



# Constraining the High-Density Equation of State with Astronomical Observations

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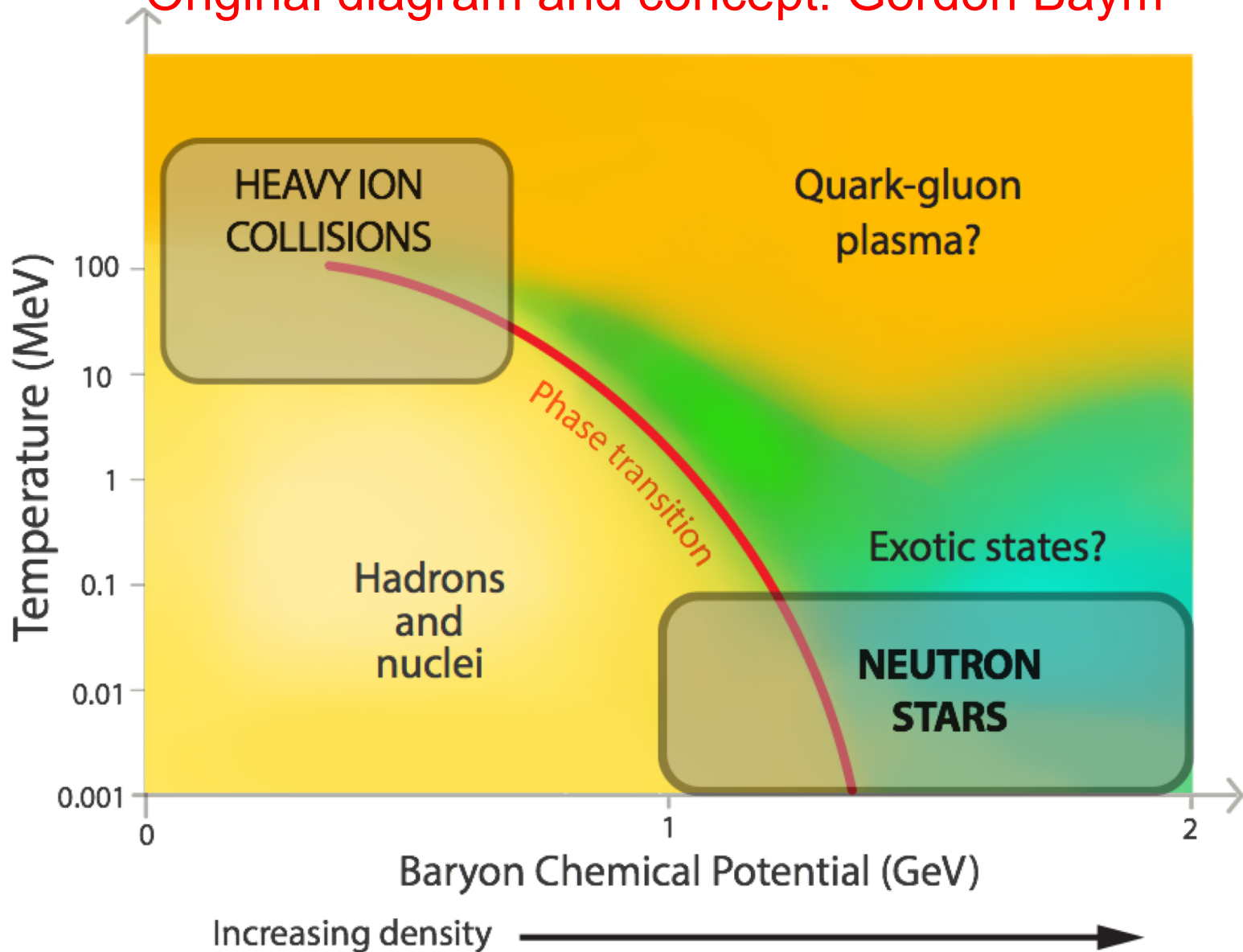
# Outline

- Neutron stars and extreme physics
- Current and future measurements
- Putting it all together: Bayesian framework
- Implications for equation of state, mass-radius relation

Based on Miller, Chirenti, Lamb 2019 (arXiv:1904.08907)

