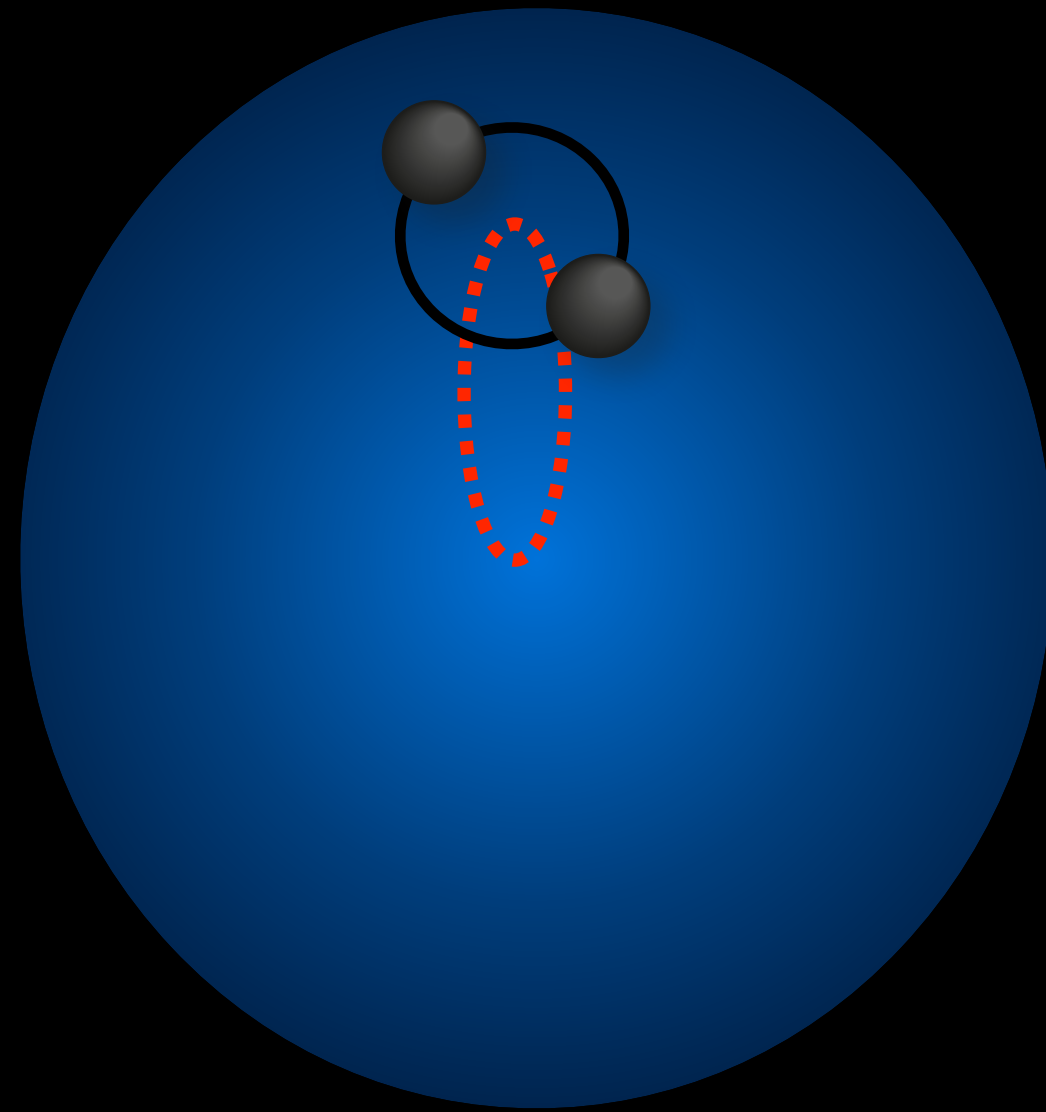
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# Chaotic Interactions

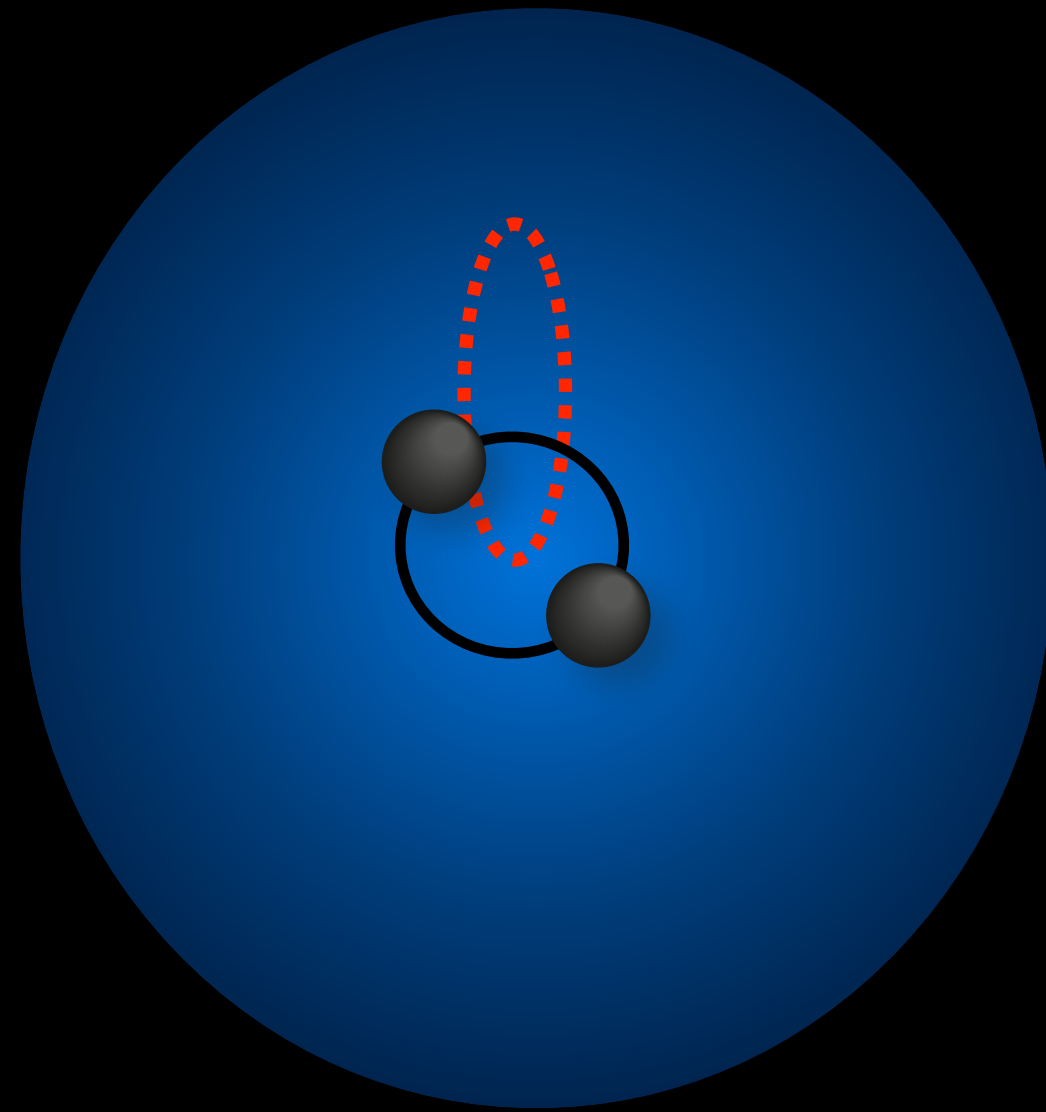
Carl Rodriguez





# Chaotic Interactions

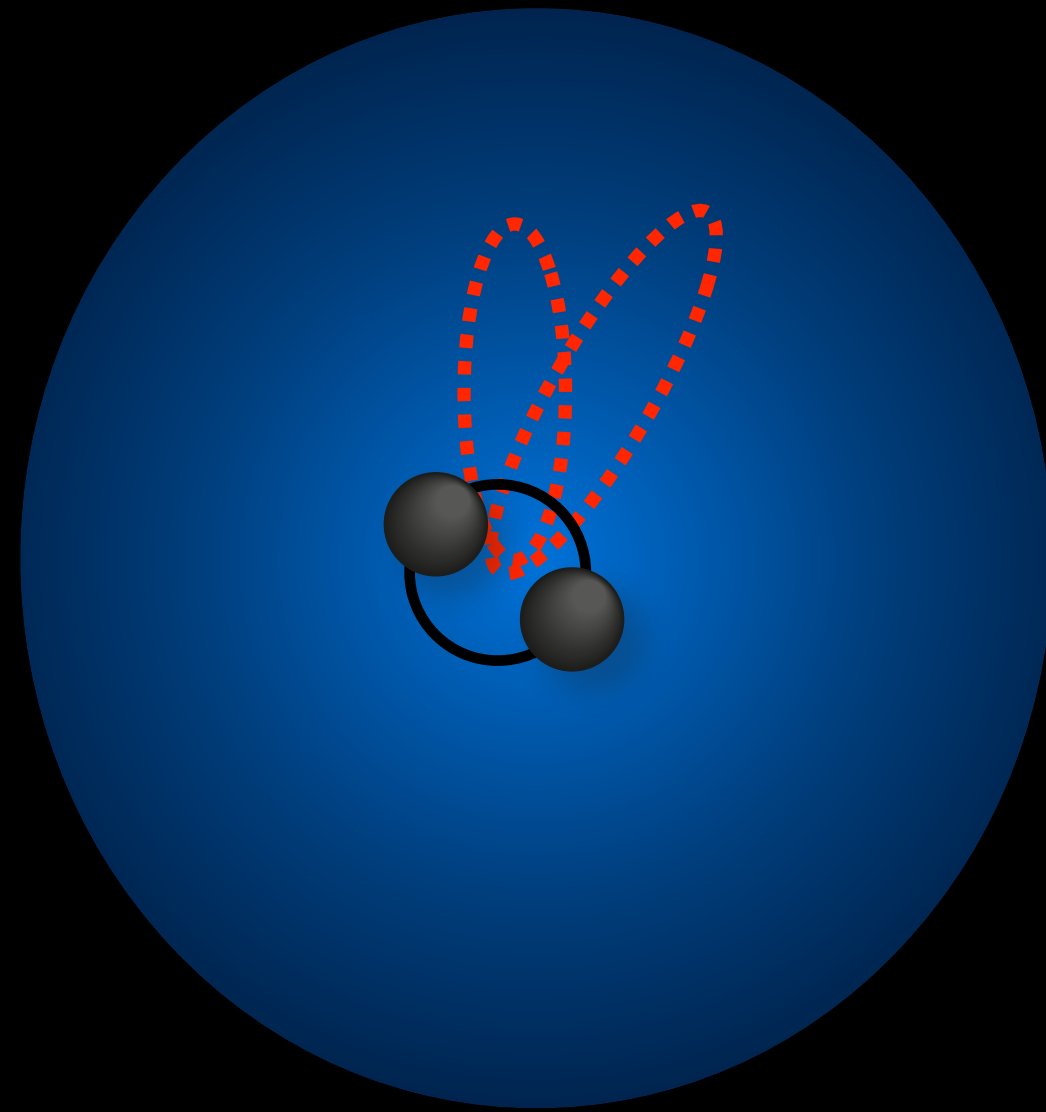
Carl Rodriguez





# Chaotic Interactions

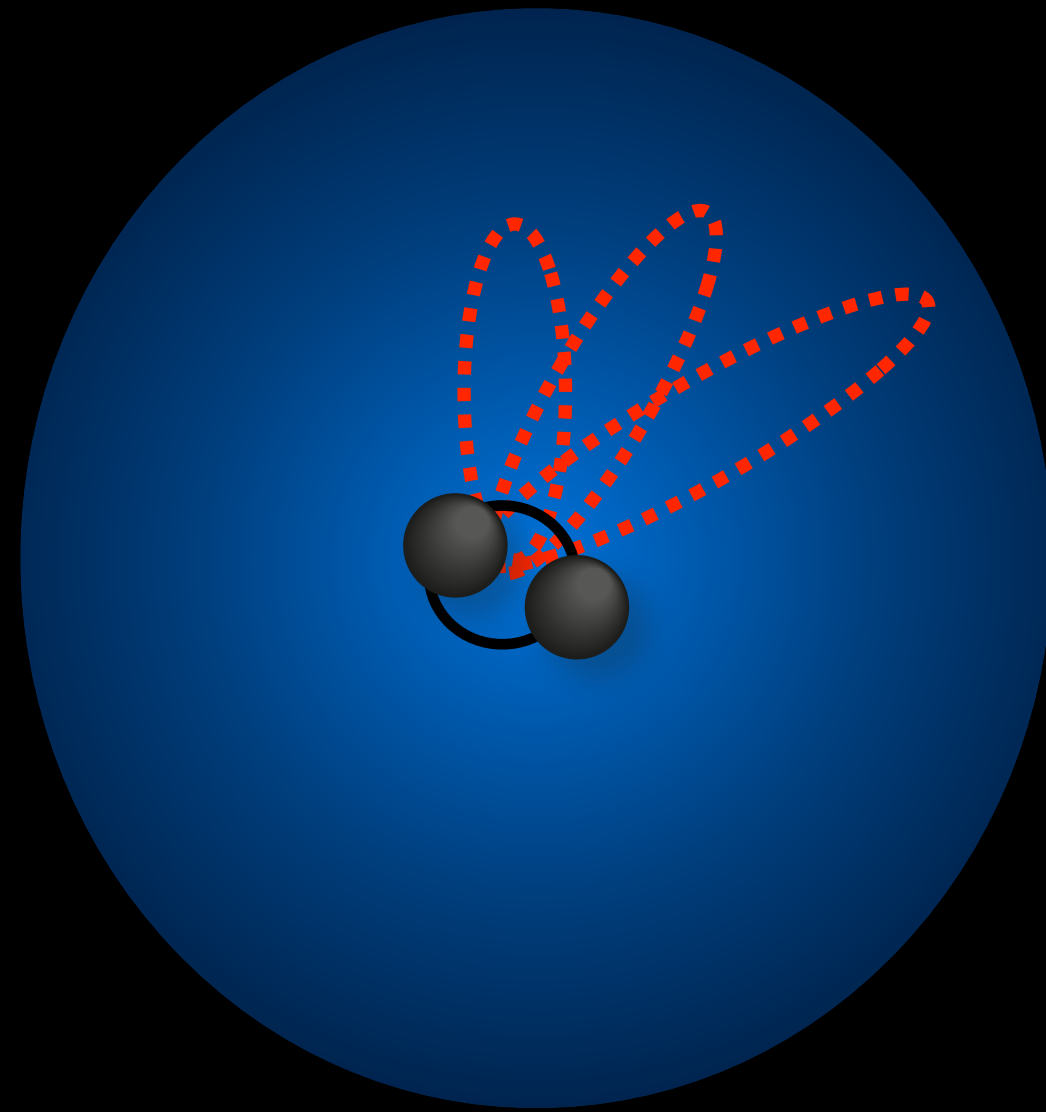
Carl • Rodriguez





# *Chaotic Interactions*

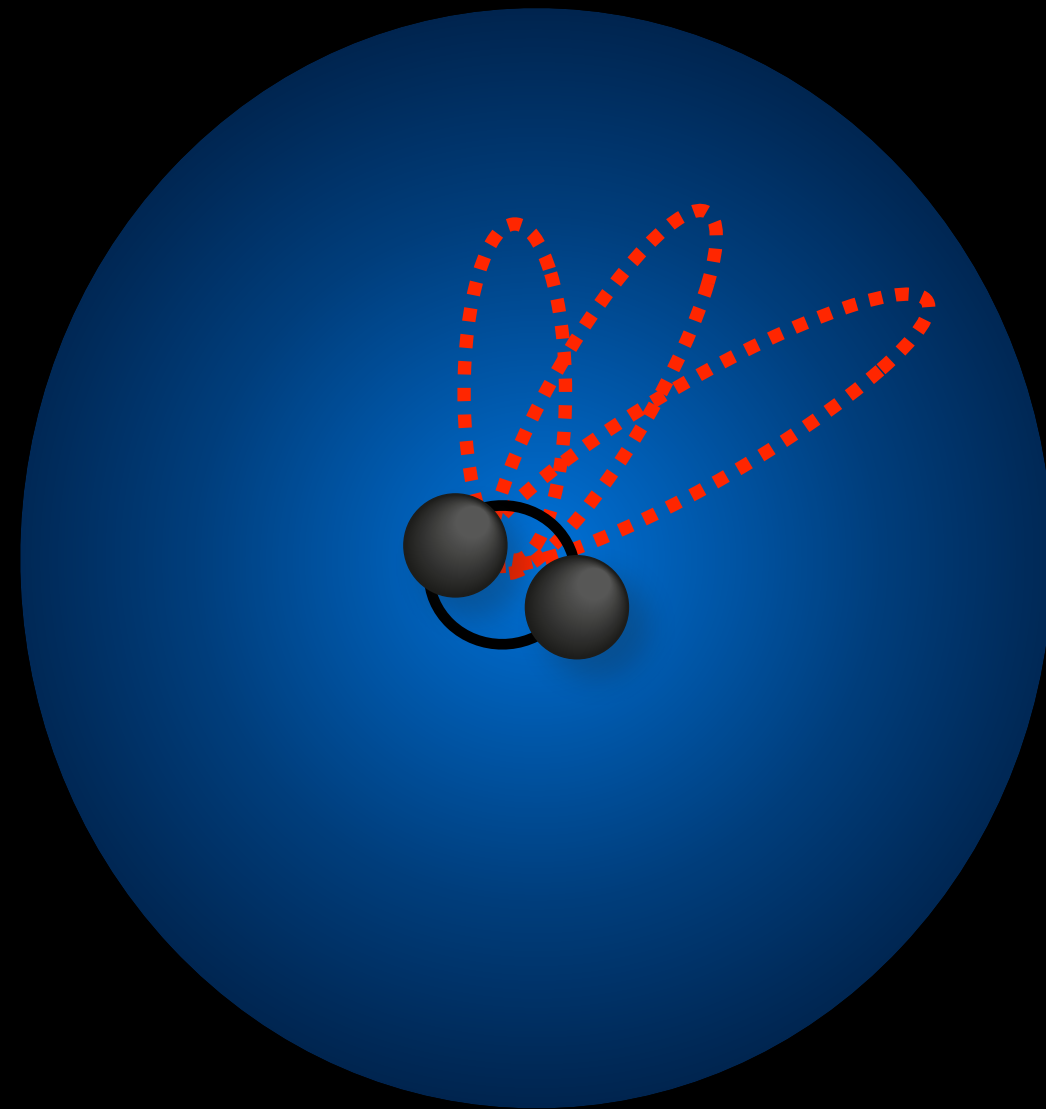
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# Merging Binaries

Carl Rodriguez

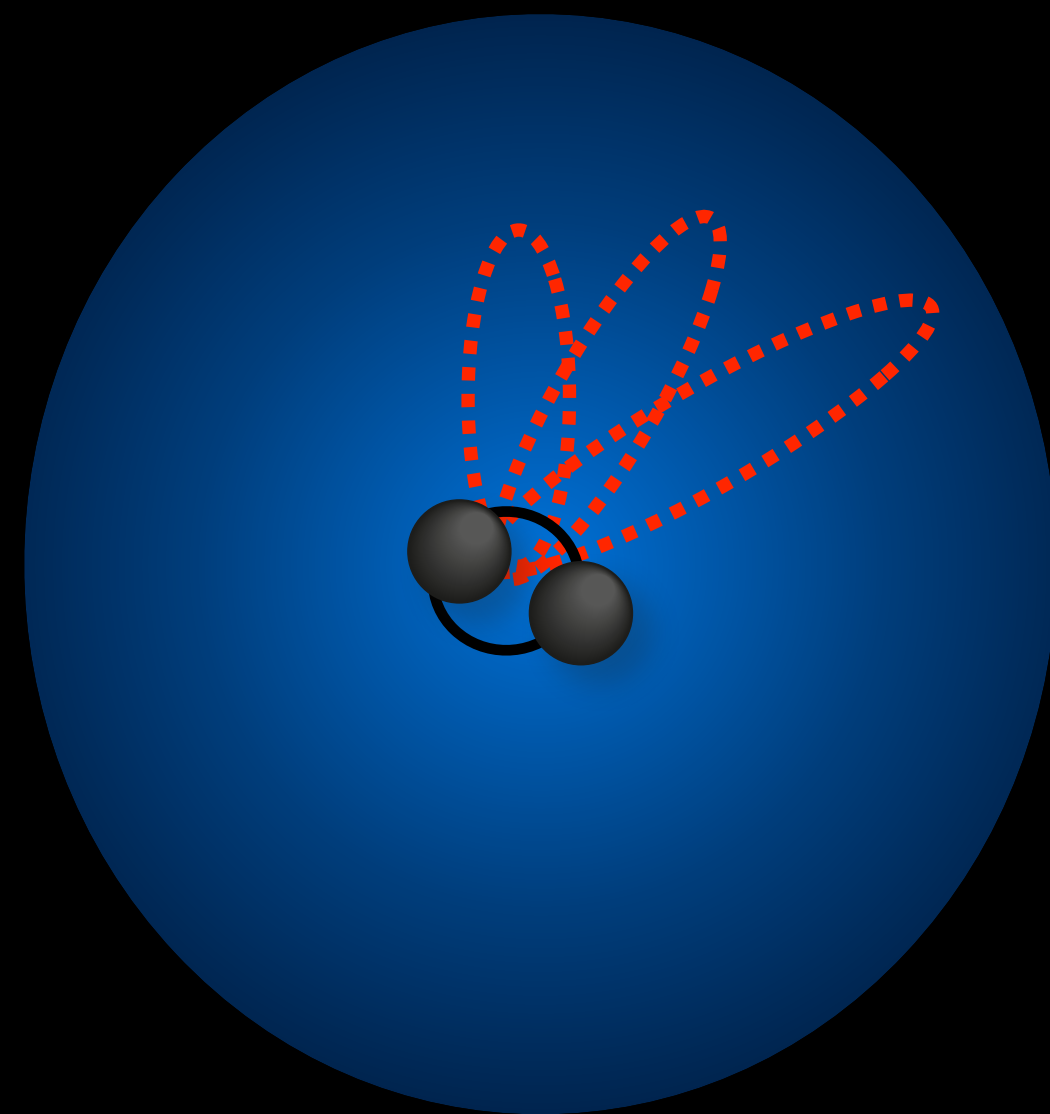


Gravitational-wave  
Merger Time

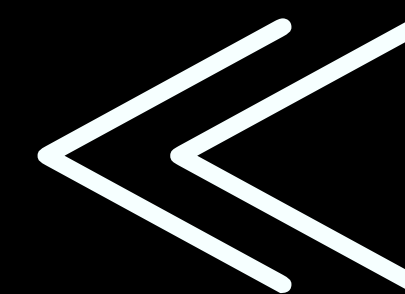
Time between  
Binary-single  
encounter



