Measuring the mechanical properties of carbon nanostructures using simple physics

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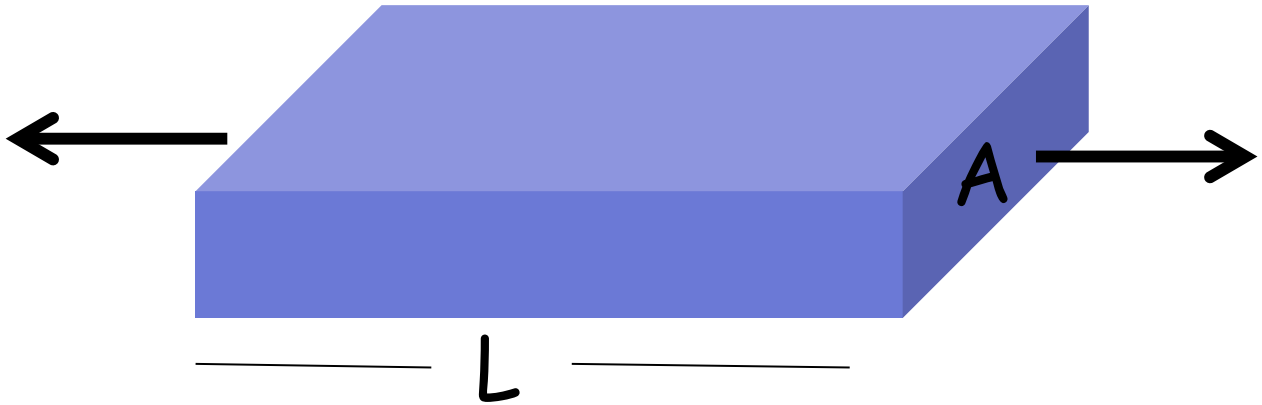
Dept. of Mechanical Engineering
Nanoscale Science and Engineering Center

Mechanical Parameters



Young's Modulus

Stress

$$\sigma = E\varepsilon$$

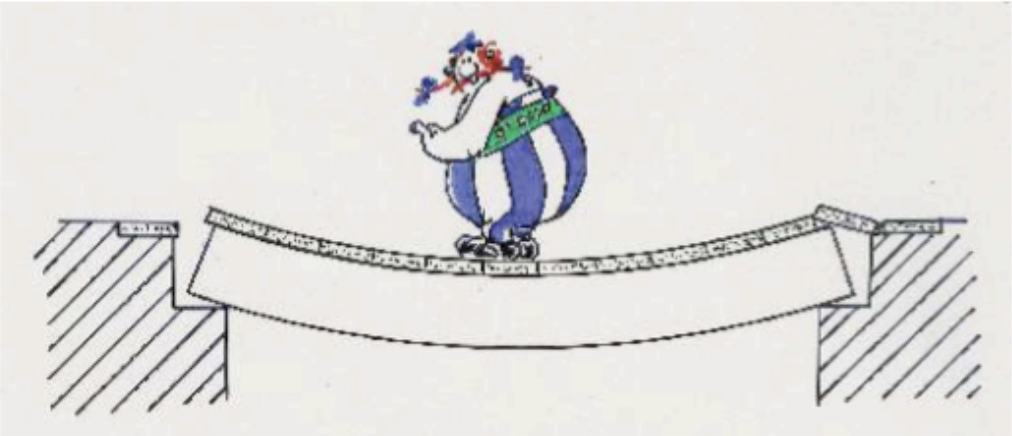
Strain

$$\frac{F}{A} = E\frac{\Delta L}{L}$$

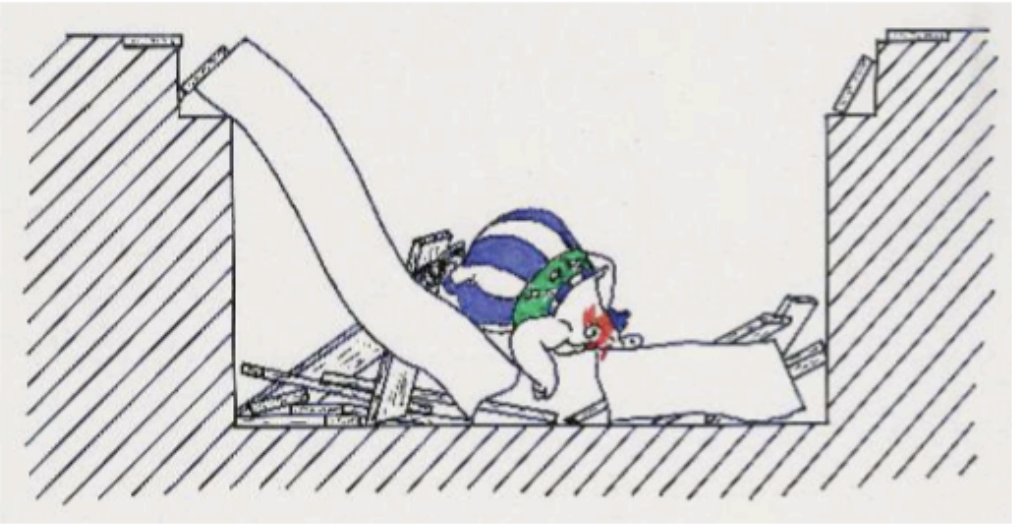
Young's Modulus: Stiffness in Elastic Regime

σ_{max} : Strength

Stiffness vs. Strength



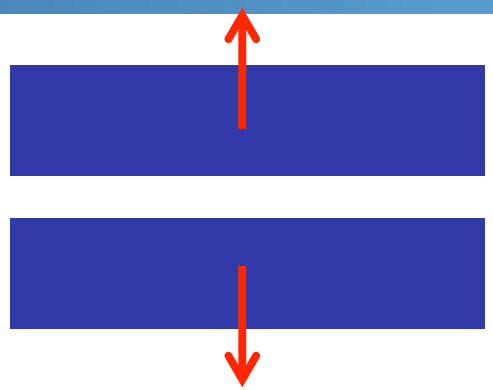
Stiffness: how much does it deform?



Strength: when does it break?

Fig. 1.1 : *Stiffness and strength.*

What determines the strength of a material?



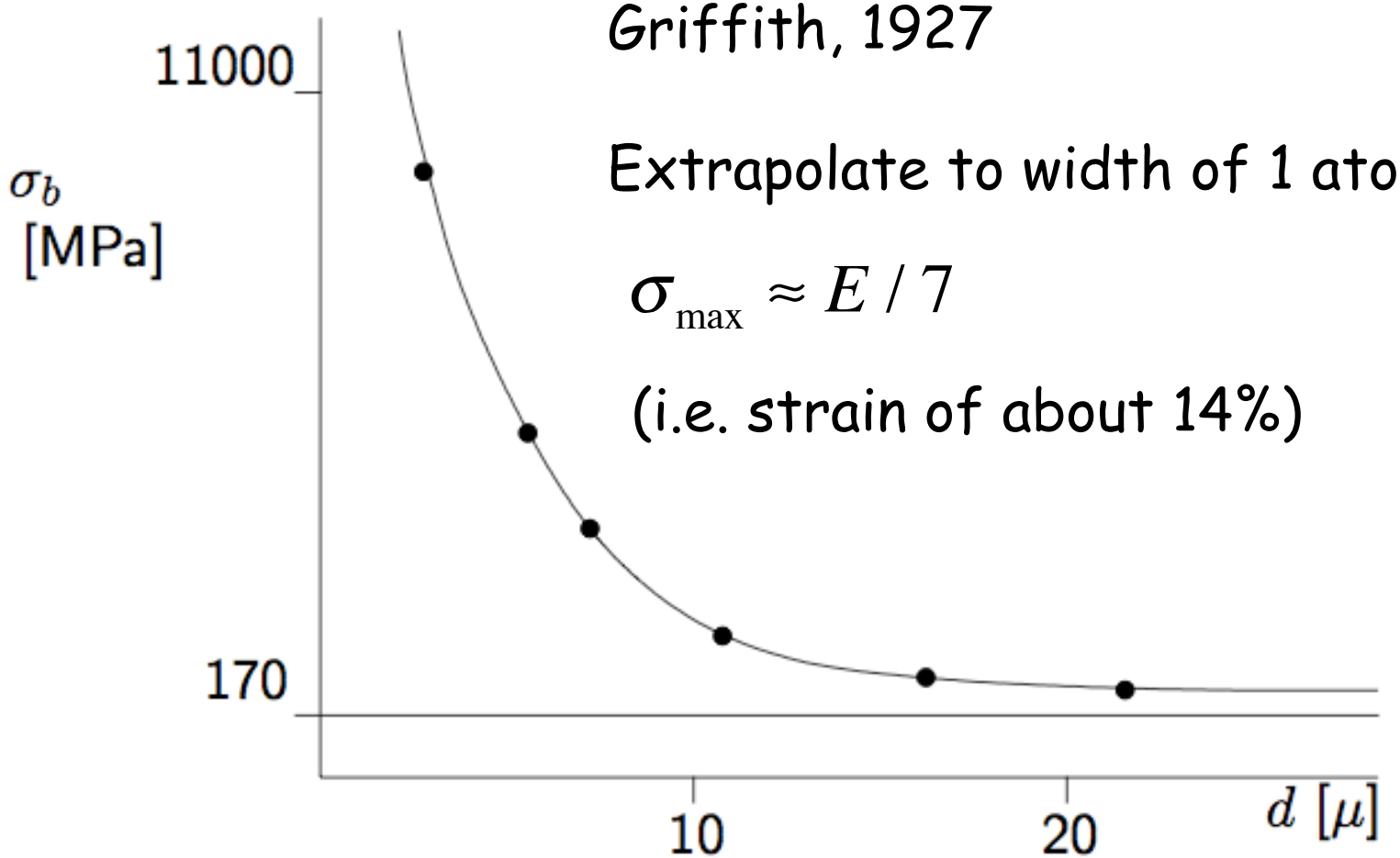
$$\sigma_{th} = \sqrt{\frac{E\gamma}{a_0}}$$

Surface Energy

Atomic Separation

	a_0 [m]	E [GPa]	σ_{th} [GPa]	σ_b [MPa]	σ_{th}/σ_b
glass	$3 * 10^{-10}$	60	14	170	82
steel	10^{-10}	210	45	250	180
silica fibers	10^{-10}	100	31	25000	1.3
iron whiskers	10^{-10}	295	54	13000	4.2
silicon whiskers	10^{-10}	165	41	6500	6.3
alumina whiskers	10^{-10}	495	70	15000	4.7
ausformed steel	10^{-10}	200	45	3000	15
piano wire	10^{-10}	200	45	2750	16.4

What determines the strength of a material?



Griffith, 1927

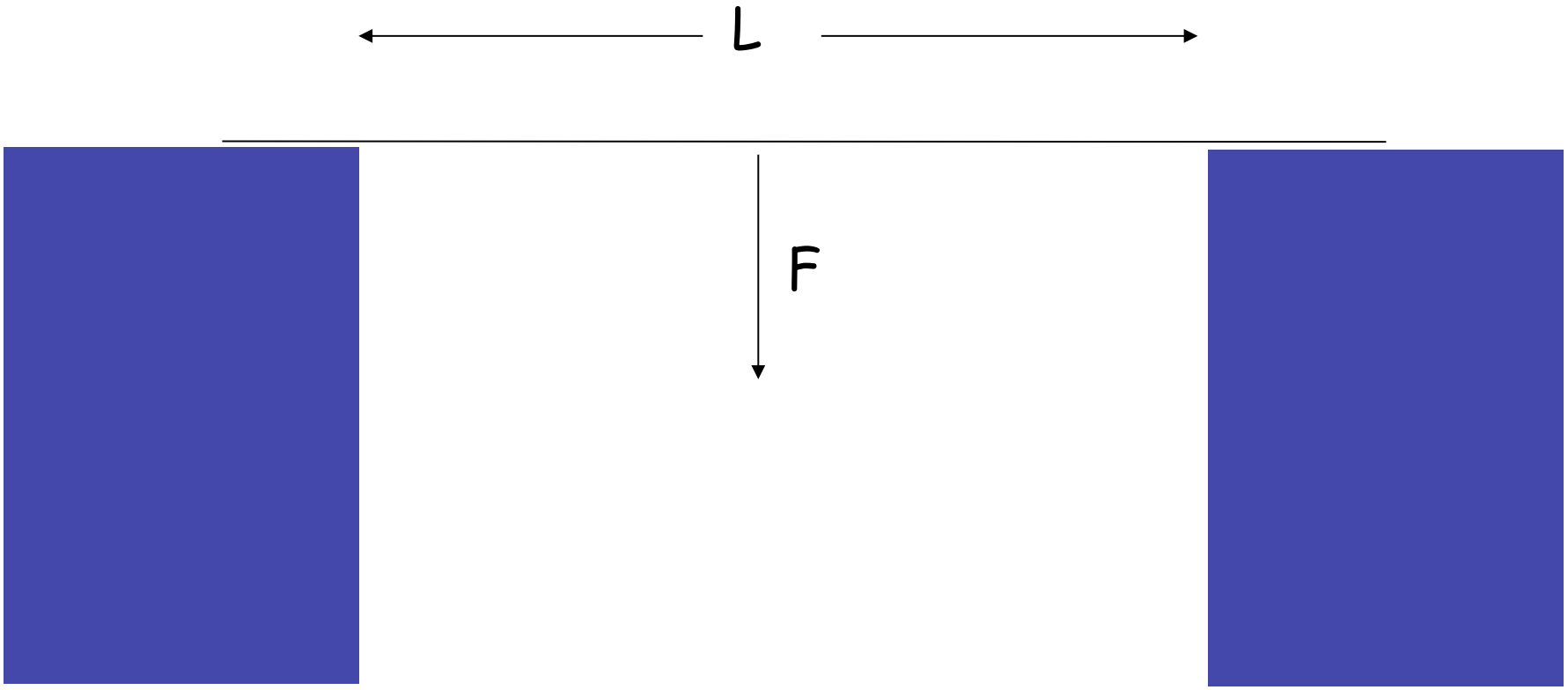
Extrapolate to width of 1 atom:

$$\sigma_{\max} \approx E / 7$$

(i.e. strain of about 14%)

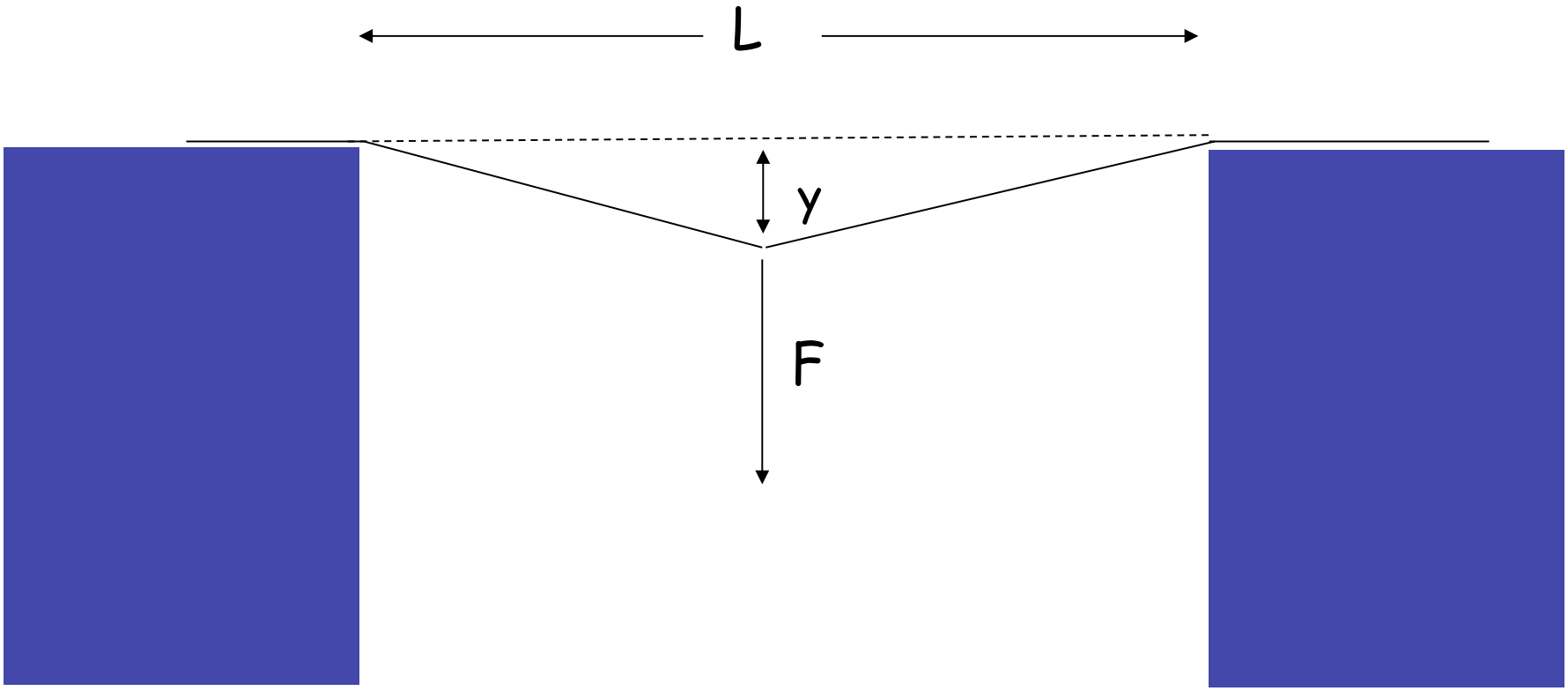
Fig. 2.9 : *Fracture strength of glass fibers in relation to their thickness.*

A Simple Physics Problem...



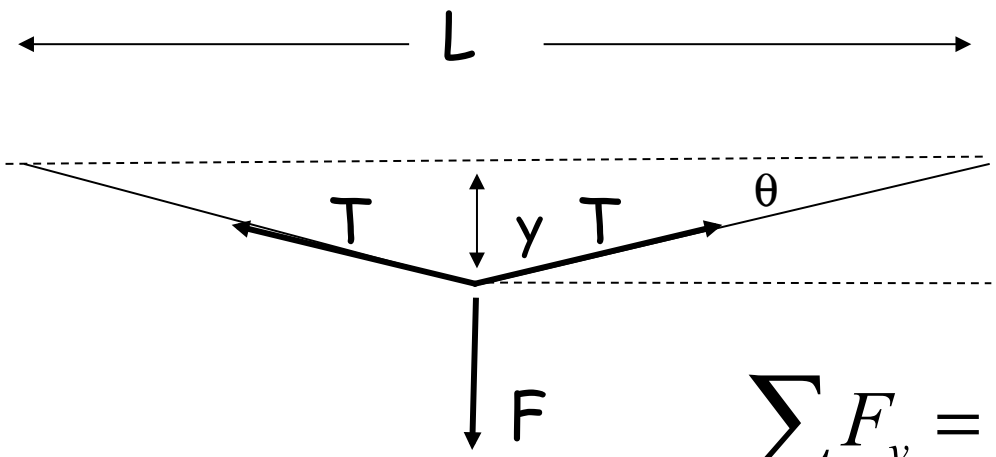
Pull a string at the center with force **F**

A Simple Physics Problem...



What is F(y)?

A Simple Physics Problem...

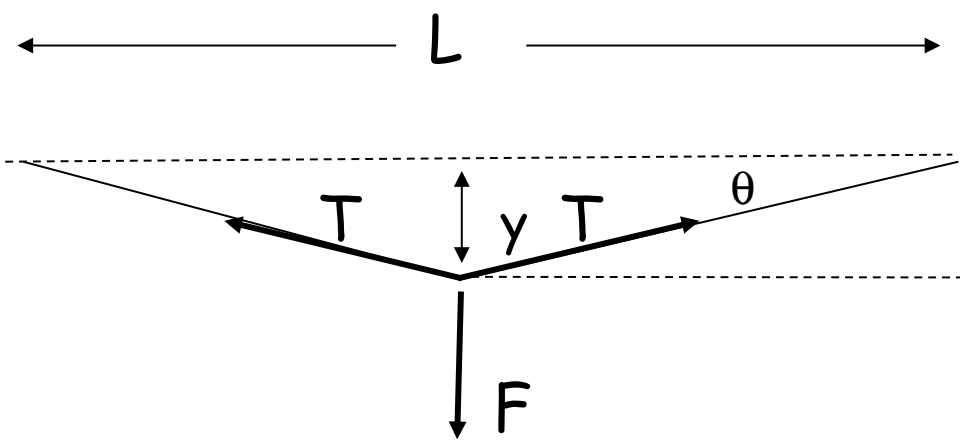


T=tension in string

$$\sum F_y = 2T\sin\theta - F = 0$$

$$\sin\theta \approx \frac{2y}{L} \qquad \Rightarrow F = 4T\frac{y}{L}$$

A Simple Physics Problem...