Figure kindly provided by Anna Watts

Original diagram and concept: Gordon Baym

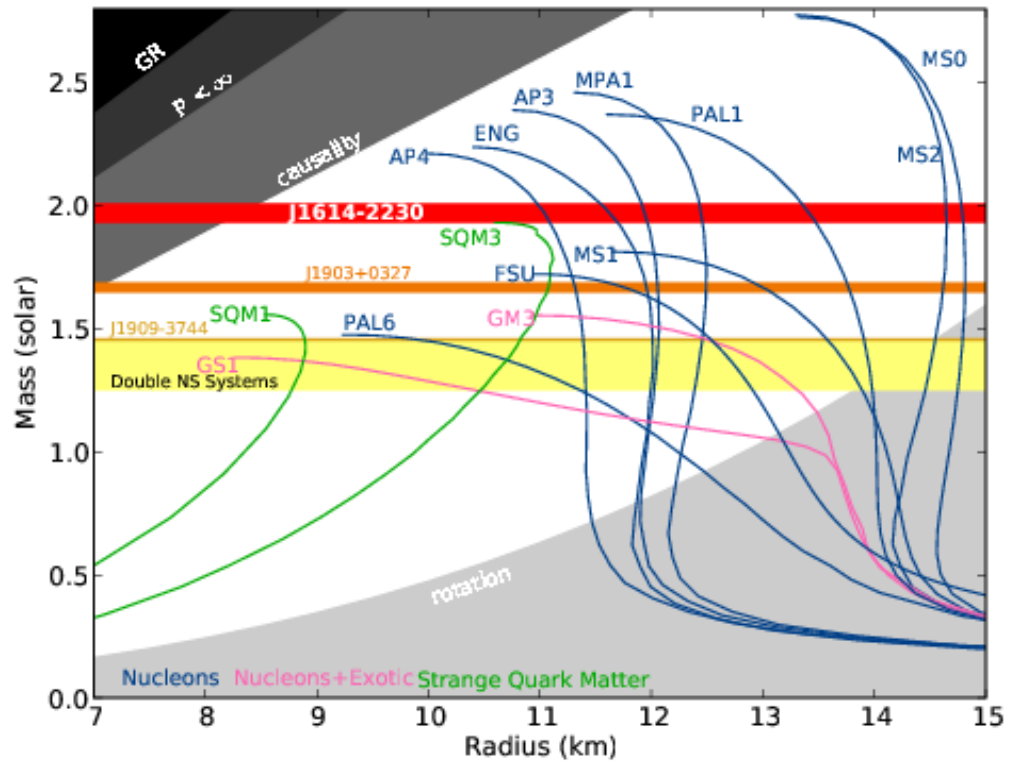


# The measurements

- What we have now:  
Masses and one tidal deformability
- What we will have soon:  
Radii and masses from NICER
- What we might have if we get lucky:  
NS moment of inertia, binding energy
- Now on to the details!

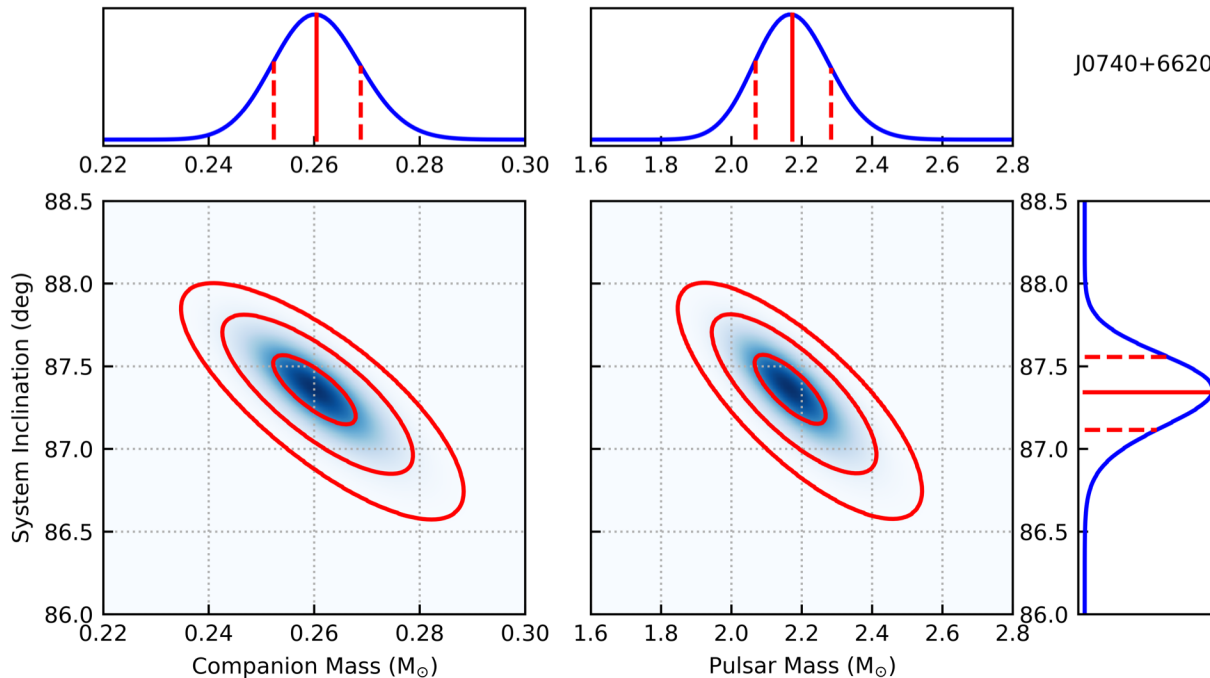
# Measurement 1: NS masses

- A given equation of state  $P(\varepsilon)$  ( $P$  is pressure,  $\varepsilon$  is total mass-energy density) predicts  $M(R)$   
**Assume equilibrium**
- Also predicts maximum mass
- Viable EOS must accommodate largest measured mass