# Merging Binaries



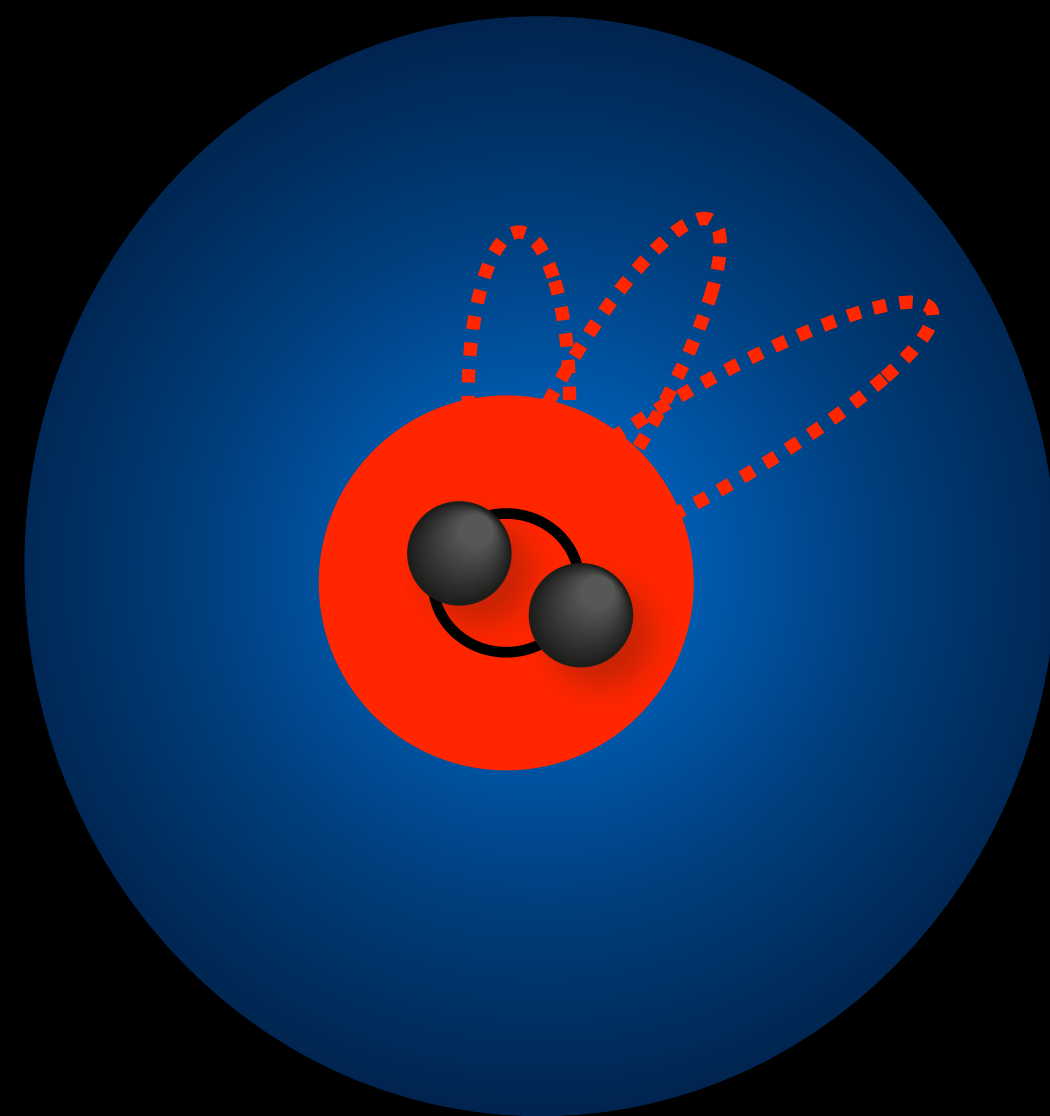
Gravitational-wave  
Merger Time



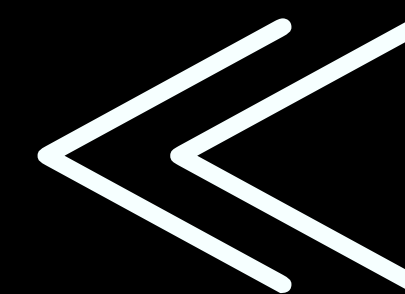
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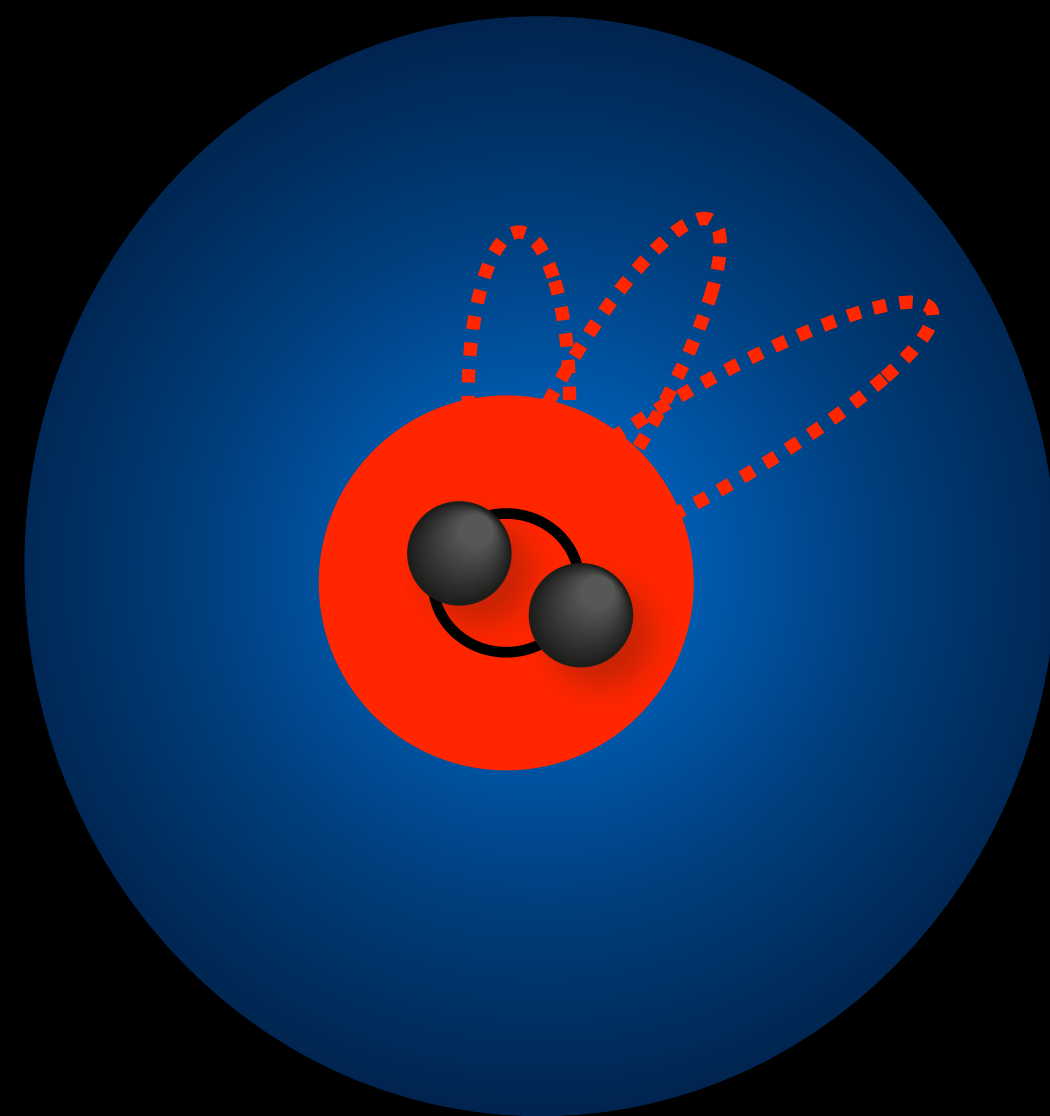
Gravitational-wave  
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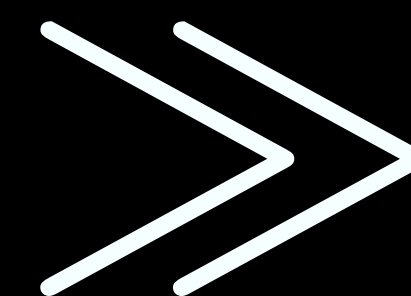
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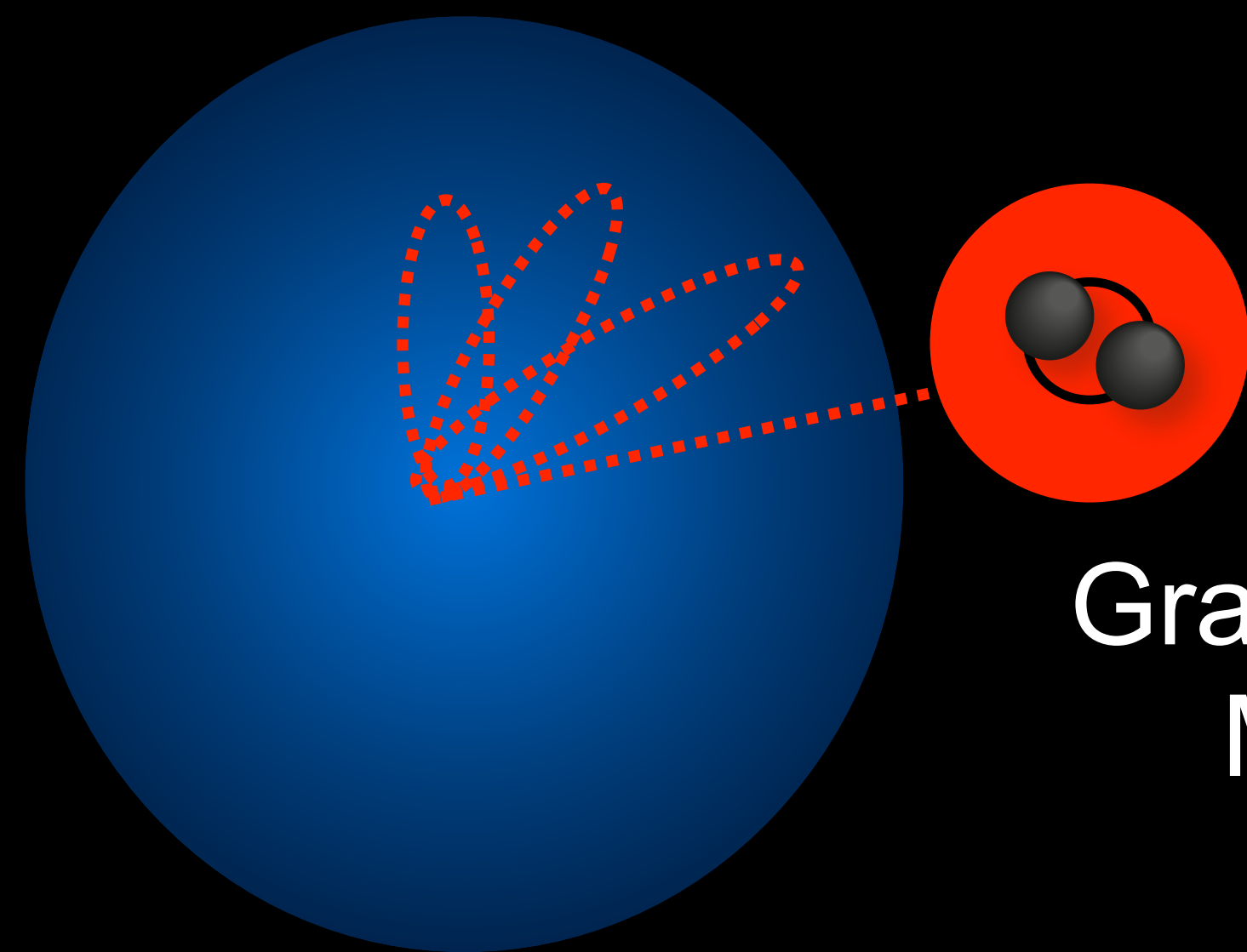
Gravitational-wave  
Merger Time



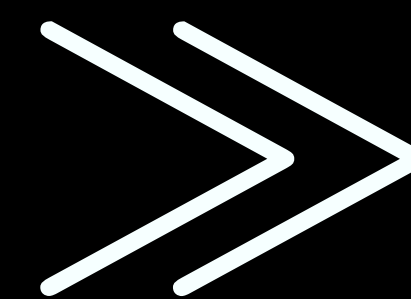
Time between  
Binary-single  
encounter



# Merging Binaries



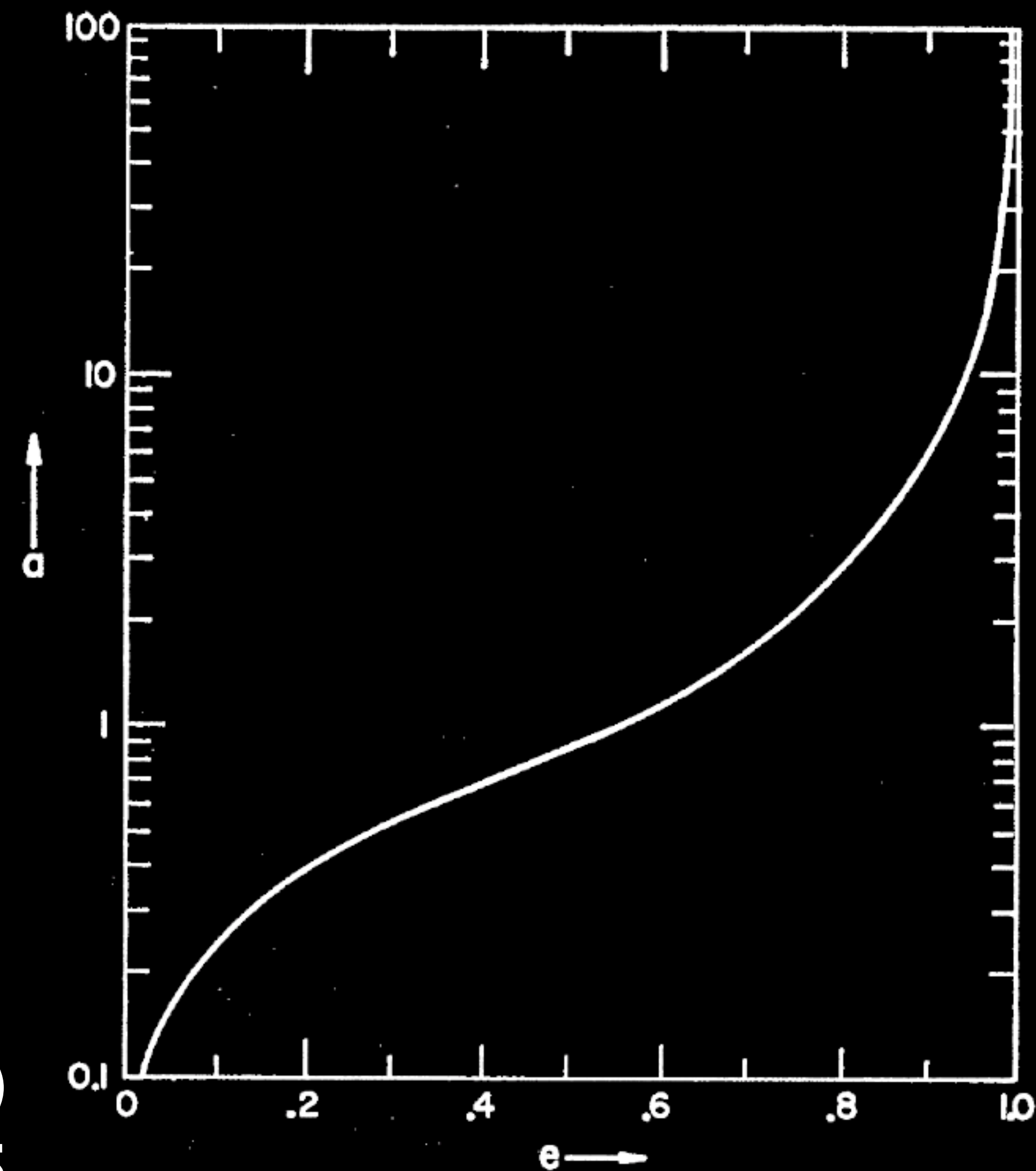
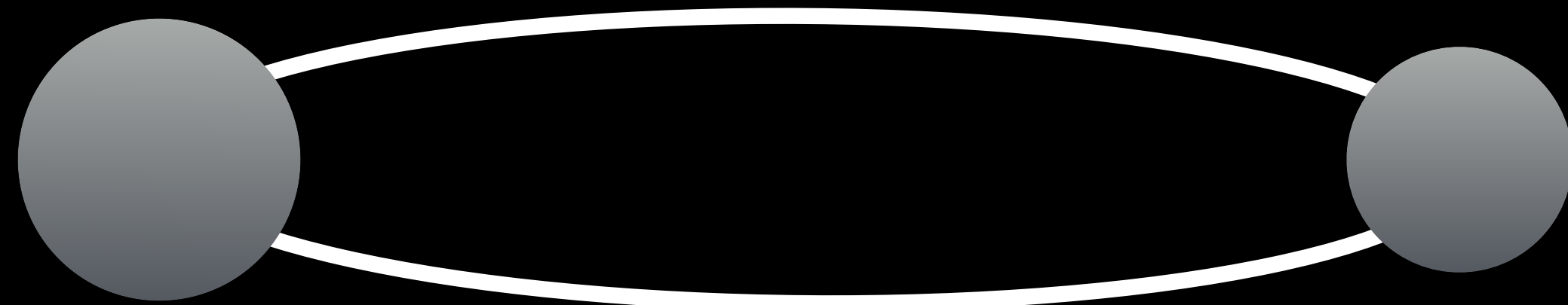
Gravitational-wave  
Merger Time



Time between  
Binary-single  
encounter



# Forming Black Hole Binaries

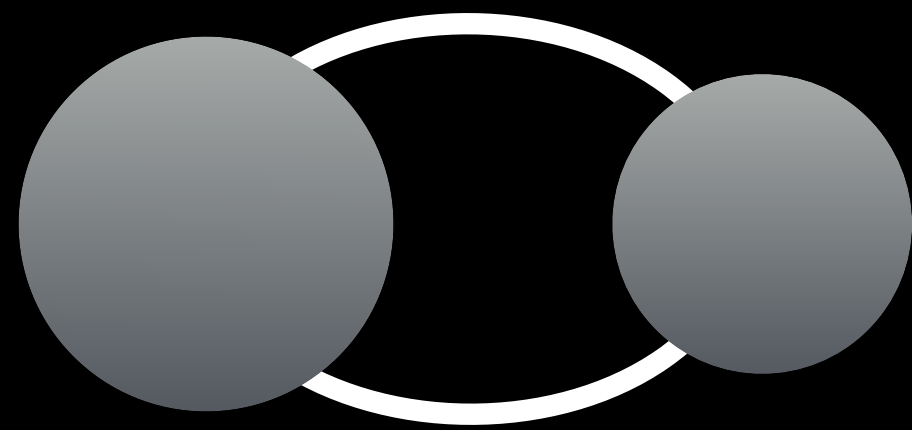


Peters & Matthews (1963)  
Phys. Rev. 131, 415

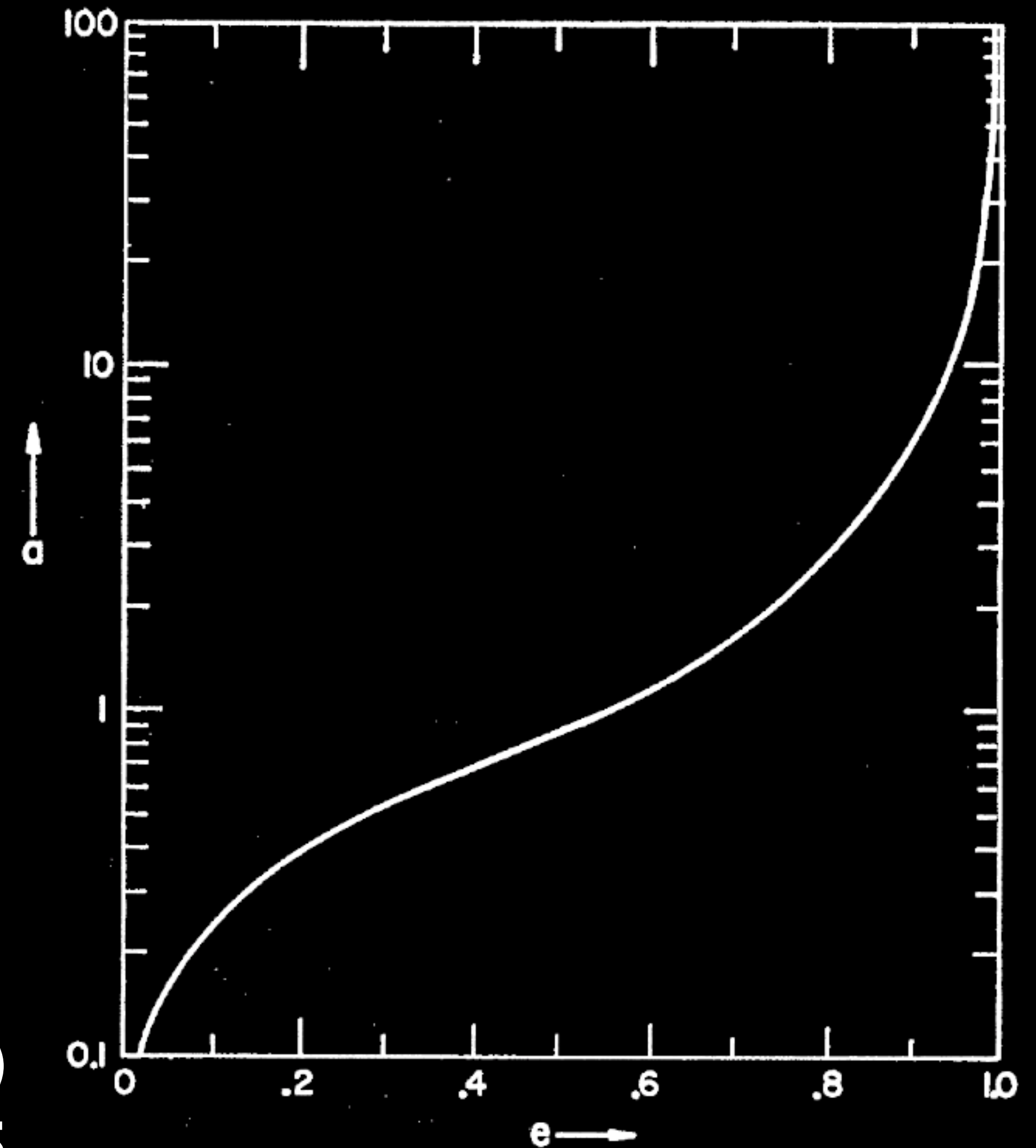


# Forming Black Hole Binaries

Carl Rodriguez

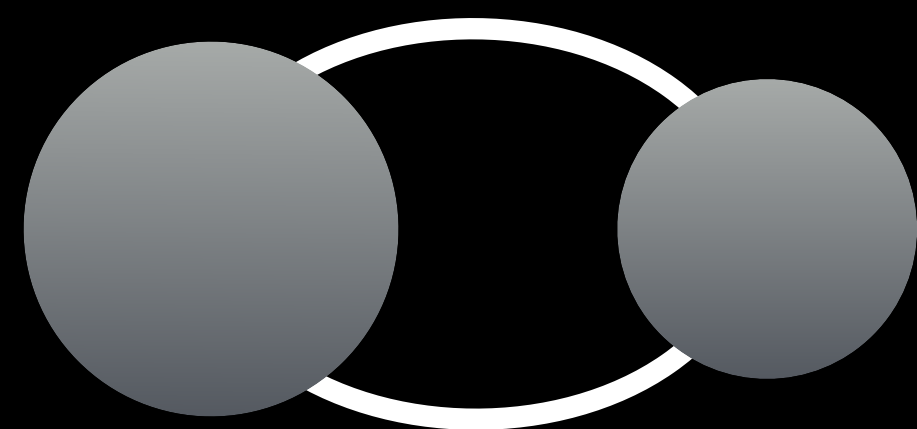


**Peters & Matthews (1963)**  
**Phys. Rev. 131, 415**

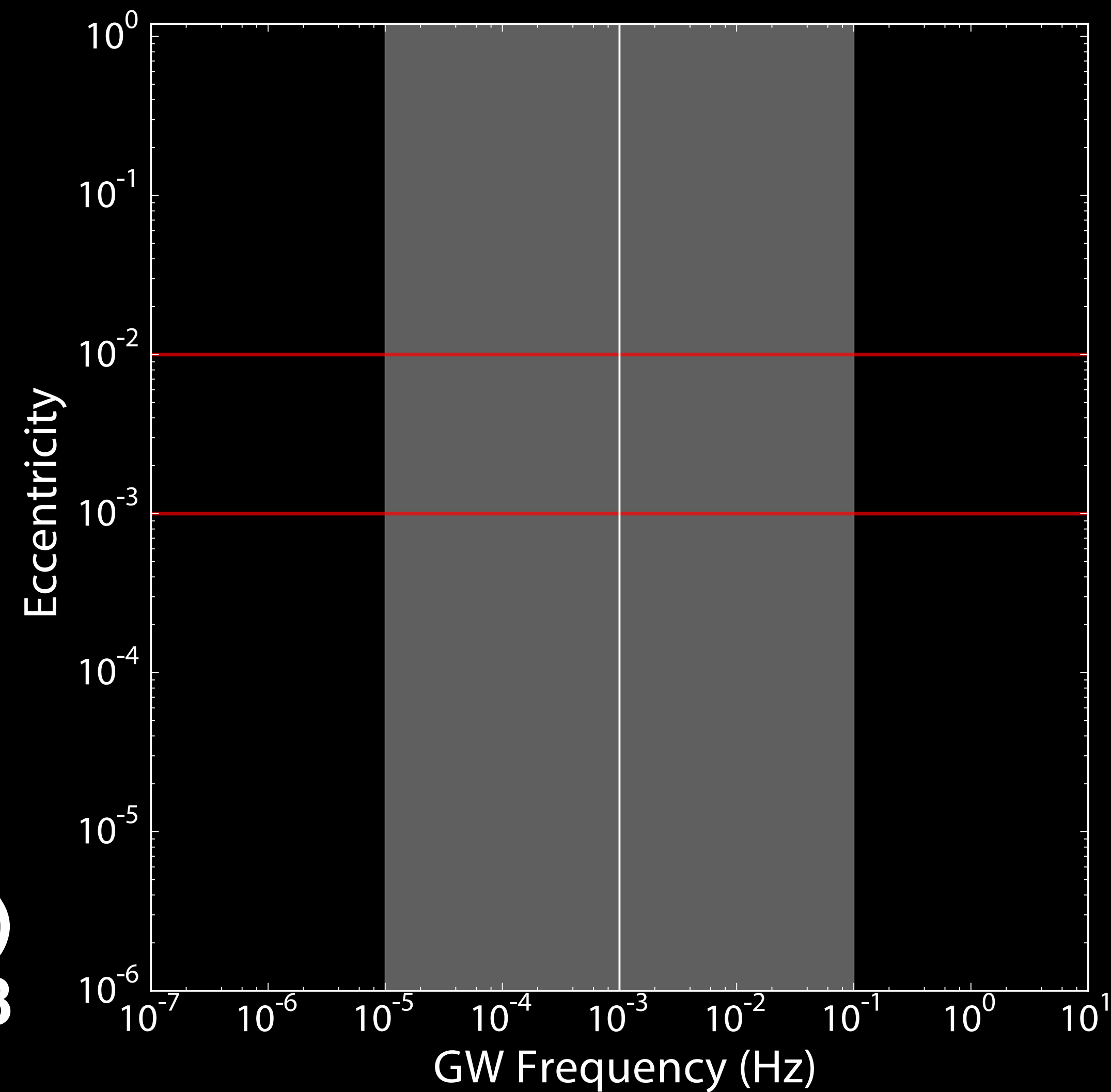




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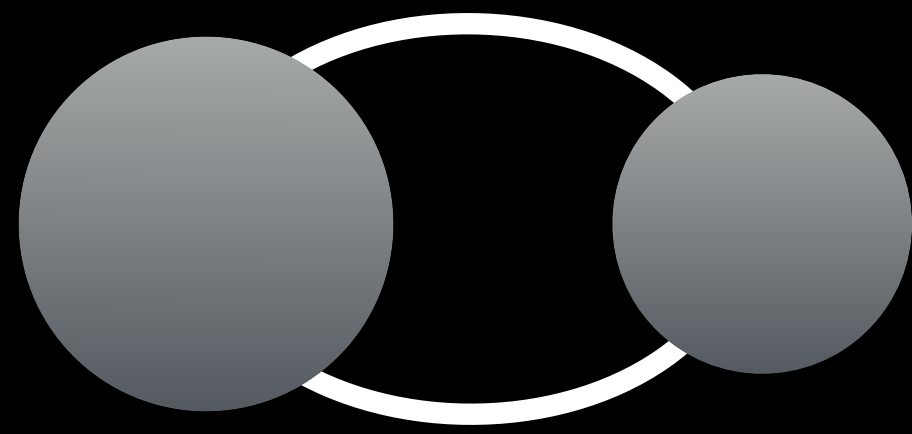


