Transport properties of graphene.

1. Effect of disorder on transport in graphene.

2. Quantum dots in graphene.
Hamiltonian $H_0$ for free electrons

$$H_0 = -i\nu \sum \nabla$$

v-velocity

$$\Sigma_{x,y} = 1^{K,K'} \otimes \tau_{x,y}^{A,B},$$

$\tau_{x,y,z}$ - Pauli matrices

A,B-sublattice space, K,K’ - valley space

Full Hamiltonian (no spin): $H = H_0 + \hat{V}$

The external potential $\hat{V}$ is generally a 4x4 matrix.

It is proportional to unity matrix only if it varies slowly on the lattice period.

Two problems:
1. $\hat{V}$ is disorder potential (full matrix)
2. $\hat{V}$ is a barrier across the graphene strip (unity matrix).
New results for 1).

a) Most general structure of $\hat{V}$ is identified (using most general symmetries).

b) Going beyond the self-consistent Born approximation (SCBA) (derivation and solution of RG equations).

c) Derivation of a supermatrix non-linear $\sigma$-model (ultimate localization but a complicated crossover).

Symmetries of the model:

1. Time reversal (TR) symmetry (exact)

$$\varphi^T(r) = (\varphi_A, \varphi_B)_{AB}; (\varphi_B^*, -\varphi_A^*)_{AB} \,_{K,K'}$$

$$\varphi^*(r) = \hat{z} \varphi(r), \quad \hat{z} = \tau_y^{A,B} \otimes \tau_y^{K,K'}$$

$$H = \hat{z}H^T \hat{z}$$
Disorder may break all the symmetries except the time reversal one!

The most general form:

\[ \hat{V}(r) = u_0(r) + \sum_{\{m,i\} = \{x,y,z\}} \hat{G}_{m,i} u_{m,i}(r), \quad \hat{G}_{m,i} = \tau^K_{m'} \otimes \tau^A_i \]

\[ u_0(r), u_m(r) \text{ are real independent random functions} \]

However, averaging must restore rotation, reflection, translation and \( C_{6v} \) symmetries!

\[ \langle \hat{V}(\mathbf{r}_1) \otimes \hat{V}(\mathbf{r}_2) \rangle = \delta(|\mathbf{r}_1 - \mathbf{r}_2|) \; \gamma_0 \; \mathbb{1} \otimes \mathbb{1} \]
\[ + \delta(|\mathbf{r}_1 - \mathbf{r}_2|) \; \gamma_{||} \; \hat{G}_{z,z} \otimes \hat{G}_{z,z} \]
\[ + \delta(|\mathbf{r}_1 - \mathbf{r}_2|) \; \gamma_{\perp} \; \sum_{j=x,y} \hat{G}_{z,j} \otimes \hat{G}_{z,j} \]
\[ + \delta(|\mathbf{r}_1 - \mathbf{r}_2|) \; \beta_{||} \; \sum_{m=x,y} \hat{G}_{m,z} \otimes \hat{G}_{m,z} \]
\[ + \delta(|\mathbf{r}_1 - \mathbf{r}_2|) \; \beta_{\perp} \; \sum_{j,m=x,y} \hat{G}_{m,j} \otimes \hat{G}_{m,j} \]

\( \gamma_0 \) is due to long range impurities (with respect to the lattice period)

\( \beta \) - intervalley scattering

Five independent real constants for the disorder.

\( \gamma_0 \) is due to long range impurities

\( \beta \) - intervalley scattering
Impurities conserving the chirality.
\[ \hat{V}(r) = -\hat{\Sigma}_z \hat{V}(r) \hat{\Sigma}_z \rightarrow \gamma_0 = \gamma_\parallel = \beta_\parallel = 0 \]

**Consequence:** One delocalized state exist at \( E=0 \)!
Minimum metallic conductivity at the center.


However: no reason for this symmetry!

Moreover, all weak coupling constants grow in the process of the renormalization!

One should go beyond SCBA.
Scattering amplitudes in perturbation theory.

\[ G_\varepsilon(p) = \frac{1}{\varepsilon - p\Sigma + i\delta} \]

\[ \int G_\varepsilon(p)G_\varepsilon(p)d^2p \propto \ln(J/\varepsilon) \]

Logarithmic contributions to the amplitudes!

However, other logarithmic contributions exist!

SCBA fails: RG treatment.
Physical quantities as integrals over supervectors $\psi$ (averaging has been performed)

$$\langle ... \rangle = \int ... \exp(-L[\psi])D\psi,$$

$$L[\psi] = L_0[\psi] + L_{\text{int}}[\psi]$$

$$L_0[\psi] = i \int \bar{\psi} \left[ \varepsilon - \hat{\mathcal{H}}_0 - \hat{\Lambda} \left( \frac{\omega}{2} + i0 \right) \right] \psi dr,$$

$$L_{\text{int}}[\psi] = \frac{1}{2} \int \left[ \gamma_0 (\bar{\psi}\psi)^2 + \Gamma_m^i \left( \bar{\psi}\hat{\mathcal{G}}_{m,i}\psi \right)^2 \right]$$

$$\Gamma_z = \gamma_z, \Gamma_{x,y}^z = \gamma_{\perp}, \Gamma_{x,y}^z = \beta_z, \Gamma_{x,y}^z = \beta_{\perp}$$

$\psi$ has 16 components for the density of states

RG treatment: $\psi = \psi_0 + \tilde{\psi}$

Integration over the fast field.