Demorest et al. 2010

# New mass: $2.17^{+0.11}_{-0.10} M_{\text{sun}}$



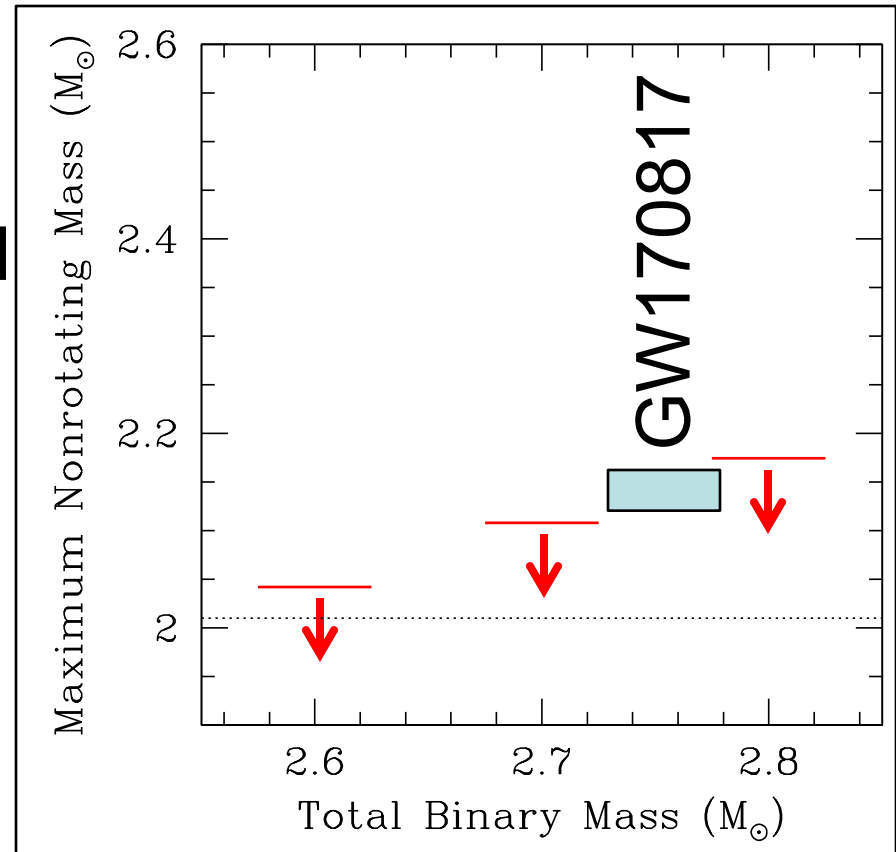
J0740+6620

Cromartie+,  
2019

- Other high masses:  $2.01 \pm 0.04 M_{\text{sun}}$ ,  
 $1.908 \pm 0.016 M_{\text{sun}}$   **$\sim 2.4 M_{\text{sun}}$  for BW?**
- Eliminates soft equations of state

# Upper Limit to NS $M_{\max}$ ?

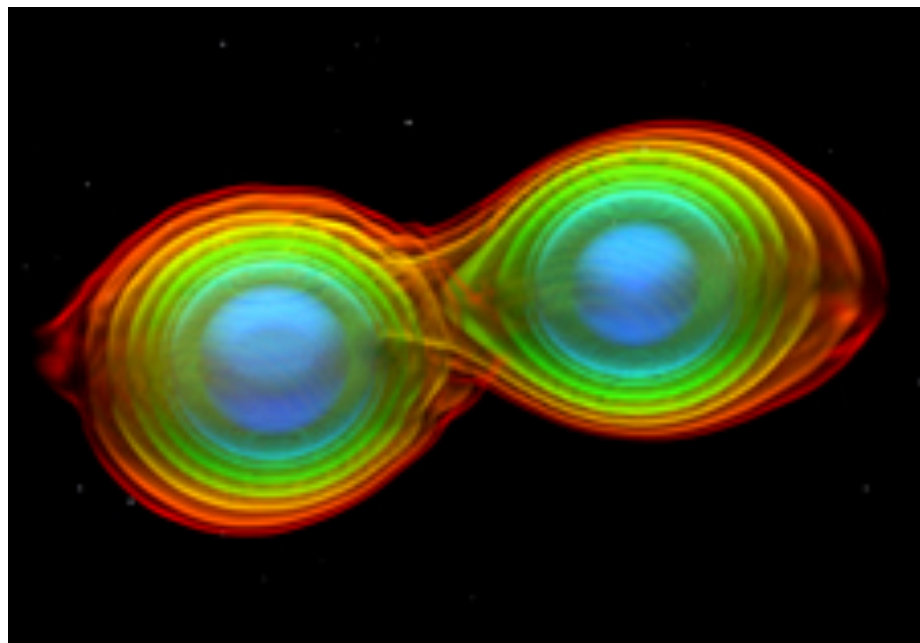
- Bauswein+13; Fryer+15; Lawrence+15
- For GW170817, we would find a limit of  $\sim 2.15 M_{\text{sun}}$   
**Margalit & Metzger 2017**  
**find  $M_{\max}=2.17 M_{\text{sun}}$**
- But we don't have clear evidence of collapse
- Similar model-dependent upper limits suggested from GRMHD simulations



Adapted from Lawrence et al. (2015)

# Meas. 2: NS tides from GW

- Tides take energy from orbit See Read talk  
Changes waveform
- A bigger NS will be deformed more
- Thus measurement of tidal deformability  $\Lambda$  gives insight into structure
- GW170817: ~soft EOS

