QED in a Pencil Trace

Andre Geim

in collaboration with
K. Novoselov, S. Morozov, F. Schedin, D. Jiang, T. Booth,
M. Katsnelson, J. Meyer & I. Grigorieva

GRAPHENE ALLOTROPES

3D
Graphite

2D
graphene

1D
Buckyballs
Carbon Nanotube

0D

PRESUMED NOT TO EXIST IN THE FREE STATE

multi-wall: 1952 to Iijima 1991
single-wall: 1993

Kroto et al 1985
Extracting a Single Plane

GRAPHITE IS STRONGLY LAYERED

SLICE DOWN TO ONE ATOMIC PLANE

individual atomic sheets: do they exist?

Free-Standing Graphene

Key: Visual Identification

AFM, low throughput, no clear signatures

OPTICS

single layer of atoms visible by naked eye

5% change in SiO$_2$, thickness important
Two Dimensional Crystallites

not just flakes but graphene crystallites

No Free Standing Monolayers Known

CHEMICAL DECOMPOSITION
up to Ø 2 µm; > 80 layers; Krishnan 1998

EPITAXIAL GROWTH
McConville 1986 (on Ni); Land 1992 (on Pt)
Affoune 2001 (on HOPG); Nagashima 1993 (on TiC)
quality & continuity (?)

SiC: Bommel 1975; Forbeaux 1998; Charrier 2002;
Berger 2004-2006; Rotenberg 2006

CHEMICAL EXFOLIATION
Dresselhauses (2002 review)
restacked and scrolled SOOT
Individual folds: Horiuchi 2004
mechanical cleavage in retrospect

Ohashi (Tanso 1997, 2000)
from 1000 down to 50 layers

**our work (Science 2004): single layer**

Philip Kim’s & Paul McEuen’s groups
(PRL 2005 & Nanoletters 2005) down to 35 layers

Nanopencil

AFM cantilever

Graphite microcrystal

Cleaved thin layer

Insulating substrate

for >10 layers,
electronic structure
of bulk graphite
(Partoen 2006)

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Other 2D Atomic Crystals

2D boron nitride in AFM

2D NbSe₂ in AFM

also,
2,3,4... layers

2D Bi₂Sr₂CaCu₂Oₓ in SEM

2D MoS₂ in optics

*PNAS 102*,
10451 (2005)
2D ATOMIC CRYSTALS

a new kind of materials
wide choice of materials properties
(electronic, mechanical, chemical, etc.)

Why 2D Crystals Exist?

stabilizing influence of substrate
extracted from 3D, remain (meta)stable
something else
“Free-Hanging” Graphene

TEM confirms single layer

one-atom-thick single-crystal fabric

single fold at the edge

Structure Of Suspended Graphene

local electron diffraction (beam Ø 250 nm)

diffraction peaks away from tilt axis become blurred

normal incidence

26° tilt
**Structure Of Suspended Graphene**

- reciprocal space
- real space

- cones in Fourier space rather than rods

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**Intrinsic Microscopic Crumpling**

- $5^\circ$ angle, and highly reproducible for different samples
- twice smaller ($\approx 2^\circ$) for bilayer graphene
- disappears for few-layer samples

- isotropic: local strain up to 1%
Intrinsic Microscopic Crumpling

atomic resolution TEM
ripple contrast appears for >1 layer

height ≈5Å; size ≈5nm;
elastic energy
few kT per ripple

Elastic Energy in 2D may stabilize 3D sheet
loose in elastic energy but gain in entropy; Nelson (1987)

Electron Transport in Graphene
Graphene Devices

- optical image
- SEM image
- design
- contacts and mesa

Electric Field Effect in Graphene

- Au contacts
- SiO₂
- Si
- 2D crystal

resistivity

graphene: currently up to 10,000 cm²/V·s at 300K

ballistic transport on submicron scale under ambient conditions
Electric Field Effect in Graphene

conductivity

\[ \sigma = n(V_g) e \mu \]

Hall effect

\[ \frac{1}{\rho_{xy}} = n e / B \]

simple behaviour;
practically constant mobility;
no trapped carriers

Quantum Oscillations in Graphene

degeneracy \( f = 4 \)
two spins & two valleys

\[ \Delta n = 4 B / \Phi_0 \]

\[ \omega_c r = \text{const} \]
Quantum Oscillations in Graphene

degeneracy $f=4$
two spins & two valleys

$\Delta n = 4B/\Phi_0$

$\rho_{xx}(kV)$

$\Delta\sigma_{xx} \propto T, \sinh\left(\frac{2\pi^2 k_B T m_c}{\hbar eB}\right)$

Band Structure of Graphene

cyclotron mass strongly depends on concentration

$E = pc^*$

$B_F = (\hbar/2\pi e)S$ and $m_c = (\hbar^2/2m)\partial S/\partial E$

experimental dependences

$B_F \sim n$ and $m_c \sim n^{1/2}$
necessitates $S \sim E(k)^2$ or $E \sim k$

$cyclotron mass$

$E = m_c v_F^2$

$v_F = 10^6 \text{ m/s } \pm 5\%$

previously seen in zero-gap 3D semiconductors (Zawadzki 1974)
CONDUCTIVITY “WITHOUT” CHARGE CARRIERS

Minimum Quantum Conductivity

no temperature dependence in the peak from 3 to 100K
Minimum Quantum Conductivity

Most theories predict \( \tau \)-times larger value of minimum conductivity than predicted by \( h/4e^2 \) per spin and valley.

\[ \sigma = \frac{e^2}{h} \quad \text{as observed in graphene} \]

Mott's argument: \( l \geq \lambda_F \)

\[ \sigma = \frac{e^2}{h} \quad k_F l \geq h \]