Young's modulus

$$T = T_0 + k\Delta L = T_0 + \frac{EA}{L}\Delta L$$

x-sectional area

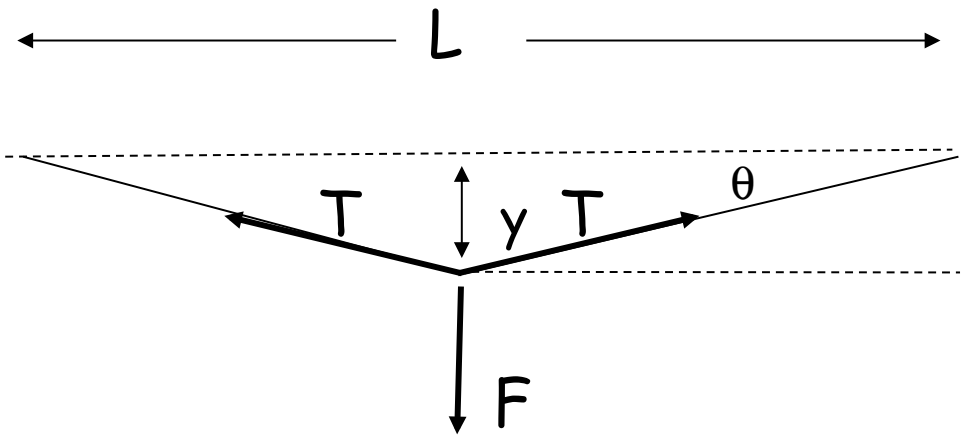
Geometry:

$$\Delta L = 2\sqrt{\left(\frac{L}{2}\right)^2 + y^2} - L = \sqrt{L^2 + 4y^2} - L$$

$$= L\sqrt{1 + \frac{4y^2}{L^2}} - L \approx L\left(1 + \frac{2y^2}{L^2}\right) - L$$

$$= \frac{2y^2}{L}$$

A Simple Physics Problem...



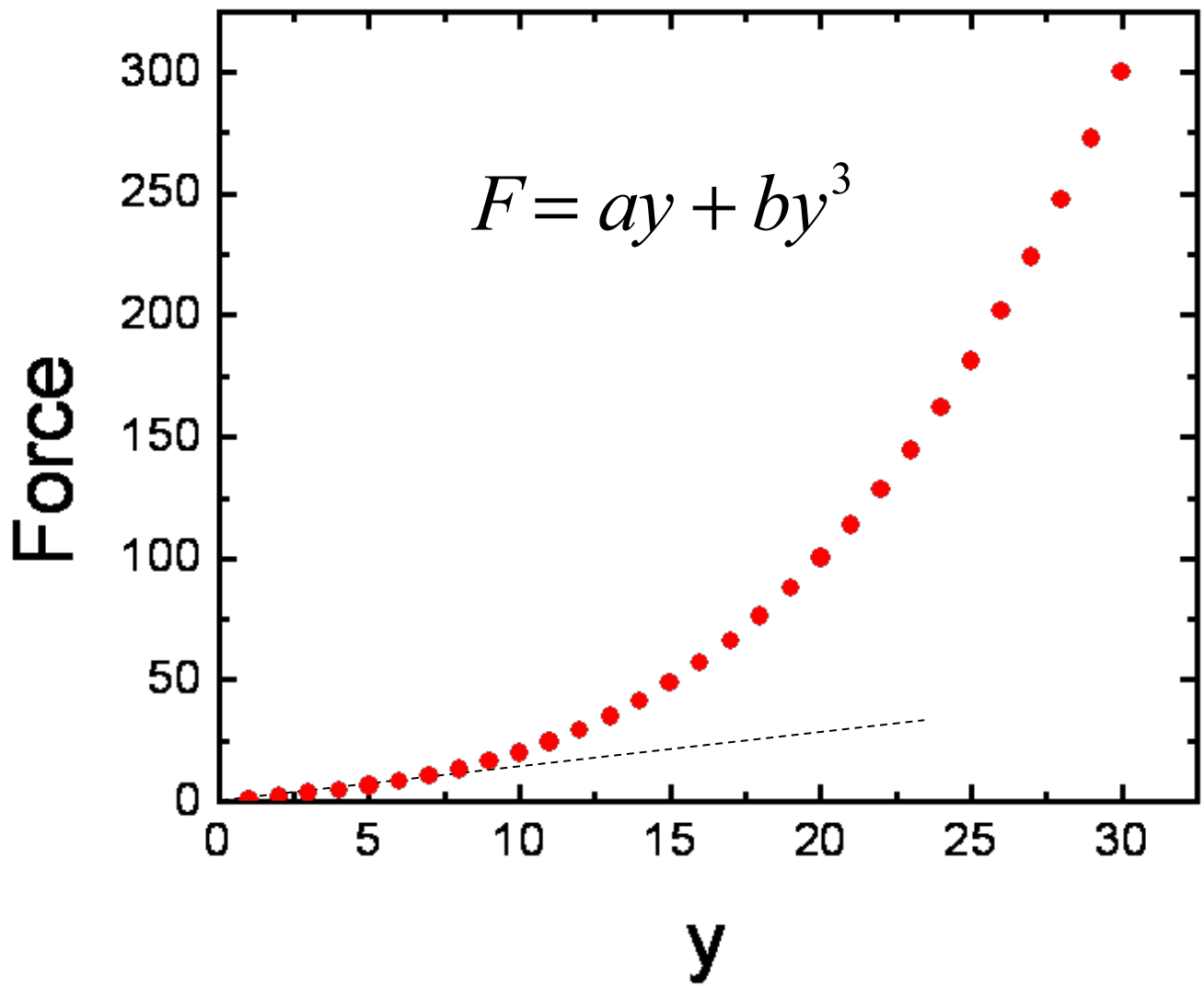
$$T = T_0 + \frac{EA}{L}\Delta L = T_0 + \frac{EA}{L}\left(\frac{2y^2}{L}\right)$$

$$F = 4\frac{T_0}{L}y + 8\frac{EA}{L^3}y^3 = ay + by^3$$

linear term

cubic term

A Simple Physics Problem...



Carbon Nanotubes

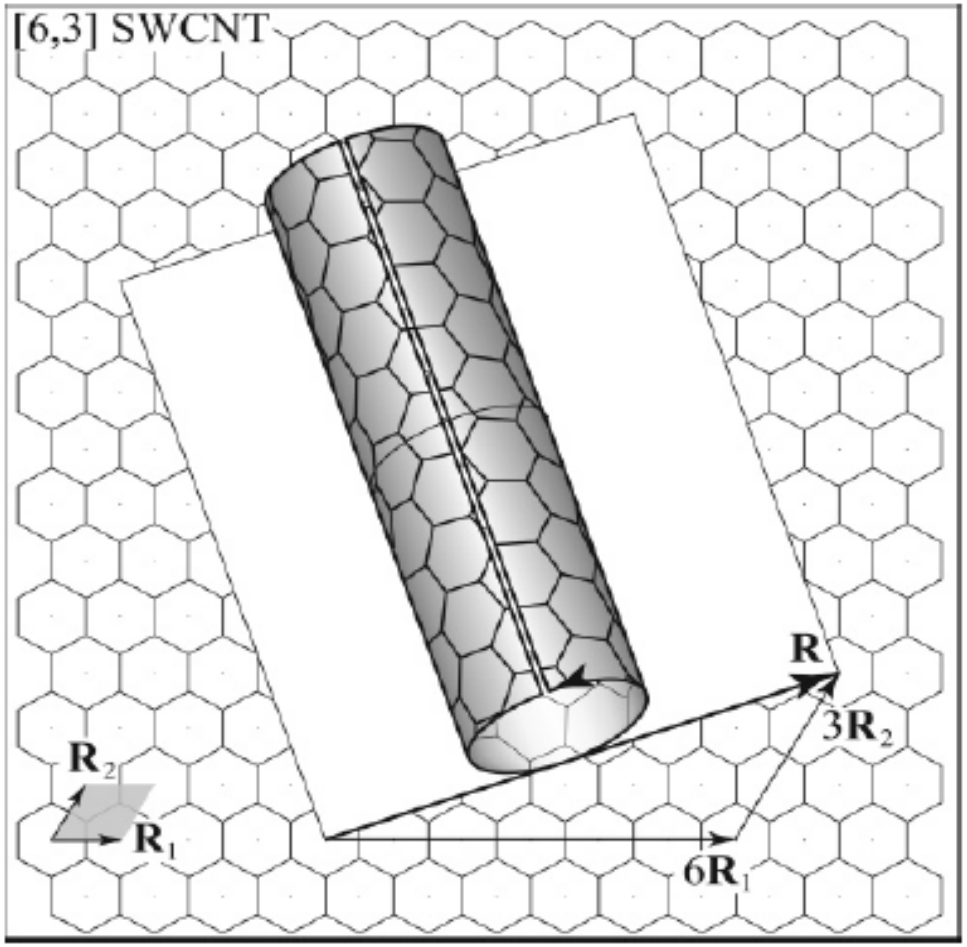
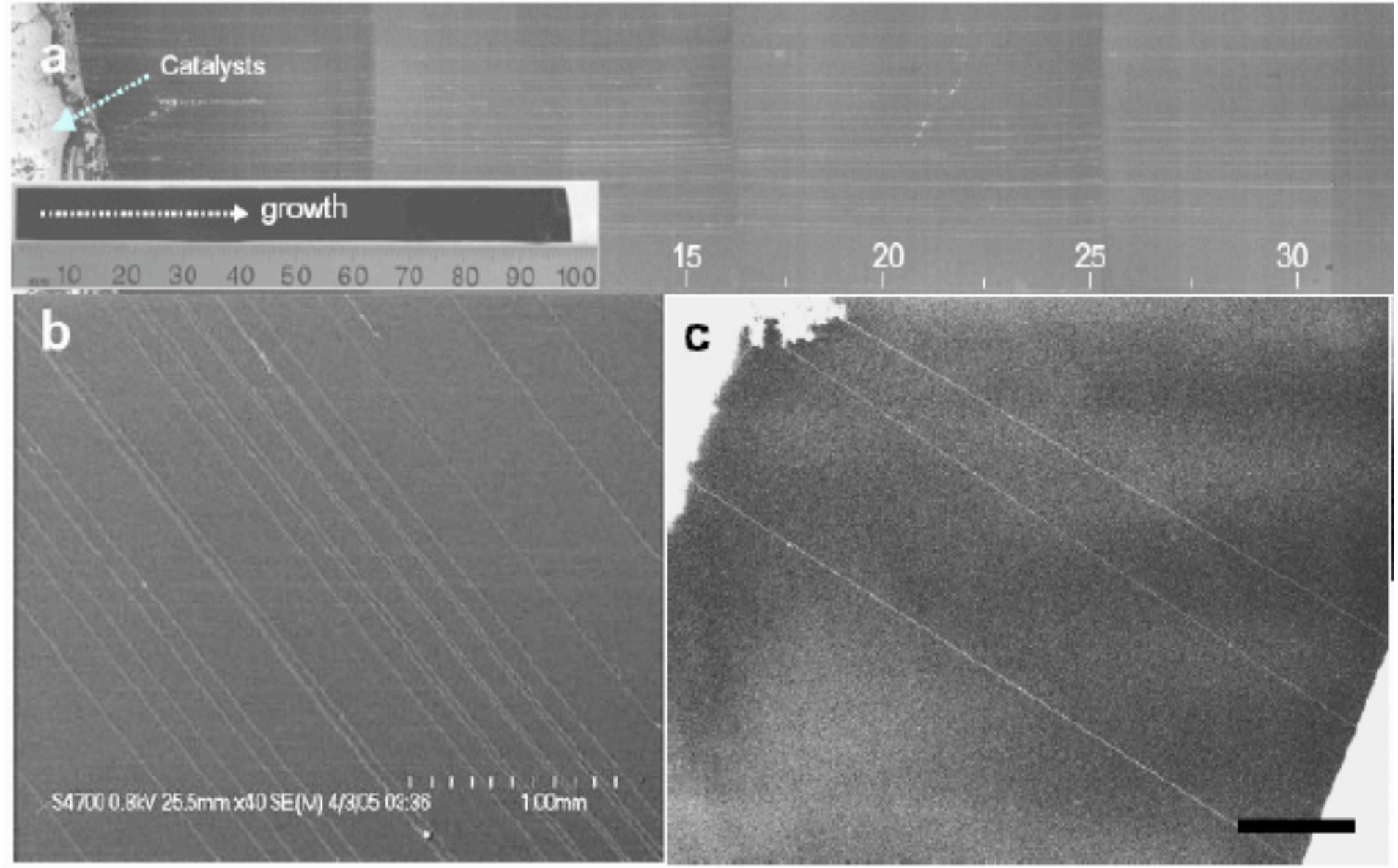


Figure 1. Mapping of the graphene sheet to form the [6,3] SWCNT with a couple of translational unit cells along the tube shown. The unit cell of the graphene lattice highlighted in gray is defined by the primitive lattice vectors $\mathbf{R}_1$ and $\mathbf{R}_2$.

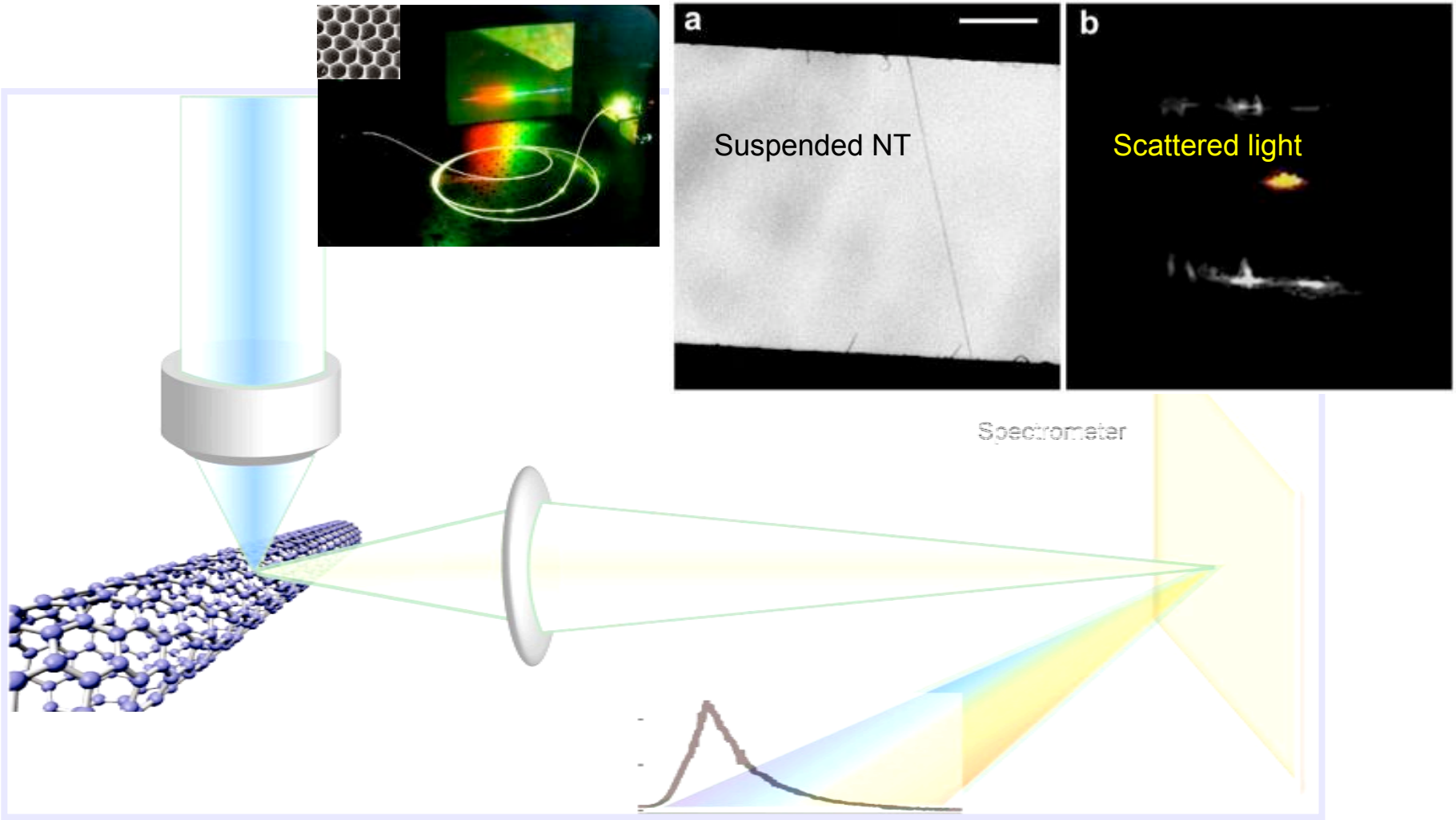
Rolled up graphene…

Should have similar mechanical properties

Carbon Nanotubes grown by CVD



Rayleigh Scattering Spectroscopy

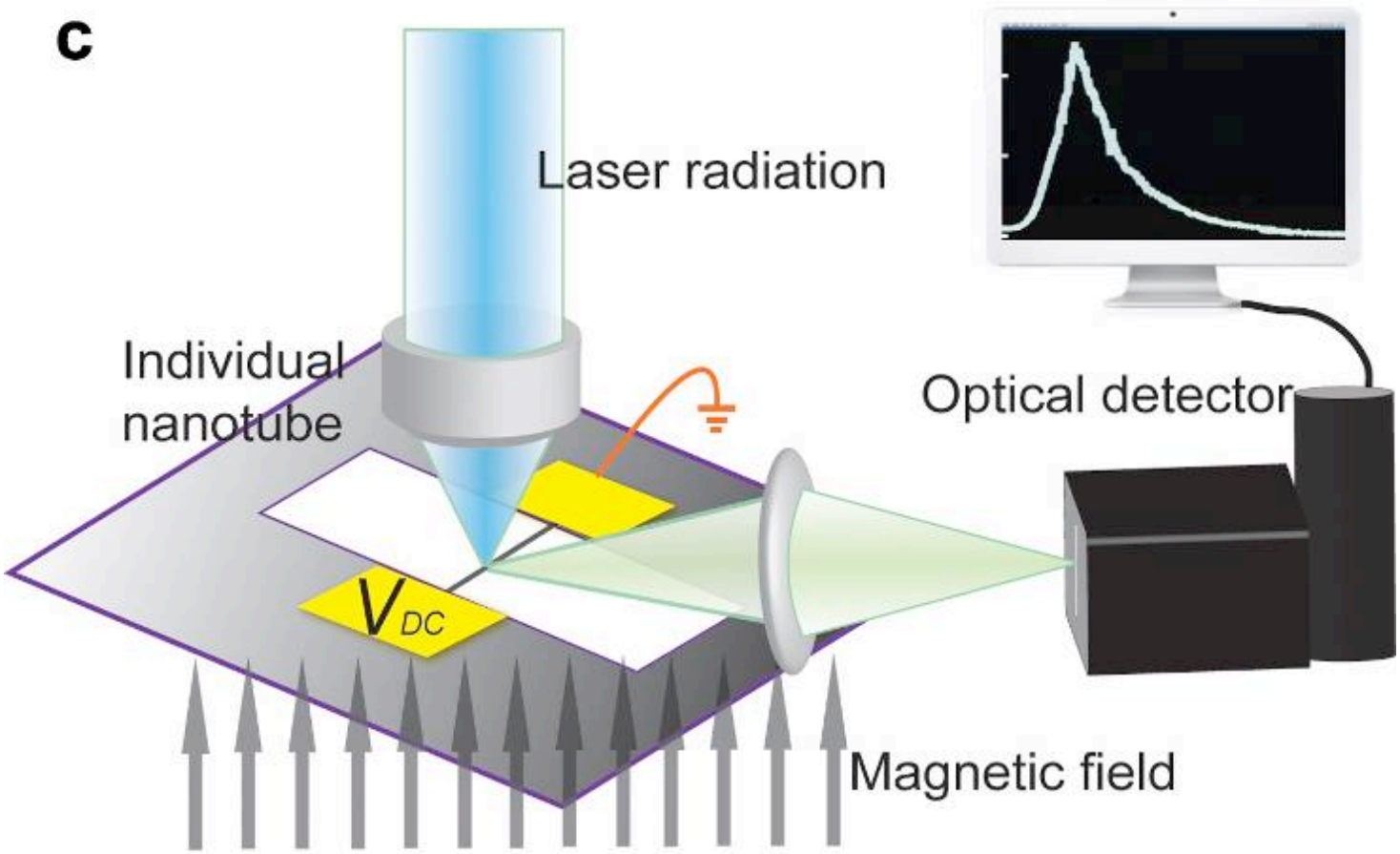


Probing Electronic Transitions in Individual Carbon Nanotubes by Rayleigh Scattering

Matthew Y. Sfeir, Feng Wang, Limin Huang, Chia-Chin Chuang, J. Hone, Stephen P. O'Brien, Tony F. Heinz, Louis E. Brus

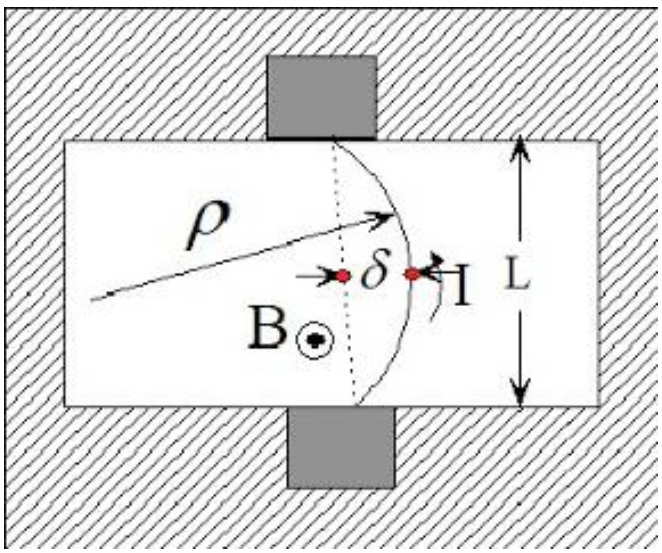
Science **306**, 1540-1543 (2004)