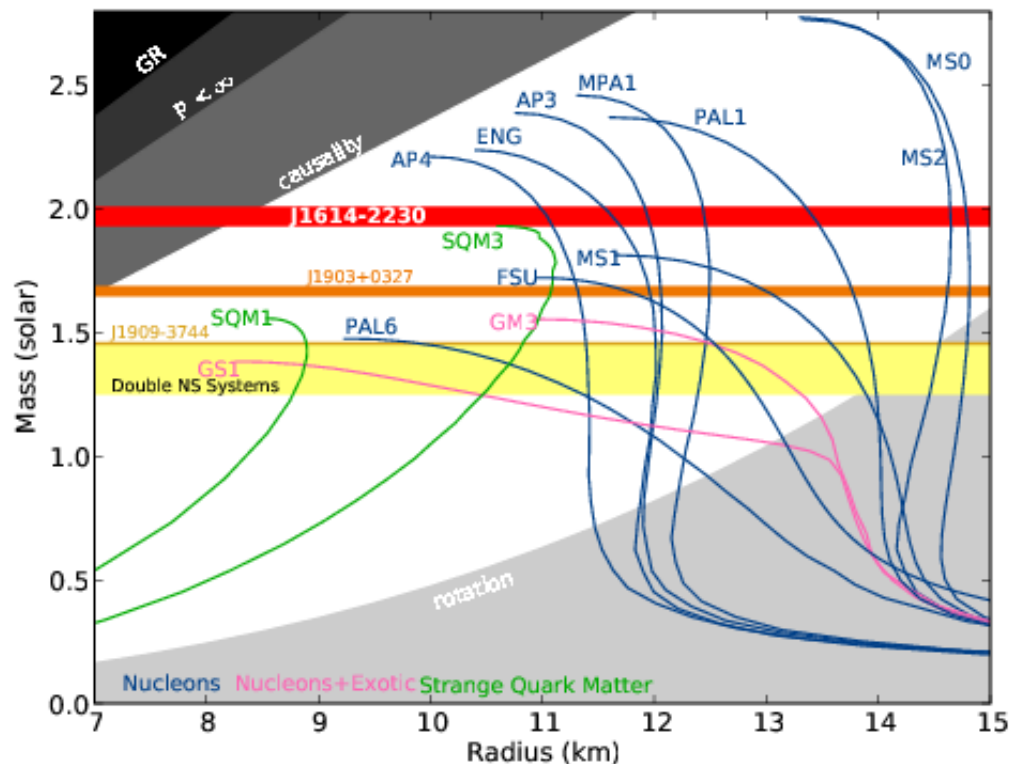


Simulation: T. Dietrich et al.  
(Albert Einstein Institute)



# Future Meas. 1: R and M

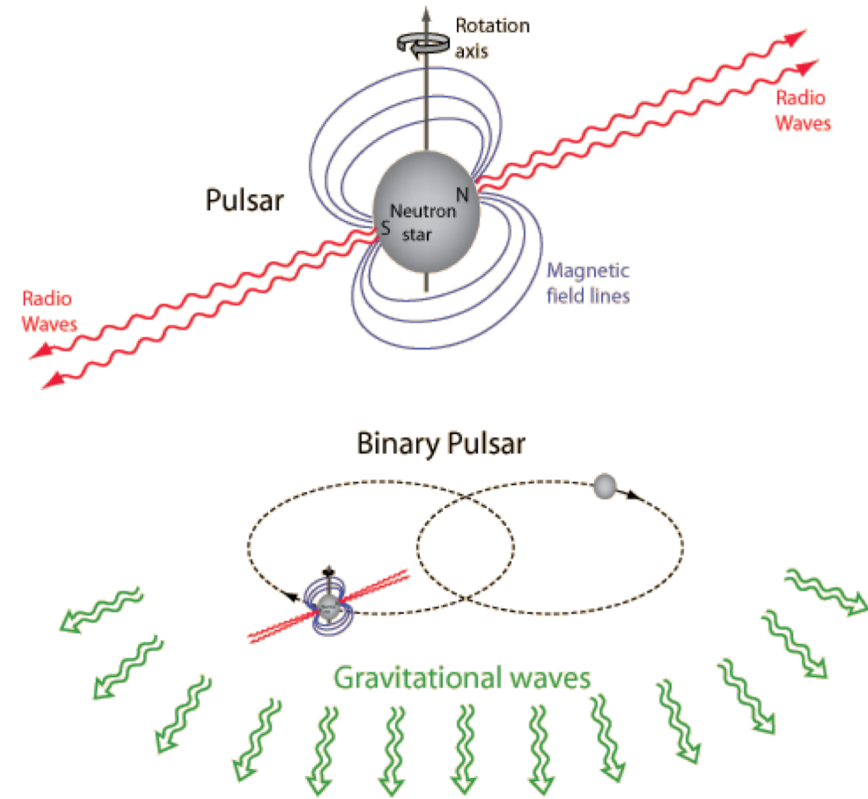
- Radius would provide great leverage  
**Wide range in models**
- But tough to measure
- All current published measurements are susceptible to huge systematic error
- NICER pulse profile modeling can help