**Breivik et al. (2016)**  
**ApJL 830, L18**

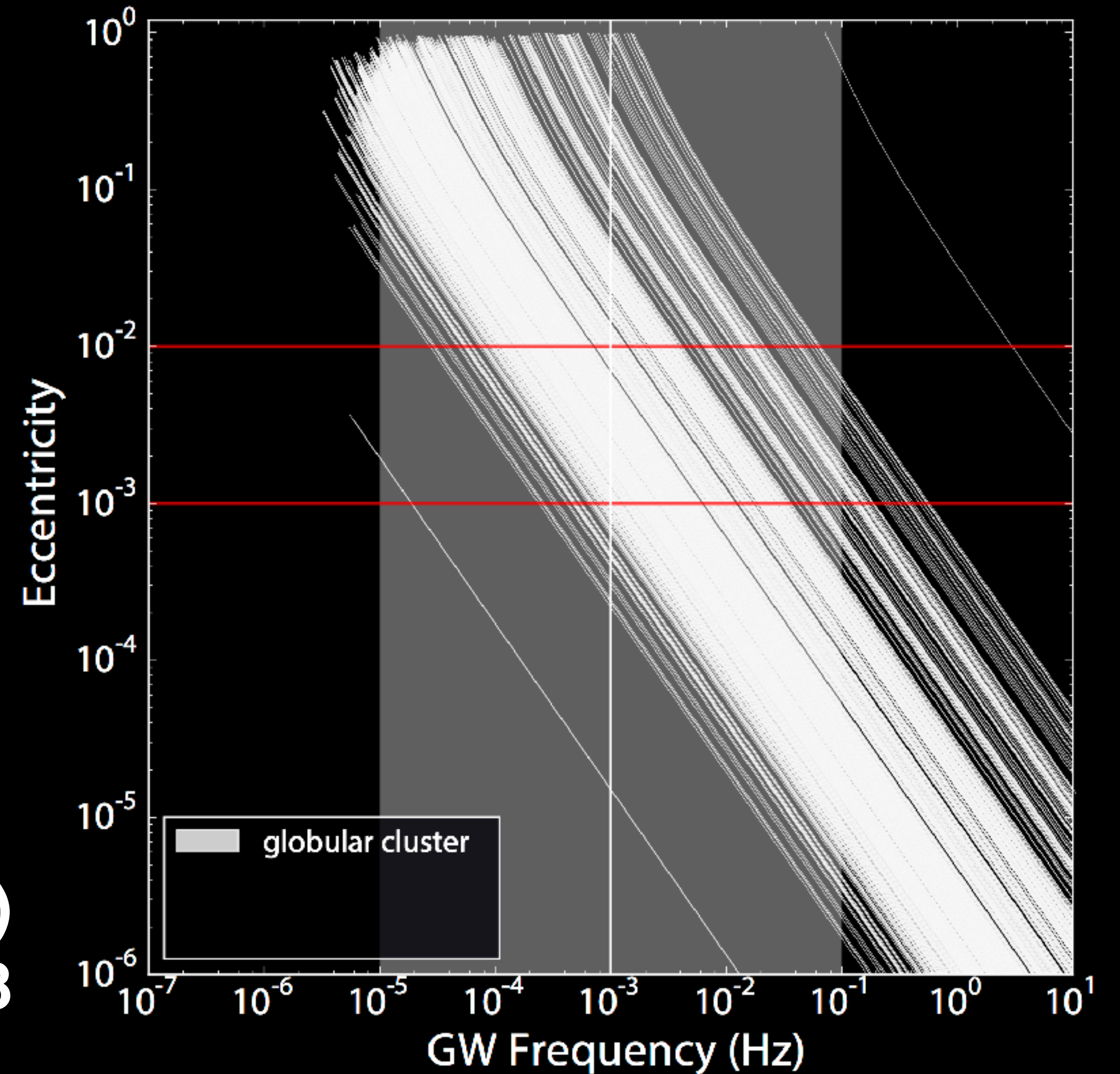




# Forming Black Hole Binaries

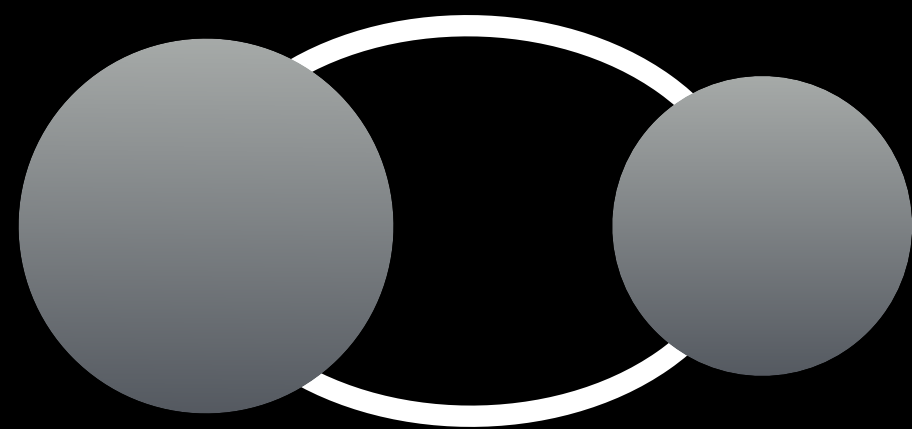


Breivik et al. (2016)  
ApJL 830, L18

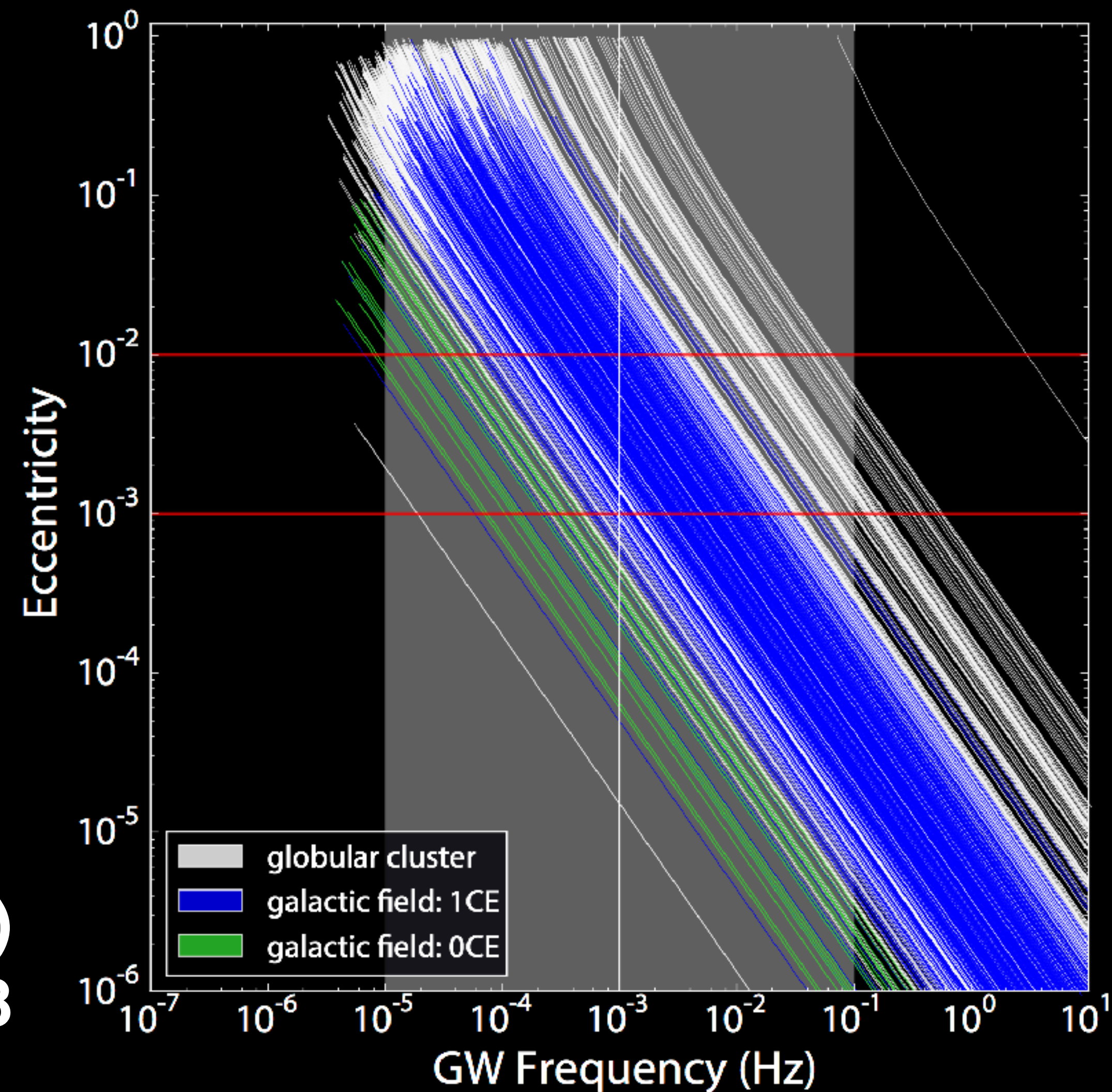




# Forming Black Hole Binaries

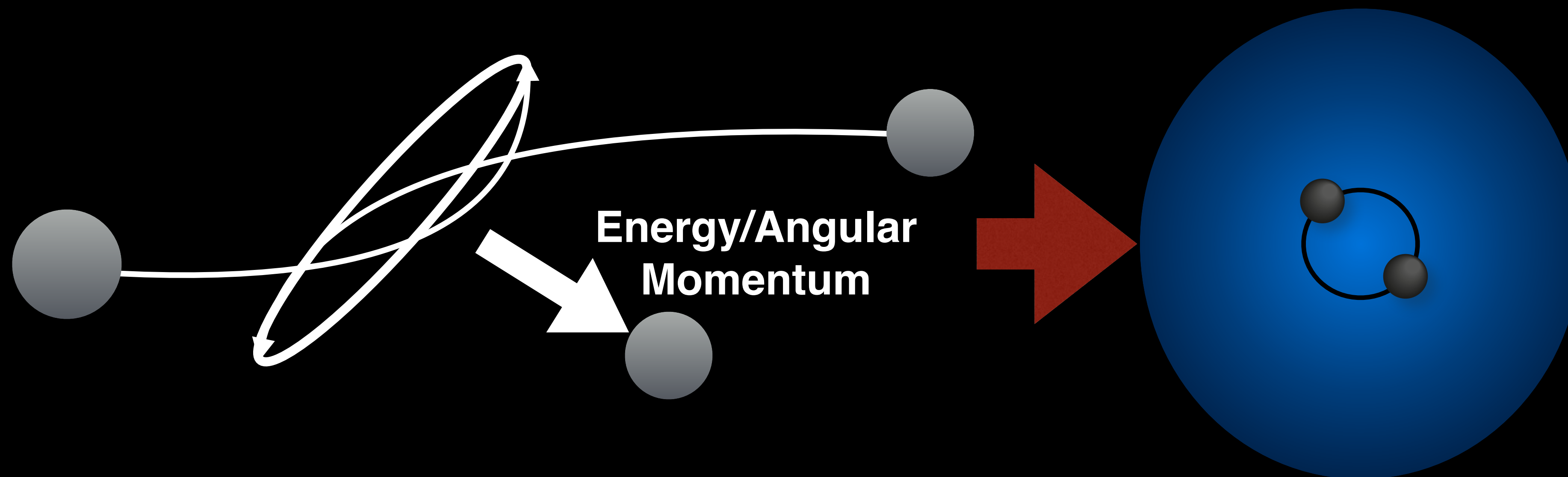


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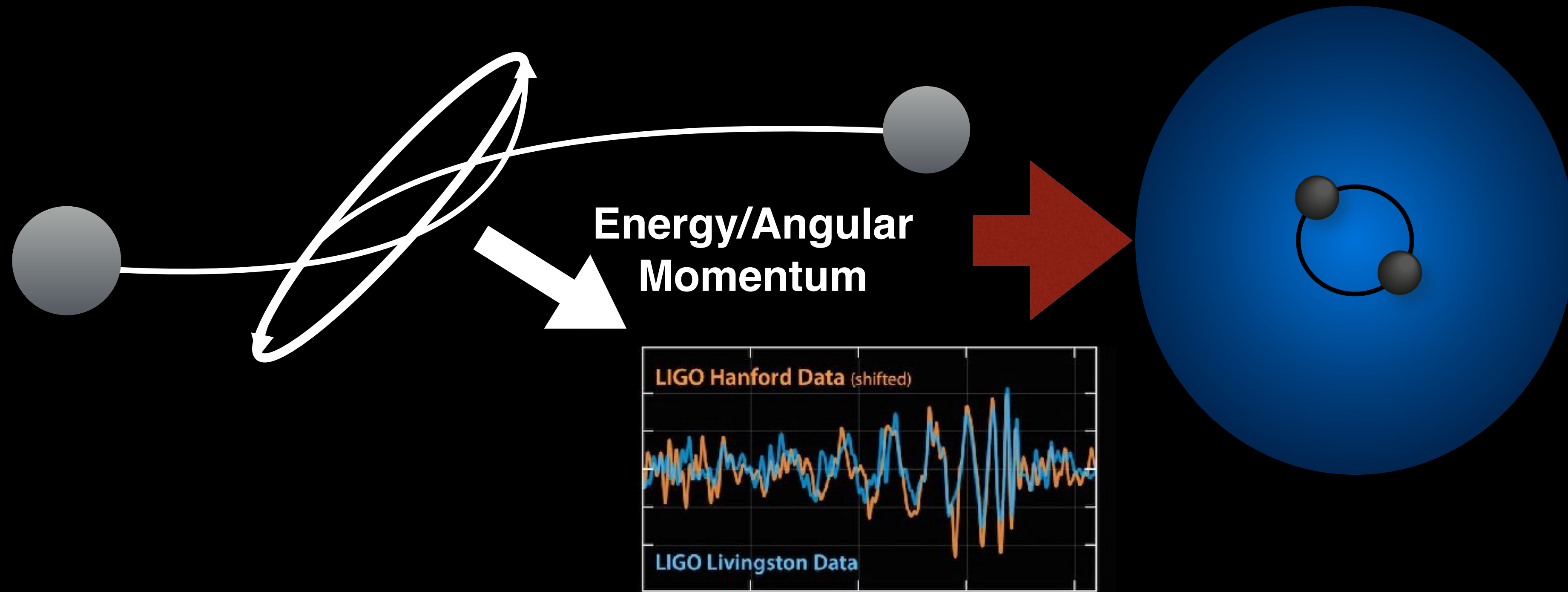


# *Eccentricity*





# Eccentricity





# Forming Black Hole Binaries

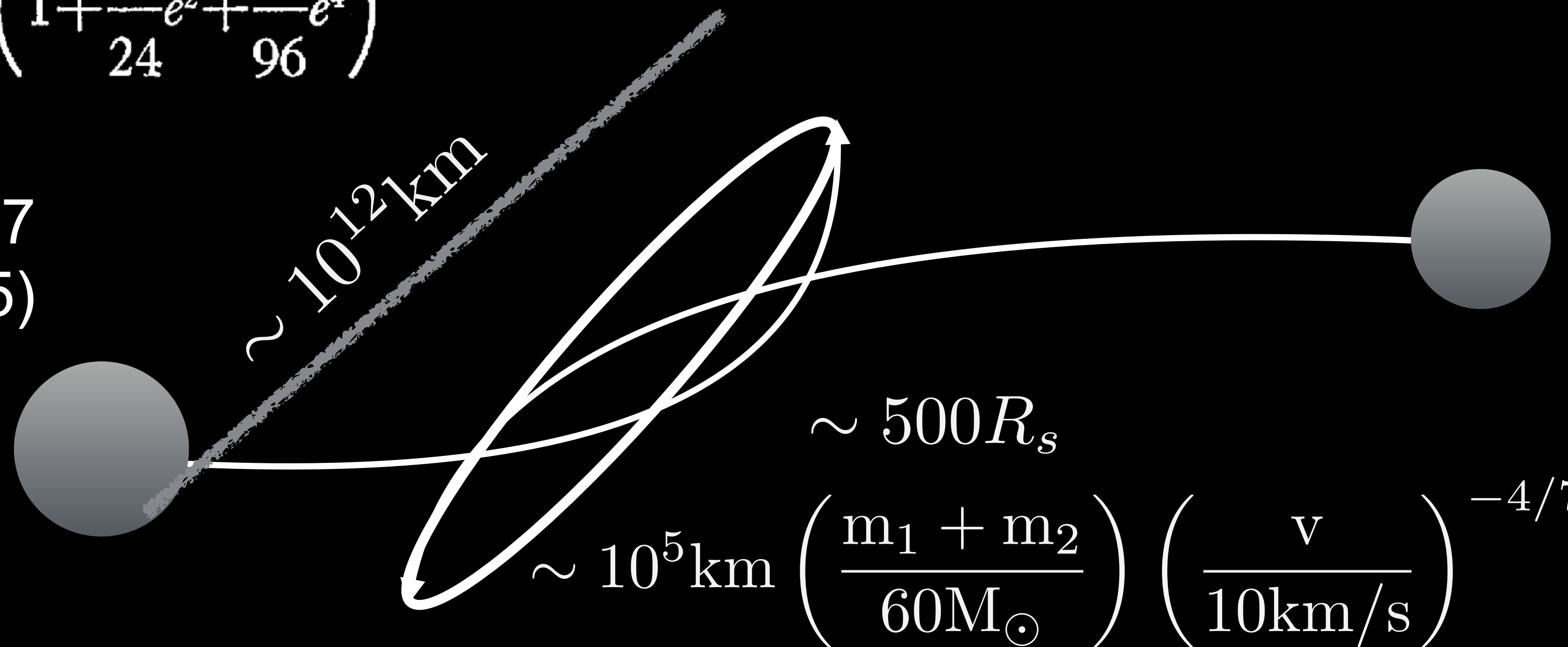
Peters & Matthews 1963

$$\langle P \rangle = \frac{32 G^4 m_1^2 m_2^2 (m_1 + m_2)}{5 c^5 a^5 (1 - e^2)^{7/2}} \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 \right)$$

Quinlan & Shapiro 1987  
(also Giersz 1985)

$$a(1 - e) \rightarrow r_p$$

$$e \rightarrow 1$$



$$\Delta E \simeq \frac{85\pi G^{7/2} M^{9/2}}{12c^5 r_p^{7/2}} > E = \frac{1}{2} M v_\infty^2$$

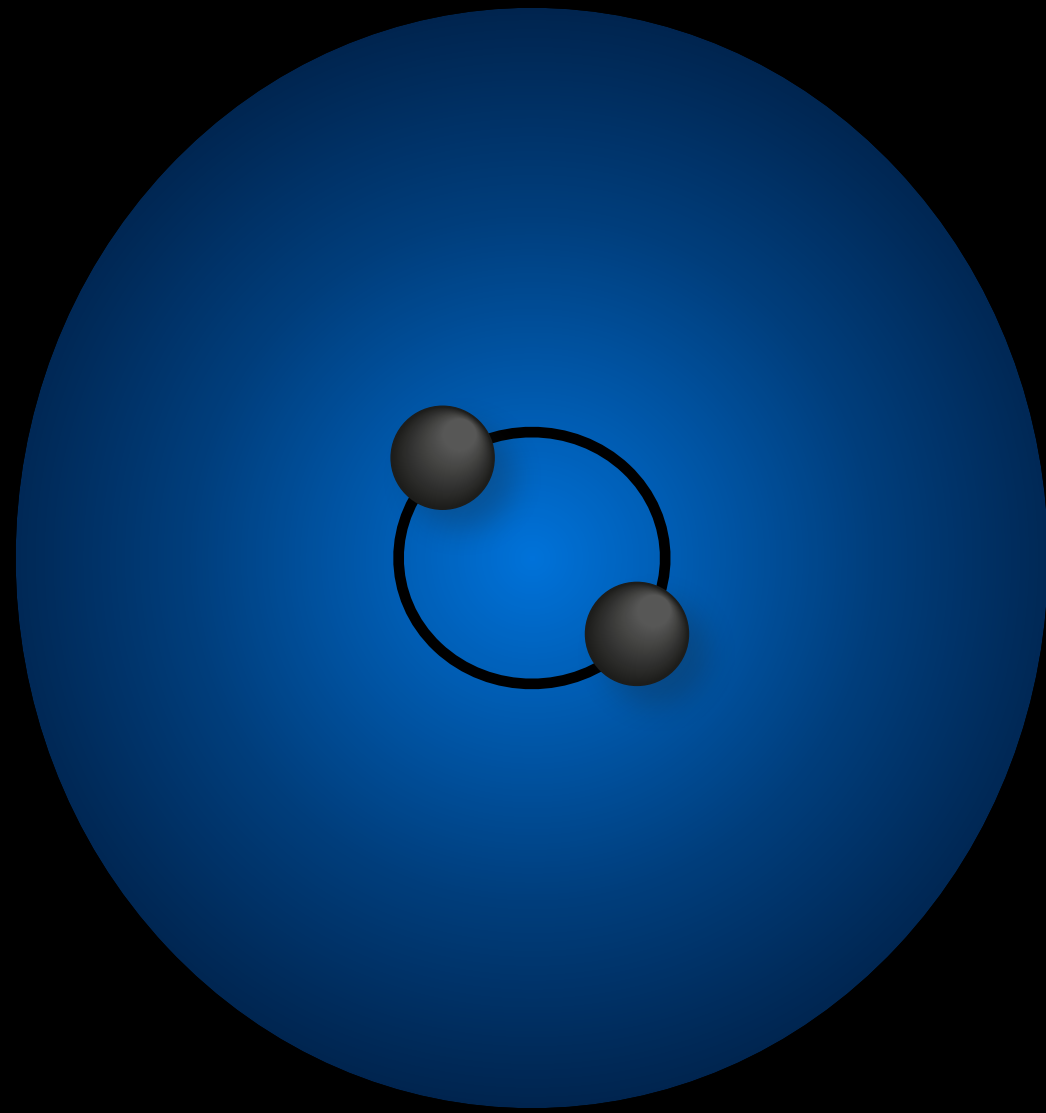
$$\sim 10^5 \text{ km} \left( \frac{m_1 + m_2}{60 M_\odot} \right) \left( \frac{v}{10 \text{ km/s}} \right)^{-4/7}$$



# Eccentricity

Lee (1993)

Gültiken, Miller, Hamilton  
(2004, 2006)



Samsing, MacLeod, Ramirez-Ruiz  
(2014, 2017)

Samsing, Ramirez-Ruiz (2017)

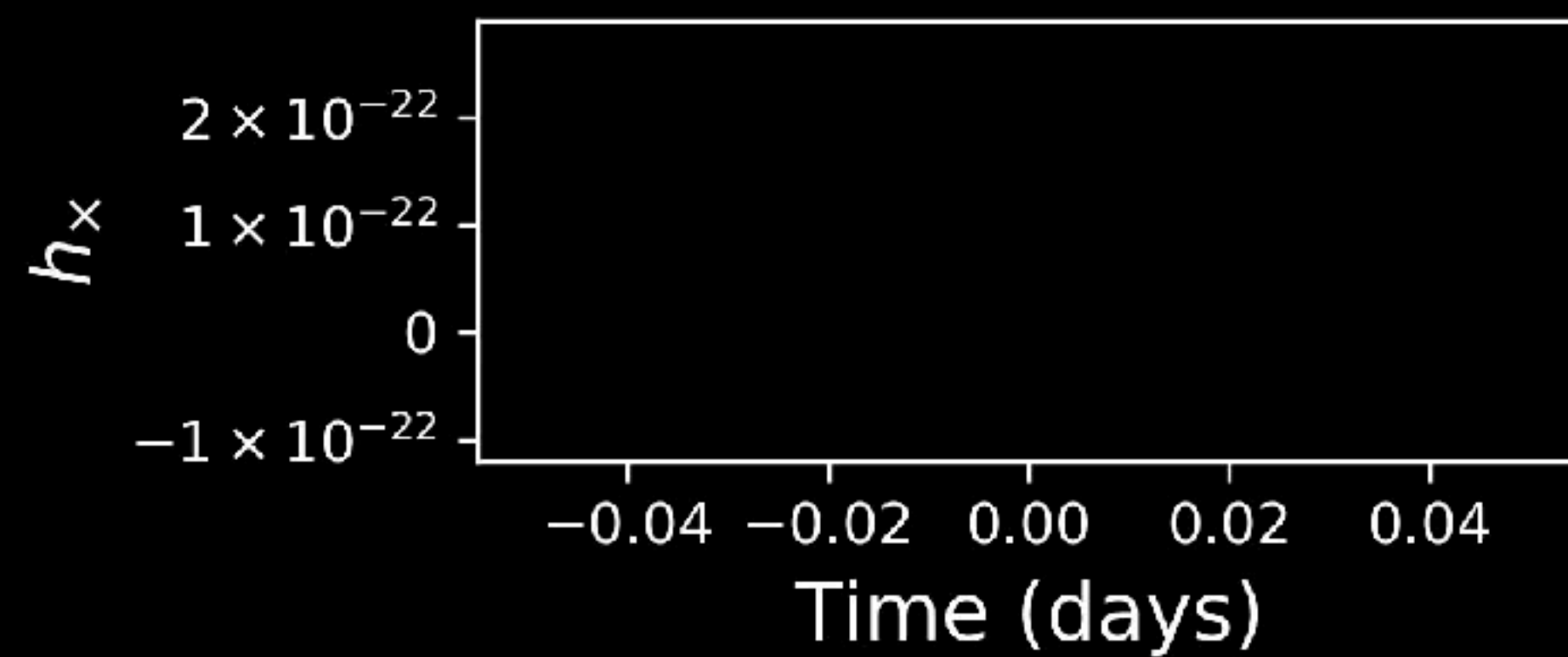
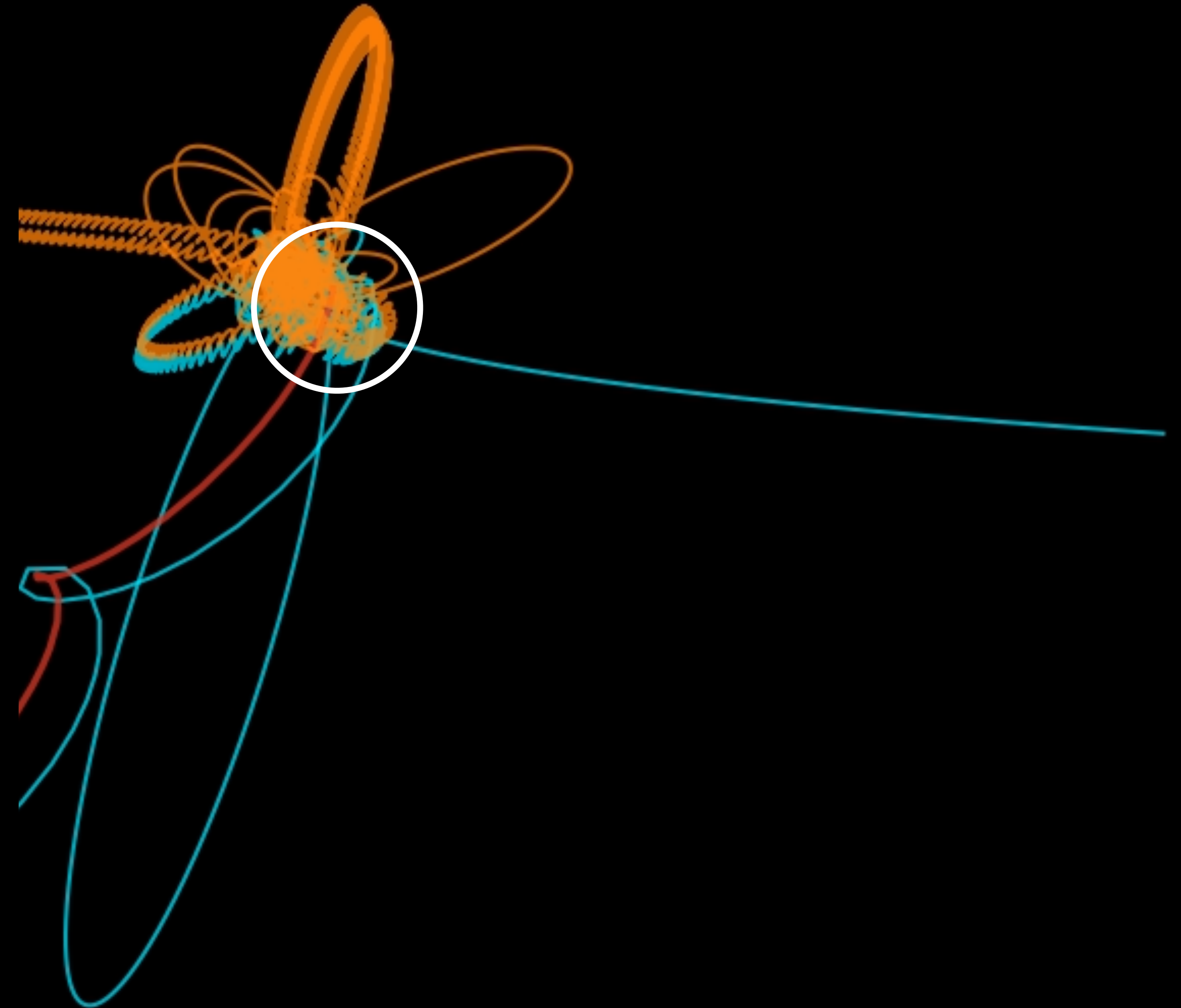
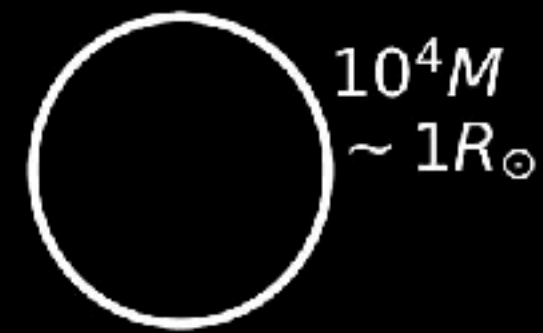
Samsing (2018)