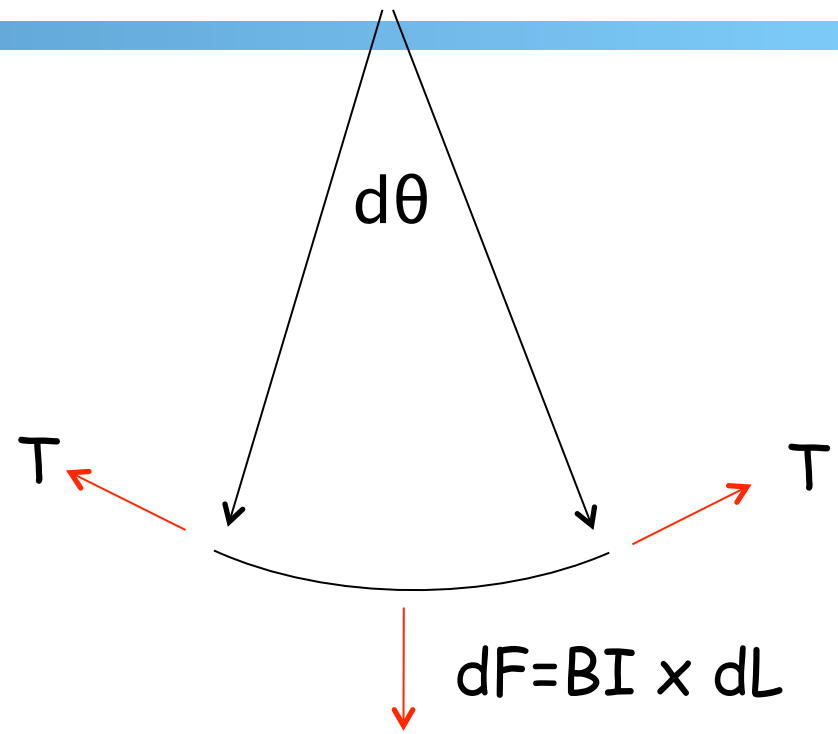
Mechanical Testing: Magnetic Displacement



Y. Wu and M. Huang

Mechanical Testing: Magnetic Displacement



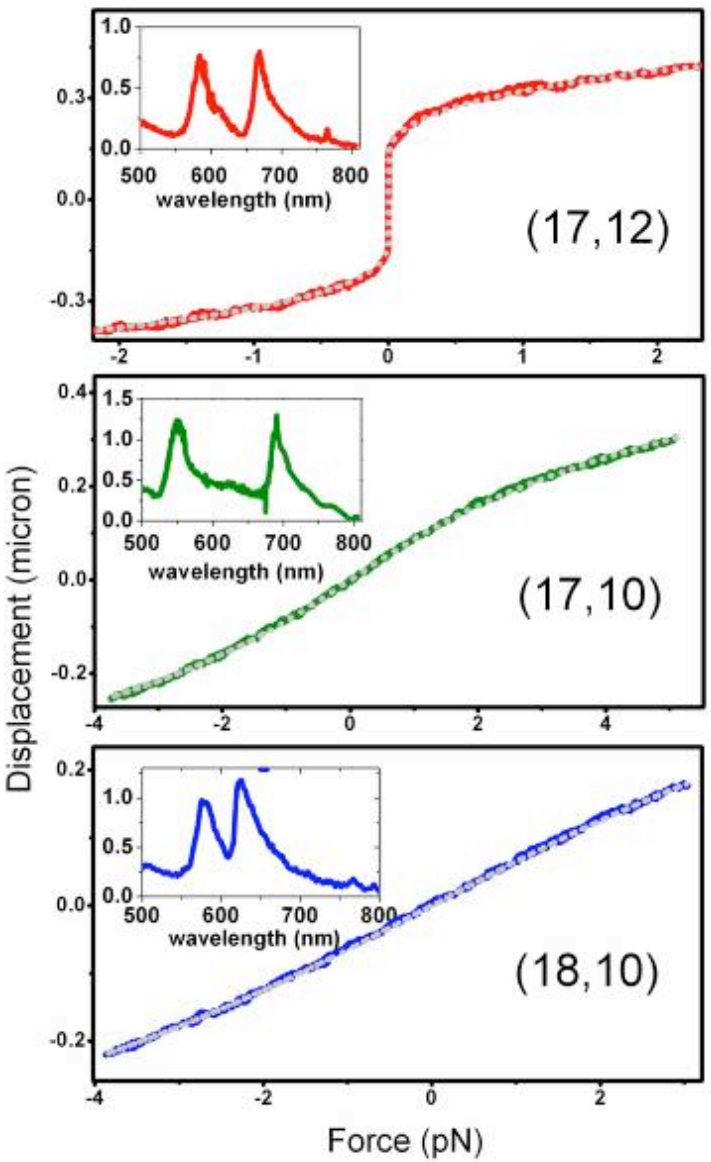


$$BIdL = 2T\sin(d\theta/2) \approx Td\theta$$

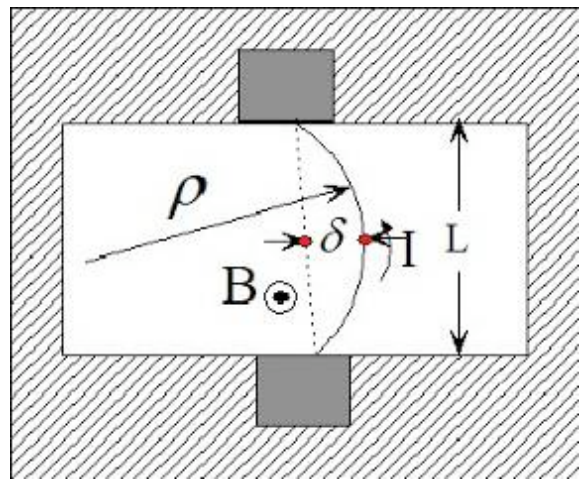
$$dL = \rho d\theta$$

$$\Rightarrow \rho = \frac{T}{BI}$$

Mechanical Testing: Magnetic Displacement



L~100 μm, d~2 nm: model as string
(no bending moment)

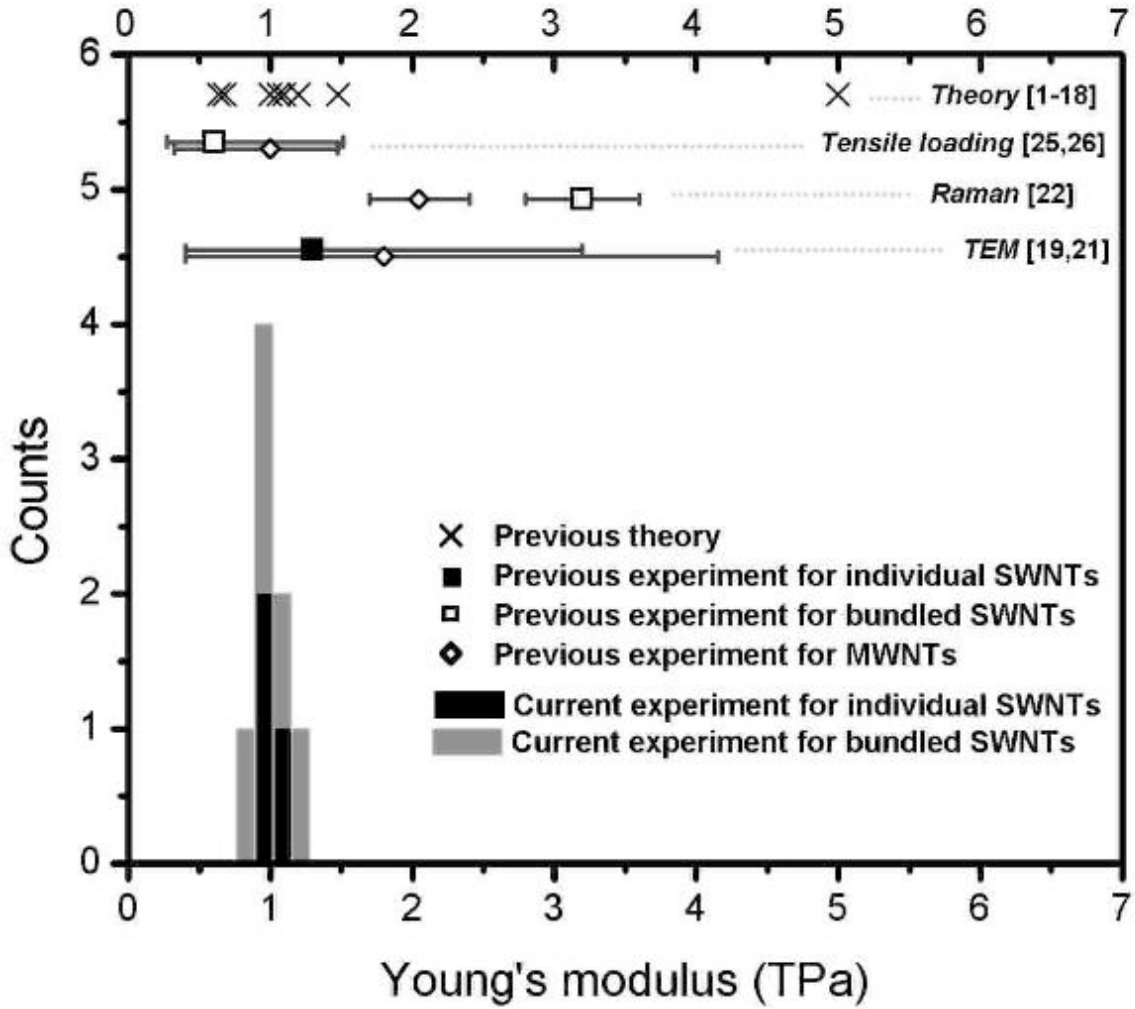


$$\rho = \frac{dl}{2d\theta} = \frac{T}{BI}$$

$$F = BIL \qquad B = 0.31T$$

$$F = 8\,A\,E\frac{\delta}{L}\cdot\max\left[0, \frac{8}{3}\left(\frac{\delta}{L}\right)^2 + \varepsilon_0\right]$$

Mechanical Testing: Magnetic Displacement



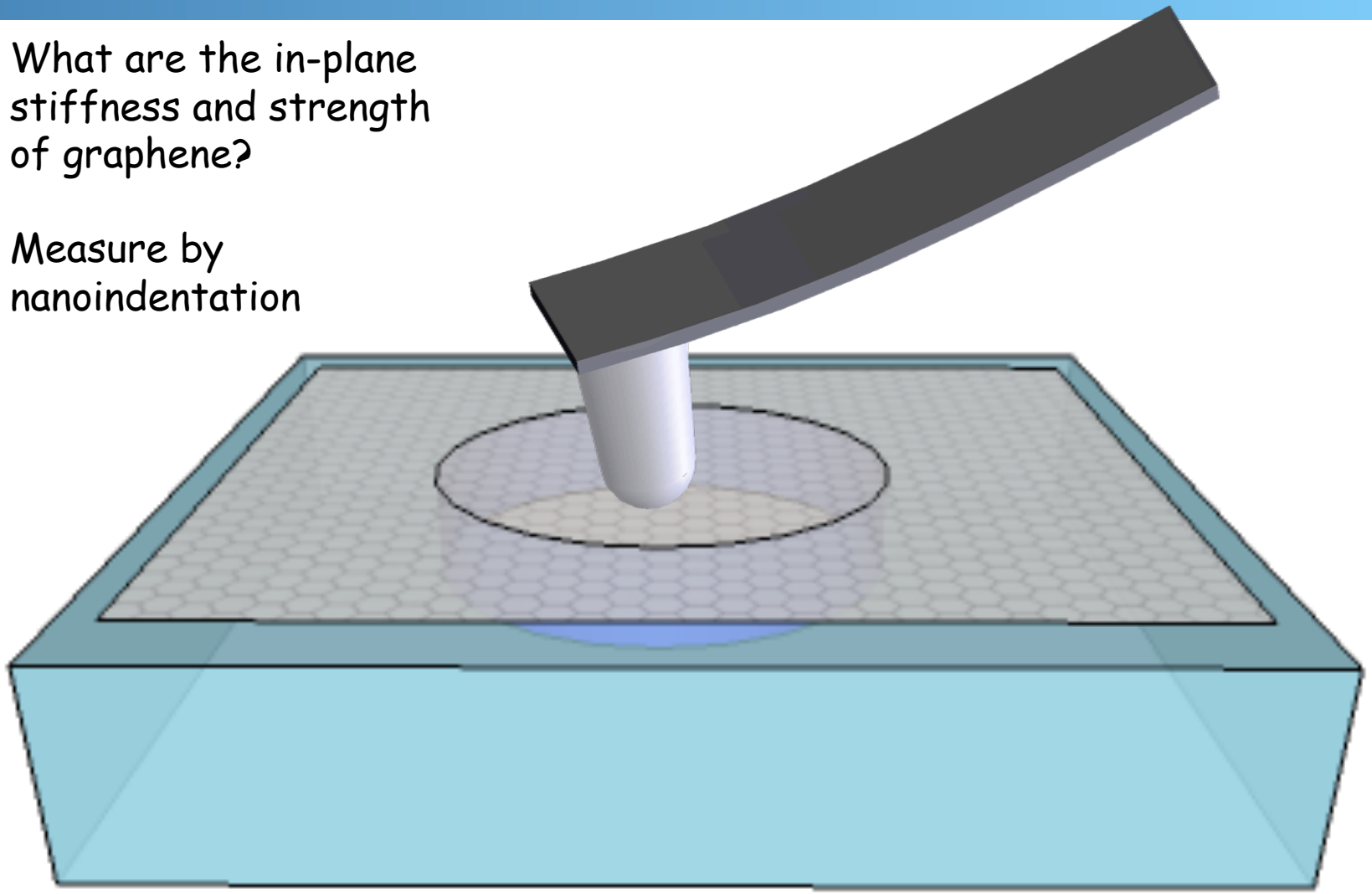
E = 1 TPa ± 16%

Knowing the nanotube structure lets us measure the stiffness precisely…

Mechanical Testing of Graphene

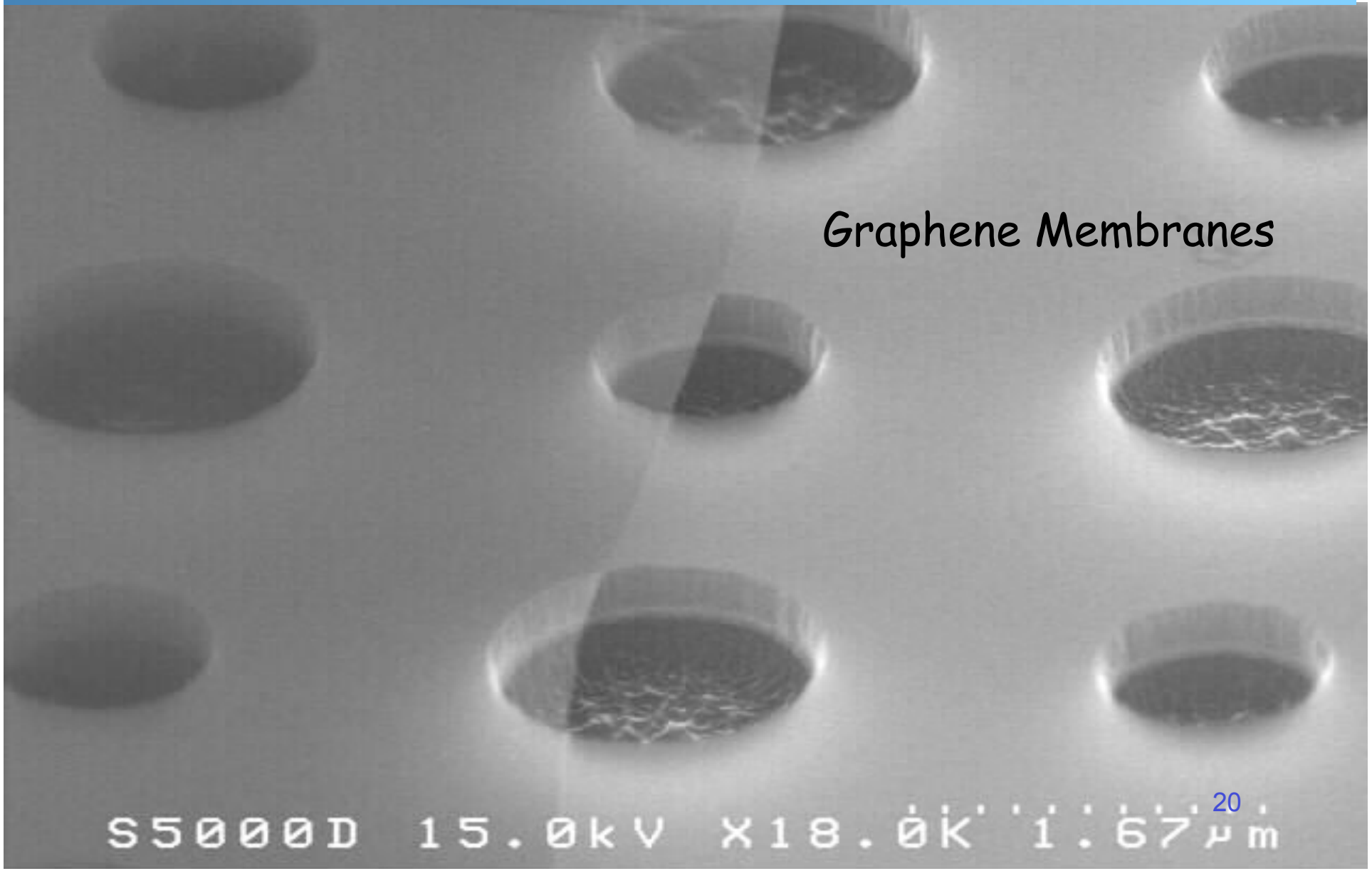
What are the in-plane stiffness and strength of graphene?

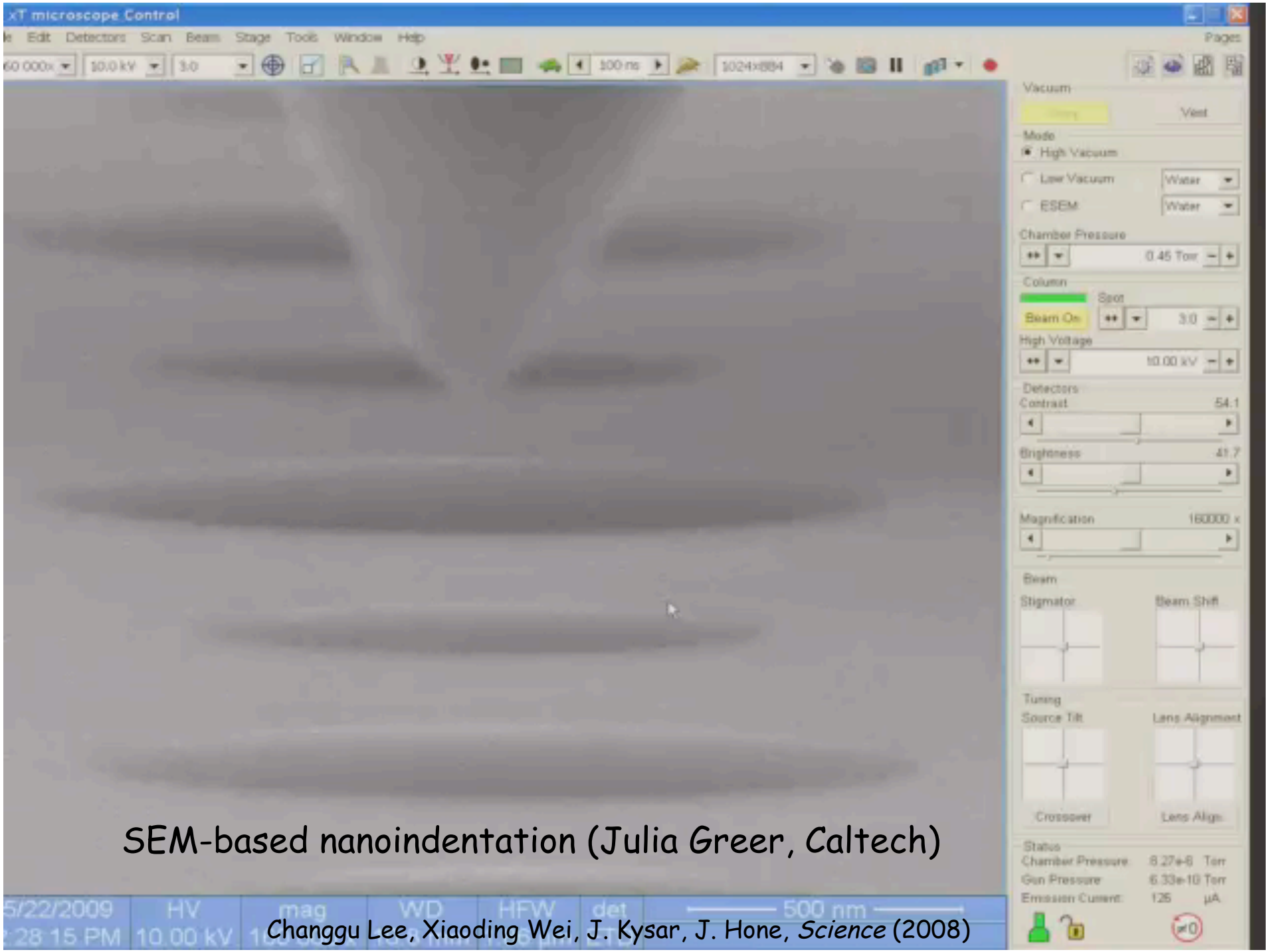
Measure by nanoindentation



Changgu Lee, Xiaoding Wei, J. Kysar, J. Hone, *Science* (2008)