Quantized resistivity NOT resistance
CHIRAL Quantum Hall Effects

Quantum Hall Effect in Graphene

quadruple degeneracy: plateaus are expected at $h/4Ne^2$

![Graphene Quantum Hall Effect Diagram]
Quantum Hall Effect in Graphene

quadruple degeneracy:
plateaus are expected at $\hbar/4Ne^2$

"half-integer" QHE

Chiral Fermions in Graphene

two sublattices $\rightarrow$ superposition of their wavefunctions $\rightarrow$ spinors (2 projections of pseudospin)
Chiral Fermions in Graphene

Dirac equation:

\[ \hat{H} = v_F \begin{pmatrix} 0 & \hat{p}_x + i\hat{p}_y \\ \hat{p}_x - i\hat{p}_y & 0 \end{pmatrix} = v_F \vec{\sigma} \cdot \vec{p} \]

Quantization of Dirac fermions:

\[ E = \pm \sqrt{2\hbar v_F^2 eBN} \]

\[ \Delta n = 4B/\phi_0 \]

McClure 1956
Semenoff 1984
Haldane 1988
Gonzalez 1993
Gusynin 2005; Peres 2006
Normal Electrons

$E_{LL} = \pm \hbar \omega_c (N + \frac{1}{2})$

Dirac Fermions

$E_{LL} = \pm v_F \sqrt{2e\hbar B} \sqrt{N}$

$\Delta n = 4B/\varphi_0$

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half-integer quantum Hall effect

$\rho_{xx} (\Omega)$

$\sigma_{xy} (4e^2/\hbar)$

relativistic analogue of the integer QHE

$n (10^{12} \text{ cm}^{-2})$

$\Delta n = 4B/\varphi_0$

also Zhang et al, ibid 201 (2005)
**room-temperature QHE**

\[ \Delta E_{LL} = v_F \sqrt{2e\hbar B} \]
\[ \Delta E_{LL}(K) = 420\sqrt{B(T)} \]

previously, only below 30K

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**BILAYER GRAPHENE**

\[ E(p) = \pm \frac{1}{2} \gamma_1 \pm \sqrt{\frac{1}{4} \gamma_1^2 + v_F^2 p^2} \]

*Nature Phys 2, 177 (2006)*
QHE in bilayer graphene

massive chiral fermions

\[ E(p) = \pm \frac{1}{2} \gamma_1 + \sqrt{4 \gamma_1^2 + v_F^2 p^2} \]

\[ \hat{F} = -\frac{1}{2m} \begin{pmatrix} 0 & (\hat{p}_x + i\hat{p}_y)^2 \\ (\hat{p}_x - i\hat{p}_y)^2 & 0 \end{pmatrix} \]

\[ E_N = \pm \hbar \omega_c \sqrt{N(N-1)} \]

McCann & Falko 2006
massive chiral fermions

\[ E(p) = \pm \frac{1}{2} \gamma_1 \pm \sqrt{\frac{1}{4} \gamma_1^2 + v_F^2 p^2} \]

\[ \hat{F} = - \frac{1}{2m} \left( \hat{p}_x + i \hat{p}_y \right) \]

\[ E_N = \pm \hbar \omega_c \sqrt{N(N-1)} \]

McCann & Falko, PRL 2006

massive Dirac Fermions

\[ E_{LL} = \pm \frac{1}{2} \hbar \omega_c \sqrt{N(N-1)} \]

Normal Electrons

\[ E_{LL} = \pm \frac{1}{2} \hbar \omega_c (N + \frac{1}{2}) \]

D(E)
Tuneable-Gap Semiconductor

\[ T = 4K \]

\[ \sigma_y (4e^2/h) \]

\[ V_g (V) \]

\[ B = 12T \]

pristine

\[ V_g \]

\[ \text{SiO}_2 \]

3.4A

\[ E \approx 1 \text{V/nm} \]

Tuneable-Gap Semiconductor

\[ T = 4K \]

\[ \sigma_y (4e^2/h) \]

\[ V_g (V) \]

\[ B = 12T \]

doped

semiconductor with electrically controlled gap up to 0.3 eV
three types of IQHE

conventional IQHE

all LL at non-zero E zero Berry phase

Dirac fermions
one LL at zero E Berry phase \( \pi \) metallic at \( \sqrt{0} \)

half-integer QHE in graphene

all LL at non-zero E zero Berry phase
three types of IQHE

- **conventional IQHE**
  - all LL at non-zero E
  - zero Berry phase

- **half-integer QHE in graphene**
  - Dirac fermions
  - one LL at zero E
  - Berry phase $\pi$
  - metallic at $v^\perp 0$

- **chiral IQHE in bilayer graphene**
  - massive Dirac fermions
  - two LLs at zero E
  - Berry phase $2\pi$
  - metallic at $v^\perp 0$

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POTENTIAL APPLICATIONS

just think of carbon nanotubes
but ... in large wafers
with fully reproducible properties

unless specifically 1D phenomena
**GRAPHENE-BASED ELECTRONICS**

- **ballistic field-effect transistor**
  - THz-frequency operation
  (despite low on-off ratio ~10 at room T)

- **chemical sensors**
  (detection of a single gas molecule!)

- **superconducting FETs**
  (Delft group)

- **spintronics with gate control**
GRAPhene DREAMS

entire SET-like circuitry carved in graphene

stable down to a single benzene ring molecular electronics but with the top-down approach

CONCLUSIONS

strictly-2D crystals do exist

“relativistic” condensed matter physics

(addressing QED phenomena such as zitterbewegung, Klein paradox, etc)

REALISTIC POSSIBILITY OF VARIOUS APPLICATIONS