Rodriguez et al. (2018 a,b)

Zevin et al. (2018)

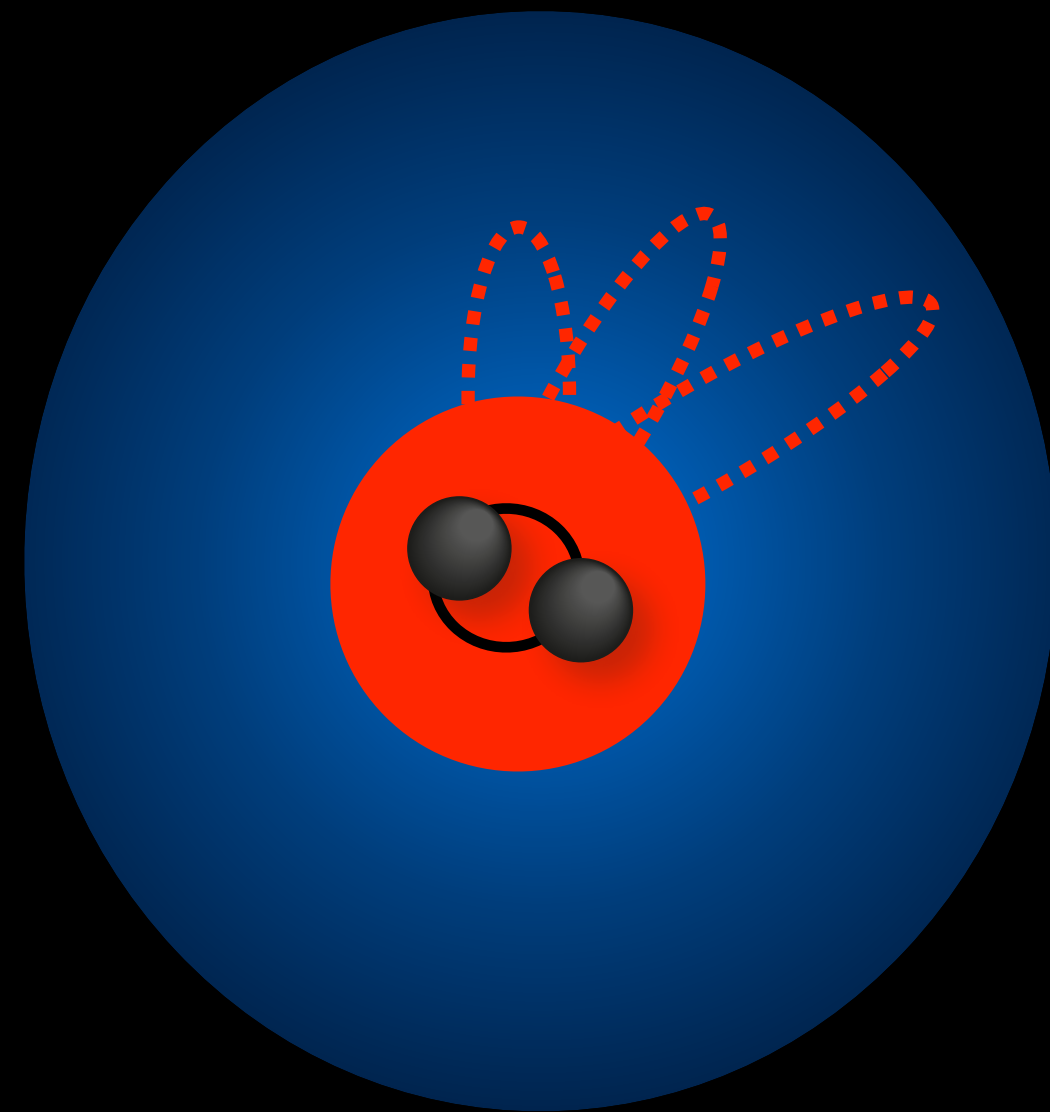


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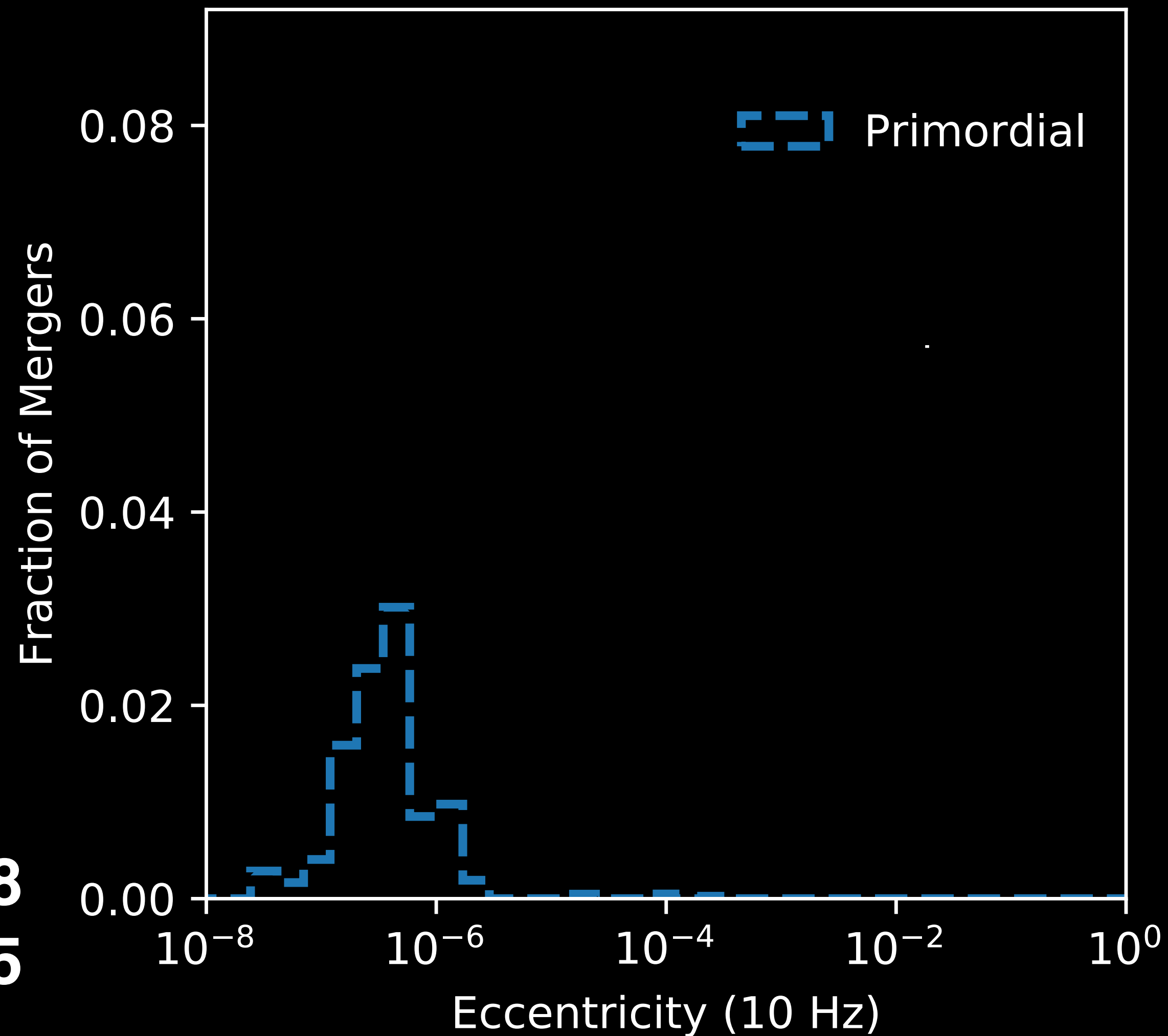


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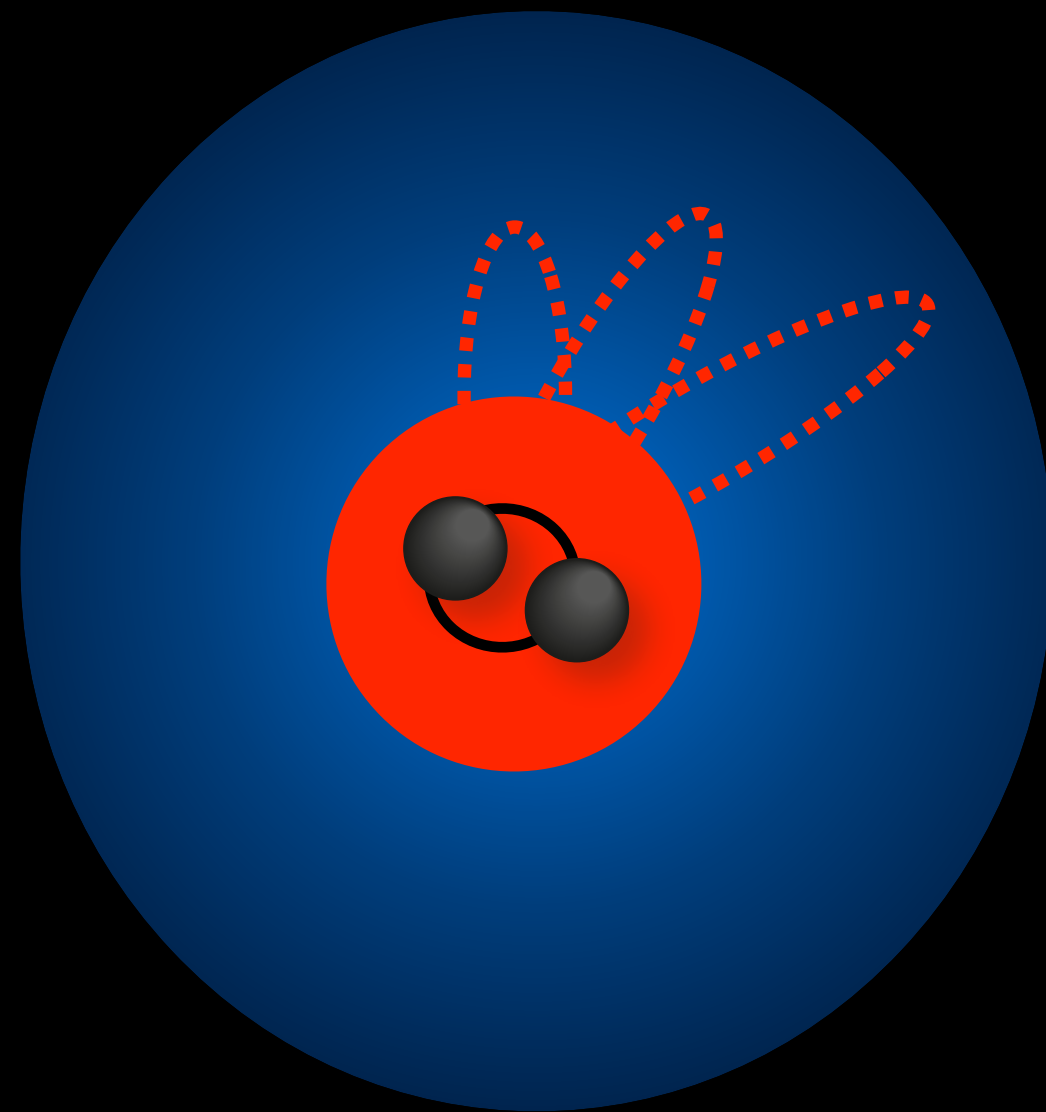
Rodriguez et al., 2018  
PRD, 123, 123005

Eccentricity Distribution (All Redshifts)



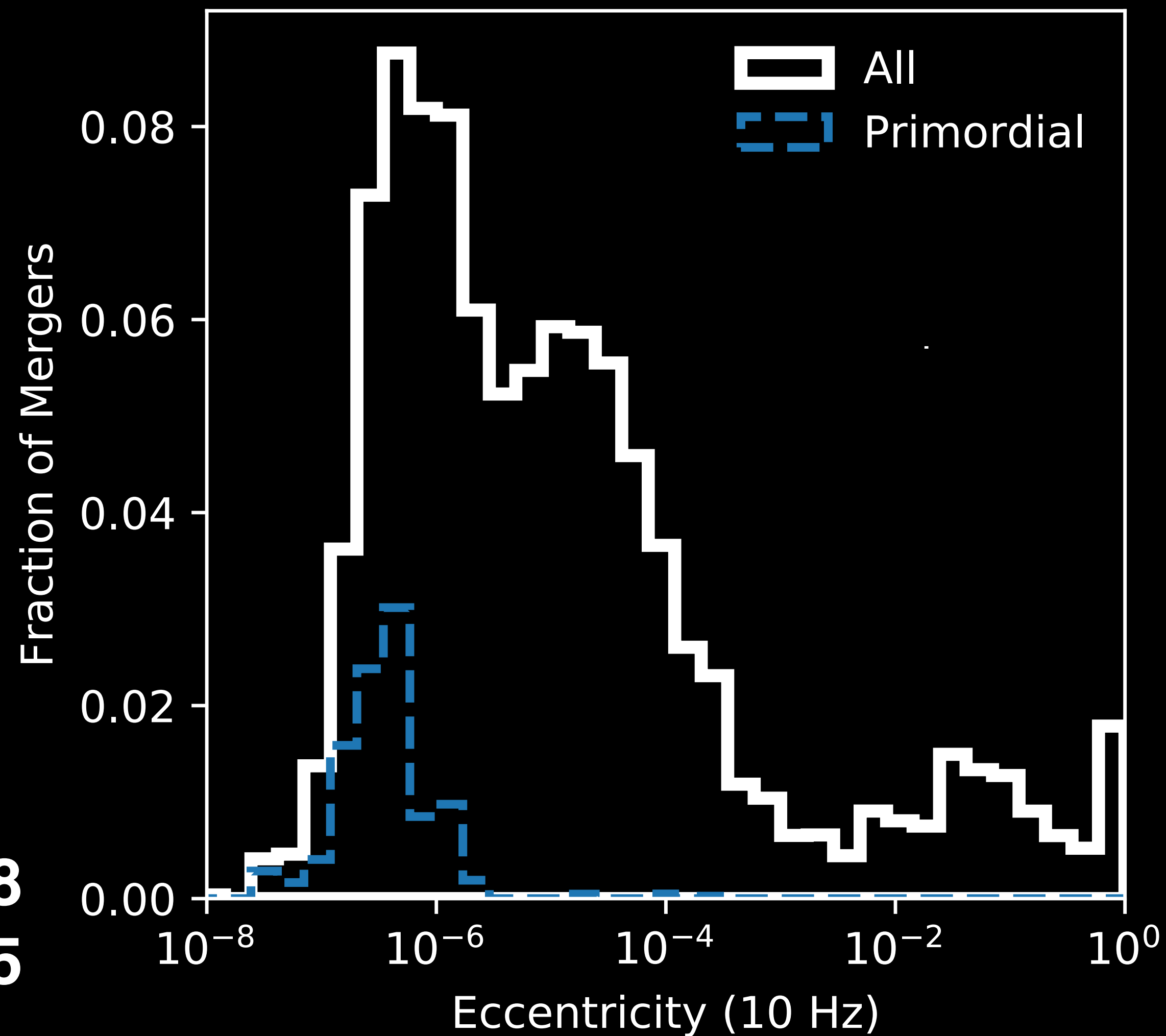


# Eccentricity



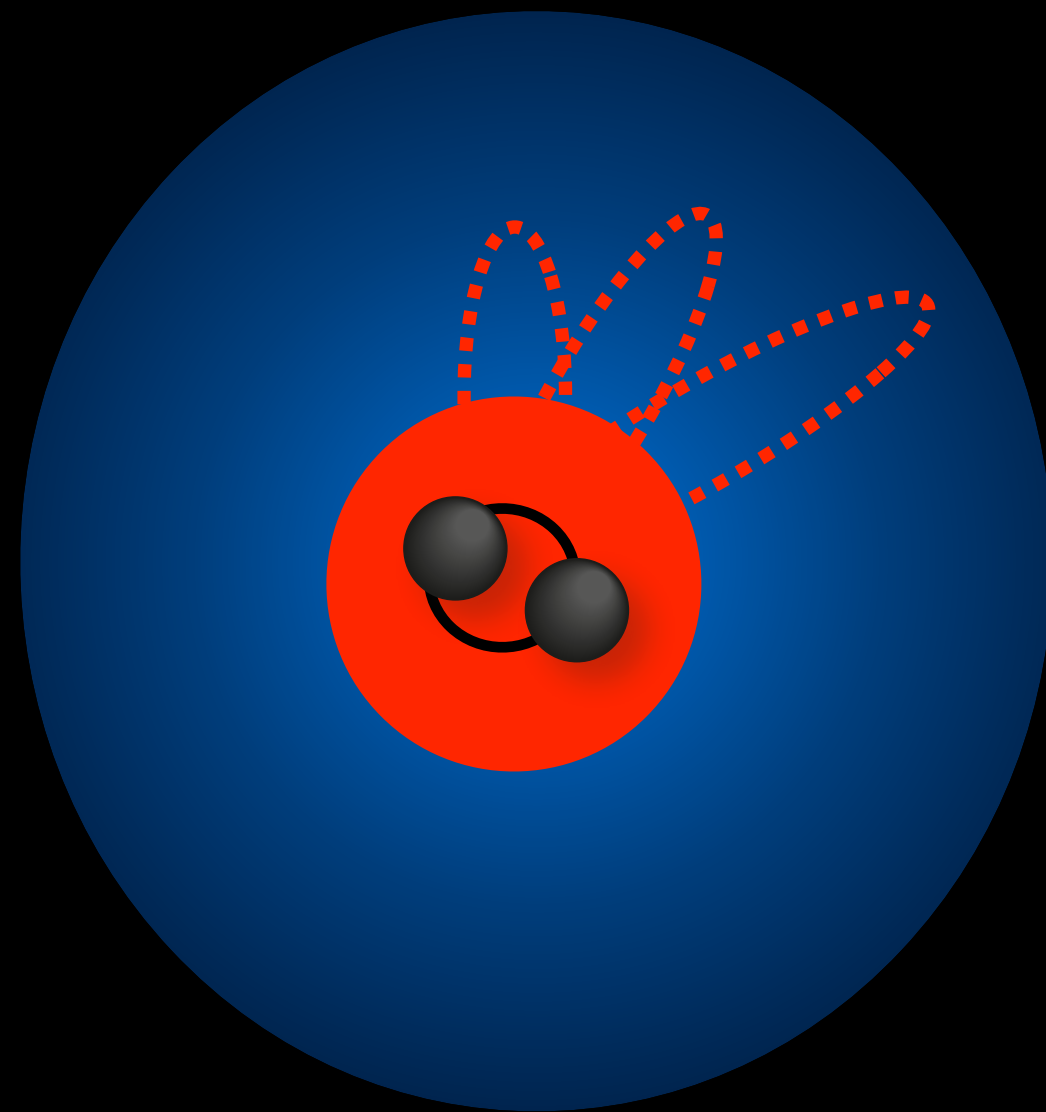
Rodriguez et al., 2018  
PRD, 123, 123005

Eccentricity Distribution (All Redshifts)



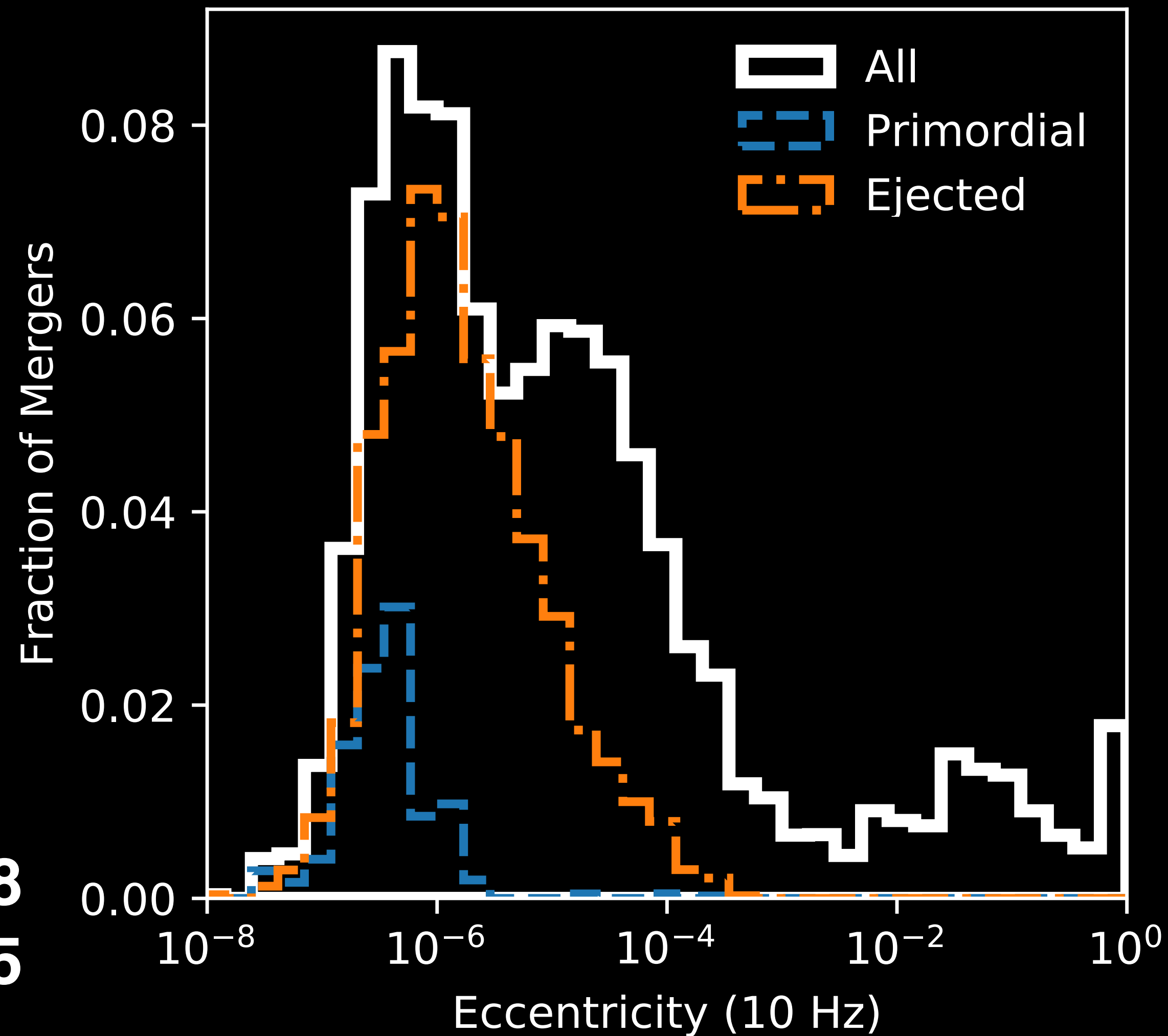


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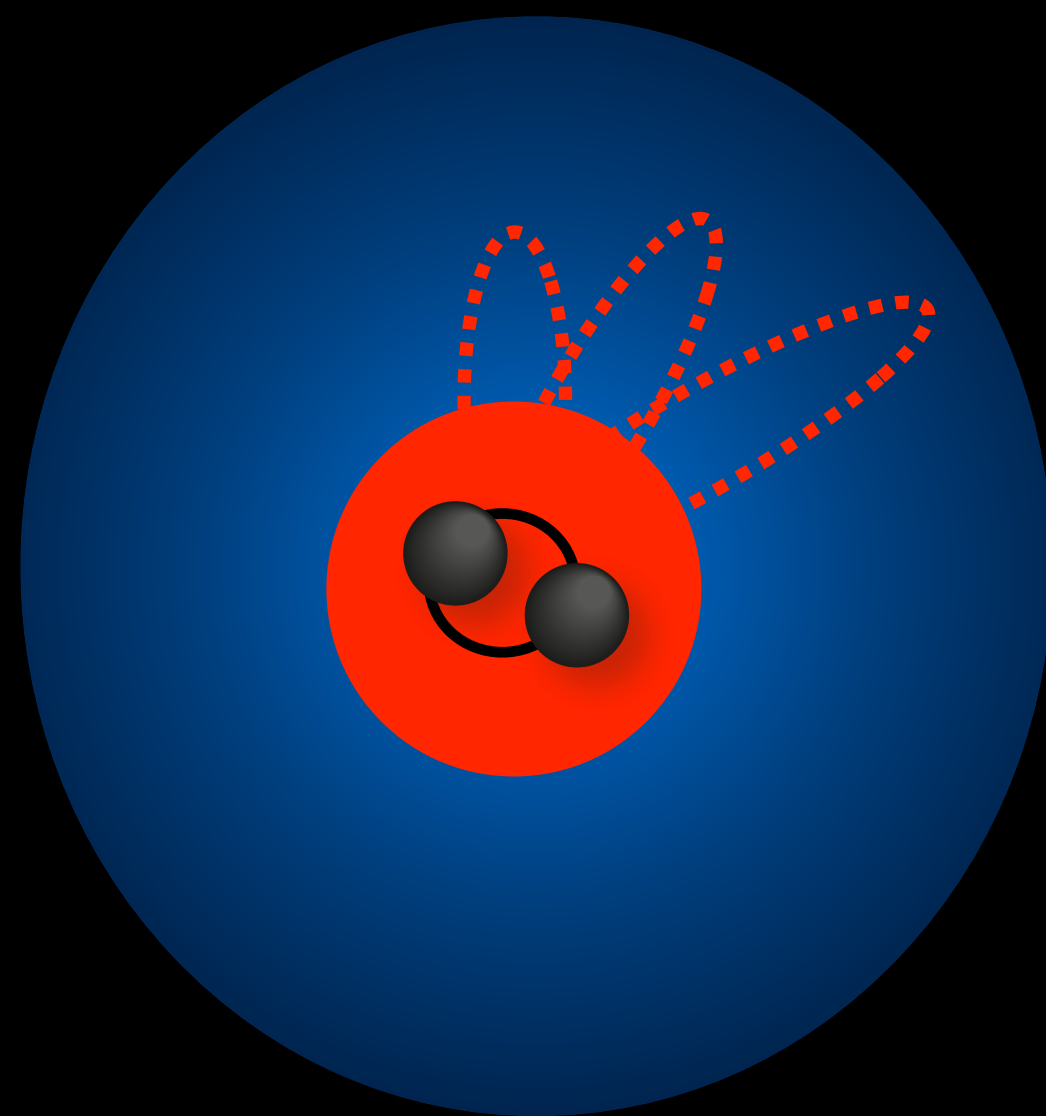
Rodriguez et al., 2018  
PRD, 123, 123005