Mechanical Testing of Graphene

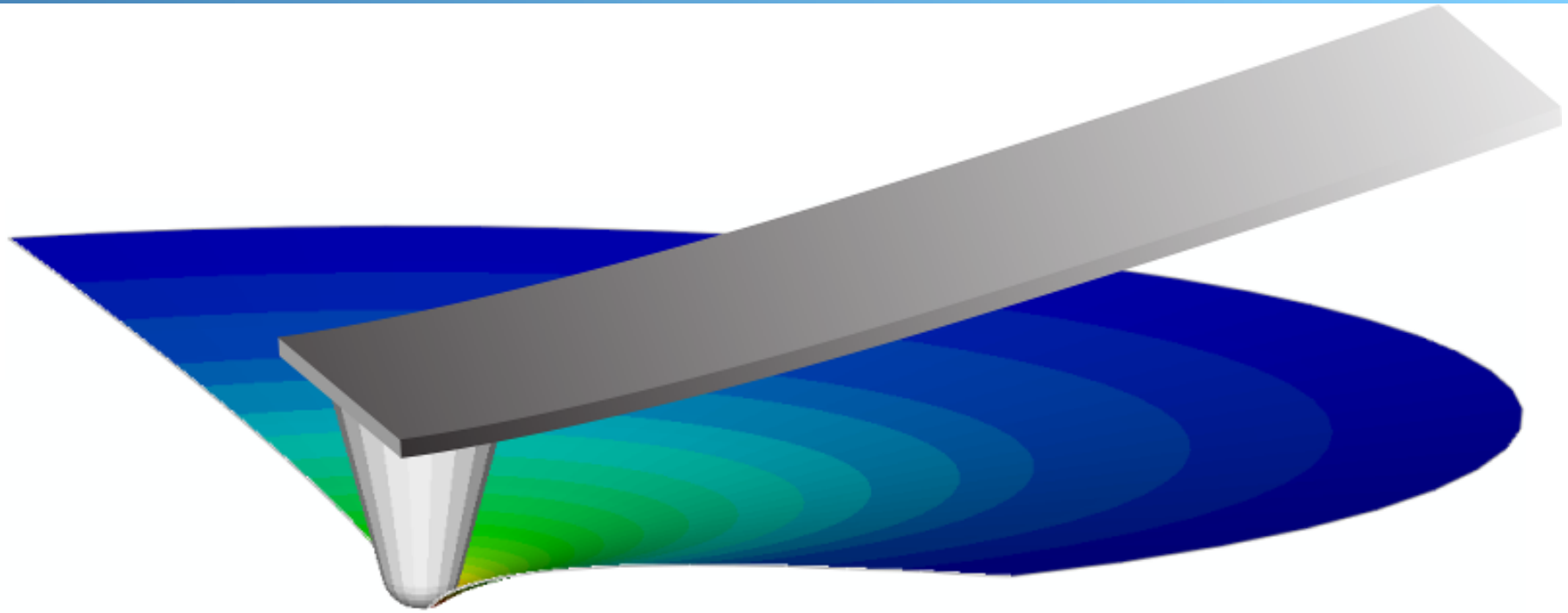




SEM-based nanoindentation (Julia Greer, Caltech)

Changgu Lee, Xiaoding Wei, J. Kysar, J. Hone, *Science* (2008)

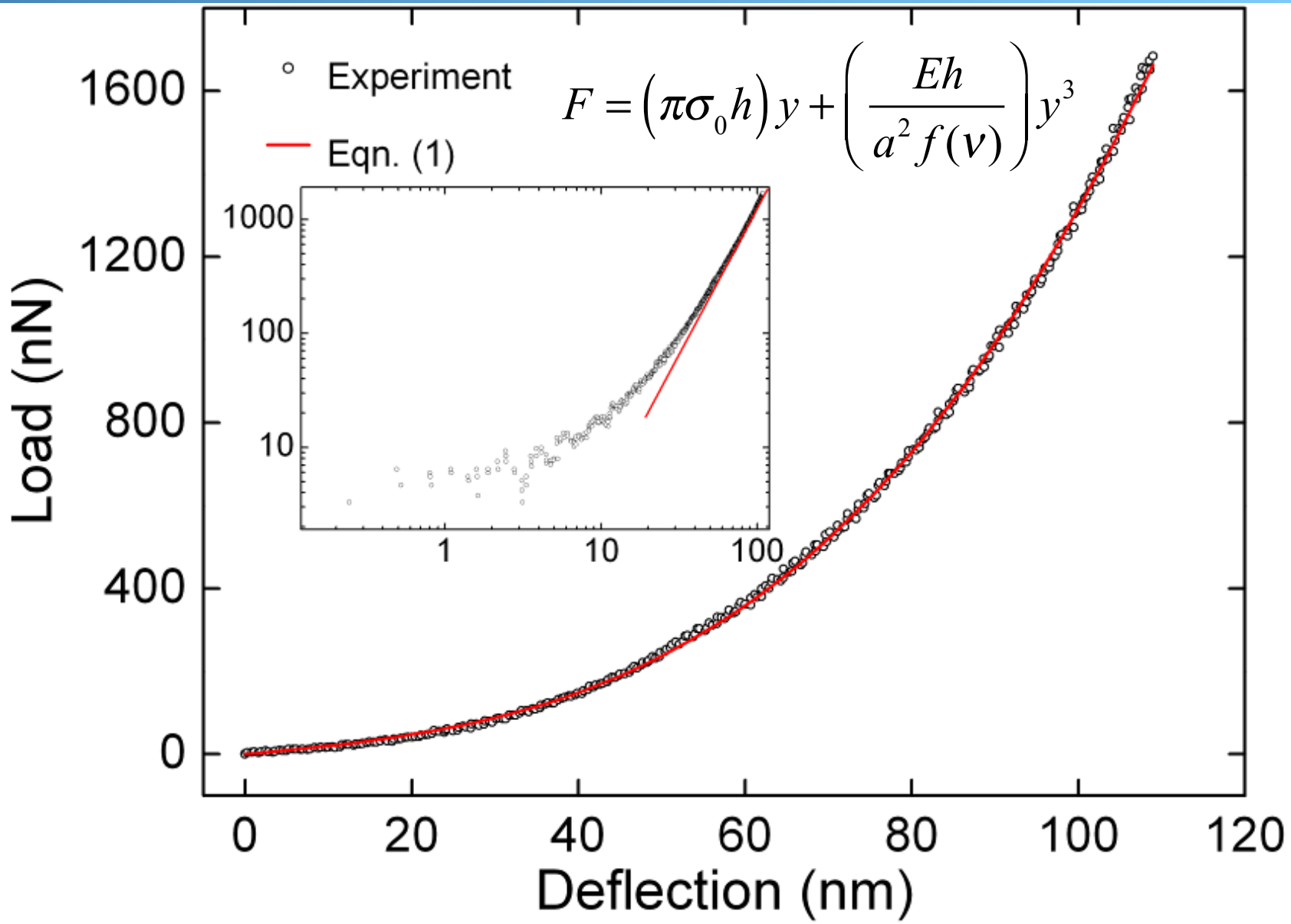
Stress distribution: FEA model



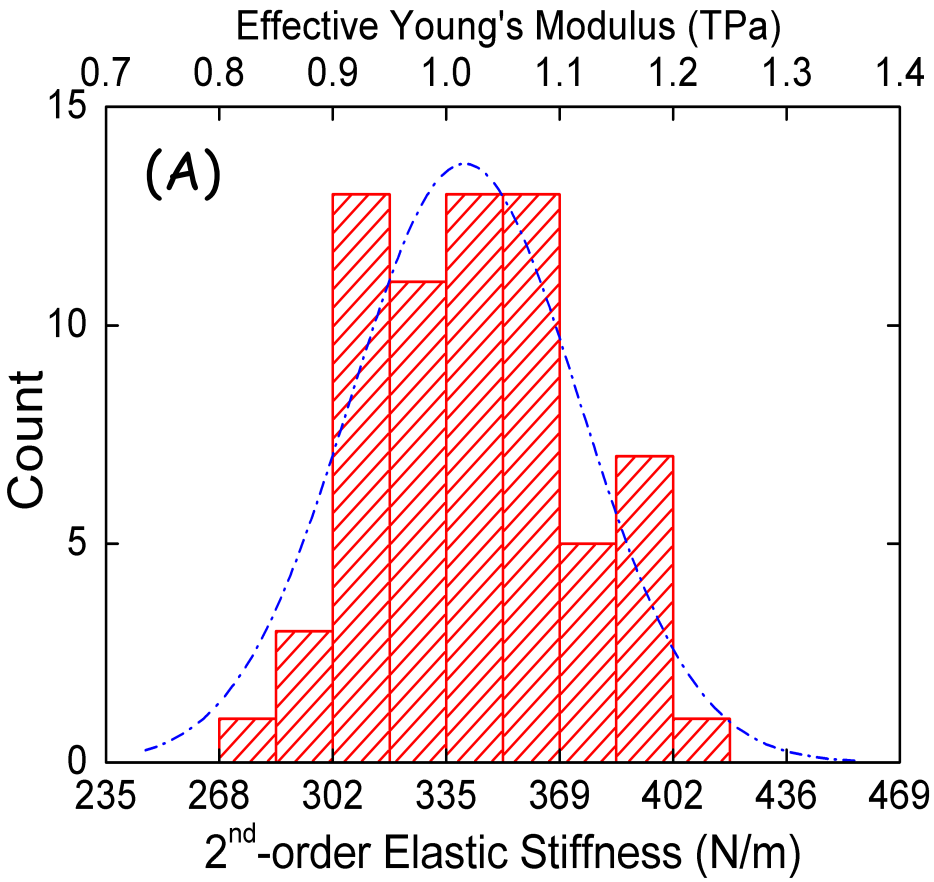
FEA modeling of indentation:

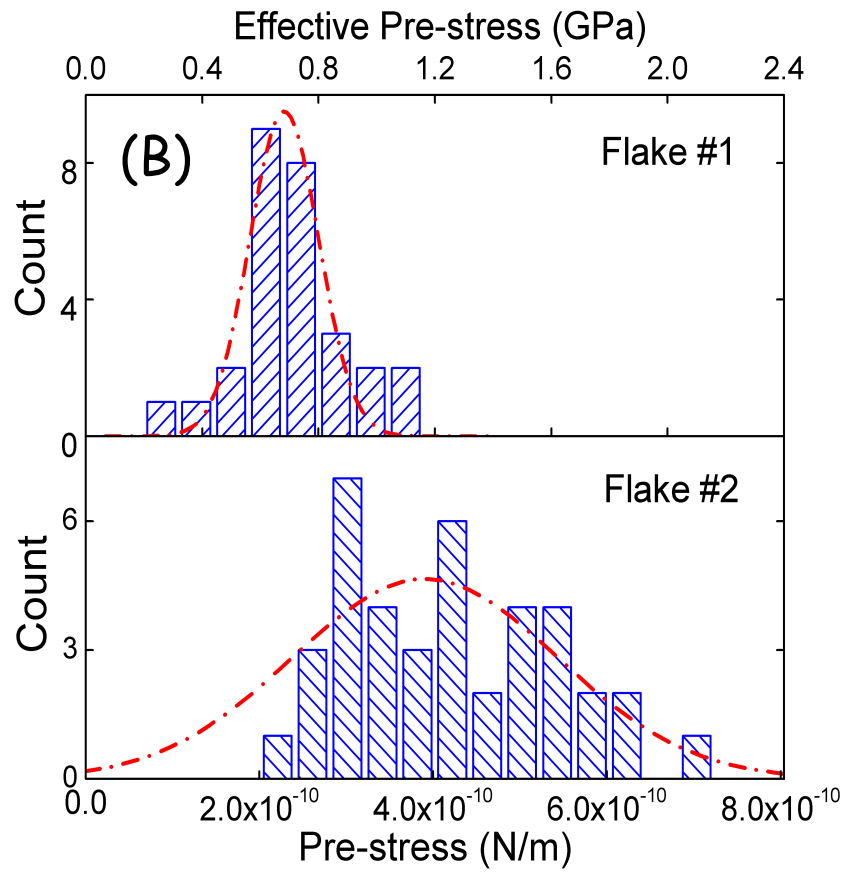
- low stress at clamping points
- most of deflection is away from tip (linear model, point loading)
- breakage will occur under tip (stress concentration)

Force – Displacement Data



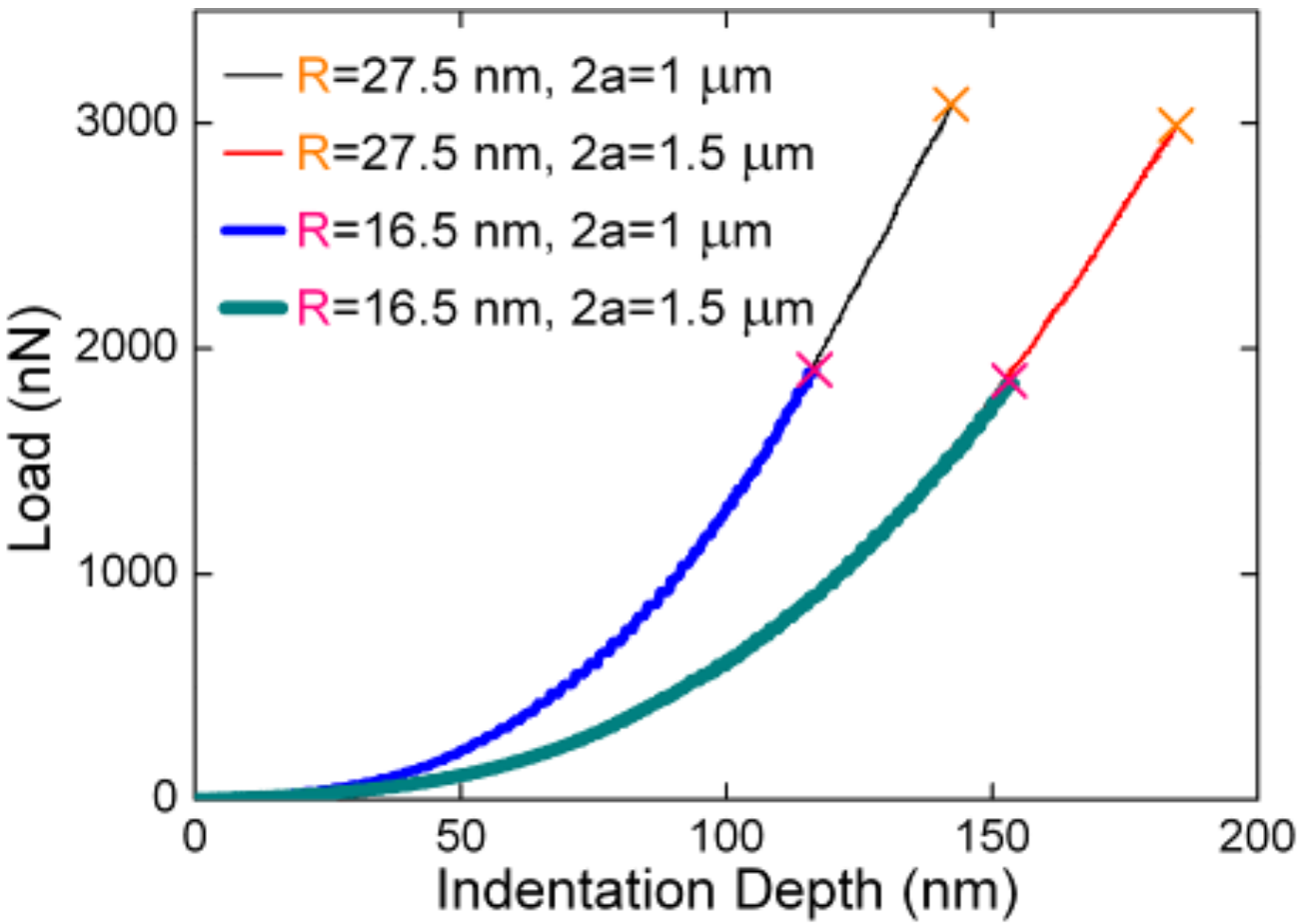
AFM Nanoindentation





Avg. Young's modulus: 1.02 TPa

Breaking the membranes



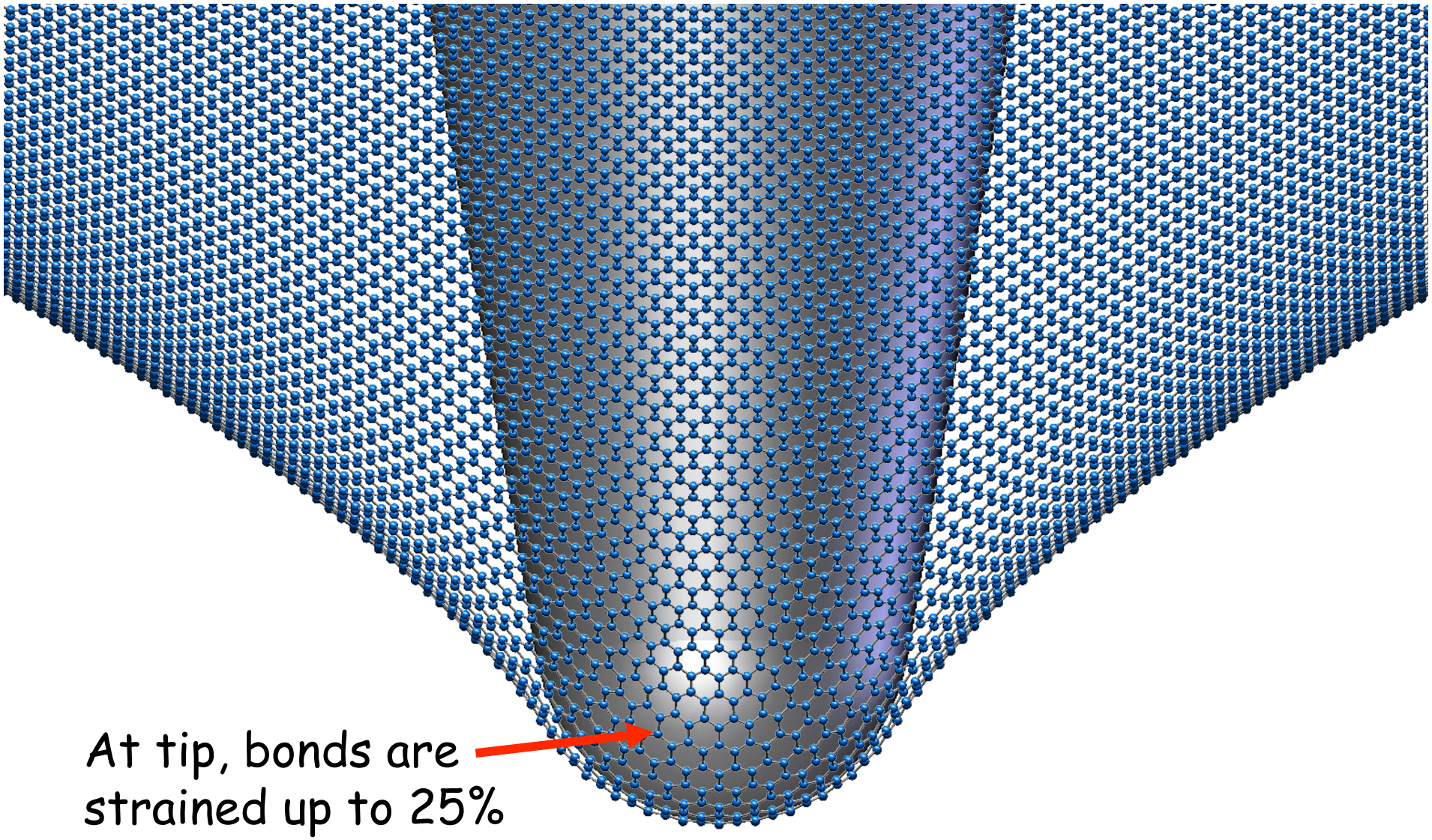
Breaking force depends only on tip radius due to stress concentration

linear elastic model:

$$\sigma_m \approx \left(\frac{P}{4\pi E h R} \right)^{\frac{1}{2}}$$

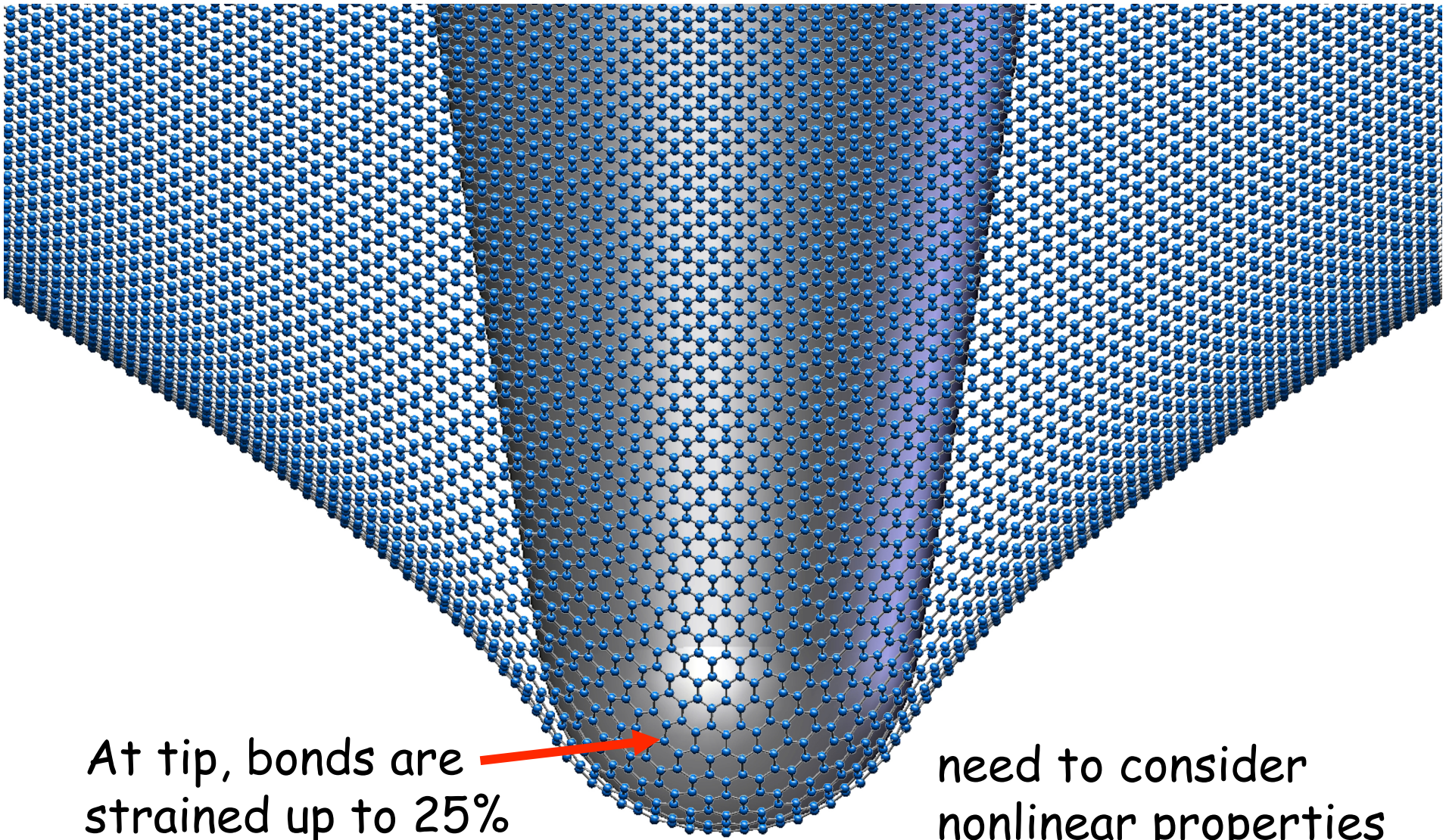
gives correct trend, but unphysically large strength...