Demorest+ 2010

# Future Meas. 2: Inertial Moment

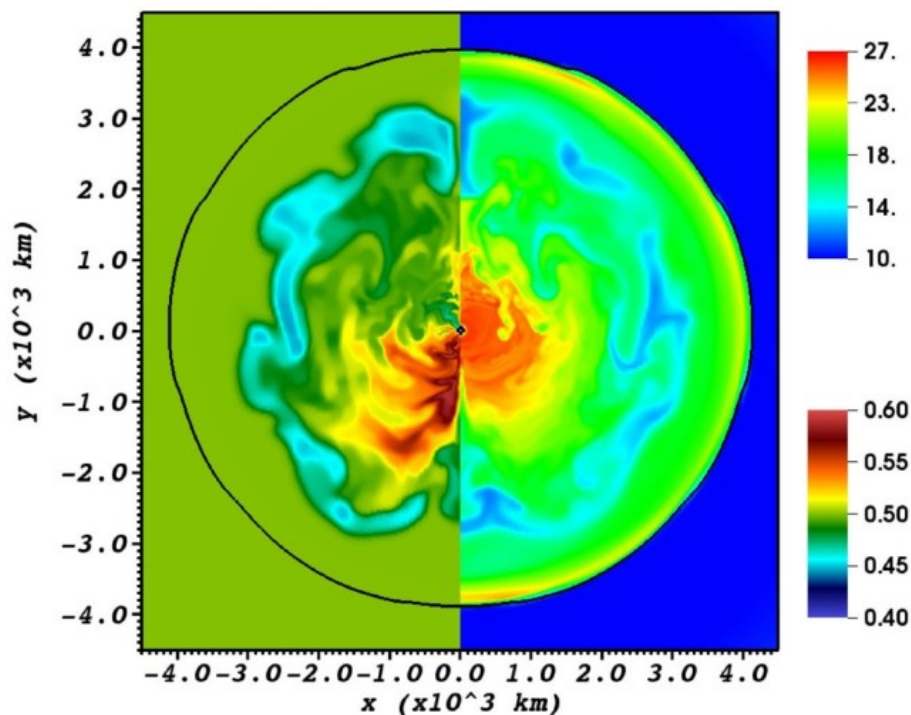
- Binary pulsars  
**First evidence of GW!**
- Highly precise masses
- Rotation of NS drags spacetime with it  
**Extra precession**
- Precession depends on angular momentum, and thus on  $I$  for known  $\Omega$
- Maybe measure in 10 yr?



<http://hyperphysics.phy-astr.gsu.edu/hbase/Astro/imgast/binpulse.gif>

# Future Meas. 3: Binding Energy

- If we know baryonic rest mass  $M_{\text{bary}}$  and grav mass  $M$ , we know NS binding energy
- Suggestion: NS from  $e^-$  capture supernovae might have known  $M_{\text{bary}}$   
**Nomoto, Podsiadlowski**
- Difficulty: can we identify which NS?



electron capture SN  
Müller et al. 2017

# Putting it all together

- There are a wide variety of measurements, of different stars, which relate in different ways to  $P(\varepsilon)$
- Putting them together requires a rigorous but practical Bayesian framework
- This is the focus of Miller, Chirenti, and Lamb 2019 (arXiv:1904.08907)

# The setup

- Pick a parametrized equation of state  
*Any will do; we use the spectral parametrization of Lindblom (2010, 2018)*
- For some constraints, no marginalization is needed (example: maximum mass)
- For others, need to marginalize (example: if have  $M, R$  likelihood, need to integrate over central densities)  
*We use uniform prior on central density*

# An example of a subtlety

- How should we incorporate a high measured mass in our constraints?
- Method used by almost everyone is to set a strict lower bound  
Often: if  $M_{\max} < 1.97 M_{\text{sun}}$  for an EOS parameter combination, that combination is ruled out; otherwise, acceptable
- But what we recommend is to use the full mass posterior

# The Results

- In the following figures, we will show the 5% to 95% credible region for pressure (represented as a function of number density,  $n$ , defined as  $\rho/m_n$ ), and radius as a function of mass, given progressive incorporation of:

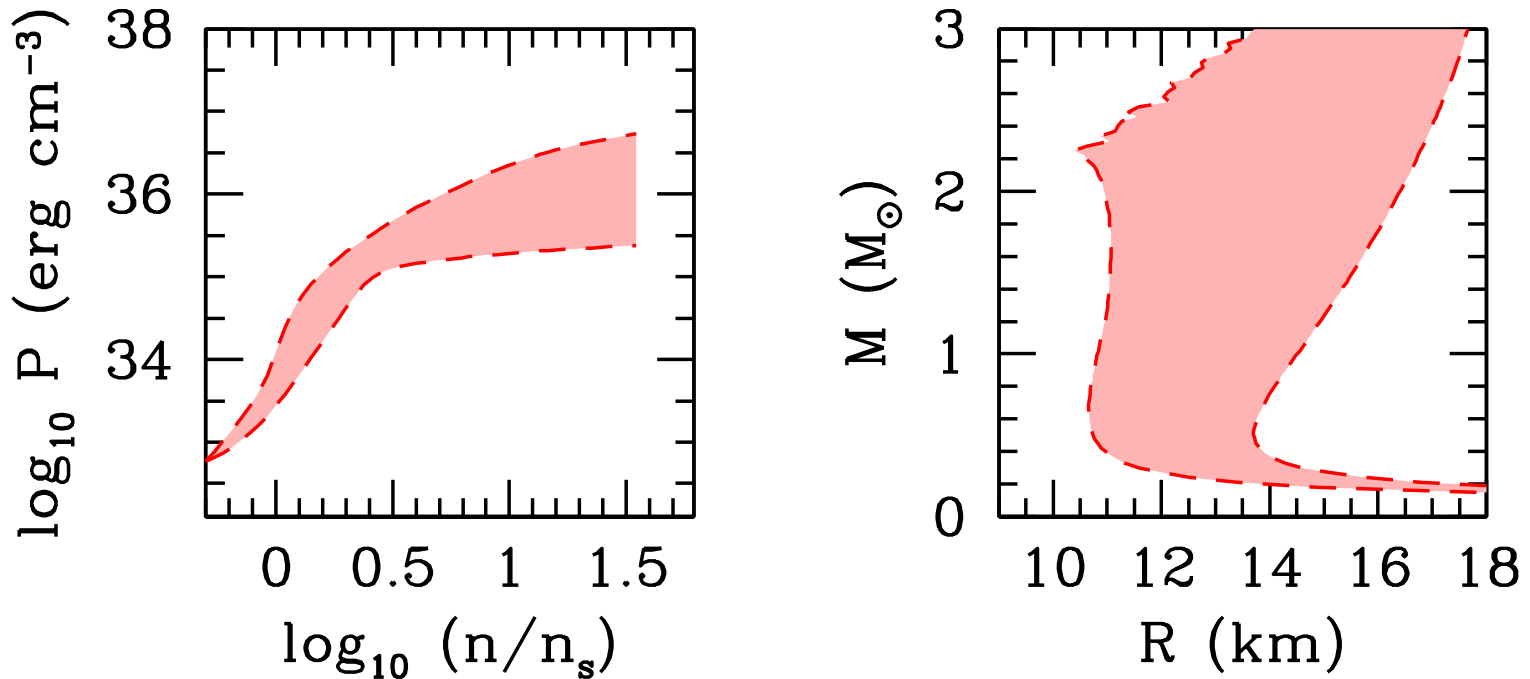
Mass, tidal deformability (in hand)

(M,R) for one star (expected from NICER)

Moment of inertia (10 years?)

Binding energy (optimistic)

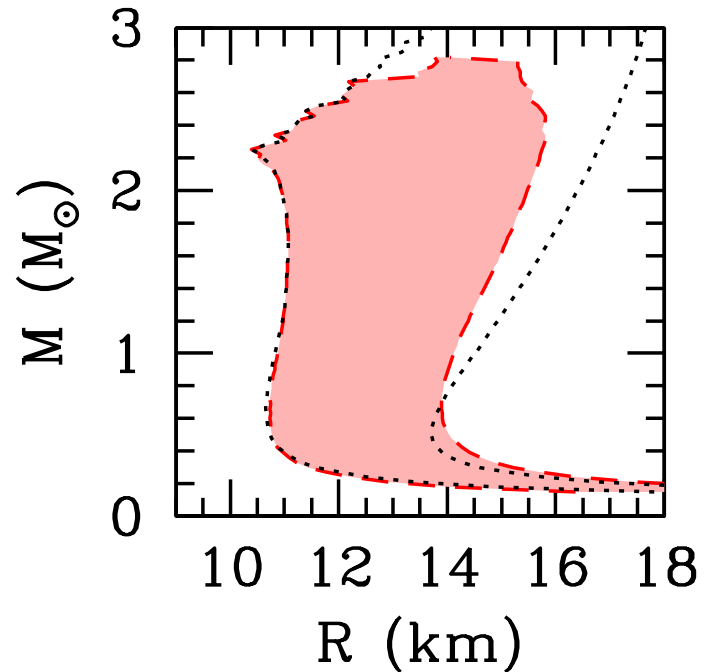
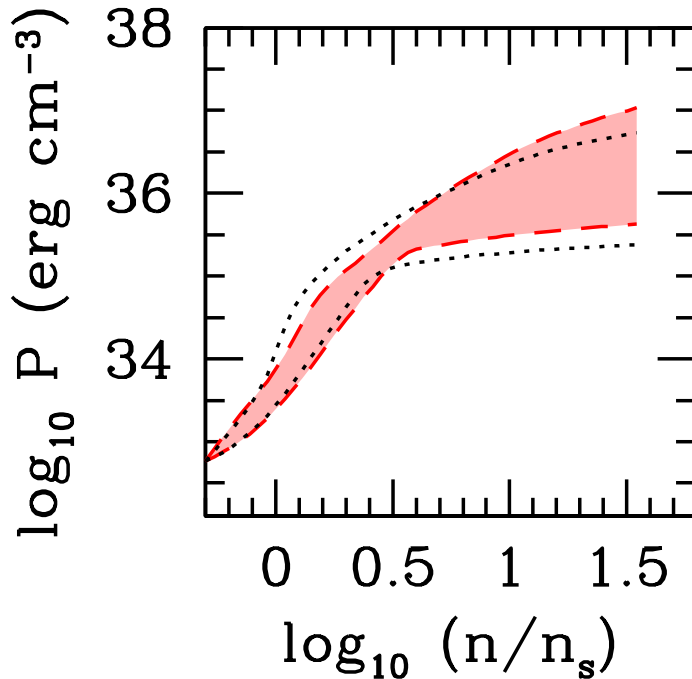
# Mass only



Use all three high-mass neutron stars;  $n_s=0.16$  fm<sup>-3</sup>  
Red region: 5% to 95% at each  $n$ ,  $M$ .  
Requirement of high masses pushes  $P$  and  $R$  high