Eccentricity Distribution (All Redshifts)



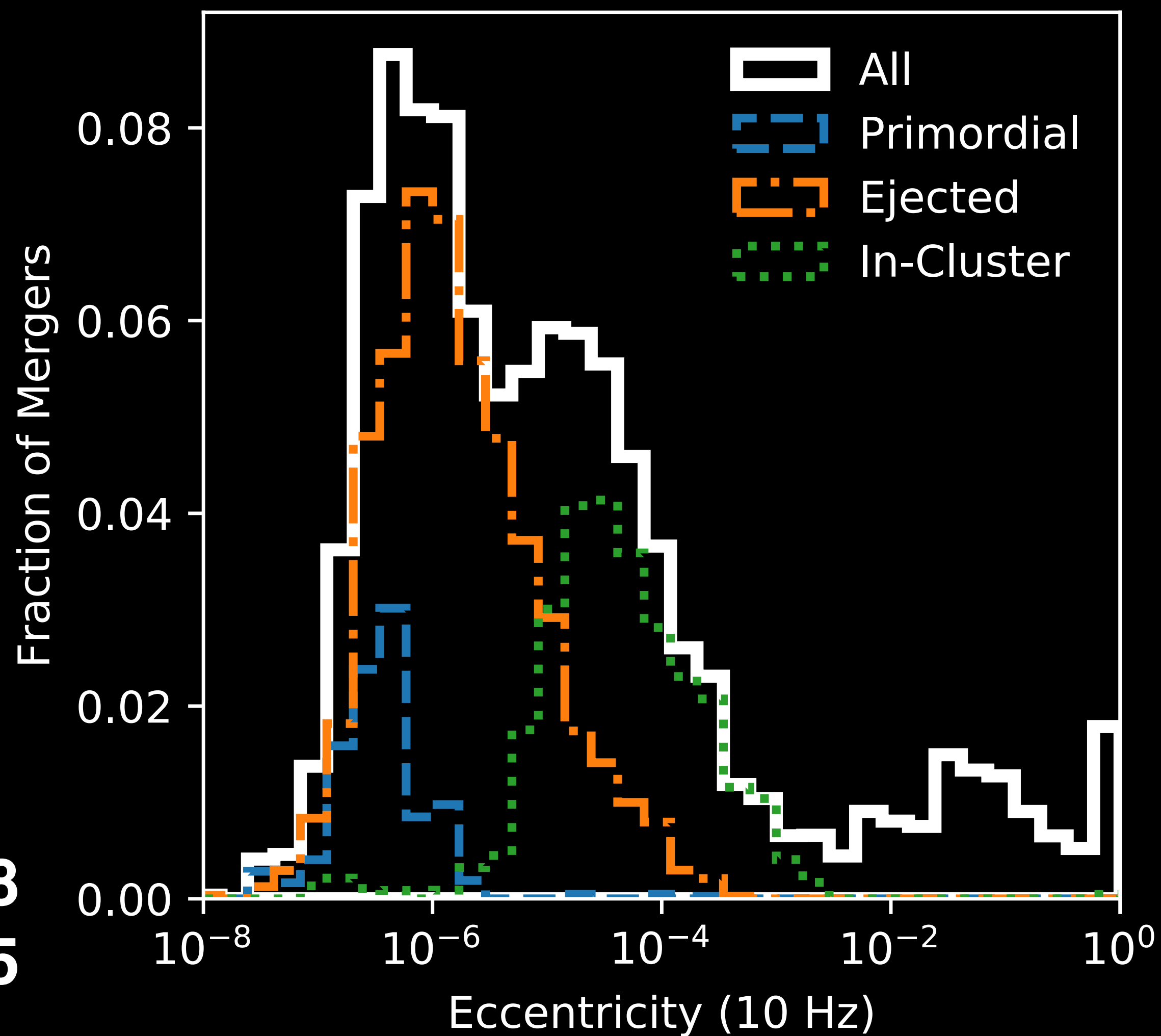


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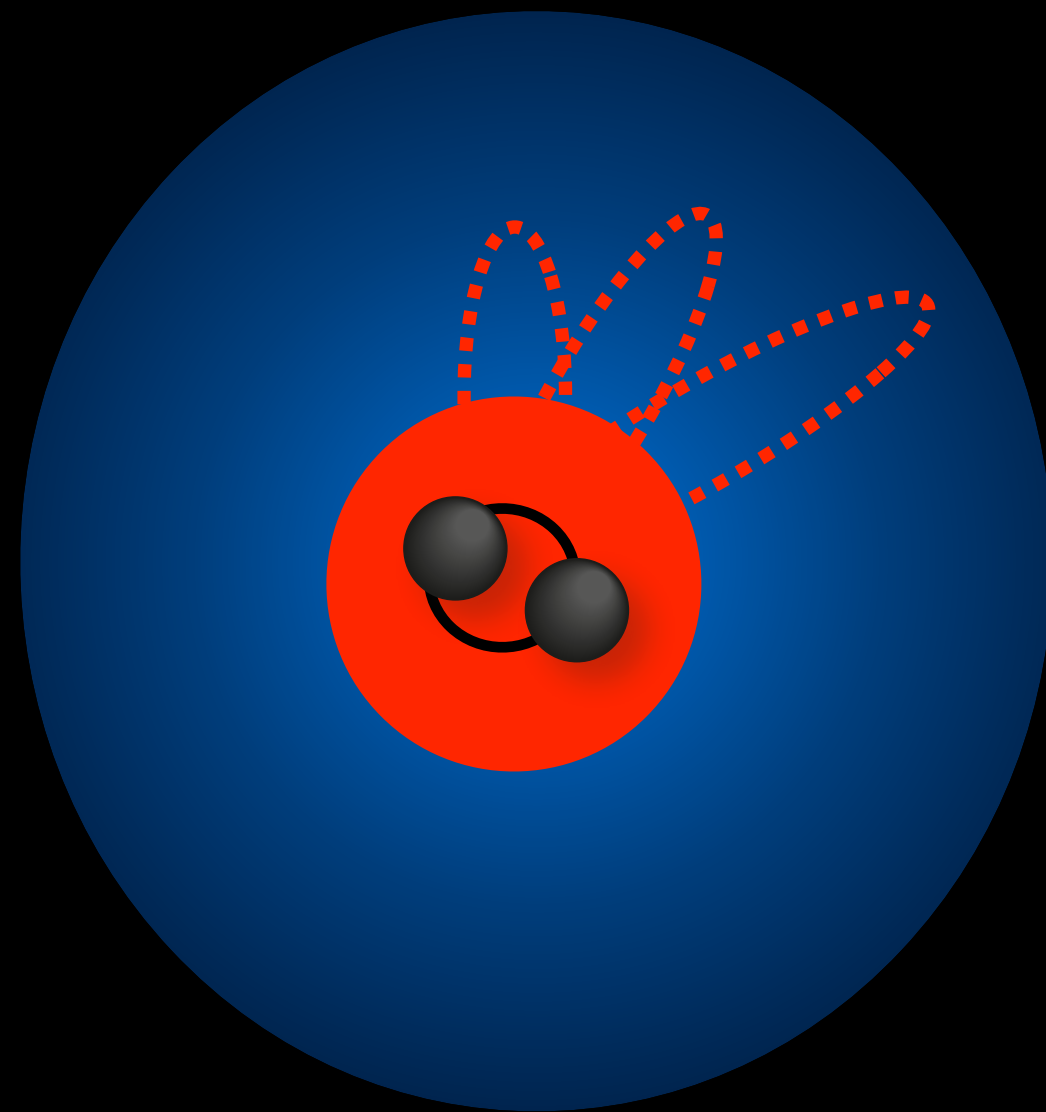
Rodriguez et al., 2018  
PRD, 123, 123005

Eccentricity Distribution (All Redshifts)



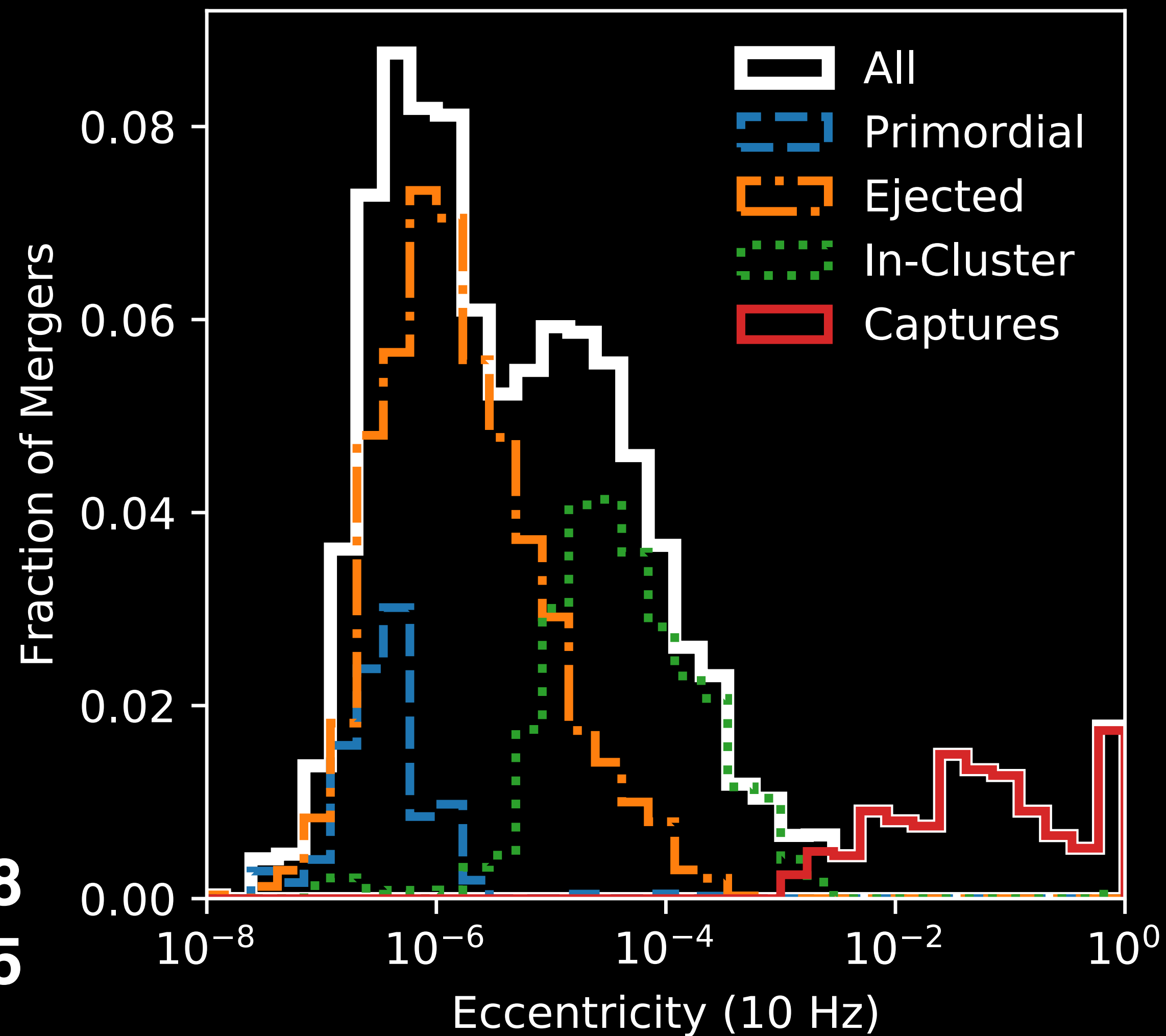


# Eccentricity



Rodriguez et al., 2018  
PRD, 123, 123005

Eccentricity Distribution (All Redshifts)





# Eccentricity

$$h_x = \frac{\cos \vartheta}{\sqrt{2}} [2h_{xy} \cos 2\varphi - (h_{xx} - h_{yy}) \sin 2\varphi],$$

$$h_{xy} = \frac{2G}{c^4 r} \frac{d^2 I_{xy}}{dt^2}, \quad h_{xx} = \frac{2G}{c^4 r} \frac{d^2 I_{xx}}{dt^2}, \quad h_{yy} = \frac{2G}{c^4 r} \frac{d^2 I_{yy}}{dt^2},$$

$$h_+ = \frac{1}{\sqrt{2}} \left\{ \frac{3 + \cos 2\vartheta}{4} [2h_{xy} \sin 2\varphi + (h_{xx} - h_{yy}) \cos 2\varphi] - \frac{1 - \cos 2\vartheta}{4} (h_{xx} + h_{yy}) \right\},$$

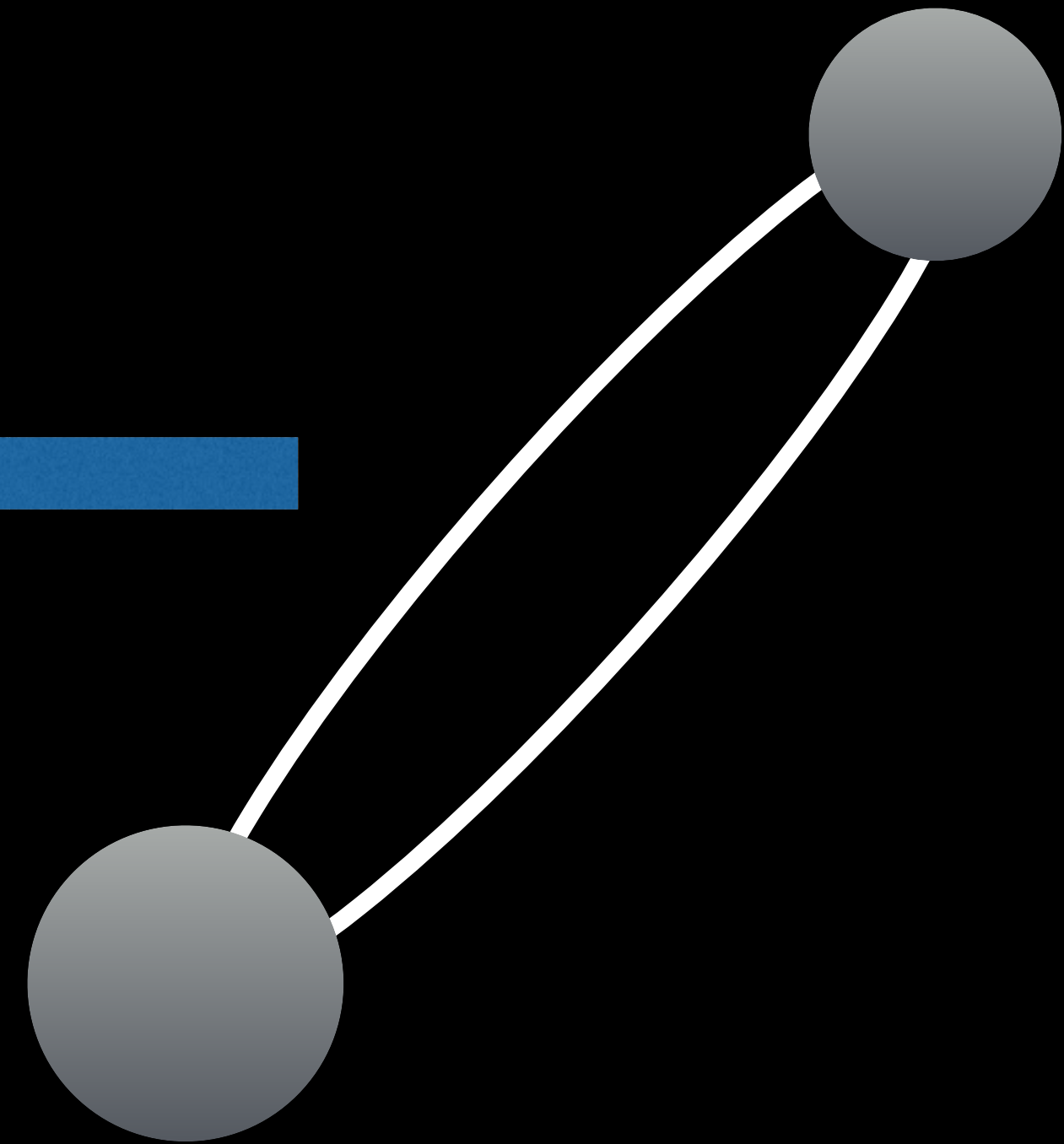
$$h_{xy} = \sum_{n=1}^{\infty} h_{xy}^{(n)} \sin \left( n \frac{2\pi}{T} t \right),$$

$$h_{x\pm y} = \sum_{n=1}^{\infty} h_{x\pm y}^{(n)} \cos \left( n \frac{2\pi}{T} t \right),$$

$$h_{xy}^{(n)} = h_0 n (1 - e^2)^{1/2} [J_{n-2}(ne) + J_{n+2}(ne) - 2J_n(ne)],$$

$$h_{x-y}^{(n)} = 2h_0 n \{ J_{n-2}(ne) - J_{n+2}(ne) - 2e[J_{n-1}(ne) - J_{n+1}(ne)] + (2/n)J_n(ne) \},$$

$$h_{x+y}^{(n)} = -4h_0 J_n(ne),$$



**Pierro et al. (2001)**  
**MNRAS 325,358**



# Eccentricity

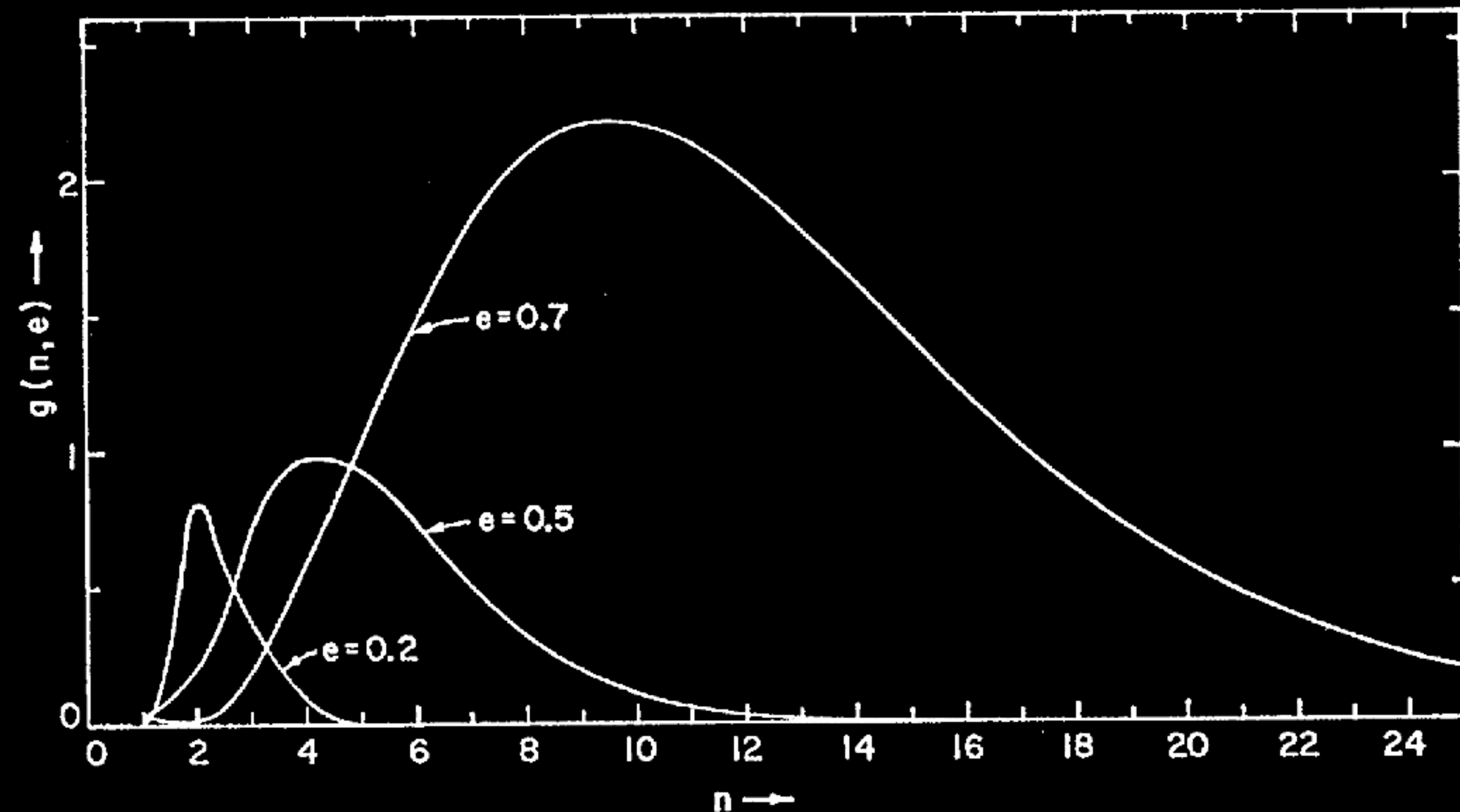
Peters & Matthews (1963)  
Phys. Rev. 131, 415

The power radiated in the  $n$ th harmonic is, from (10) and (18),

$$P(n) = \frac{32 G^4 m_1^2 m_2^2 (m_1 + m_2)}{5 c^5 a^5} g(n, e), \quad (19)$$

where

$$g(n, e) = \frac{n^4}{32} \left\{ \begin{aligned} & \left[ J_{n-2}(ne) - 2eJ_{n-1}(ne) \right. \\ & \left. + \frac{2}{n} J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne) \right]^2 \\ & + (1-e^2) [J_{n-2}(ne) - 2J_n(ne) + J_{n+2}(ne)]^2 \\ & \left. + \frac{4}{3n^2} [J_n(ne)]^2 \right\}. \quad (20) \end{aligned} \right.$$



Wen (2003)  
ApJ 598, 413

$$f_{\text{GW}}^m(e_1) = \frac{\sqrt{GM_1}}{\pi} (1 + e_1)^{1.1954} \frac{1}{(a_1 \epsilon)^{1.5}}$$



## ON THE ECCENTRICITY DISTRIBUTION OF COALESCING BLACK HOLE BINARIES DRIVEN BY THE KOZAI MECHANISM IN GLOBULAR CLUSTERS

LINQING WEN

Division of Physics, Mathematics, and Astronomy, California Institute of Technology, MS 103-33, Pasadena, CA 91125;

lwen@ligo.caltech.edu

*Received 2002 November 13; accepted 2003 August 1*

### ABSTRACT

In a globular cluster, hierarchical triple black hole systems can be produced through binary-binary interaction. It has been proposed recently that the Kozai mechanism could drive the inner binary of the triple system to merge before it is interrupted by interactions with other field stars. We investigate qualitatively and numerically the evolution of the eccentricities in these binaries under gravitational radiation (GR) reaction. We predict that  $\sim 30\%$  of the systems will possess eccentricities greater than 0.1 when their emitted gravitational waves pass through 10 Hz frequency. The implications for gravitational wave detection, especially the relevance to data analyses for broadband laser interferometer gravitational wave detectors, are discussed.