Breaking Force



At tip, bonds are strained up to 25%

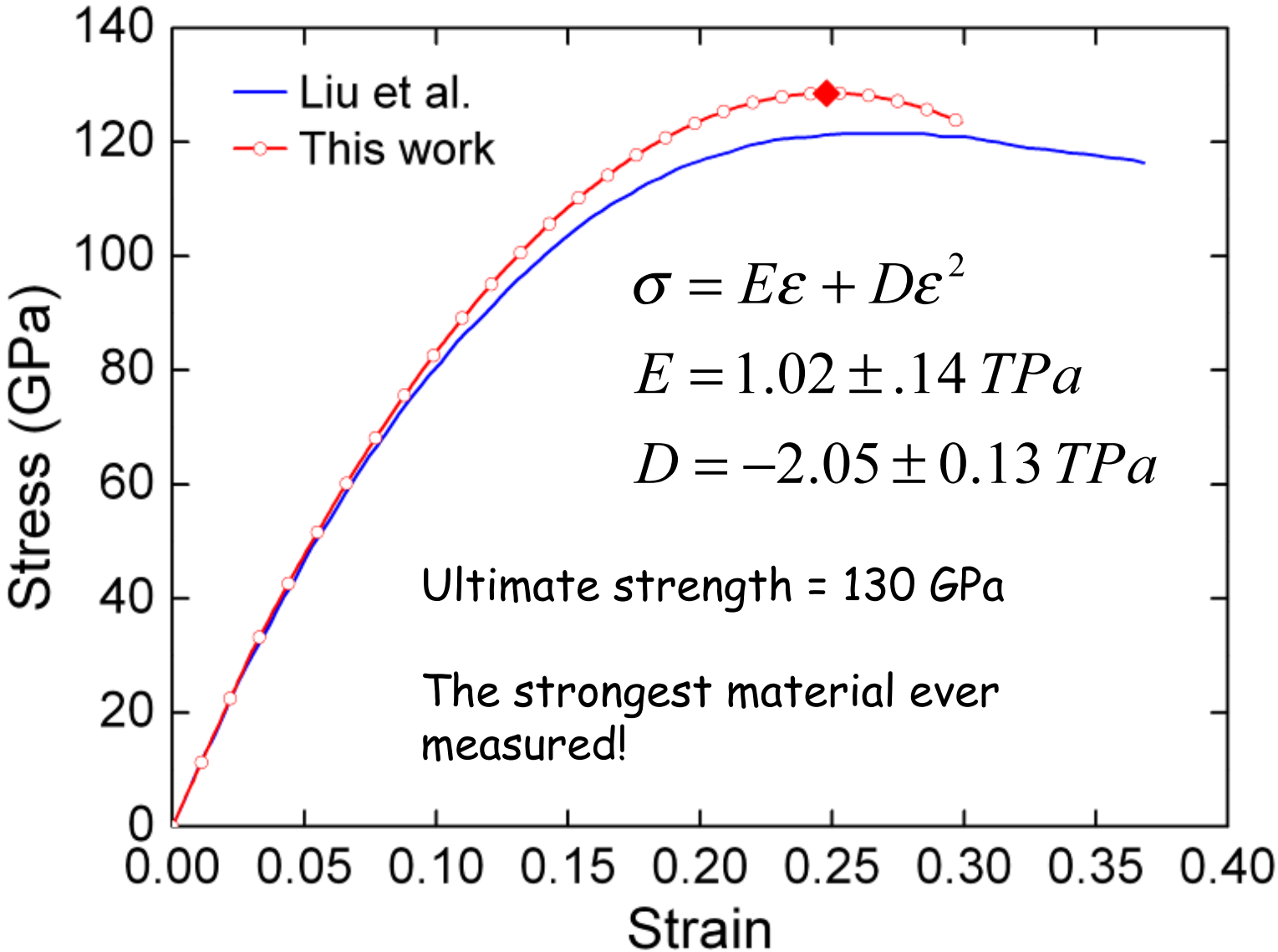
Breaking Force



At tip, bonds are strained up to 25%

need to consider <u>nonlinear</u> properties

Non-linear model



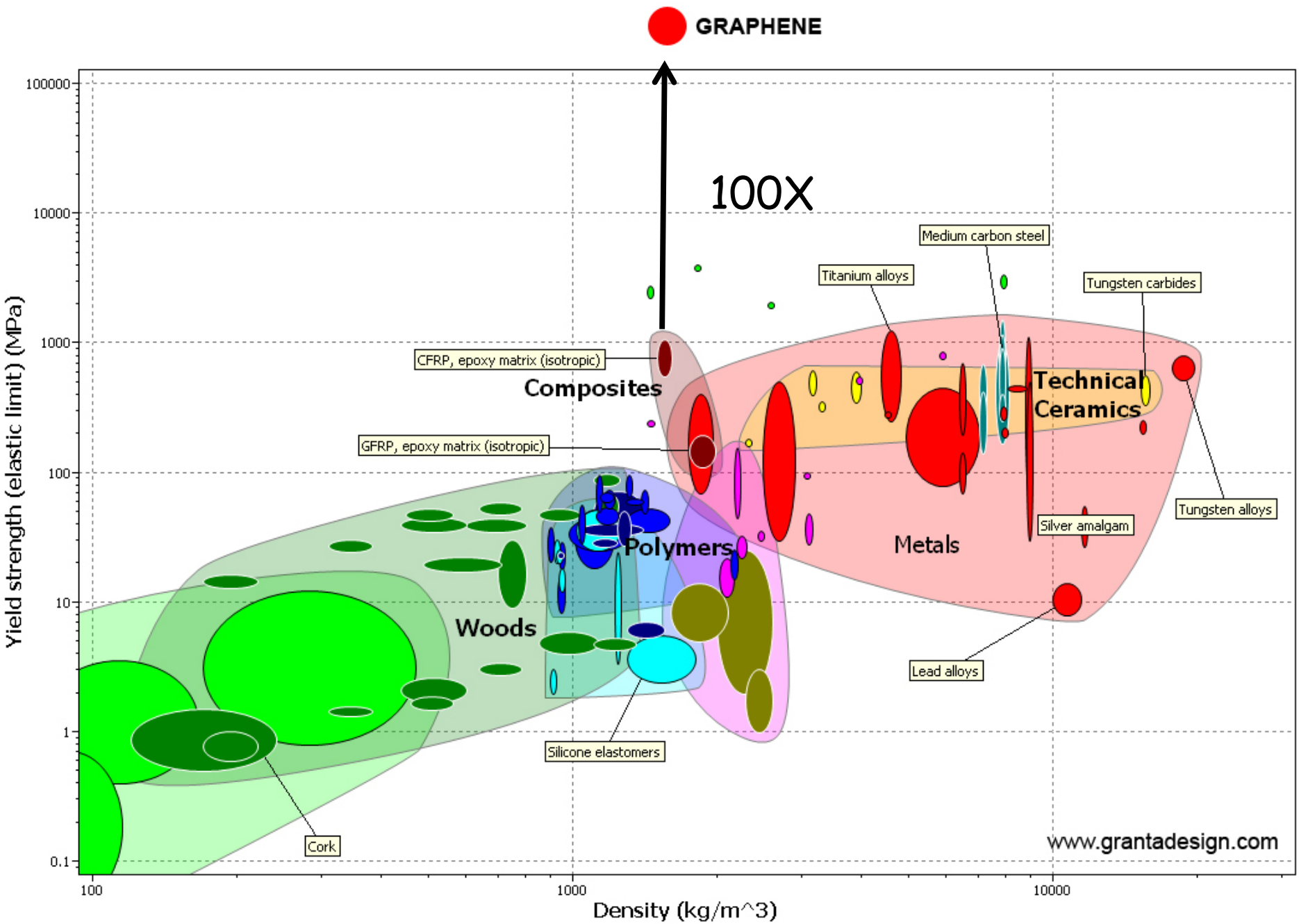
$$\sigma = E\varepsilon + D\varepsilon^2$$

$$E = 1.02 \pm .14\ TPa$$

$$D = -2.05 \pm 0.13\ TPa$$

Ultimate strength = 130 GPa

The strongest material ever measured!


GRAPHENE
100X
Yield strength (elastic limit) (MPa)
100000
10000
1000
100
10
1
0.1
Density (kg/m^3)
100
1000
10000
Medium carbon steel
Titanium alloys
Tungsten carbides
CFRP, epoxy matrix (isotropic)
Composites
Technical Ceramics
GFRP, epoxy matrix (isotropic)
Tungsten alloys
Silver amalgam
Polymers
Metals
Woods
Lead alloys
Silicone elastomers
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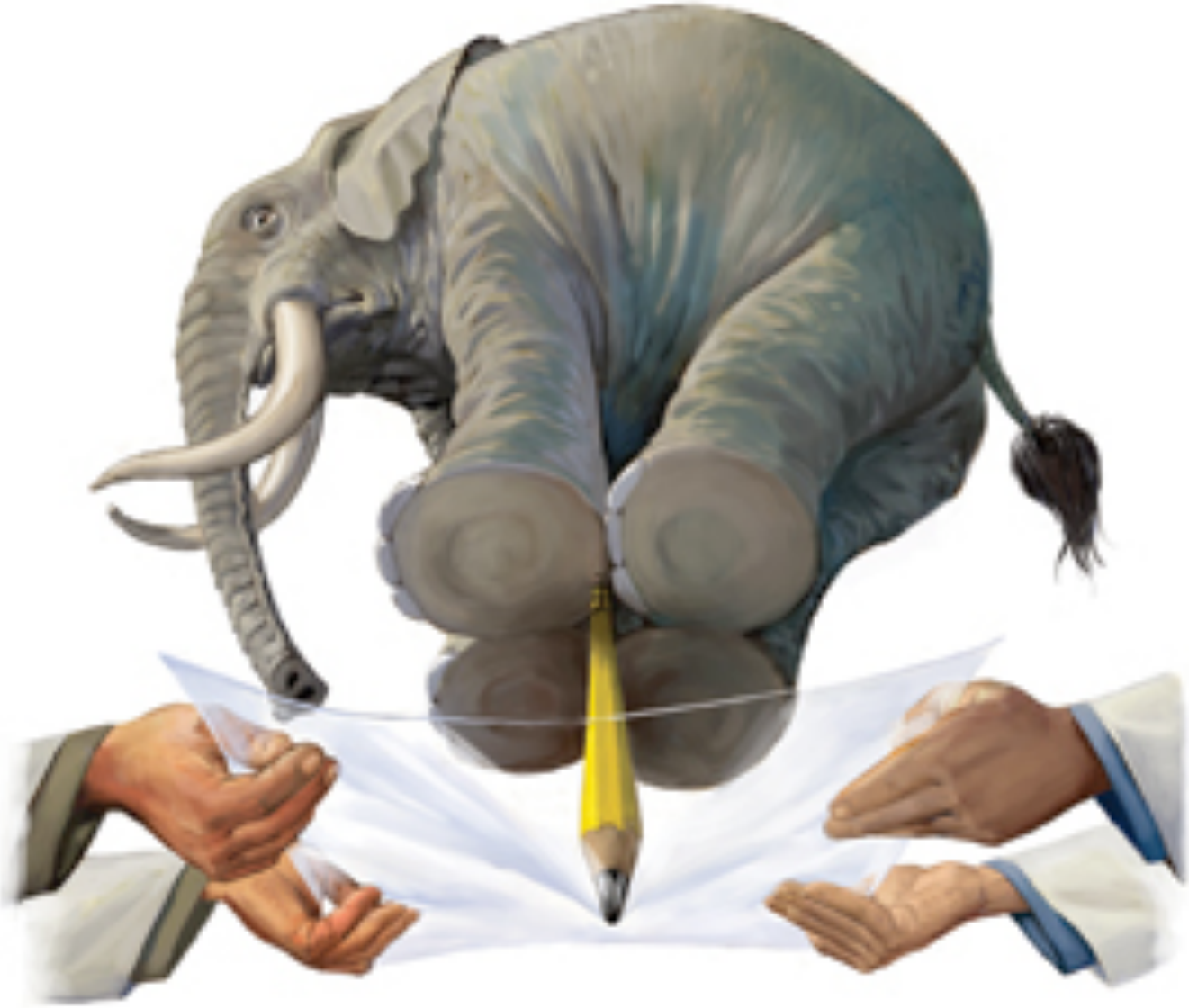
See Inside

Elephant Illustrates Important Point

A statement about a material raises heavy issues

By Steve Mirsky | November 7, 2011 | 11

The tweet, posted on September 1, 2011, by @qikipedia, read in its entirety: "It would take an elephant, balanced on a pencil to break through a sheet of graphene the thickness of cling film." Some detective work revealed that the statement originated with mechanical engineering professor James Hone of Columbia University, who said in 2008, "Our research establishes graphene as the strongest material ever measured, some 200 times stronger than structural steel. It would take an elephant, balanced on a pencil, to break through a sheet of graphene the thickness of Saran Wrap."



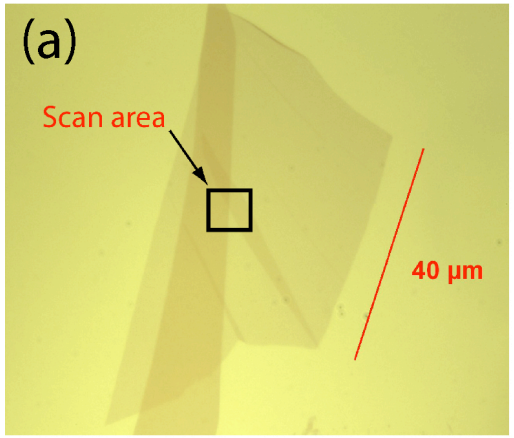
Important Questions…

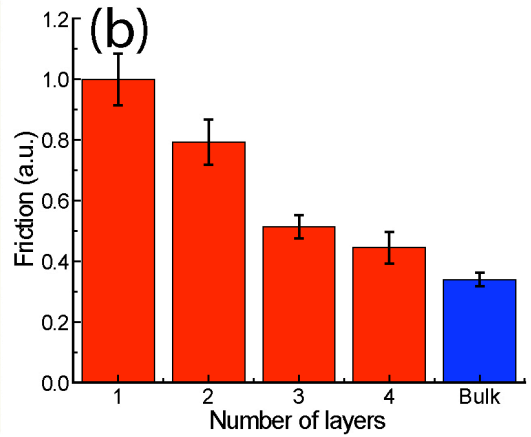
What is the pencil made of? You can't expect a regular old pencil to carry the weight of an elephant.

How do you get the elephant onto the pencil? Wait a second, back up. Is it an African elephant, weighing in at, say, 15,000 pounds, or is it the more diminutive Asian elephant, tipping the scales at a more manageable 10,000 pounds?

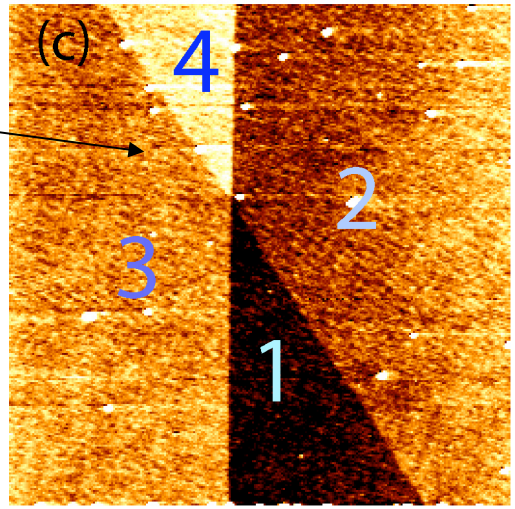
A baby Asian elephant is going to be the easiest choice to get onto the pencil. As it approaches the graphene, do the researchers play Henry Mancini's "Baby Elephant Walk"? If not, why not? These opportunities don't come along every day.

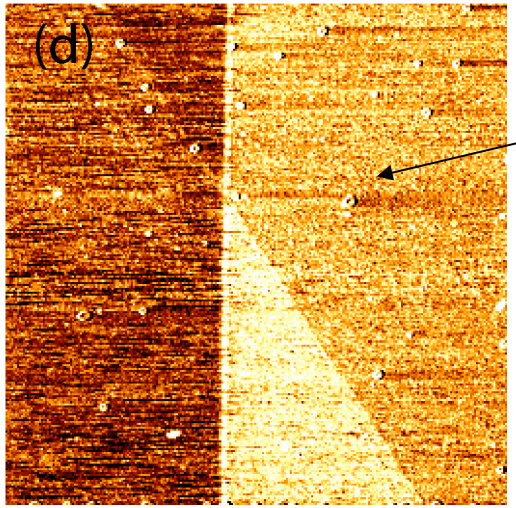
Graphene Friction





topography



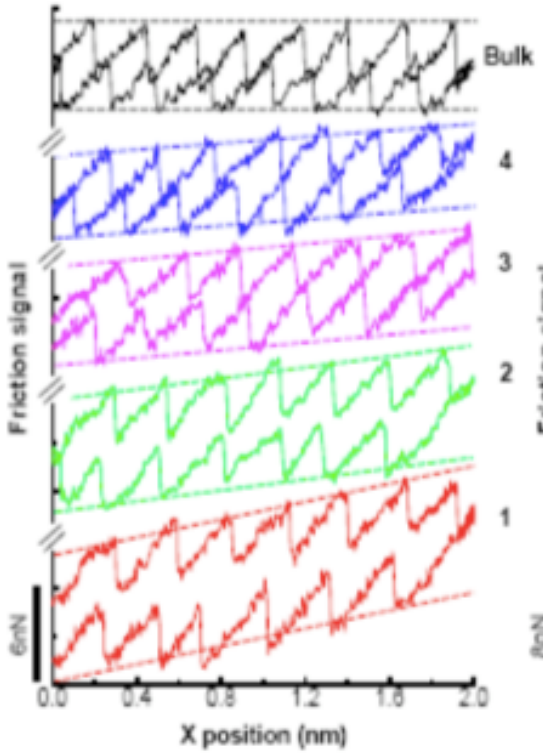


friction

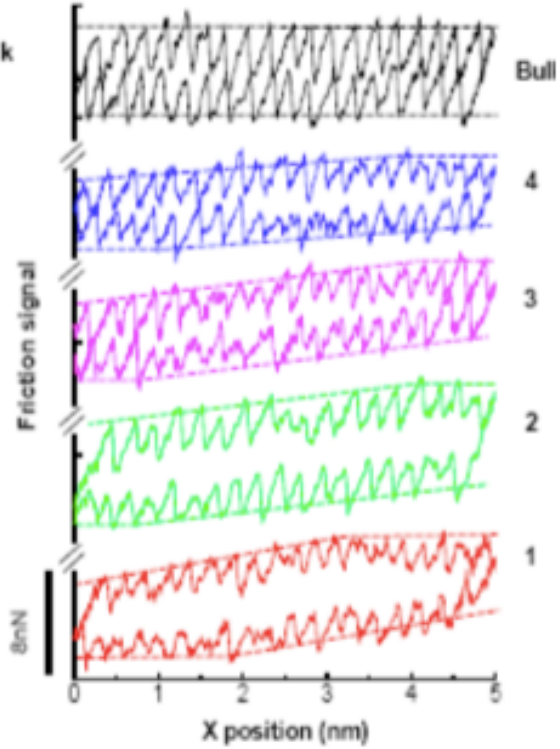
friction increases as the number of layers decreases…