Reproducing the form of the Lagrangian: RG equations
Assumption: \( \gamma_0 \gtrsim \gamma_{\parallel, \perp}, \beta_{\parallel, \perp} \)

New coupling constants:

\[
\begin{align*}
    g_{\parallel} &= \gamma_{\parallel} + 2\gamma_{\perp} & g_{\perp} &= \beta_{\parallel} + 2\beta_{\perp} \\
    \delta g_{\parallel} &= \gamma_{\parallel} - \gamma_{\perp} & \delta g_{\perp} &= \beta_{\parallel} - \beta_{\perp}
\end{align*}
\]

Equations:

\[
\begin{align*}
    2\pi v \partial_t v &= - (\gamma_0 + g_{\parallel} + 2g_{\perp}) \\
    9\pi v^2 \partial_t \gamma_0 &\approx 2 \left( g_{\parallel}^2 + 2g_{\perp}^2 \right) \\
    9\pi v^2 \partial_t g_{\parallel} &\approx -8g_{\parallel}^2 - 20g_{\parallel}g_{\perp} + 14g_{\perp}^2 \\
    9\pi v^2 \partial_t g_{\perp} &\approx 4g_{\parallel}g_{\perp} - 18g_{\perp}^2 \\
    \pi v^2 \partial_t \delta g_{\parallel, \perp} &\approx -3\gamma_0 \delta g_{\parallel, \perp}
\end{align*}
\]

Absence of intervalley scattering is not stable
Solution of the RG equations:

\[ \nu(\varepsilon) = \left( \frac{\gamma_0}{\pi} \ln \frac{|\tilde{\varepsilon}|}{\varepsilon_0} \right)^{1/2}, \quad \gamma_0(\varepsilon) = \gamma_0 + O \left( \frac{g_{||}(\varepsilon)}{\gamma_0} \right) \]

\[ g_{||}(\varepsilon) \approx g_{\perp}(\varepsilon) \approx \frac{9\gamma_0}{14 \ln \left[ t^* / \ln \frac{|\tilde{\varepsilon}|}{\varepsilon_0} \right]} \]

\[ \varepsilon_0 \approx J \exp \left( - \frac{\pi \nu_0^2}{\gamma_0} \right) \]

\[ \tilde{\varepsilon} = \max(\varepsilon, \varepsilon_0) \]

\( \varepsilon_0 \) - is the energy at which the 1-loop approximation breaks down

All coupling constants are important. No chance for moving to chiral disorder!
Physical quantities with the “ultraviolet” logarithmic renormalization

**Diffusion coefficient:**

\[ D(\epsilon) = \frac{v^2 \tau_{tr}(\epsilon)}{2}; \quad \frac{1}{\tau_{tr}(\epsilon)} = \frac{\pi \gamma_0 \nu(\epsilon)}{4} \]

**Density of states:**

\[ \nu(\epsilon) = \frac{|\vec{\epsilon}|}{\pi \hbar^2 v^2(\epsilon)} \]

**Conductivity:**

\[ \sigma(\epsilon) = 2e^2 \nu(\epsilon) D(\epsilon) = \frac{4e^2}{\pi^2 \hbar} \ln \left( \frac{|\vec{\epsilon}|}{\epsilon_0} \right) \]

However, this is not the end of the story: localization effects!
Non-linear supermatrix $\sigma$-model for describing localization effects


$$\langle O \rangle = \int O(\hat{Q}) \exp \left( -F \left[ \hat{Q} \right] \right) \mathcal{D}\hat{Q}$$

For any correlation function $O$

$$1 = \int \exp \left( -F \left[ \hat{Q} \right] \right) \mathcal{D}\hat{Q}$$

Due to supersymmetry

For graphene: $Q$ are 16x16 supermatrices (unity in the sublattice space), $Q^2 = 1$
Free energy functional \( F \)

\[
F = \frac{\pi \hbar \nu(\epsilon)}{16} \text{Str} \int \left\{ D(\epsilon) \left( \nabla \hat{Q} \right)^2 + 2i \omega \Lambda \hat{Q} \right.
\]

\[
- \frac{\pi \nu(\epsilon) g_{\parallel}(\epsilon)}{4} \left[ \hat{\rho}_z, \hat{Q} \right]^2 - \frac{\pi \nu(\epsilon) g_{\perp}(\epsilon)}{4} \left( \left[ \hat{\rho}_x, \hat{Q} \right]^2 + \left[ \hat{\rho}_y, \hat{Q} \right]^2 \right) \left\} dr; \right.
\]

\[
\hat{\rho}_j = \tau_j^{KK'} \otimes \mathbb{1}^{eh} \otimes \mathbb{1}^{AR} \otimes \mathbb{1}^g; \quad \hat{\Lambda} = \mathbb{1}^{KK'} \otimes \mathbb{1}^{eh} \otimes \tau_z^{AR} \otimes \mathbb{1}^g;
\]