**Wen (2003)**  
**ApJ 598, 413**

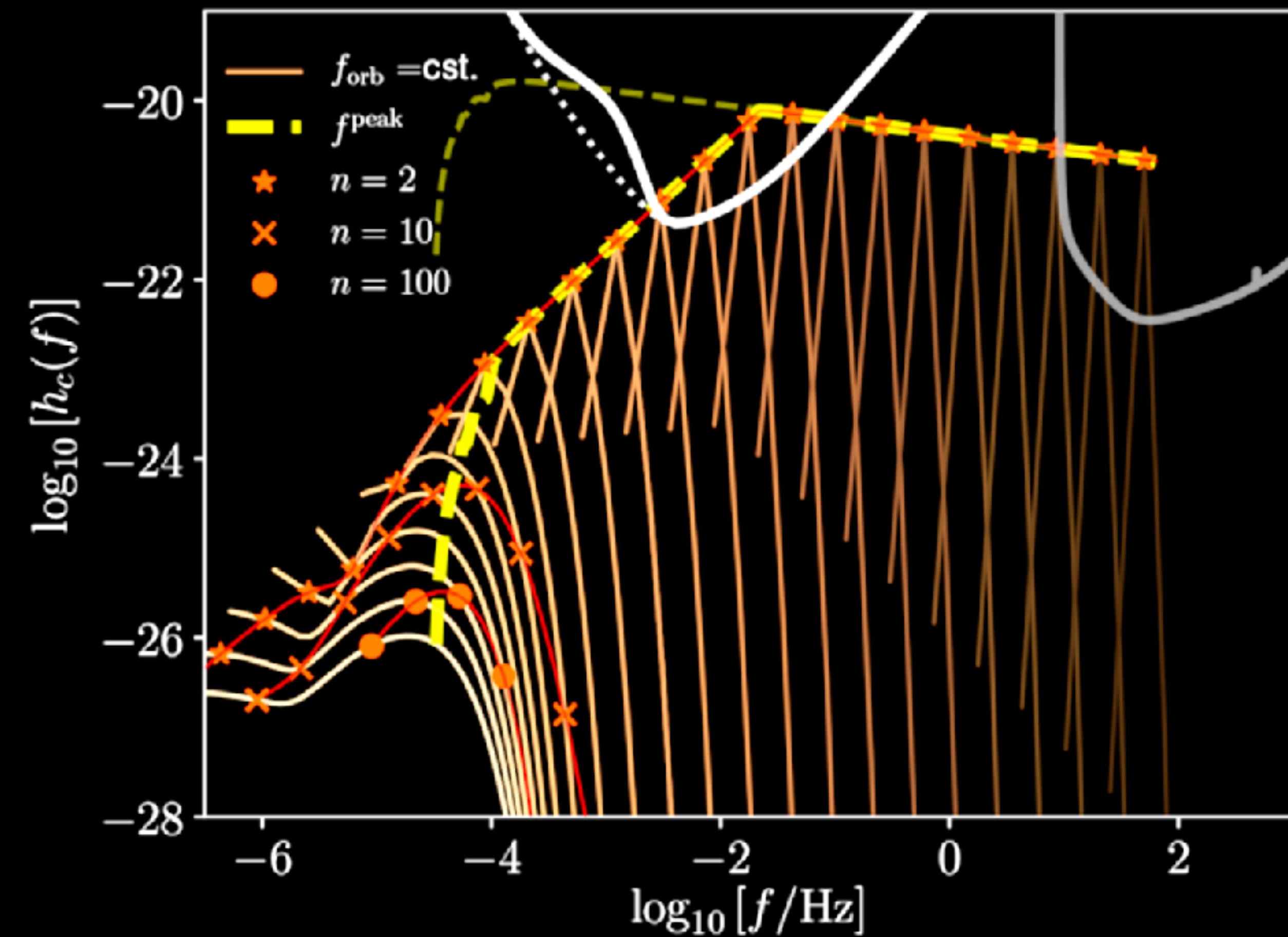
$$f_{\text{GW}}^m(e_1) = \frac{\sqrt{GM_1}}{\pi} (1 + e_1)^{1.1954} \frac{1}{(a_1 \epsilon)^{1.5}}$$

3)  
5



# Eccentricity

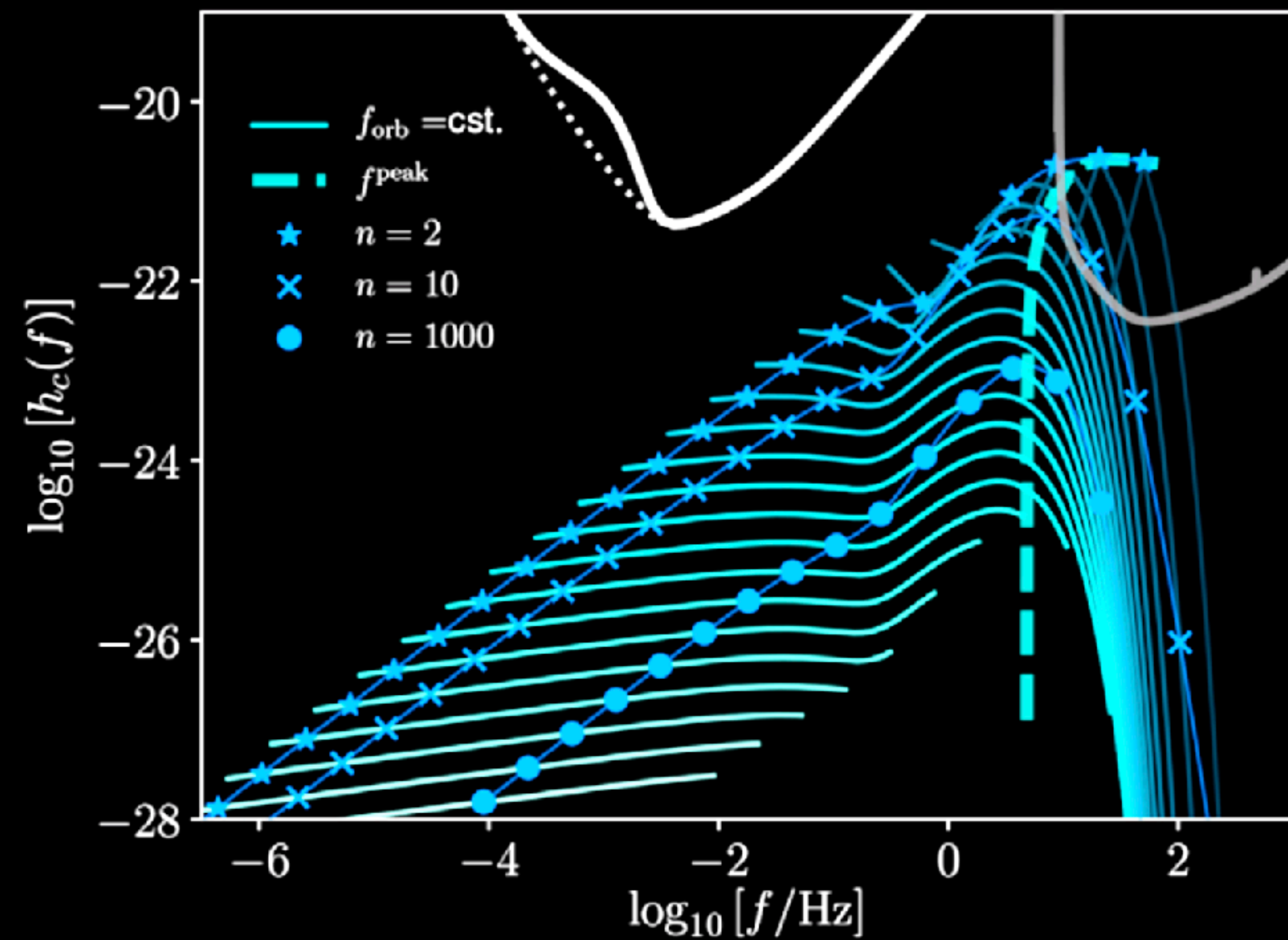
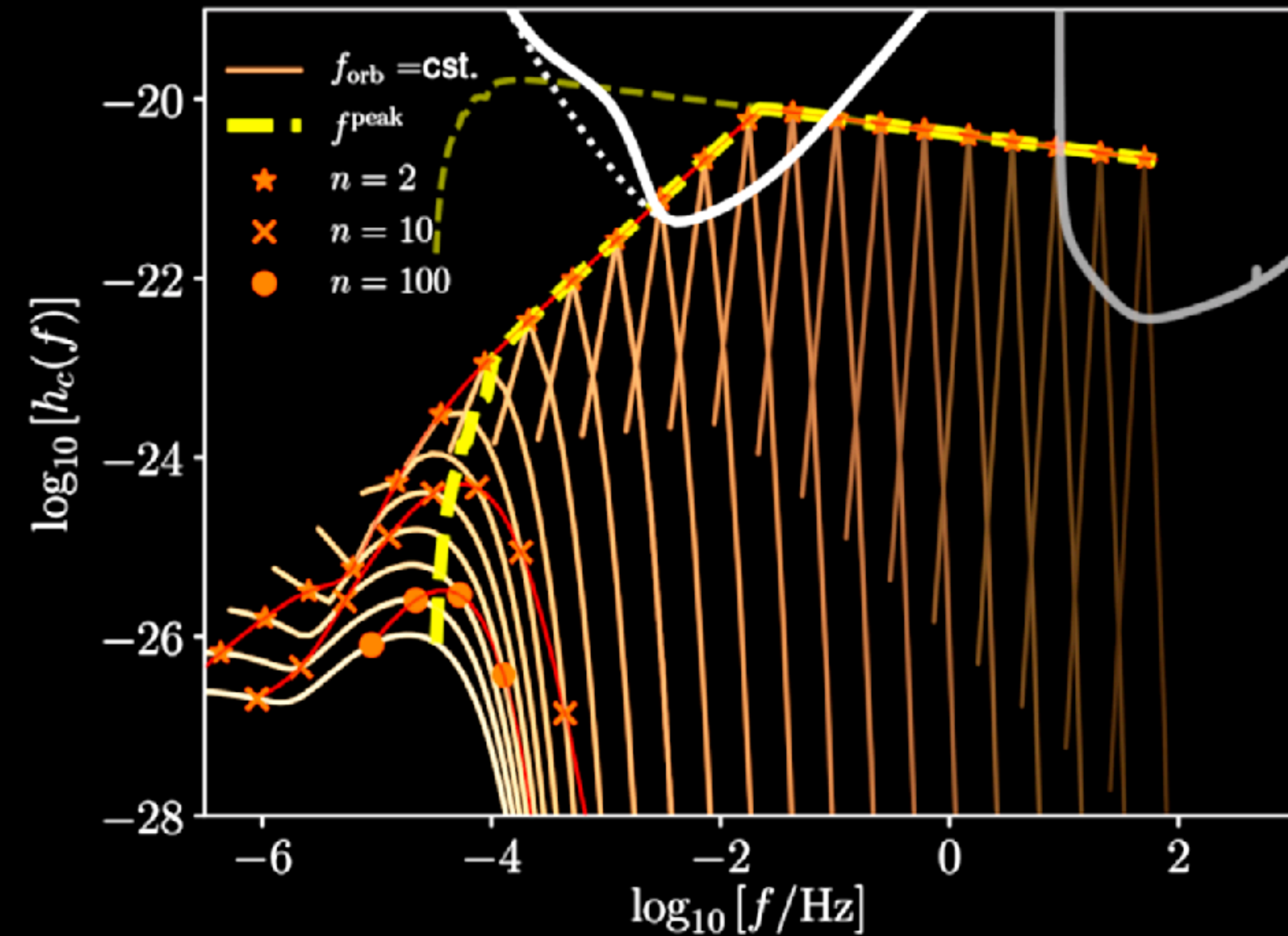
D'Orazio & Samsing (2018)  
MNRAS 481, 4775





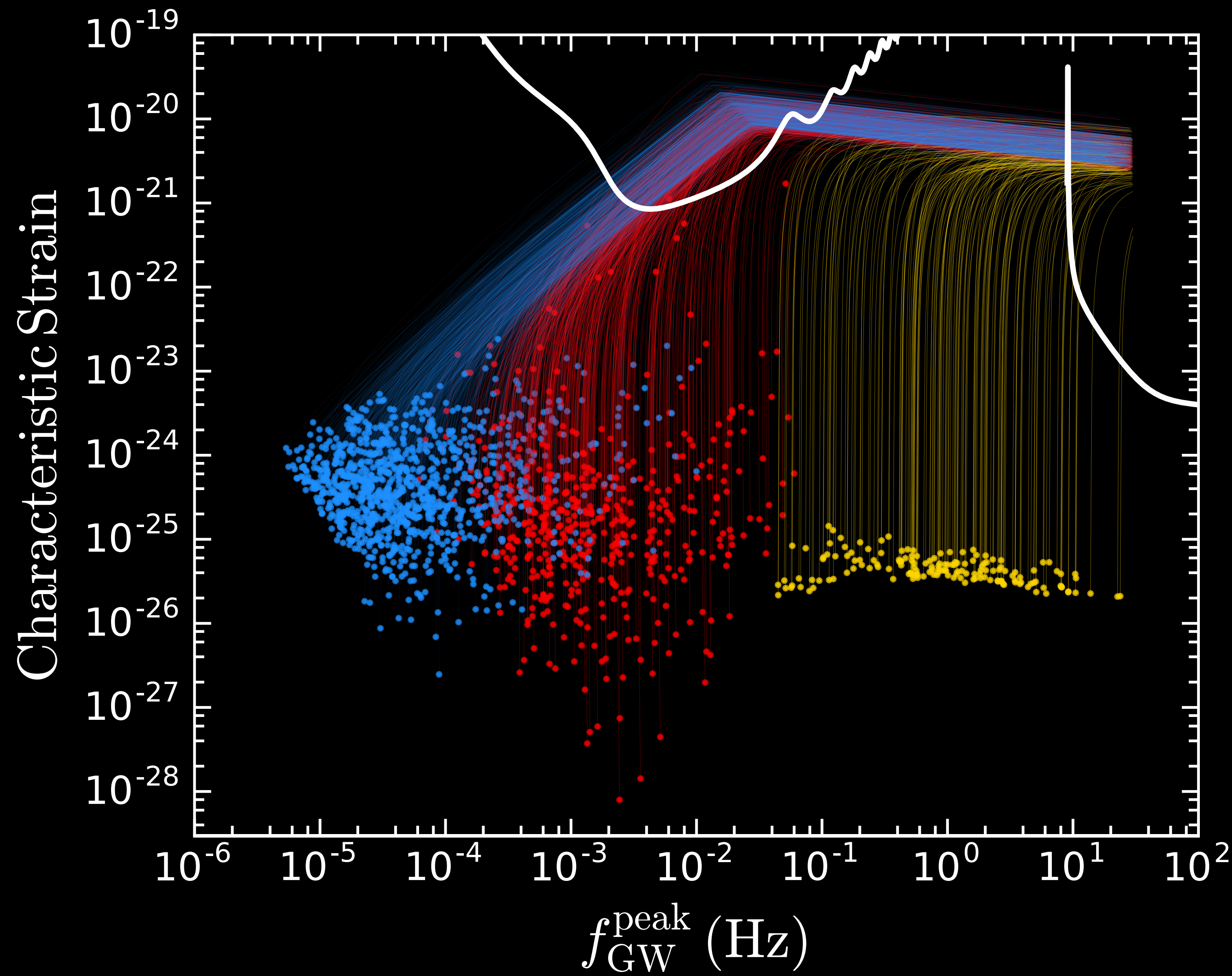
# Eccentricity

D'Orazio & Samsing (2018)  
MNRAS 481, 4775





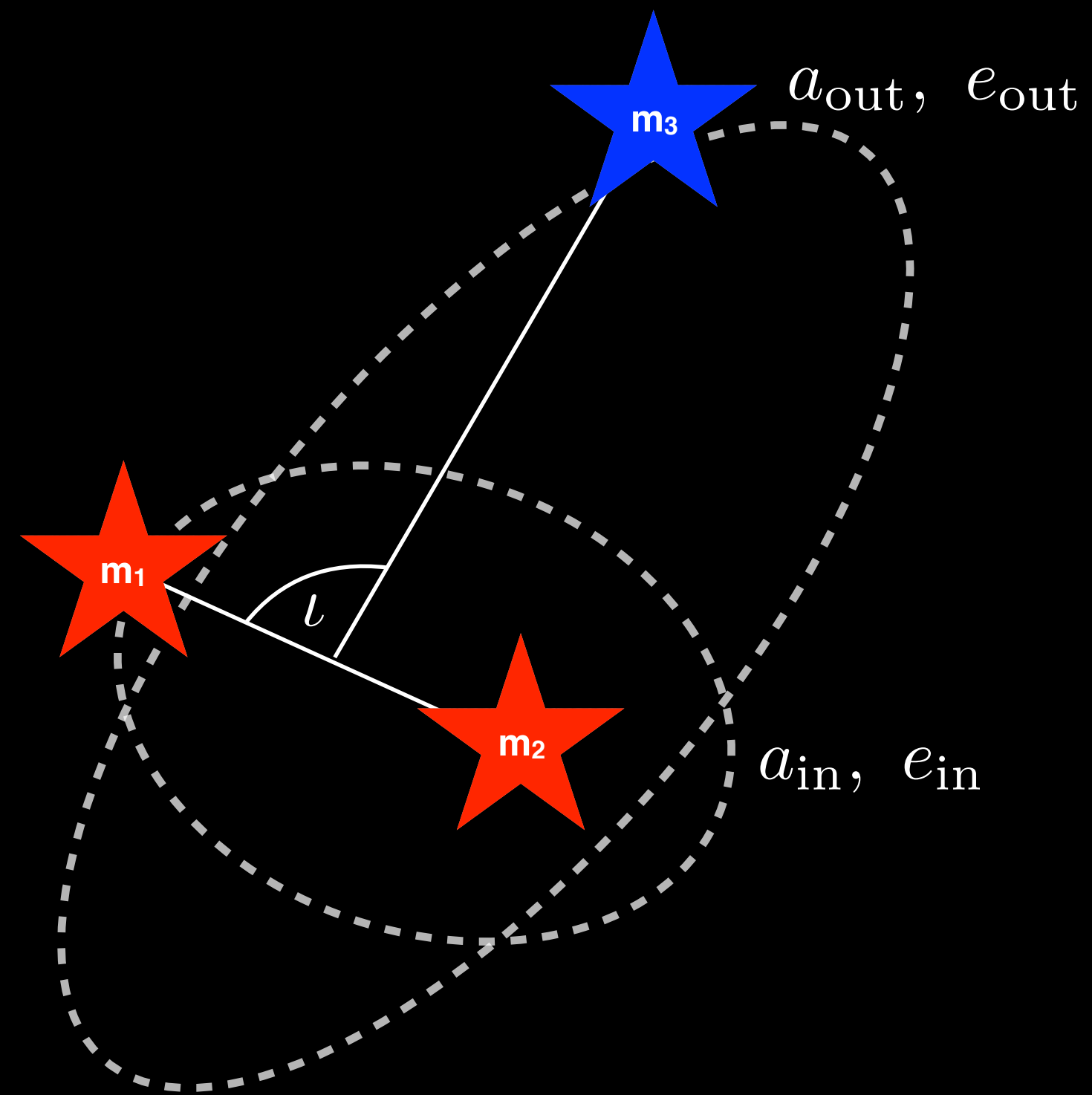
# Eccentricity



**Kremer et al., (2019)**  
**Phys. Rev. D 99, 063003**

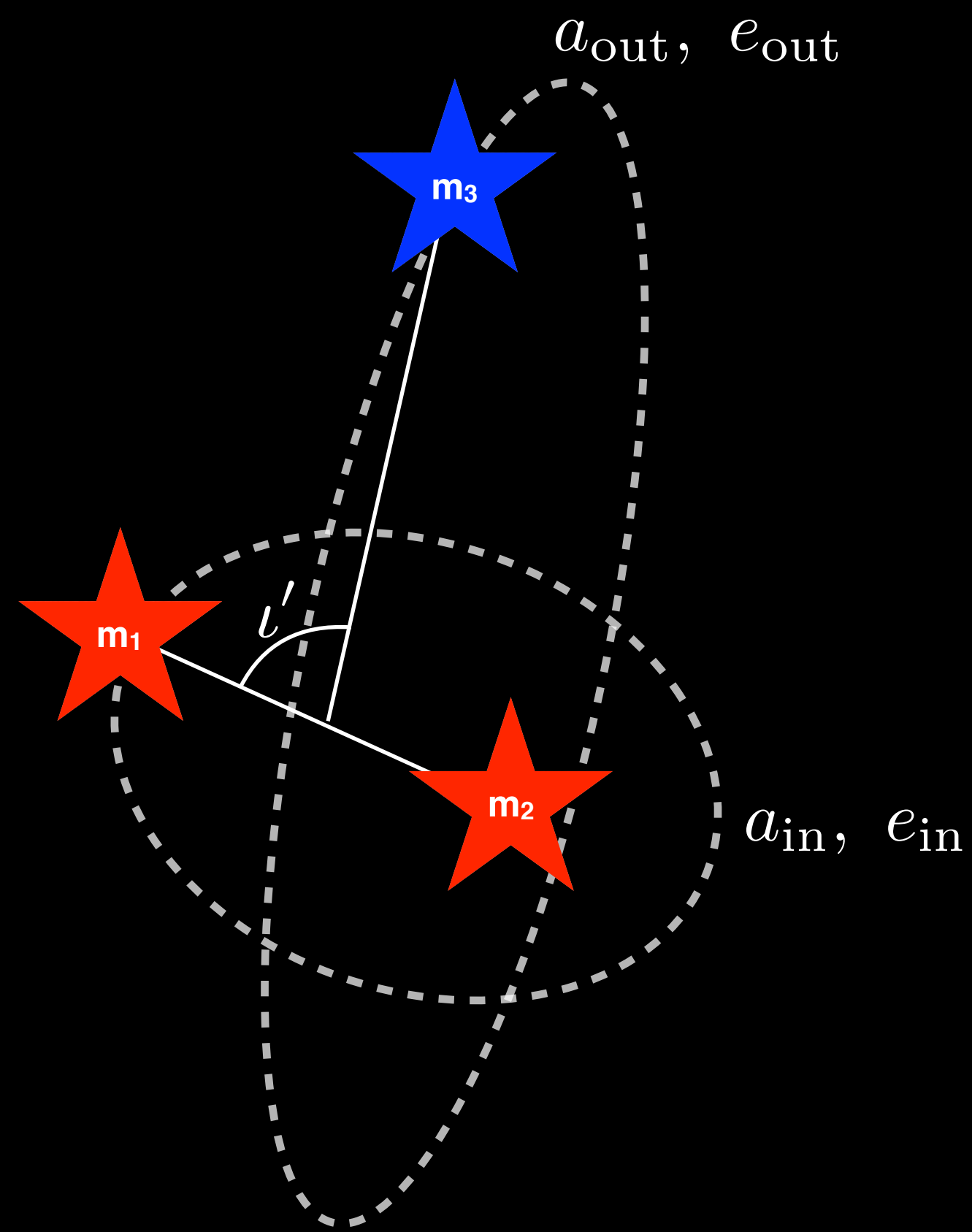


# How?



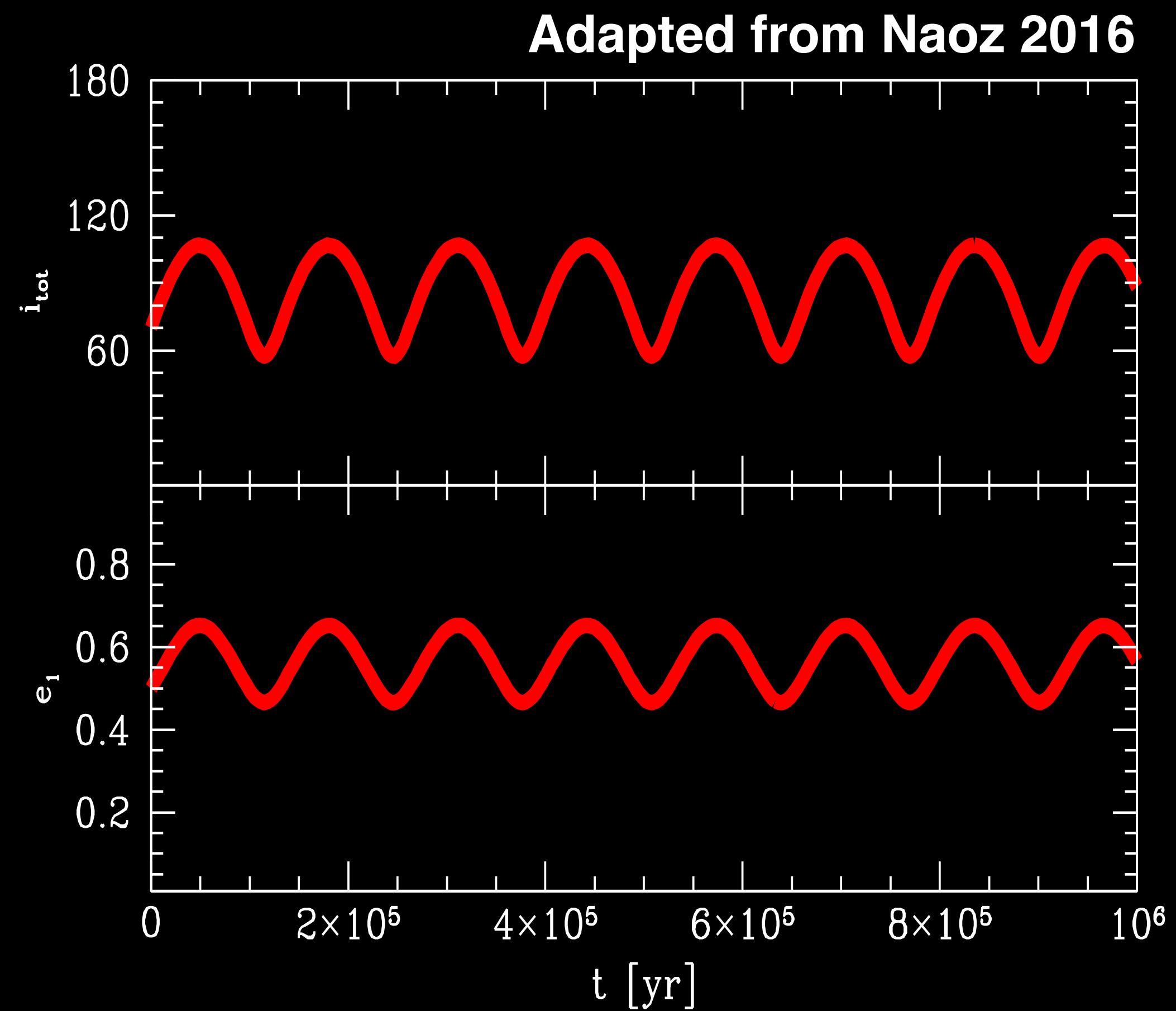
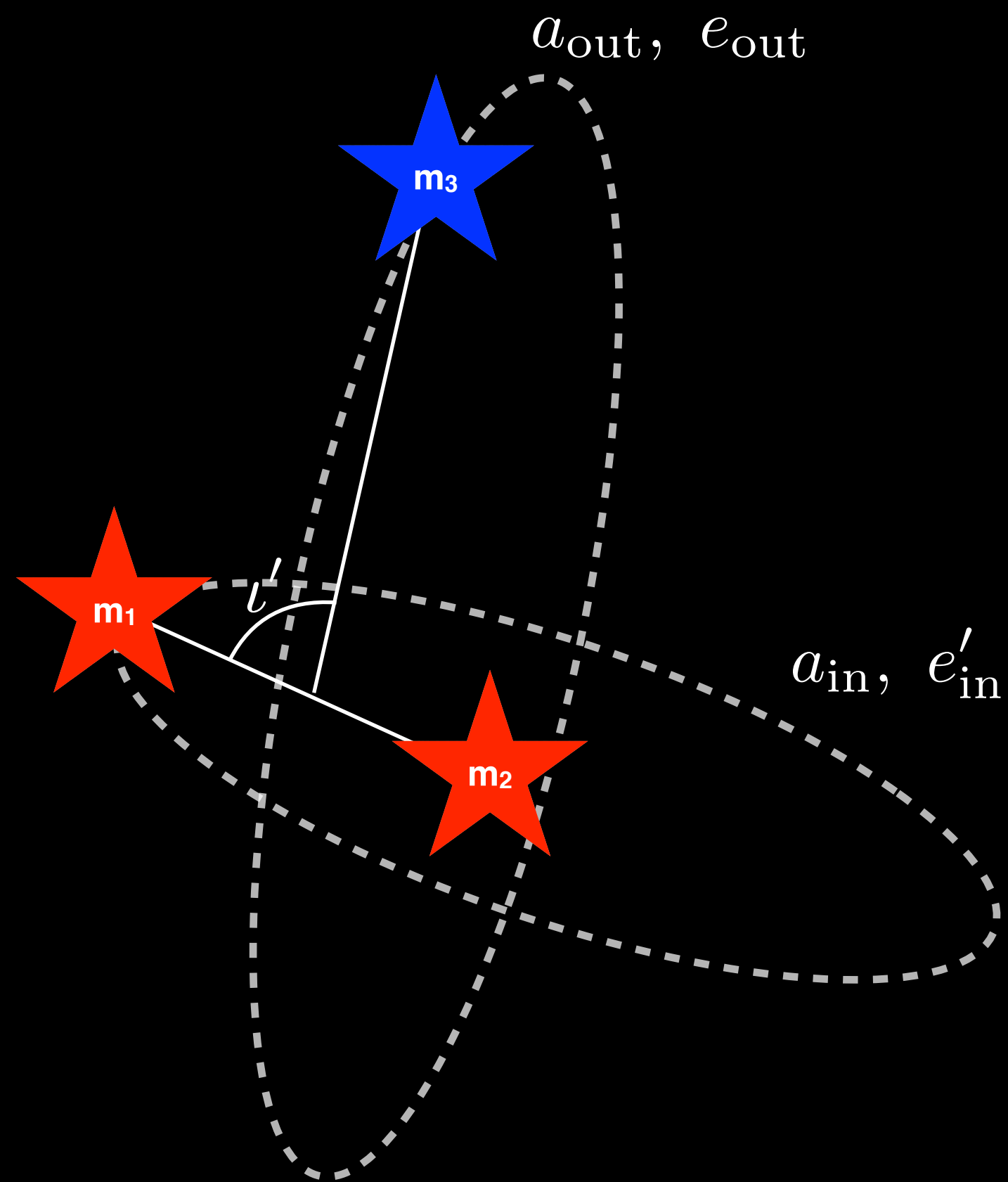