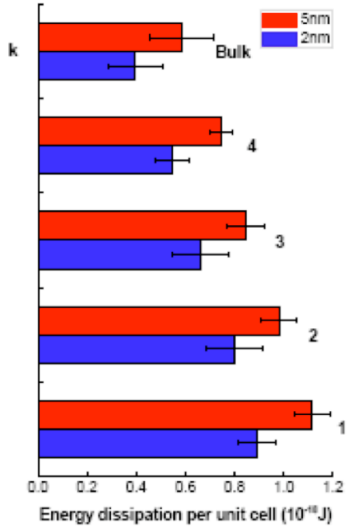
Stick-slip friction loops

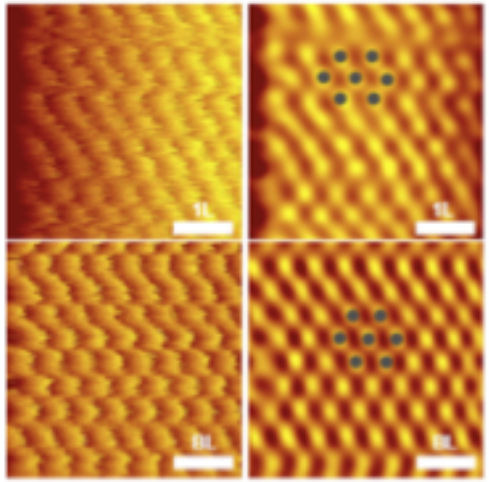
2 nm scan



5 nm scan







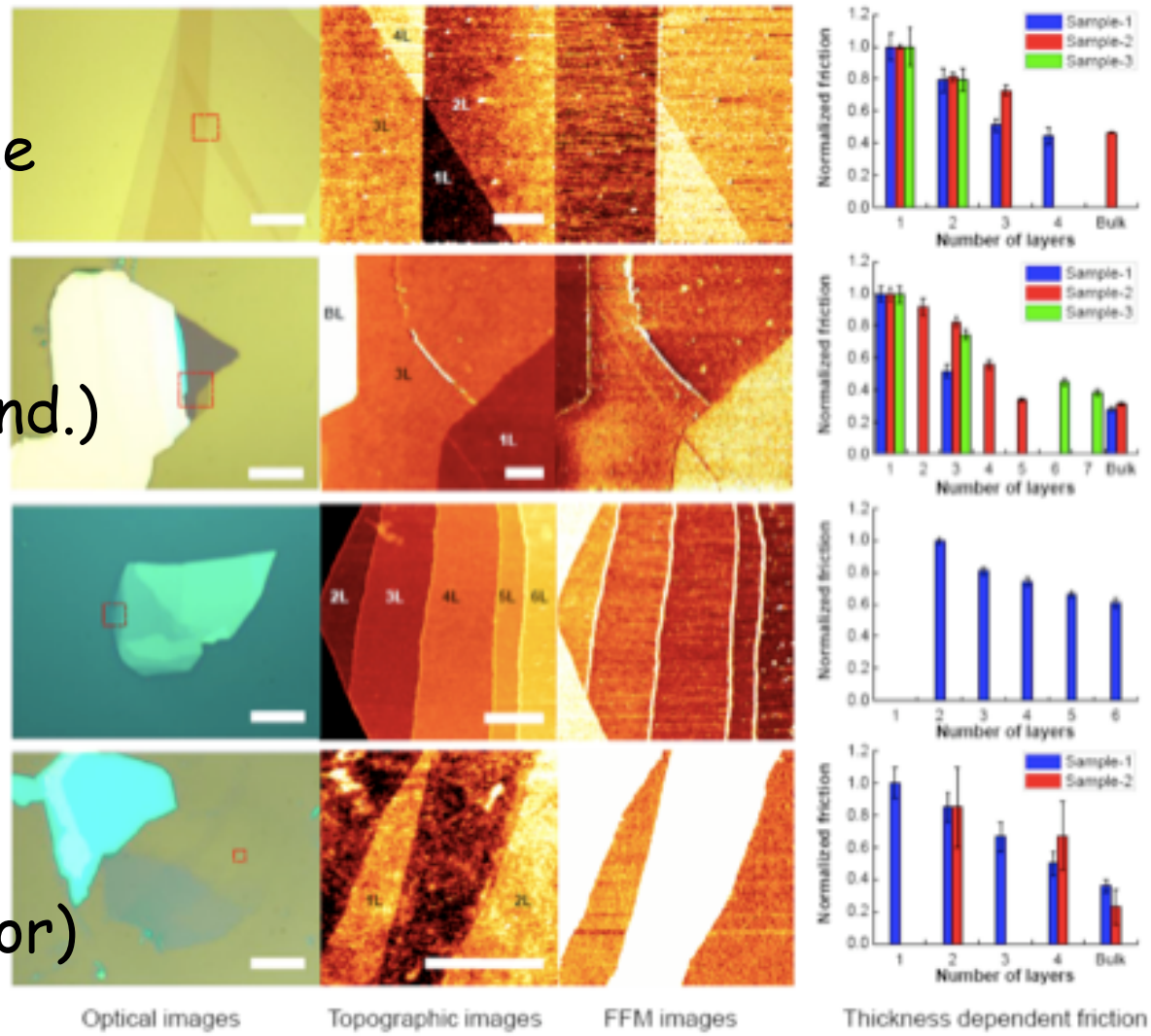
Friction in Layered Materials

Graphene

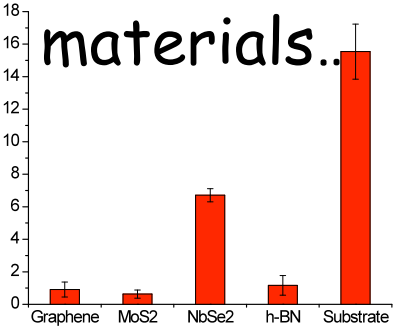
MoS_2 (Semicond.)

$NbSe_2$ (Metal)

h-BN (Insulator)



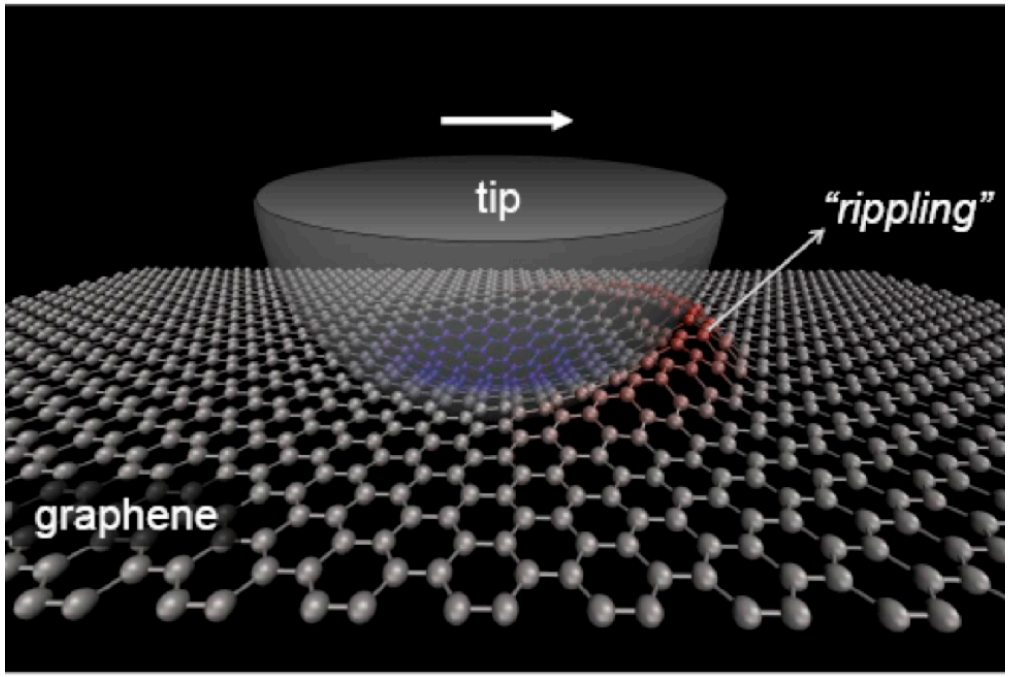
Thinner = higher friction For all 4 materials..

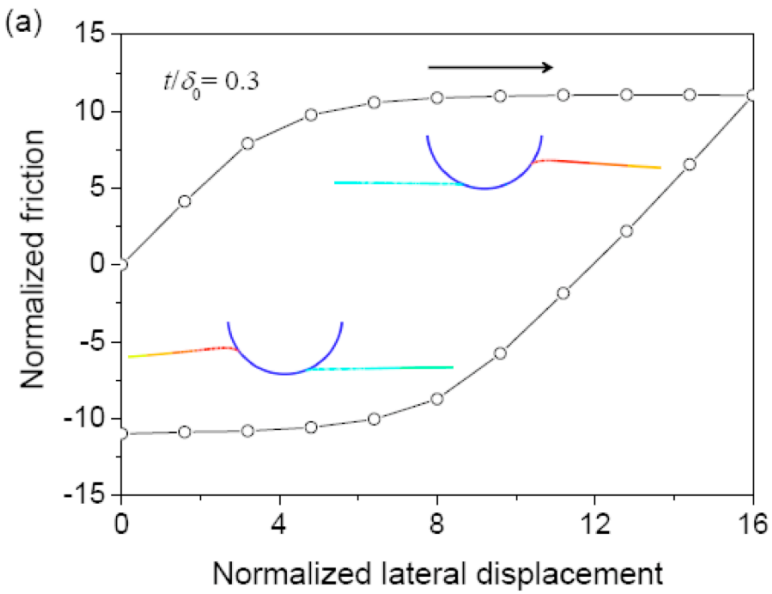


Friction magnitude smallest for graphene and MoS_2

Changgu Lee, Qunyung Li, William Kalb, Xin-Zhou Liu, Helmuth Berger, Robert Carpick, JH, *Science* 328, 76 (2010)

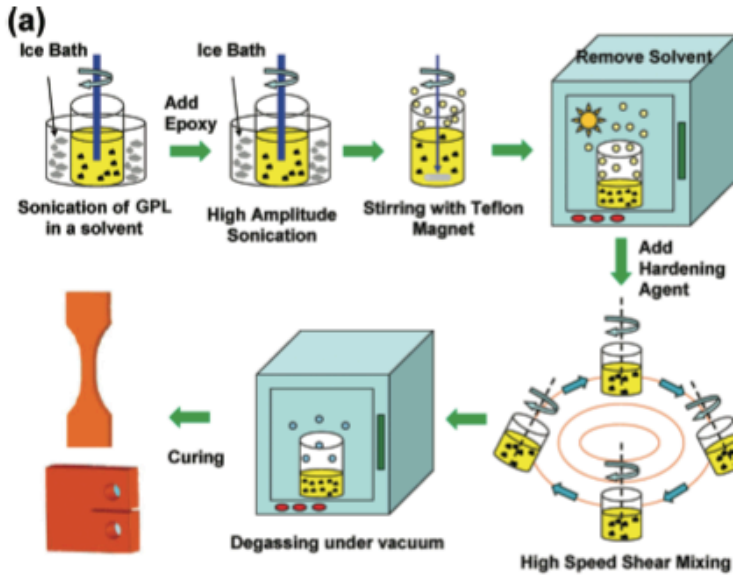
Simple model…

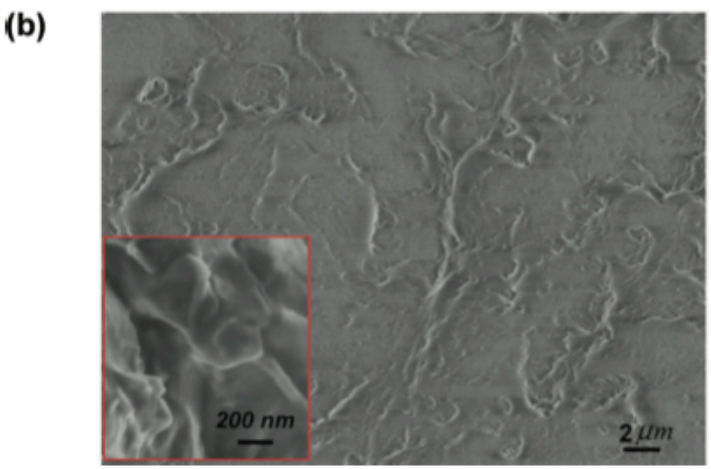


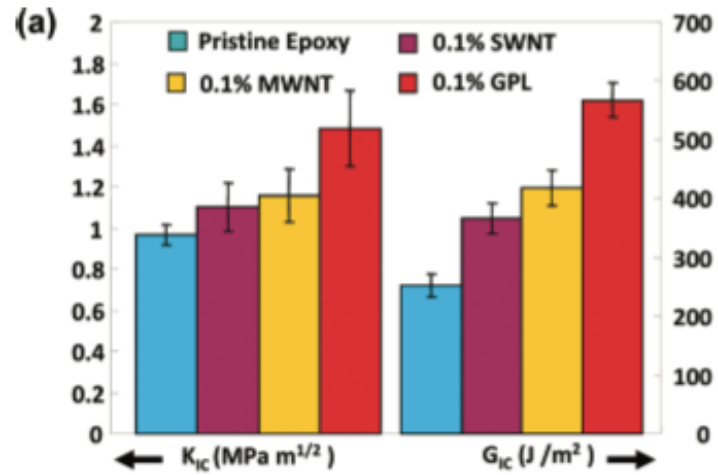


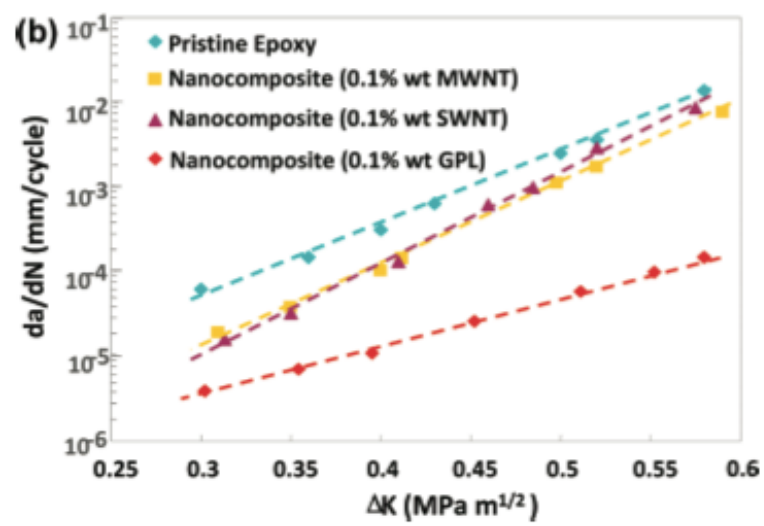
Some applications….

Composite Materials



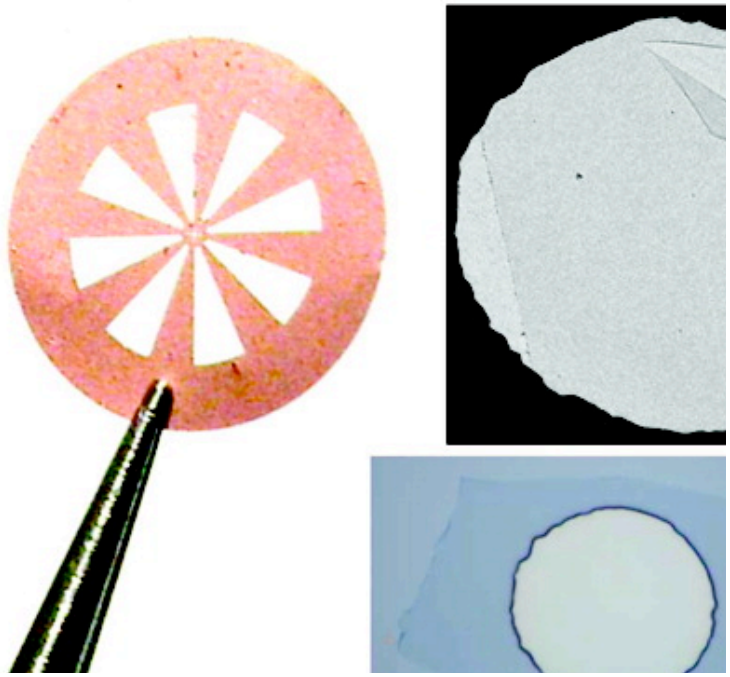




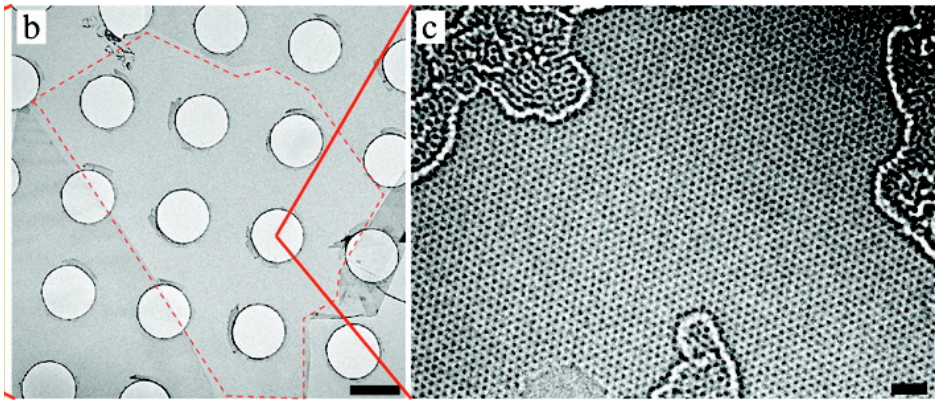


Raffiee et al, ACS Nano (2009)

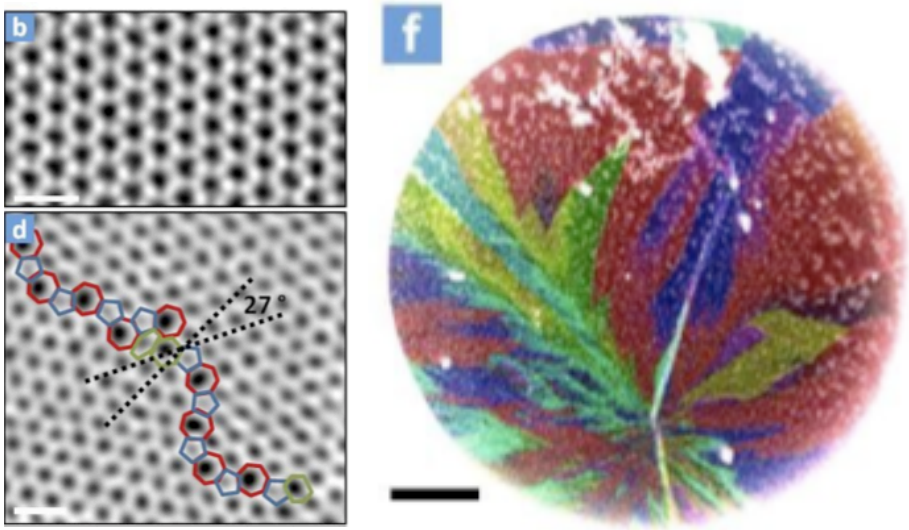
Free-Standing Membranes



Tim J. Booth; Peter Blake; Rahul R. Nair; Da Jiang; Ernie W. Hill; Ursel Bangert; Andrew Bleloch; Mhairi Gass; Kostya S. Novoselov; M. I. Katsnelson; A. K. Geim; *Nano Lett.* **2008,** 8, 2442-2446.