All the constants should be taken from the solution of the RG equations!

\[
D(\epsilon) = \frac{v(\epsilon)^2 \tau_{tr}(\epsilon)}{2} \quad \frac{1}{\tau_{tr}(\epsilon)} = \frac{\pi \gamma_0 \nu(\epsilon)}{4}
\]

\[
v(\epsilon) = \left( \frac{\gamma_0}{\pi} \ln \frac{|\tilde{\epsilon}|}{\epsilon_0} \right)^{1/2} \quad \gamma_0(\epsilon) = \gamma_0 + \mathcal{O} \left( \frac{g_{\parallel}(\epsilon)}{\gamma_0} \right)
\]

\[
g_{\parallel}(\epsilon) \approx g_{\perp}(\epsilon) \approx \frac{g_{\parallel}(\epsilon)}{14 \ln \left[ t^* / \ln |\tilde{\epsilon}| / \epsilon_0 \right]}, \quad \tilde{\epsilon} = \max(\epsilon, \#\epsilon_0)
\]
1) If only diagonal disorder ($\chi_0$) is present:

$$F = \frac{\pi \nu(\varepsilon)}{16} \text{Str} \int \left[ D(\varepsilon)(\nabla Q)^2 + 2i \omega \Lambda Q \right] dr$$

$$\hat{Q} = \hat{C} \hat{Q}^T \hat{C}^T; \quad \hat{C} = i \tau_y^{K K'} \otimes 1^A R \otimes \left( \tau_{-}^{e h} \otimes 1^g - \tau_{+}^{e h} \otimes \tau_{z}^{g} \right)$$

The symmetry of $Q$ corresponds to 2 replicas of the symplectic ensemble. ➡️ Antilocalization!

The symplectic symmetry was first noticed by Suzuura and Ando (2002)

However, the terms with $\rho_i$ break the symmetry and the ensemble becomes orthogonal!

At large distances: $$\hat{Q} = 1^A K K' \otimes \hat{Q}_o$$

-is 8x8 supermatrix of the orthogonal symmetry: localization!
Perturbation theory: first order gives the weak localization correction.

\[ \Delta \sigma_{WL} = \frac{e^2}{2\pi^2 \hbar} \left[ -\ln \frac{\tau_\phi}{\tau_{tr}} + 2 \ln \frac{\tau_2}{\tau_{tr}} + \ln \frac{\tau_3}{\tau_{tr}} \right] \]

\[ \frac{1}{\tau_2} = \frac{1}{\tau_\phi} + \frac{1}{\tau_\perp}; \quad \frac{1}{\tau_3} = \frac{1}{\tau_\phi} + \frac{1}{2\tau_\parallel} + \frac{1}{\tau_\perp}; \quad \tau_\phi \text{-inelastic dephasing time} \]

Agrees with McCann et al (2006)

Ultraviolet logarithmic renormalization

\[ \frac{\tau_{tr}}{\tau_1} \approx \frac{\tau_{tr}}{\tau_2} \approx \frac{36}{7 \ln[t^*/\ln |\tilde{\varepsilon}/\varepsilon_0|]} \]
Qualitative picture (from Aleiner and Efetov (2006)).

FIG. 2: Schematic dependence, $\sigma(T)$, fot the undoped graphene and for $\tau_\phi^{-1} \propto T$. 
How to make a quantum dot in graphene?
Silvestrov and Efetov (2007)

Peculiarity of graphene with respect to conventional 2D electron gases: One barrier is sufficient!

Avoiding the reflectionless penetration (Klein paradox) through the barrier: making a strip with a barrier across it.

Transversal quantization removes the problem of the confinement!
Real space

Applied electrostatic potential

Sufficient to make the quantum dot!

Quasiclassical treatment complemented by numerics.
Conductance as a function of the energy (gate voltage)

\[ V = -\left(\frac{x}{x_0}\right)^2 U/2. \]

\[ \xi = \left[\hbar cx_0^2/U\right]^{1/3}, \quad \varepsilon_0 = \hbar c/\xi. \]

FIG. 1: Upper curve: Conductance of the graphene quantum dot as a function of the Fermi energy for the metallic armchair edges for \( L = 4\xi \), Eq. \( (4) \). The lower curve: the contribution to the conductance from the transmission channels with \( p_y = \pm \pi \hbar/L \). All calculations are carried out for the zero temperature, \( T = 0 \).
Fabrication of quantum dots in graphene: essential detail for quantum computer! Experiments are being carried out!

FIG. 2: Examples of trajectories described in the text drawn on the $x, p_x$ plane (arbitrary units). Solid lines show the trajectories with $\varepsilon < 0$ either bouncing inside the barrier, or reflected by it from the left/right. Tunnelling events between the bounded and unbounded trajectories are shown schematically (t). Thick dashed lines show the trajectories with $\varepsilon > 0$ either transmitted for $|p_y| < \varepsilon/c$ (open channels) or reflected for $|p_y| > \varepsilon/c$ (closed channels).