# How?



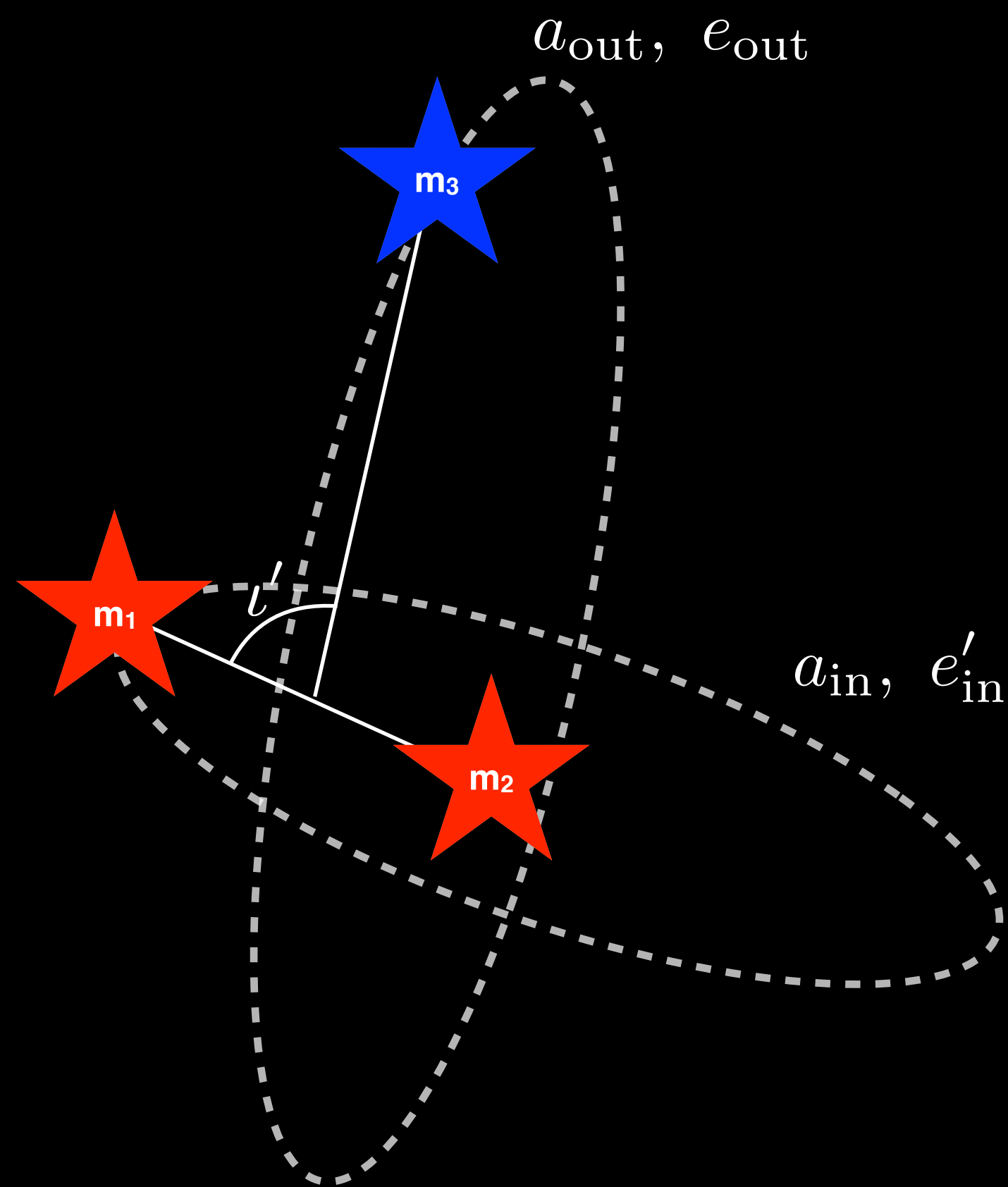


# How?

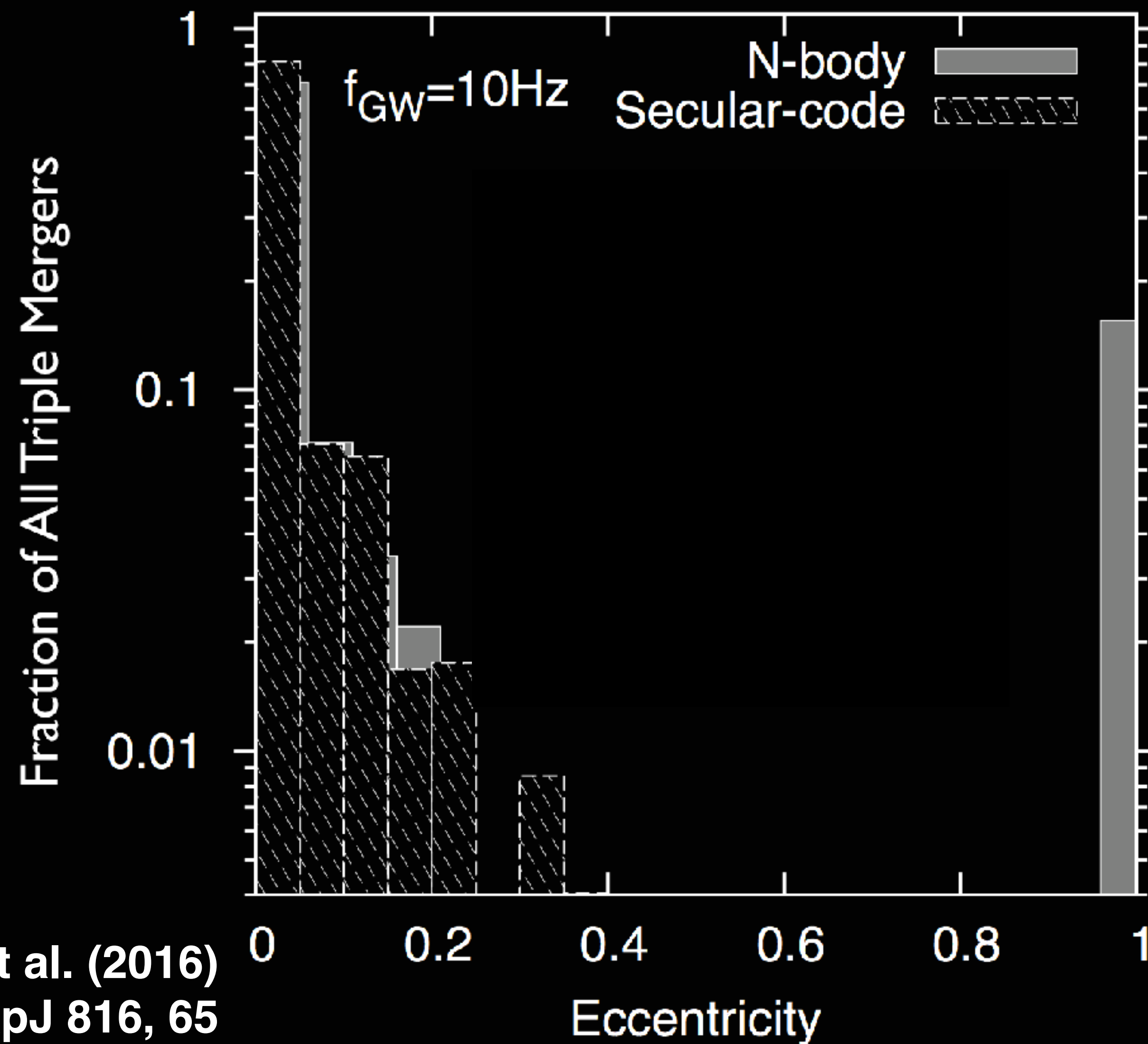




# How?

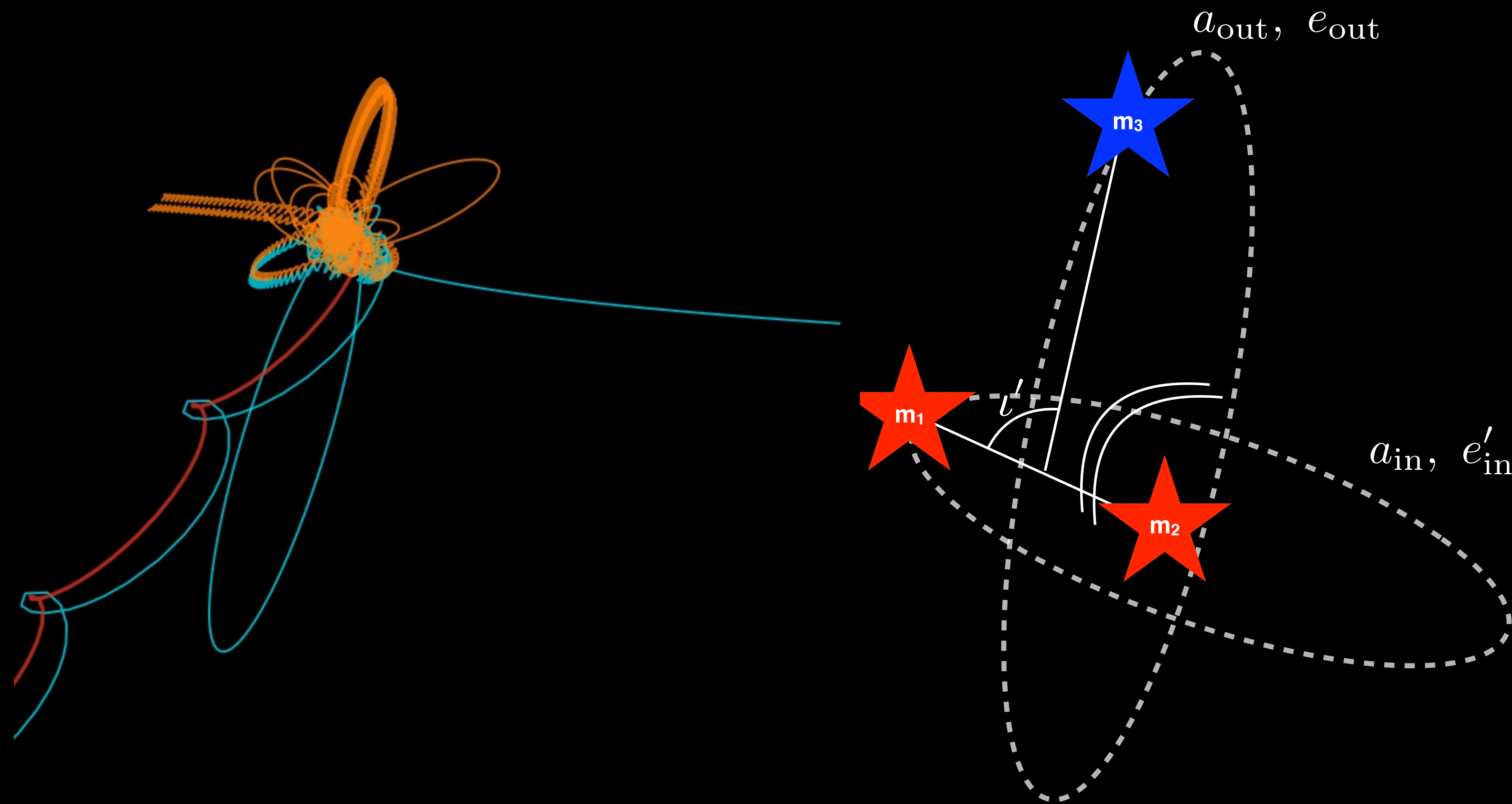


Antonini, et al. (2016)  
ApJ 816, 65





# How?



## Scatterings

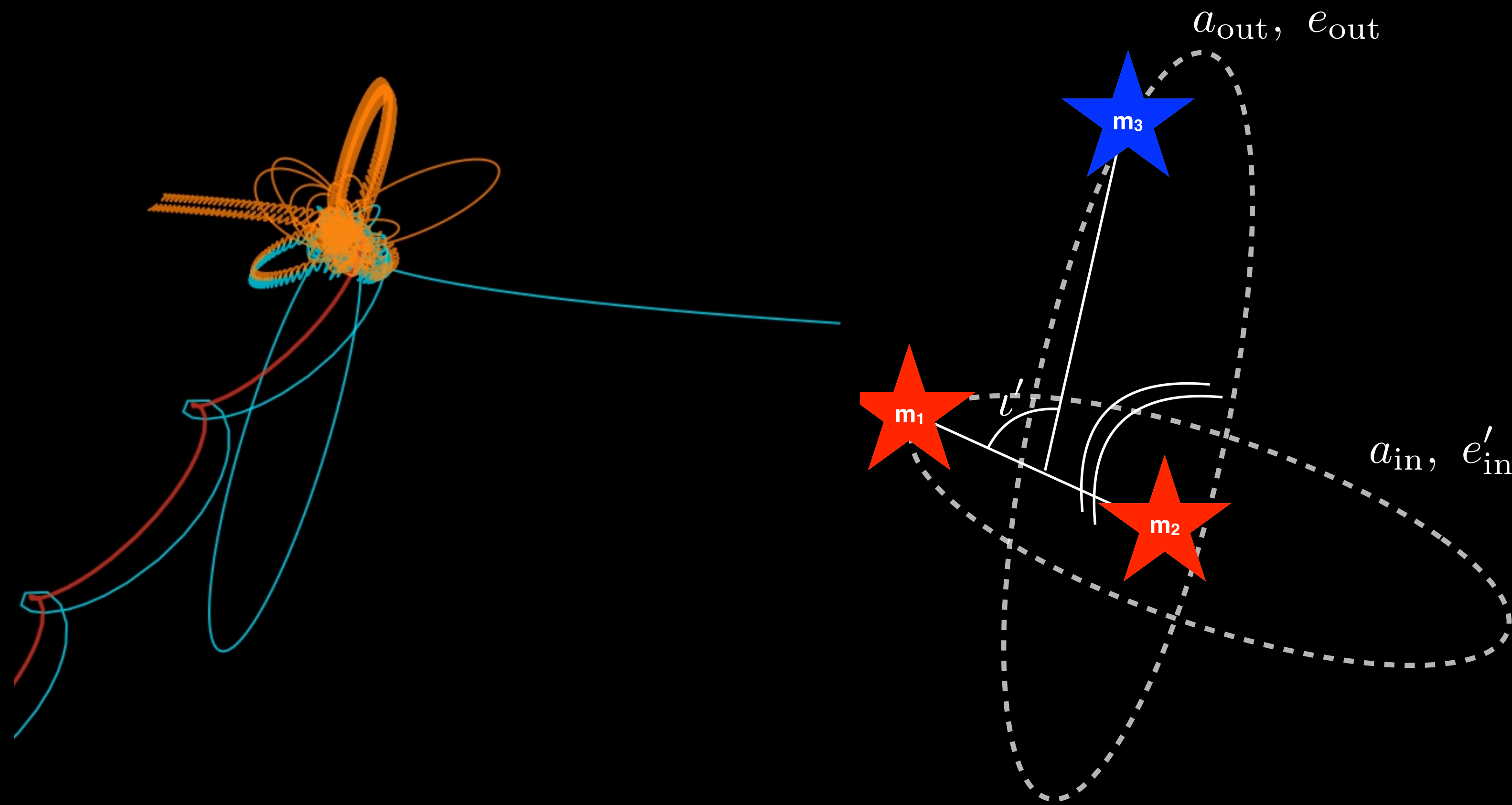
Clusters (Open/GC)  
Galaxies

## Kozai

Field Triples  
SMBH / BBH



# How?



## Scatterings

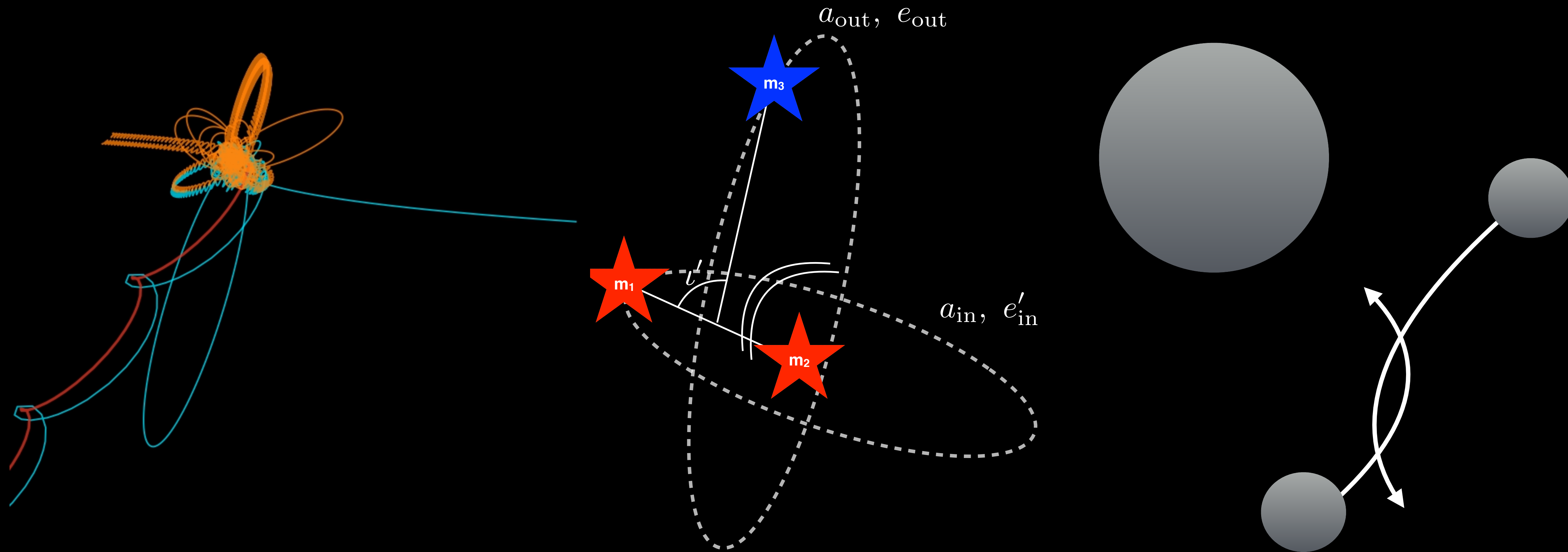
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# How?