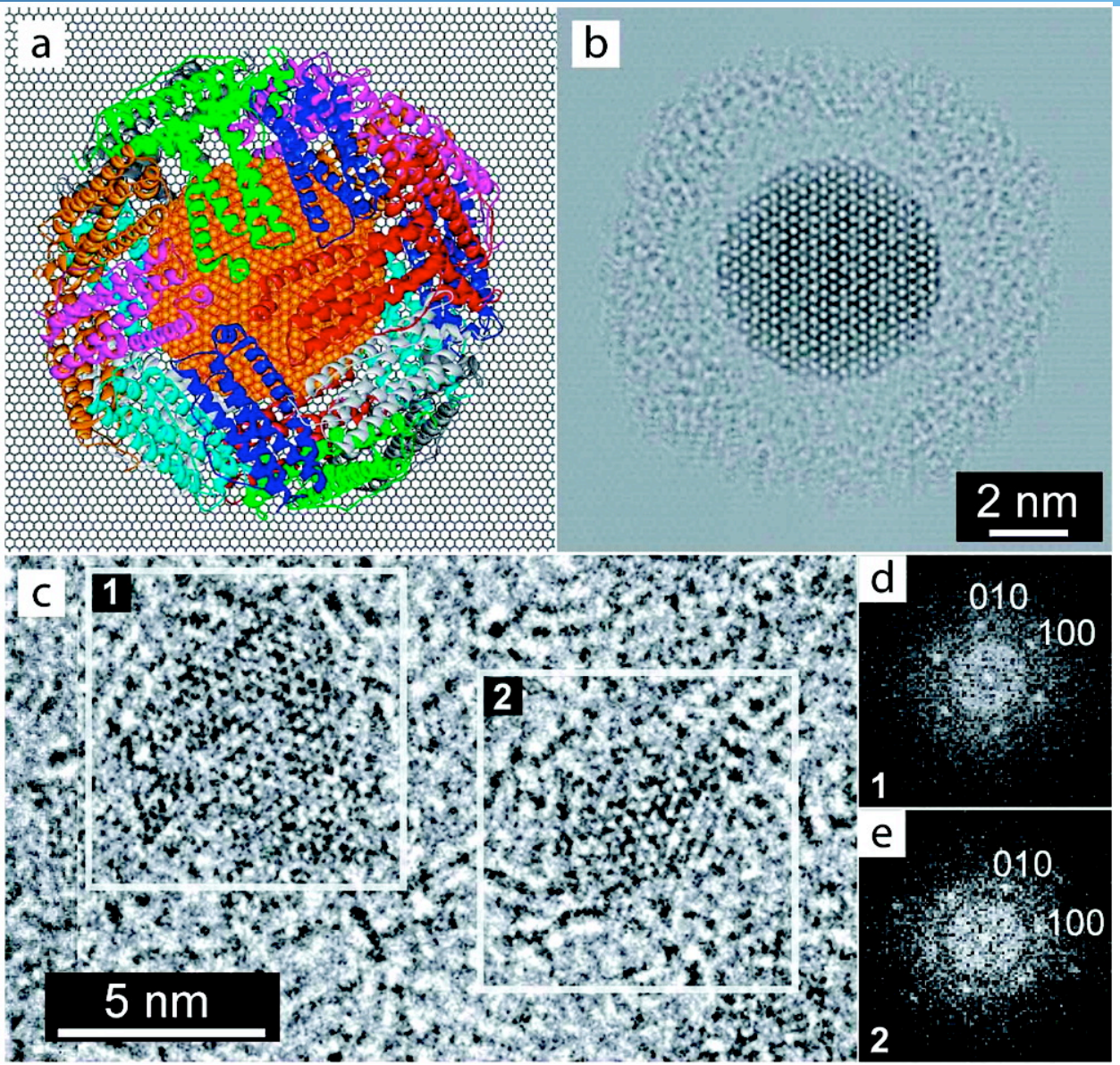
Jannik C. Meyer; C. Kisielowski; R. Erni; Marta D. Rossell; M. F. Crommie; A. Zettl; *Nano Lett.* **2008,** 8, 3582-3586



McEuen group, in press

TEM-compatible membranes: direct imaging of graphene lattice and defects / grains

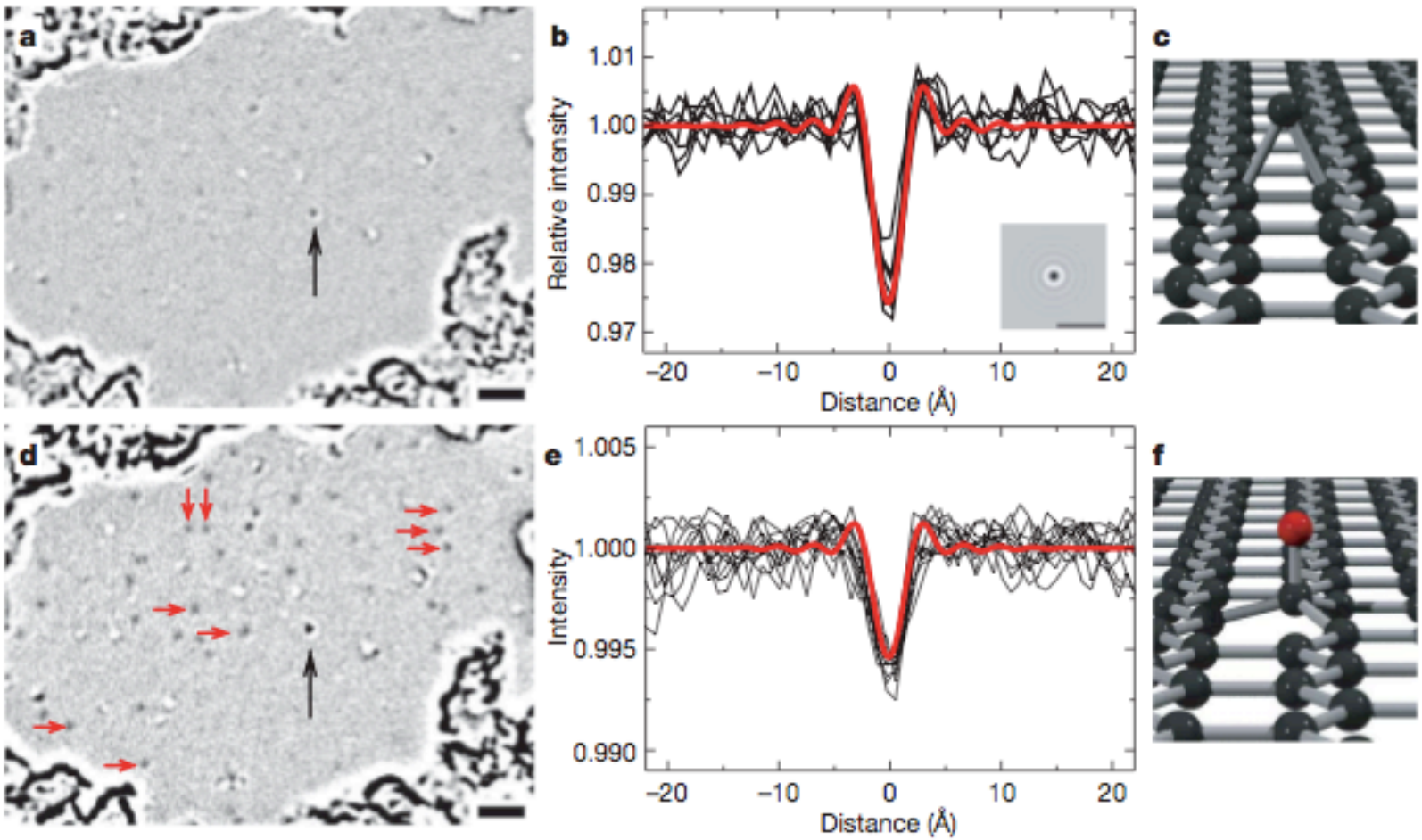
Free-Standing Membranes



TEM imaging of proteins / biomolecules

Neil R. Wilson; Priyanka A. Pandey; Richard Beanland; Robert J. Young; Ian A. Kinloch; Lei Gong; Zheng Liu; Kazu Suenaga; Jonathan P. Rourke; Stephen J. York; Jeremy Sloan; *ACS Nano* **2009**, 3, 2547-2556.

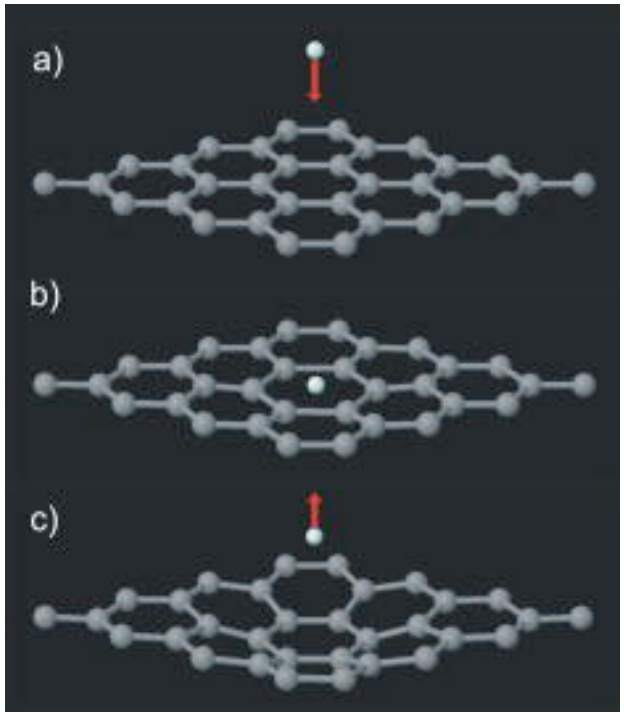
Free-Standing Membranes



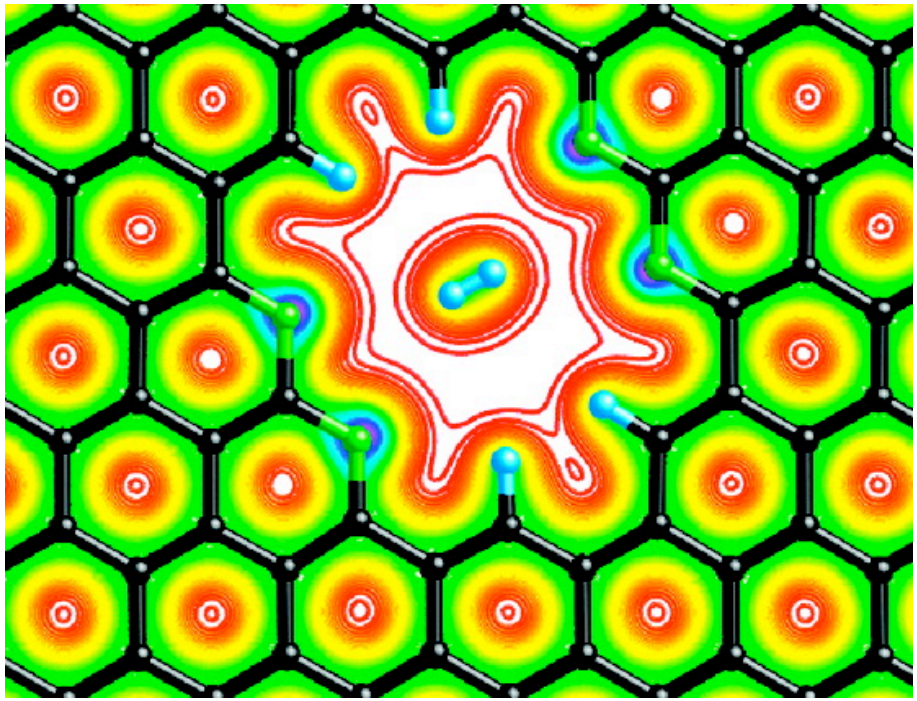
Meyer and Zettl, Nature 2008

TEM imaging of single hydrogens!

Gas Trapping and Separation

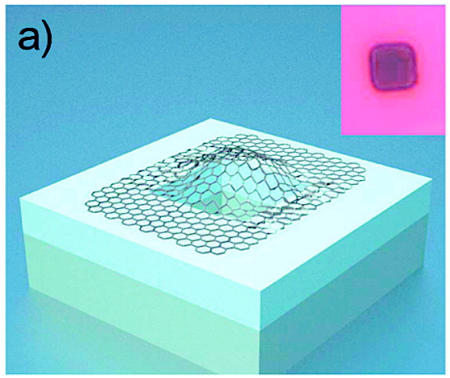


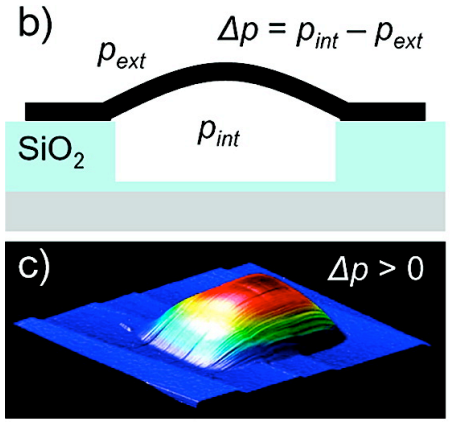
Appl. Phys. Lett. 93, 193107 (2008)

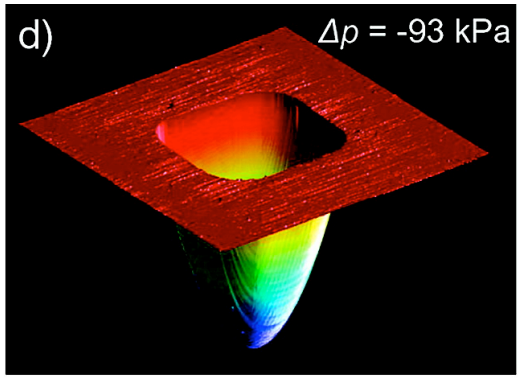


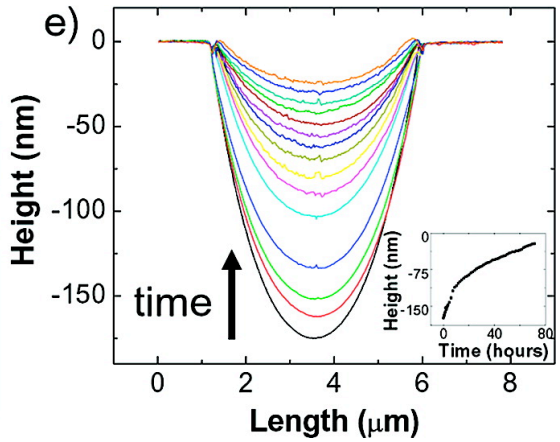
De-en Jiang; Valentino R. Cooper; Sheng Dai; *Nano Lett.* **2009,** 9, 4019-4024.

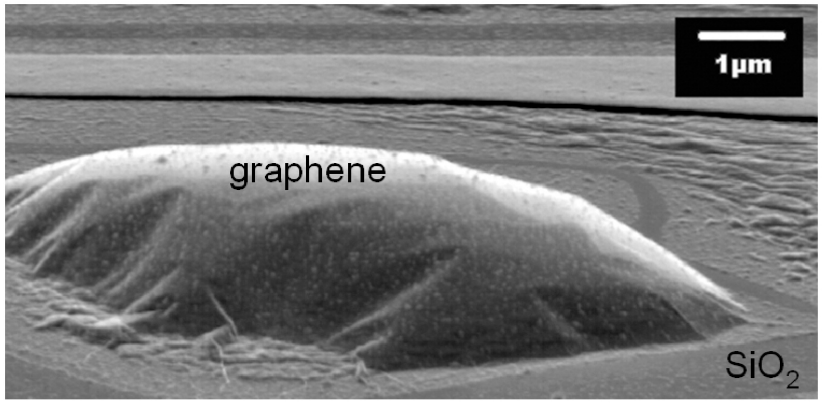
Graphene 'Bubbles'

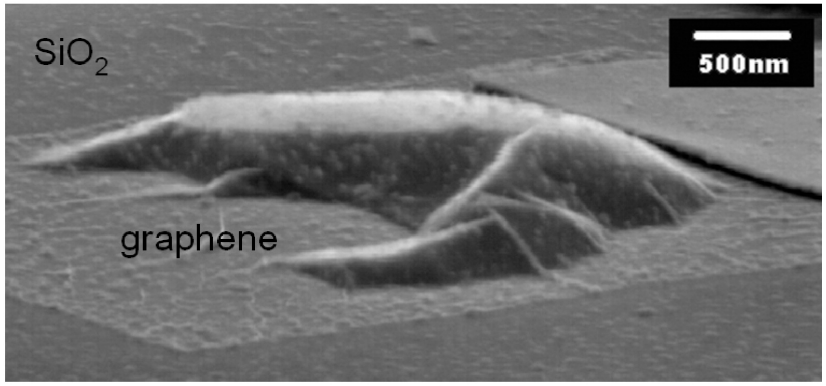








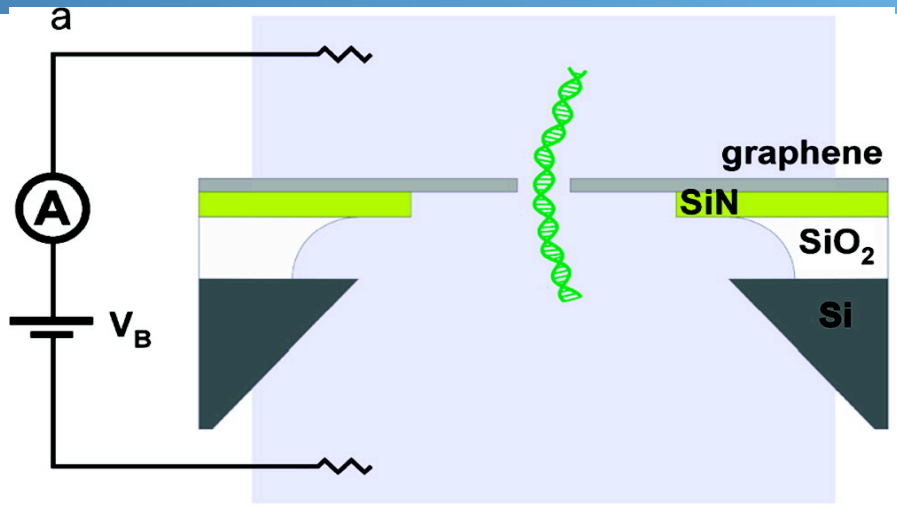


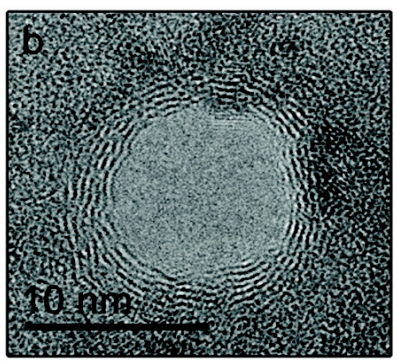


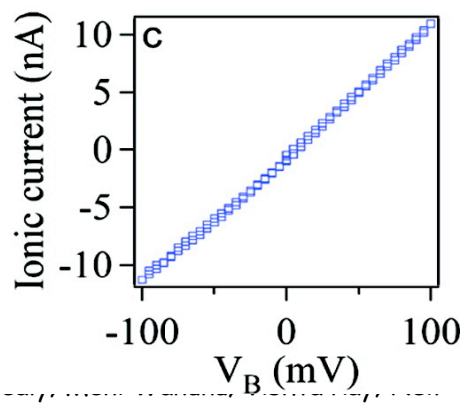
J. Scott Bunch; Scott S. Verbridge; Jonathan S. Alden; Arend M. van der Zande; Jeevak M. Parpia; Harold G. Craighead; Paul L. McEuen; *Nano Lett.* **2008,** 8, 2458-2462.

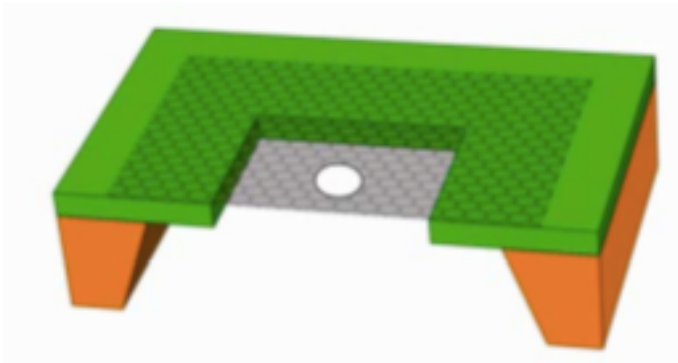
E. Stolyarova; D. Stolyarov; K. Bolotin; S. Ryu; L. Liu; K. T. Rim; M. Klima; M. Hybertsen; I. Pogorelsky; I. Pavlishin; K. Kusche; J. Hone; P. Kim; H. L. Stormer; V. Yakimenko; G. Flynn; *Nano Lett.* **2009,** 9, 332-337.

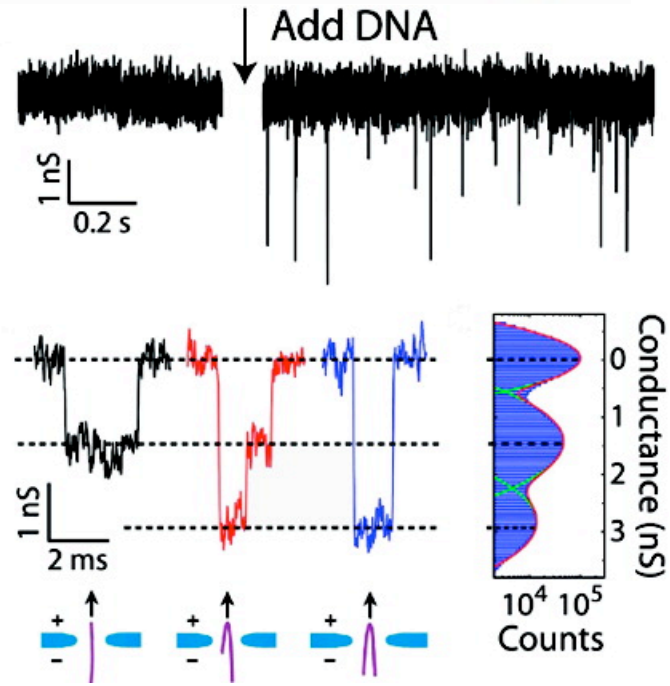
Nanopores: DNA sequencing









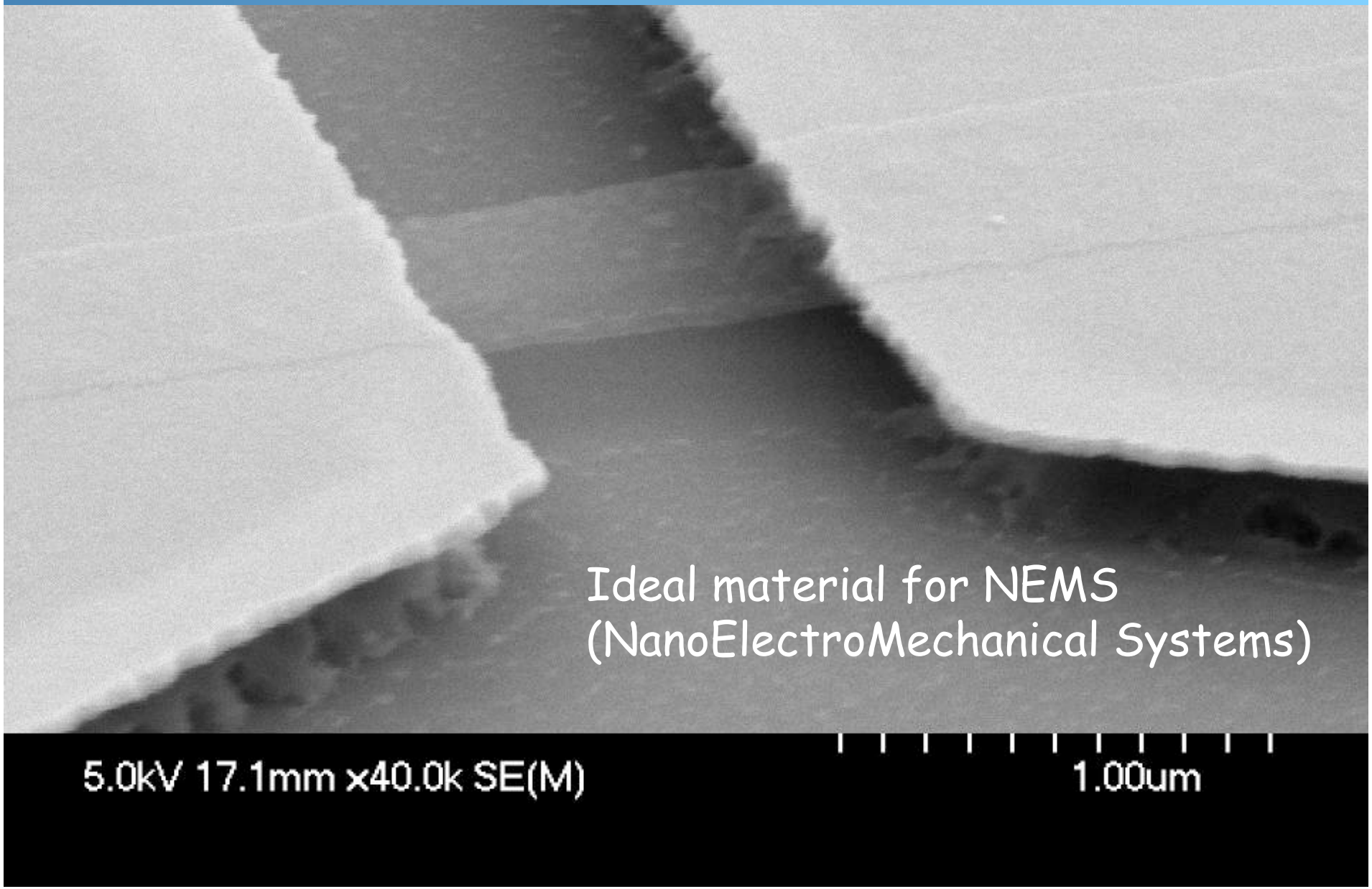


Peterman; John Bartel; Michael D. Fischbein; Kimberly Venta; Zhengtang Luo; A. T. Charlie Johnson; Marija Drndić; *Nano Lett.* **2010,** 10, 2915-2921.