# Mass and tidal deformability

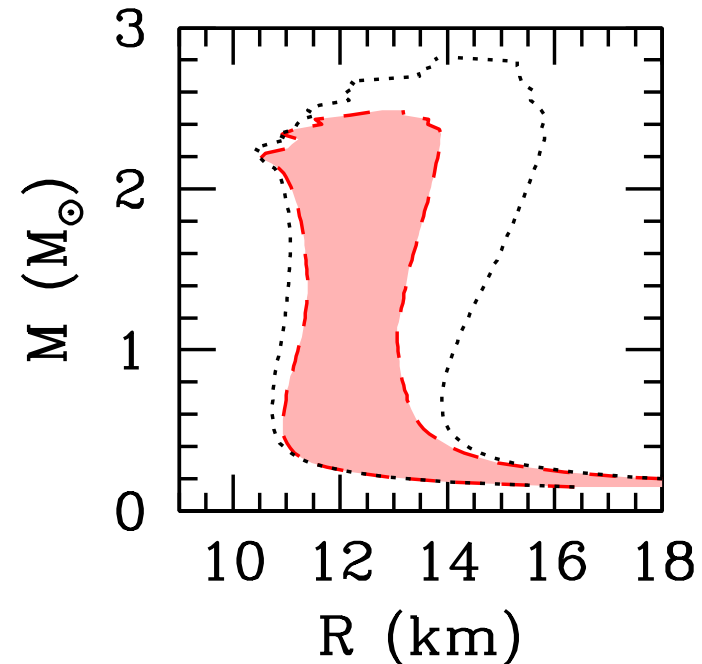
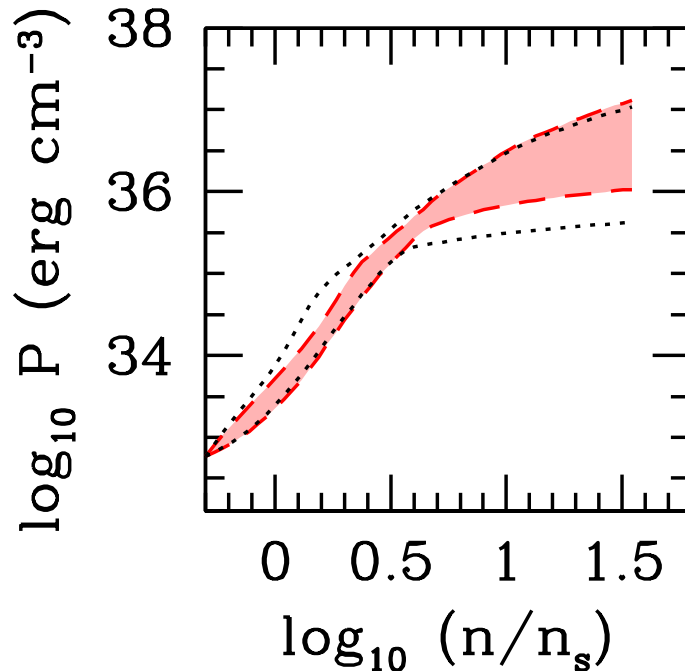


Include LIGO GW170817  $\Lambda$  measurement

Dotted lines are mass-only 5%, 95% credible regions

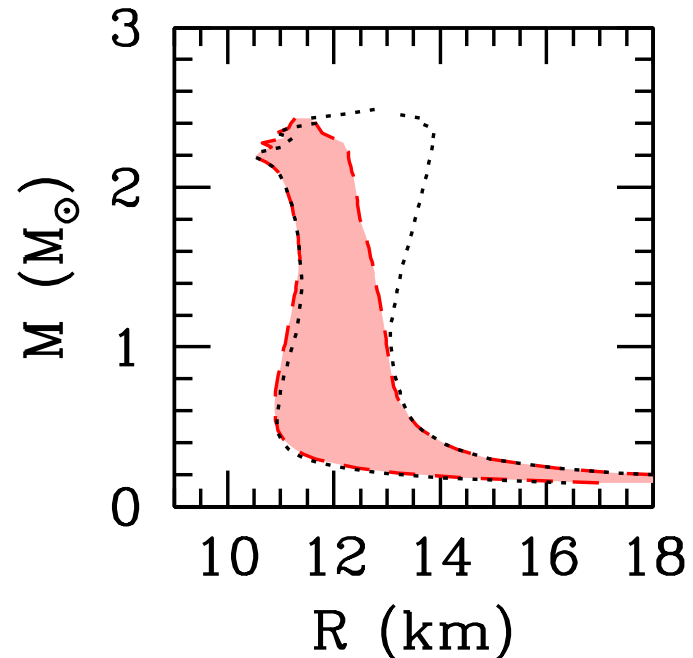
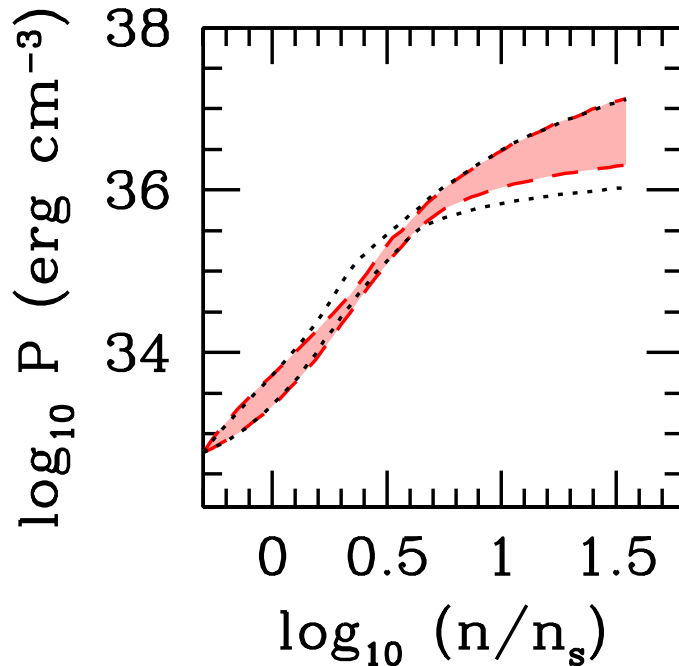
Low  $\Lambda$  pushes  $P$  lower for low  $n$ , pushes  $R$  lower