## Scatterings

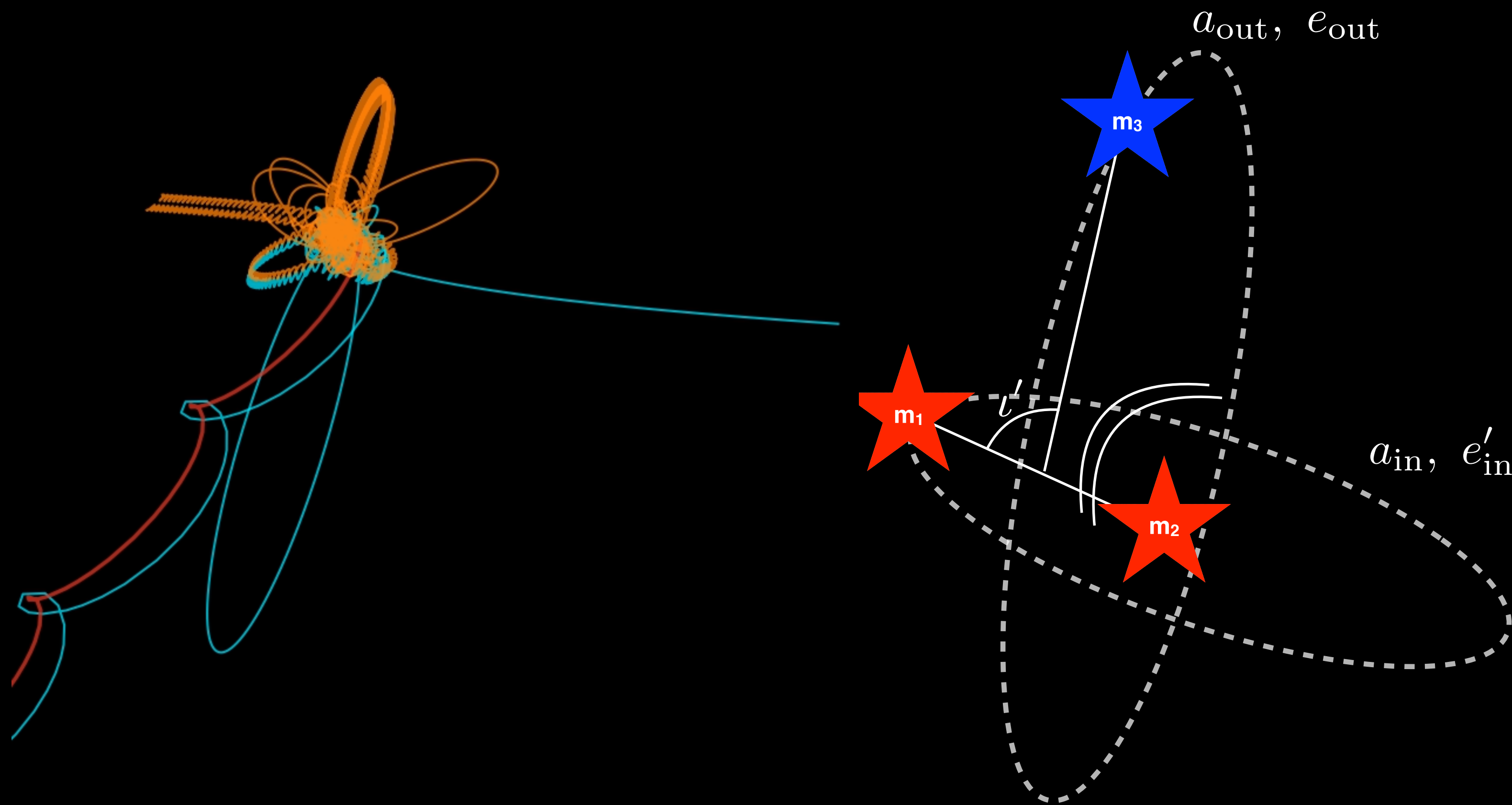
Clusters (Open/GC)  
Galaxies

## Kozai

Field Triples  
SMBH / BBH



# How?

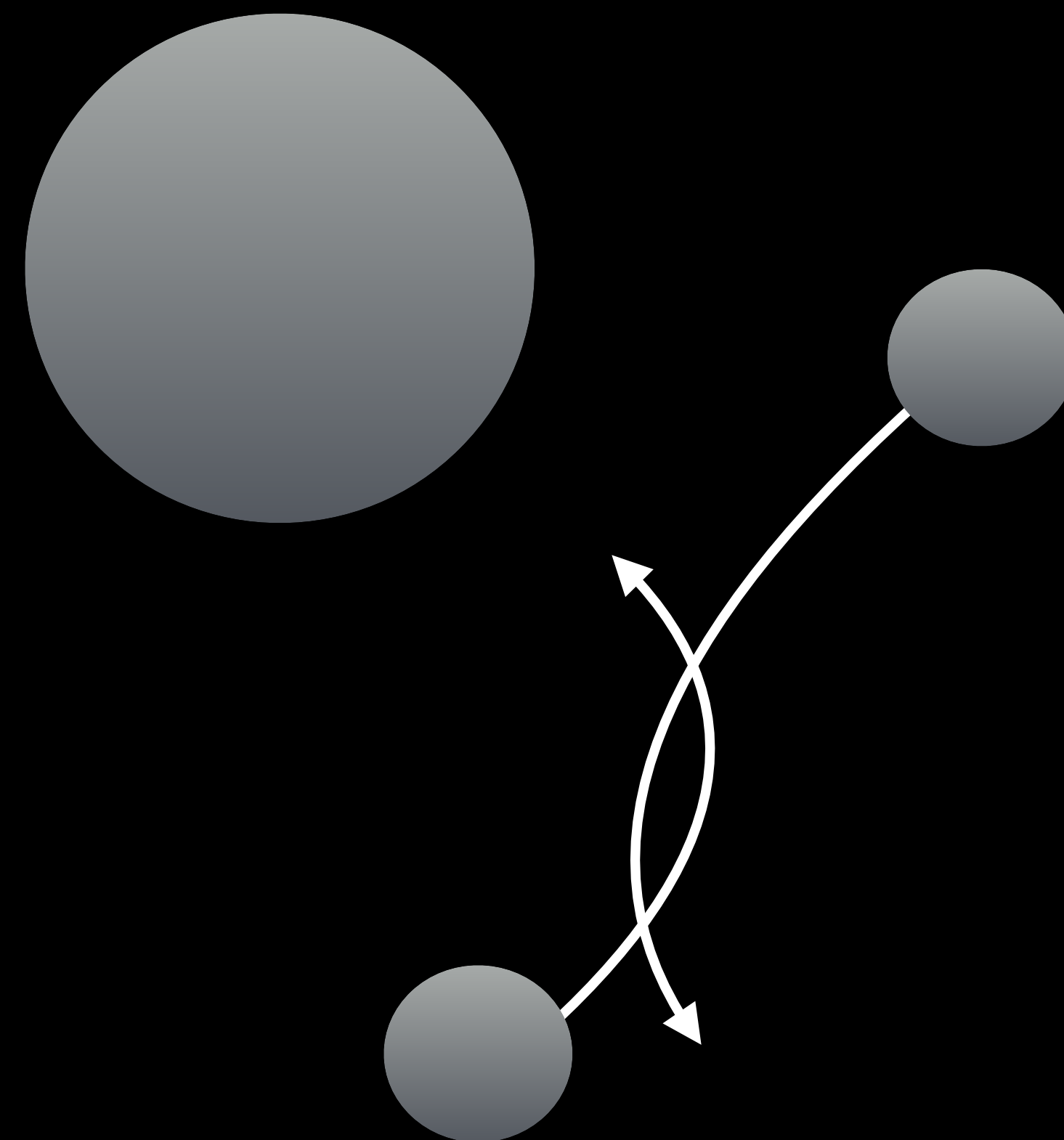


## Scatterings

Clusters (Open/GC)  
Galaxies

## Kozai

Field Triples  
SMBH / BBH



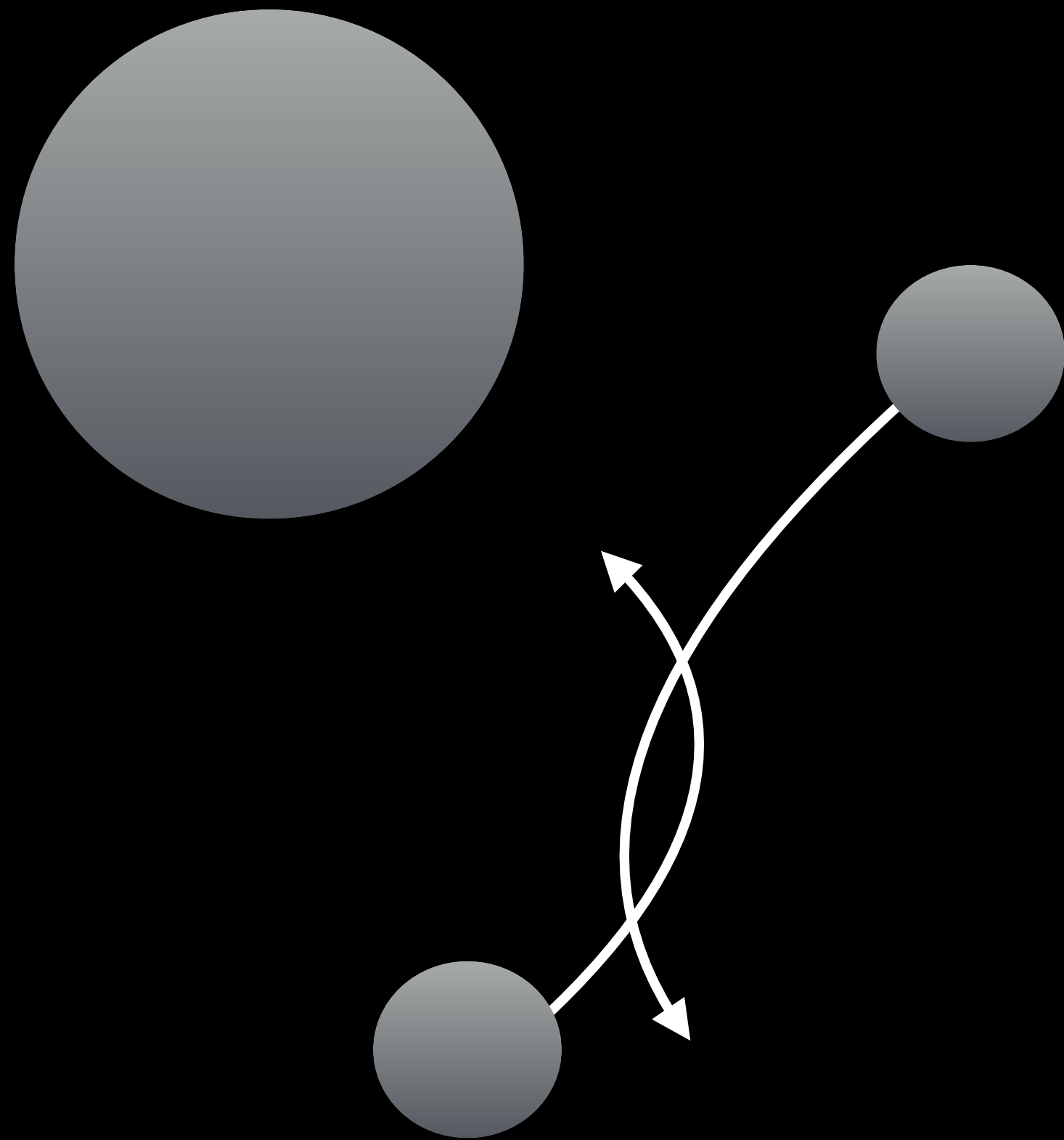
## Single-Single

GCs (not really)  
SMBHs



# How?

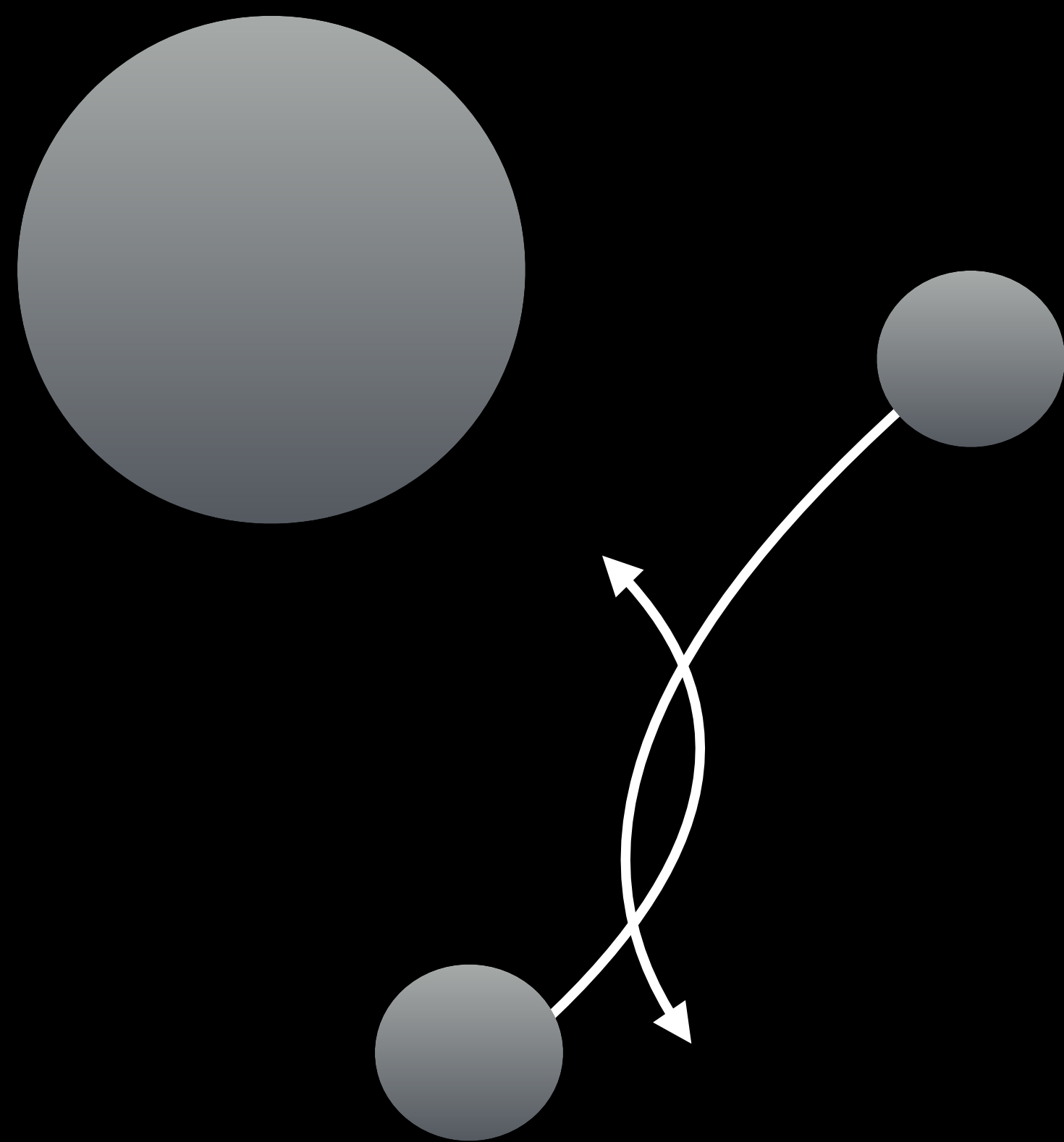
Carl Rodriguez



O'leary, Kocsis, Loeb (2009)  
MNRAS 395, 2127

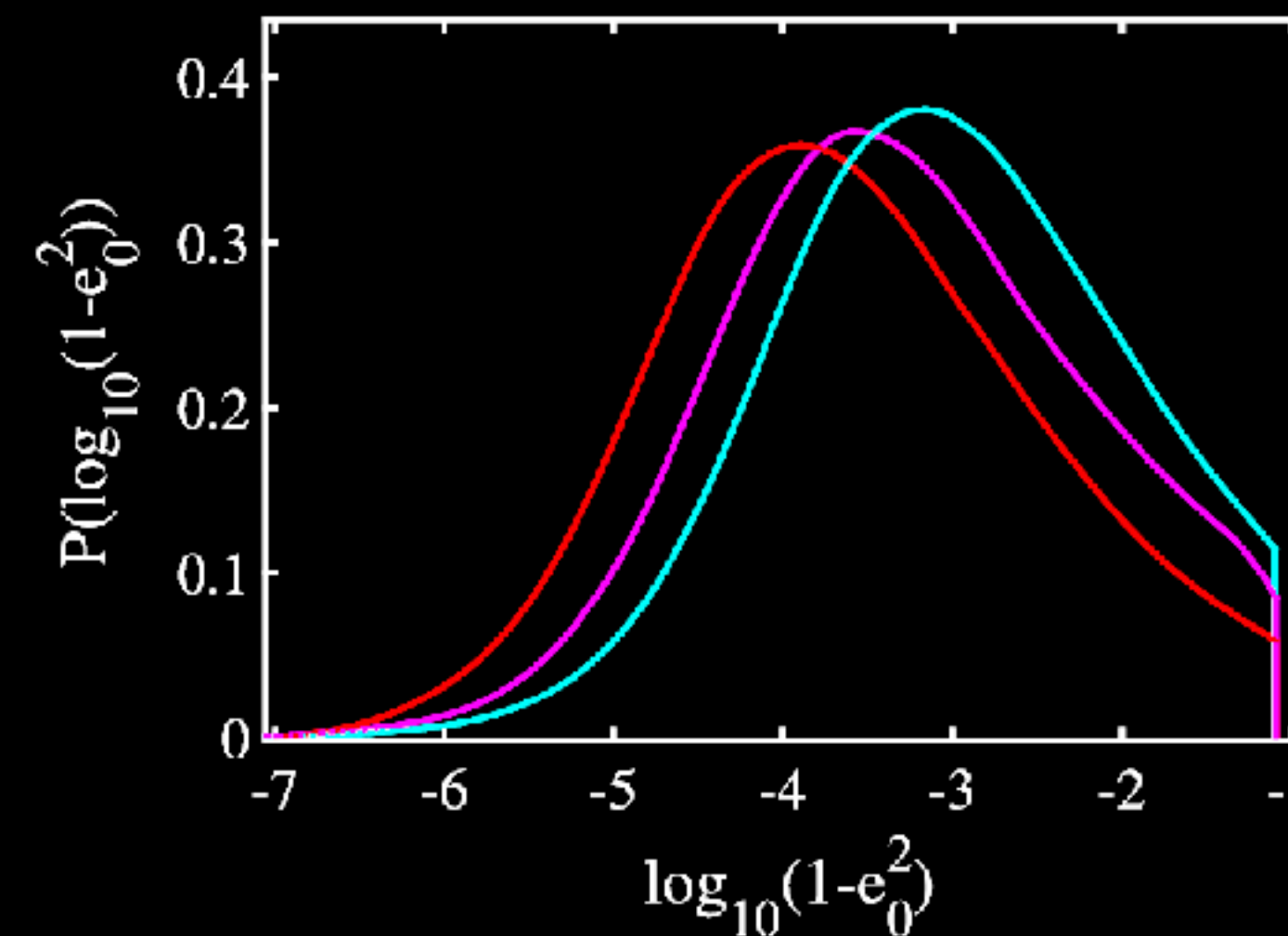
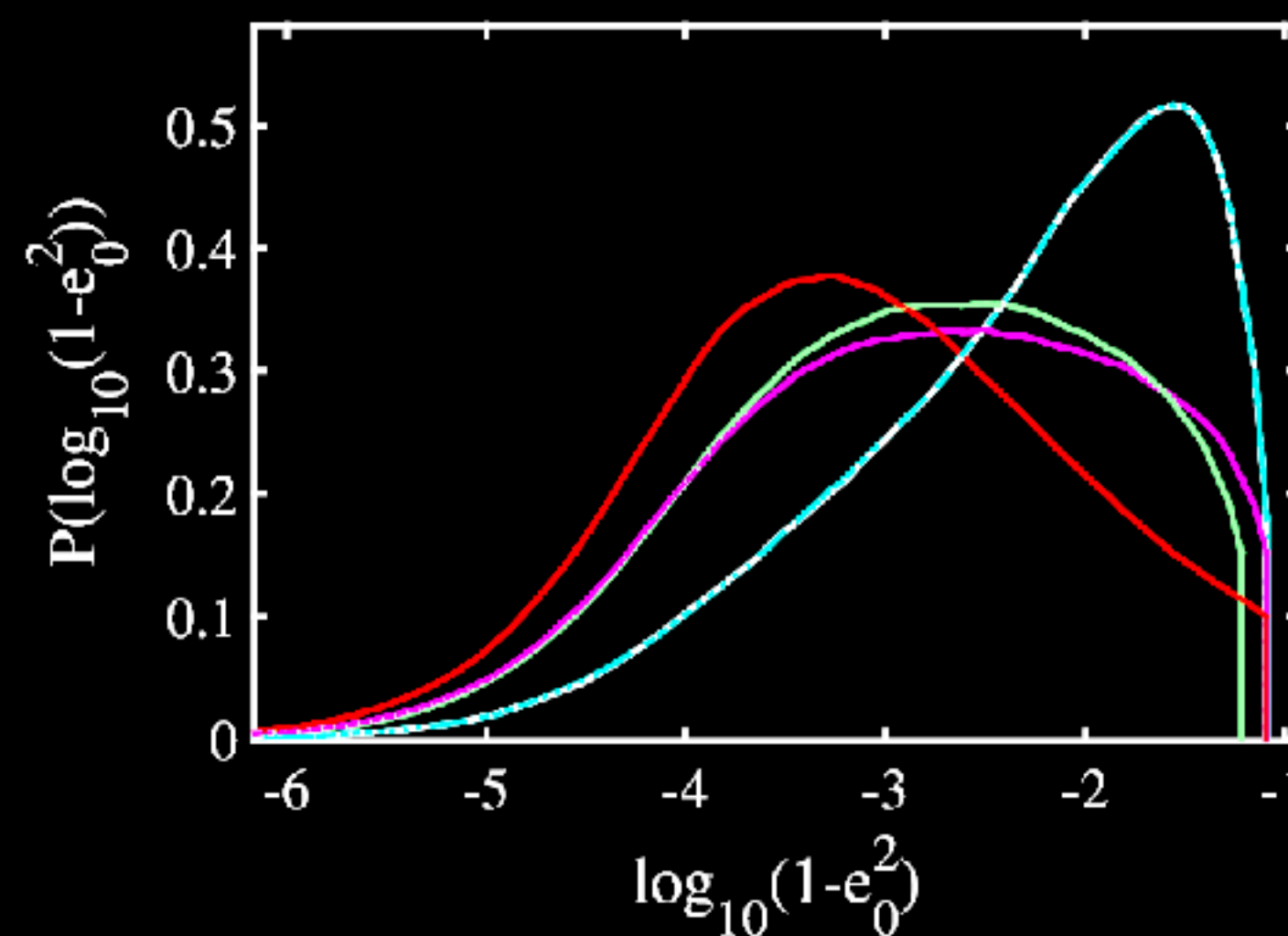
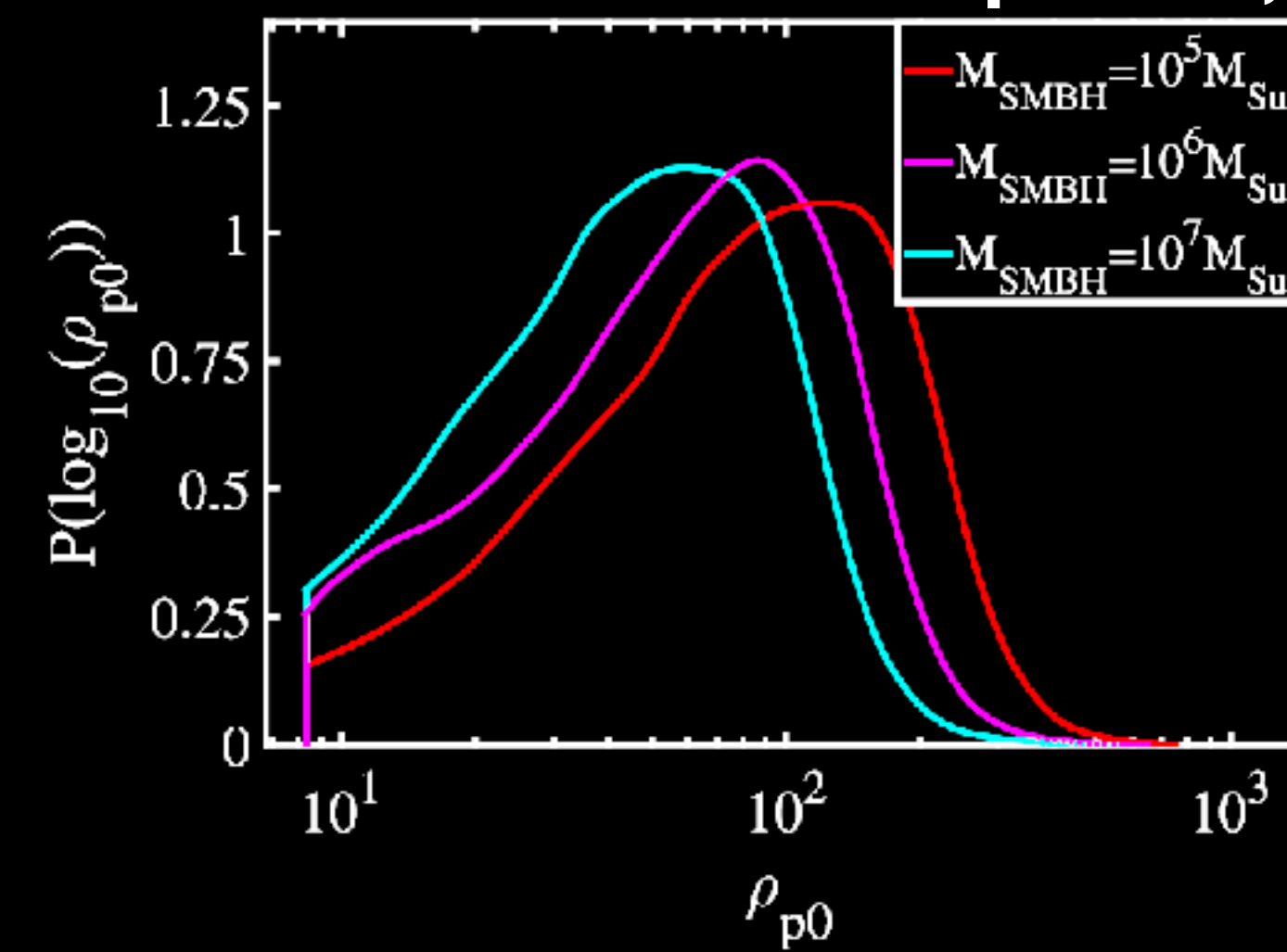
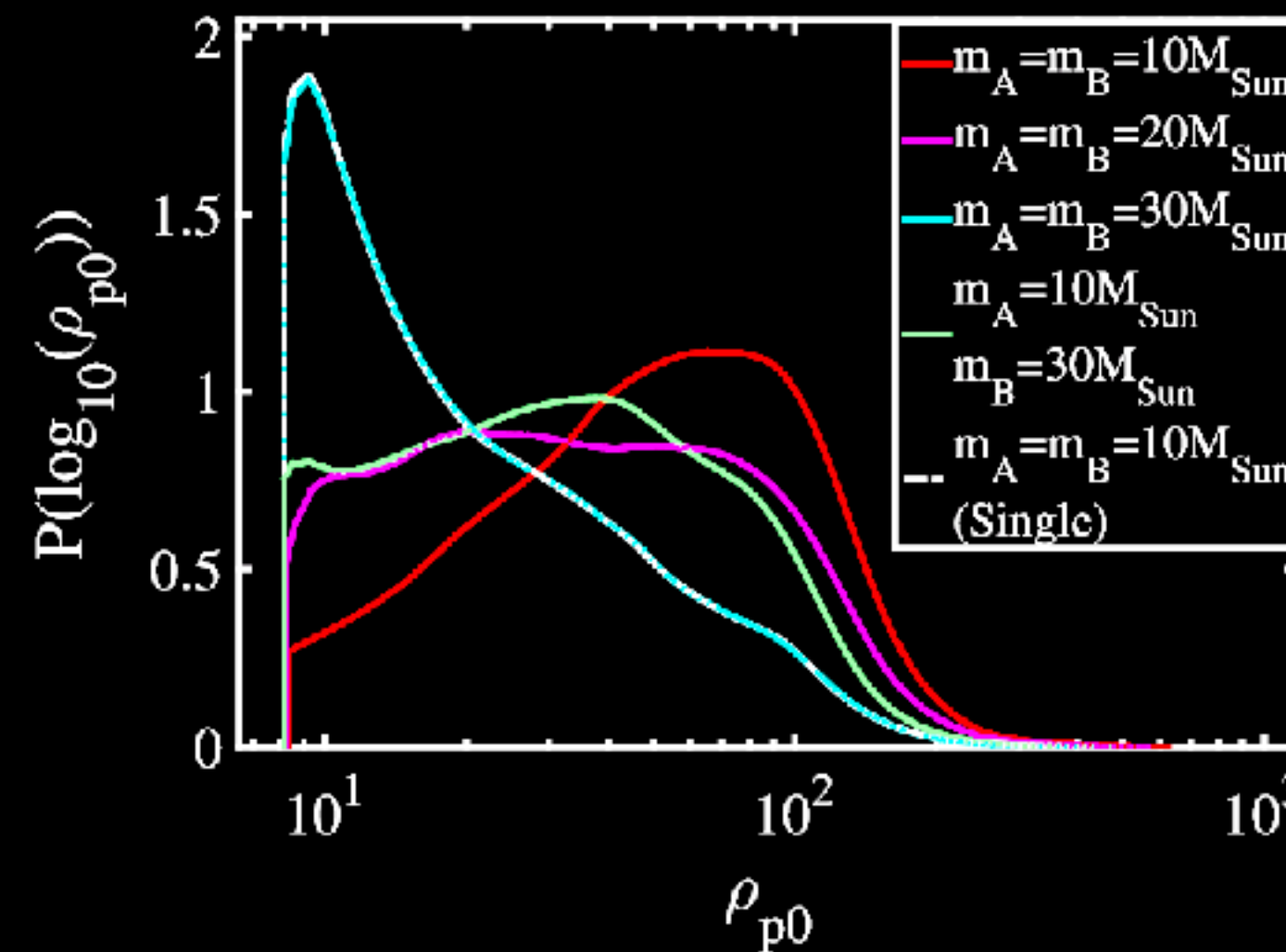


# How?



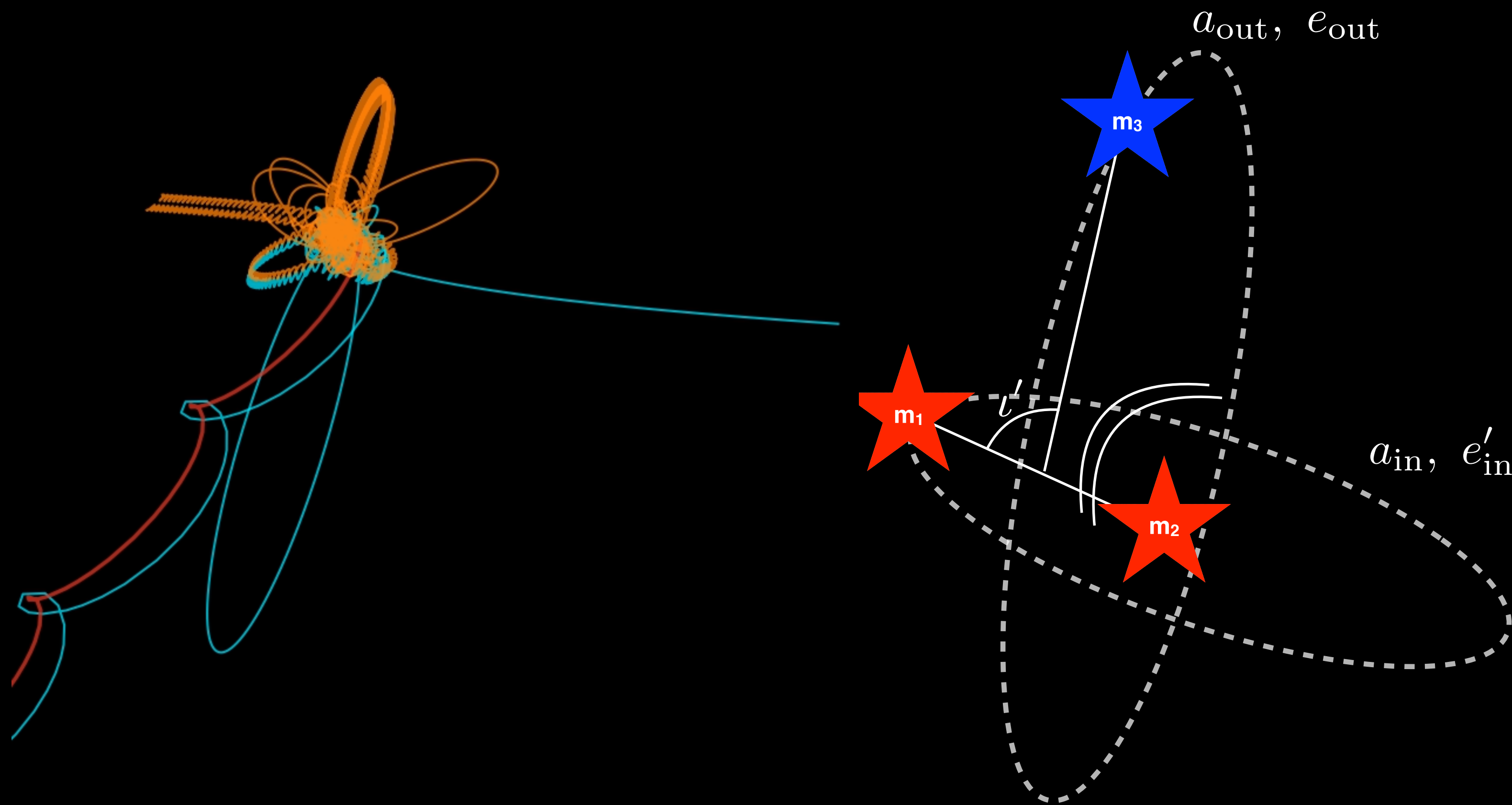
O'leary, Kocsis, Loeb (2009)  
MNRAS 395, 2127

Gondán et al. (2018)  
ApJ 860, 5





# How?



## Scatterings

Clusters (Open/GC)  
Galaxies

## Kozai

Field Triples  
SMBH / BBH

## Single-Single

GCs (not really)  
SMBHs