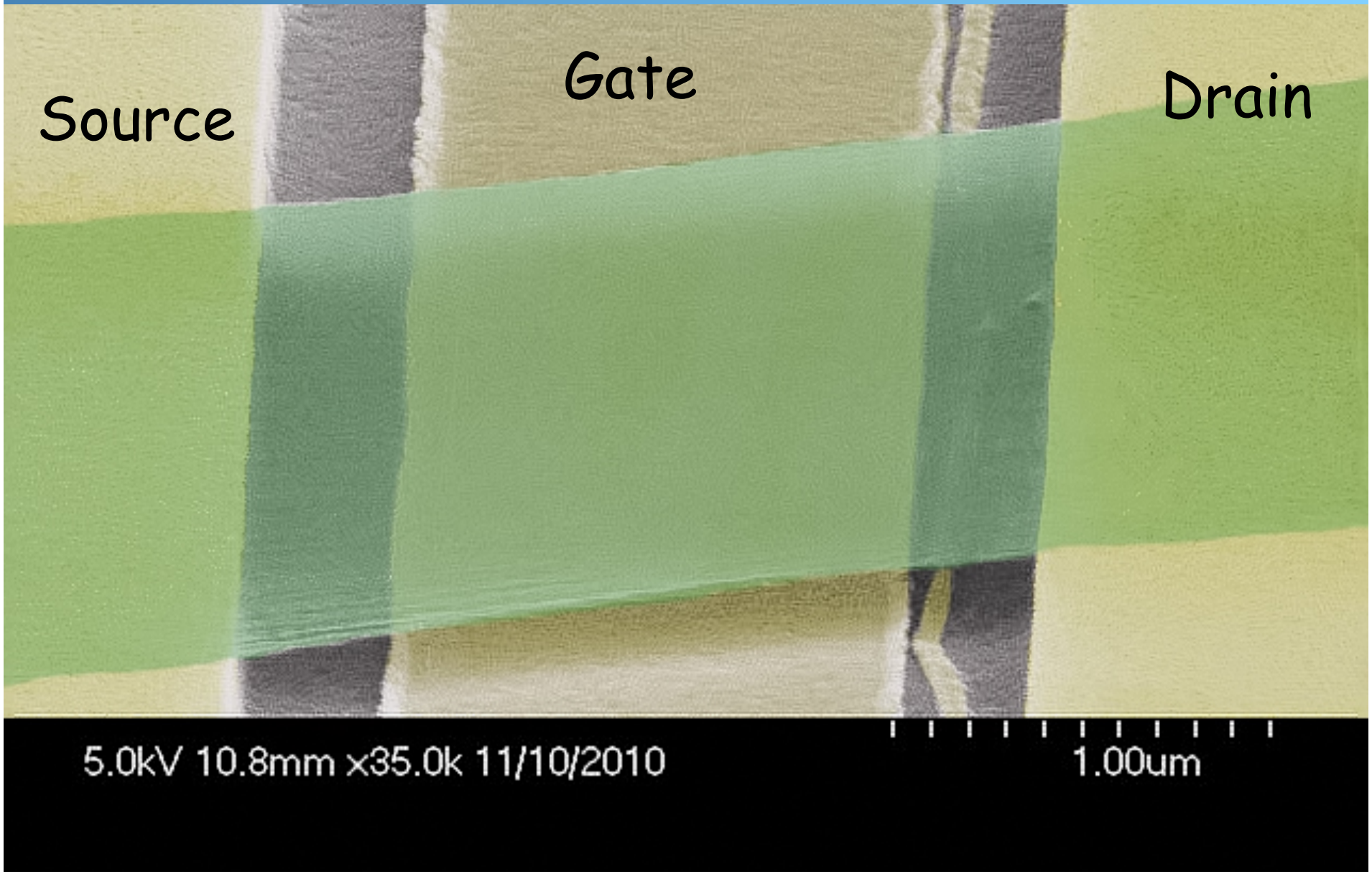
Grégory F. Schneider; Stefan W. Kowalczyk; Victor E. Calado; Grégory Pandraud; Henny W. Zandbergen; Lieven M. K. Vandersypen; Cees Dekker; *Nano Lett.* **2010,** 10, 3163-3167.

Golovchenko et al, Nature 2010

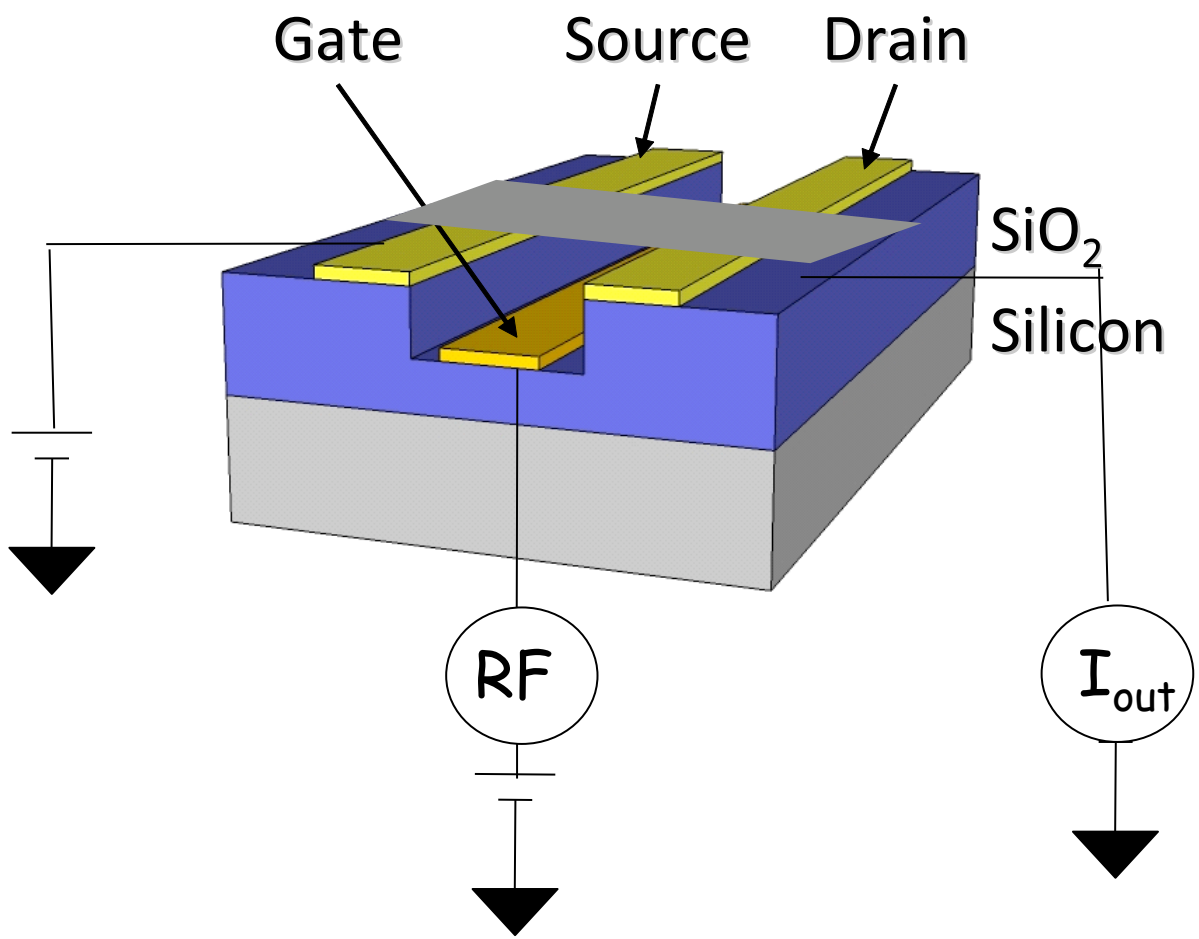
Suspended Graphene



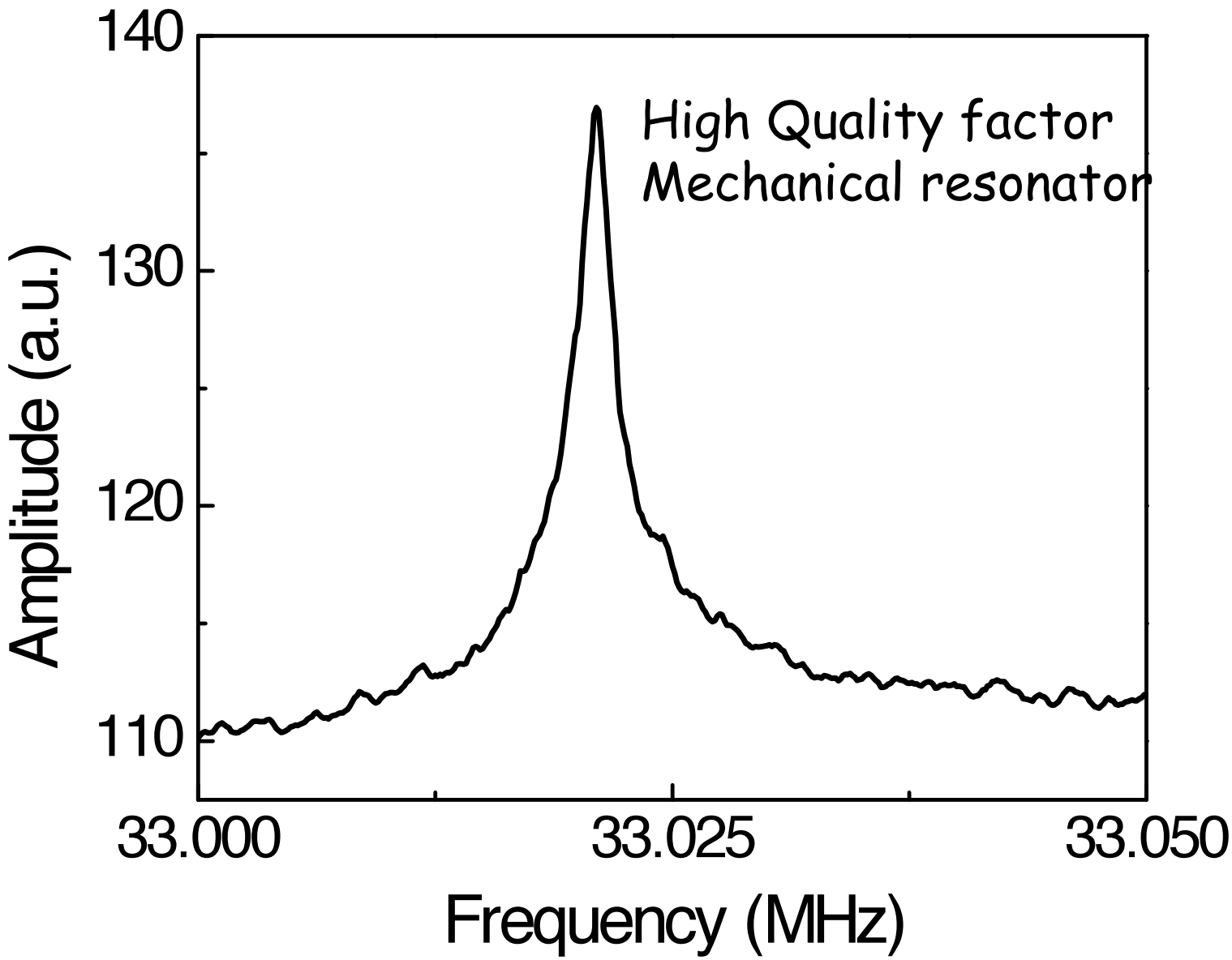
Graphene Mechanical Resonators



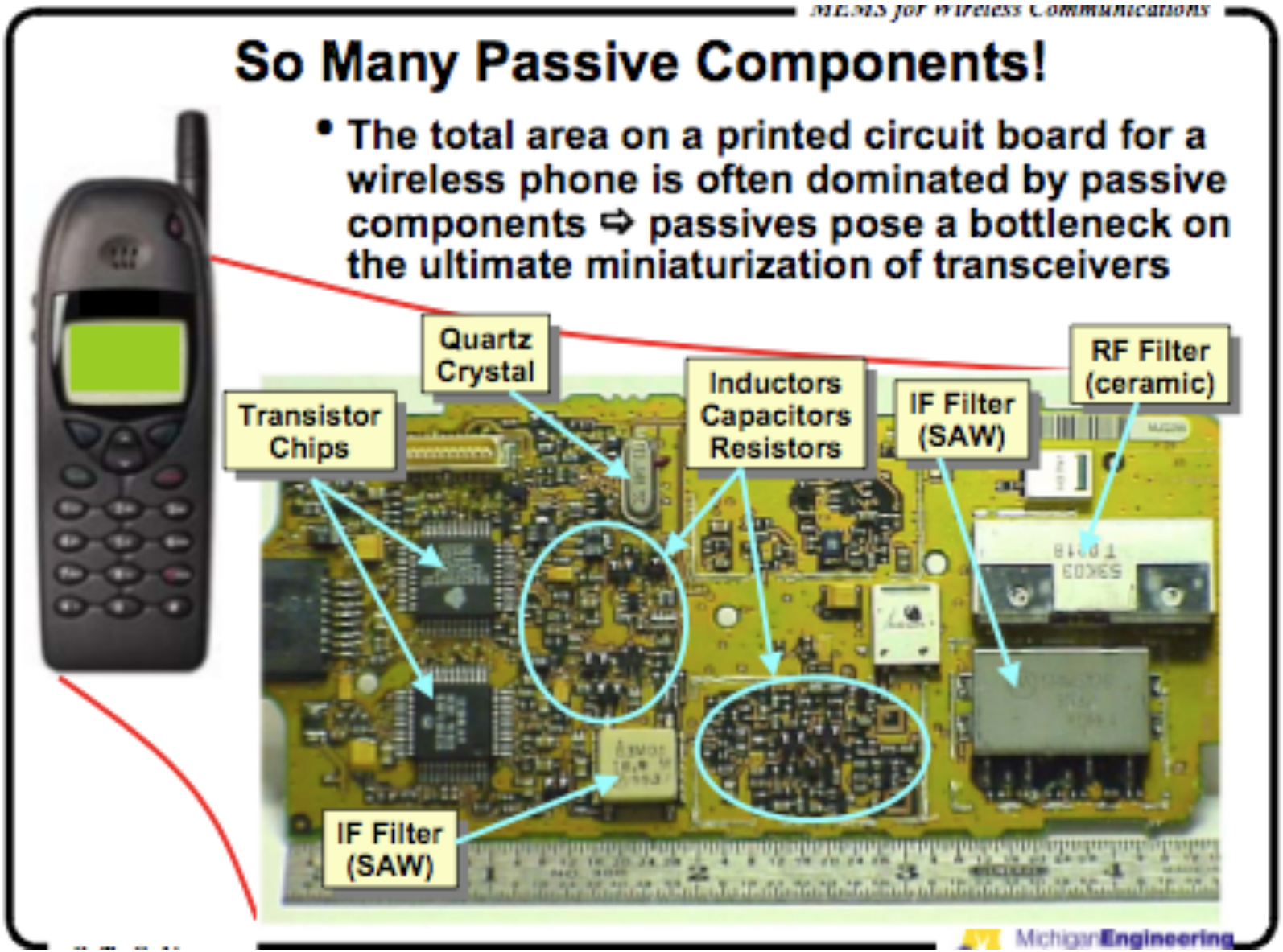
Graphene Mechanical Resonators



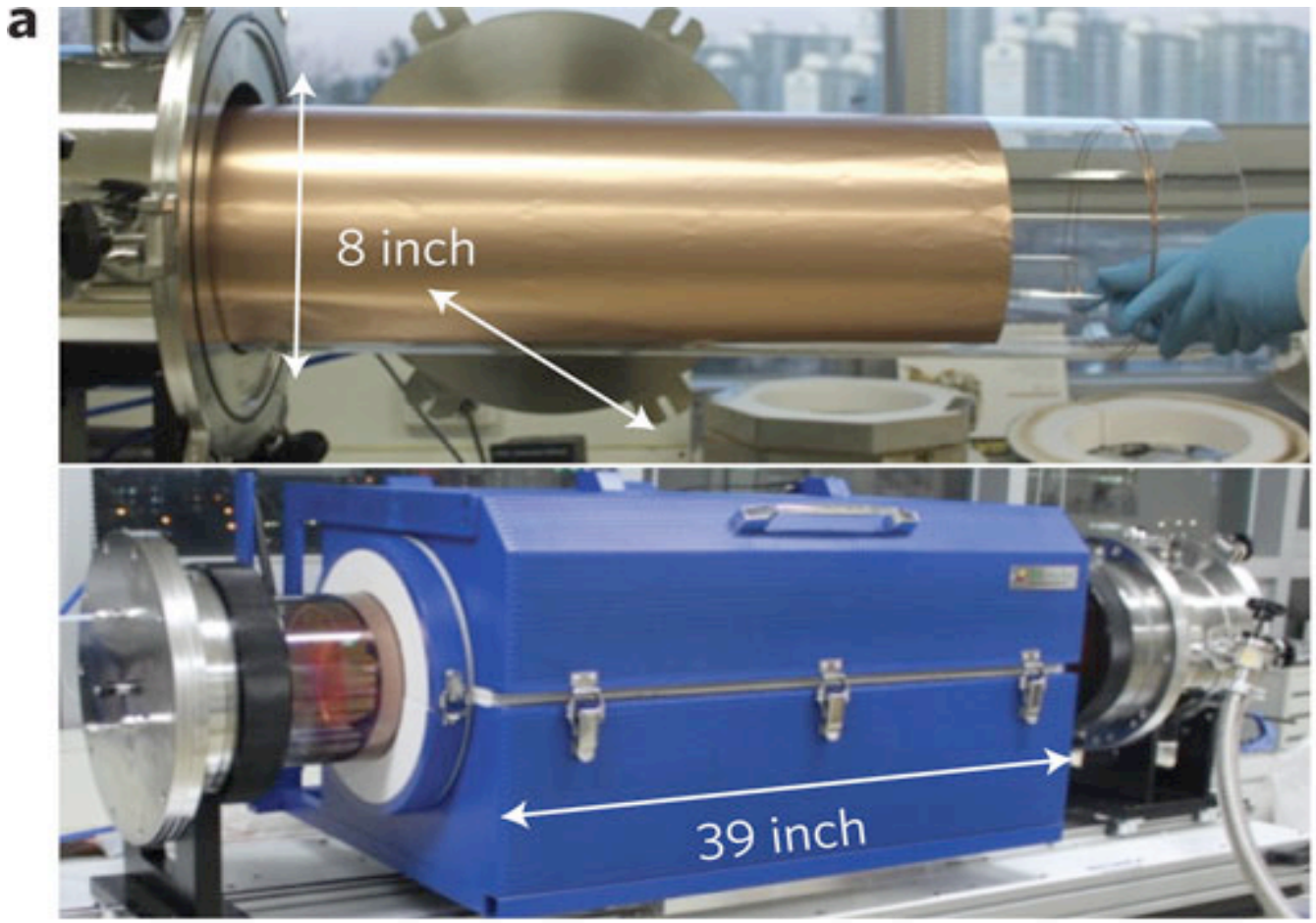
Graphene Mechanical Resonators



Mechanical Signal Processing



CVD Growth of Graphene



R. Ruoff, UT

B.H. Hong, SKKU (Korea)

At 1000 °C, methane (CH_4) becomes graphene on surface of copper