# M, $\Lambda$ , and (R,M)



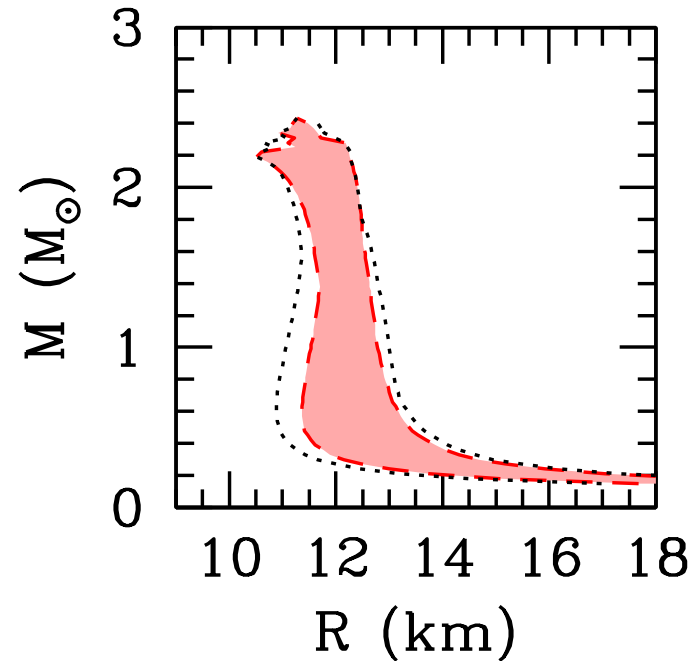
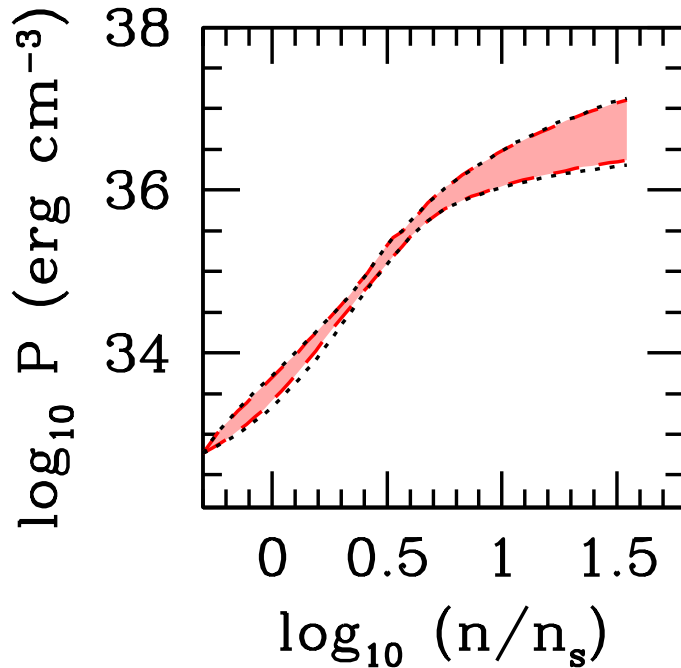
Assume 5% precision on  $(12 \text{ km}, 1.4 M_{\text{sun}})$  for one star  
Dotted lines are  $M + \Lambda$  5%, 95% credible regions  
(M,R) measurement naturally tightens R

# M, $\Lambda$ , (R,M), and I



Assume 10% fractional precision on one  $1.338 M_{\text{sun}}$  star  
Dotted lines are  $M+\Lambda+(R,M)$  credible regions  
I measurement tightens EOS,  $R(M)$  significantly

# $M$ , $\Lambda$ , $(R, M)$ , $I$ , and $E_{\text{bind}}$



Assume  $\Delta M_{\text{bary}} = 0.005 M_{\text{sun}}$  for measured  $M = 1.249 M_{\text{sun}}$

Dotted lines are  $M+L+(R, M)+I$  credible regions

$E_{\text{bind}}$  measurement reduces low- $n$  uncertainty

# Conclusions

- High neutron star masses and low tidal deformabilities are already squeezing allowed equation of state
- Mass and radius measurements from NICER will be crucial
- Succession of NS-NS events seen in GW will give us enormous information
- Finally, a data-rich era for NS interiors!