Freie Universität Berlin

Tunneling density of states of graphene

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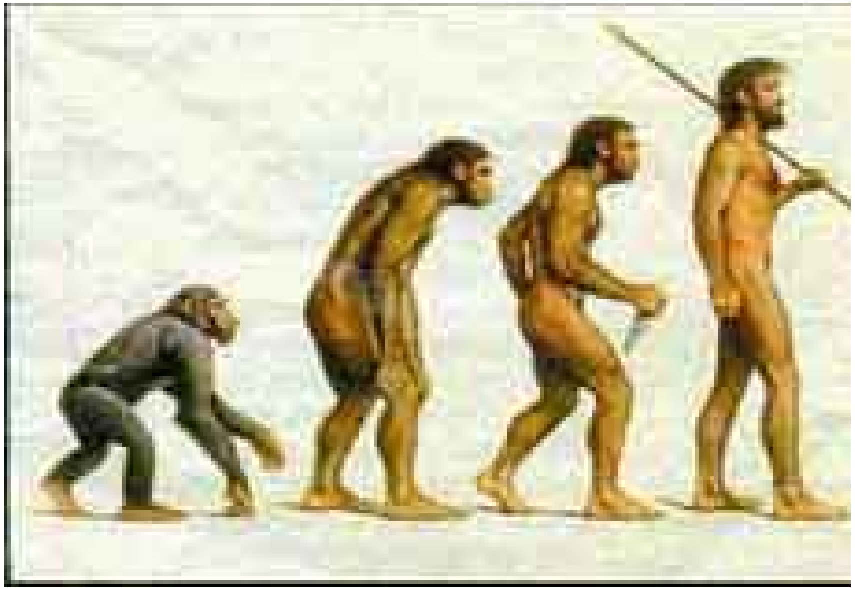
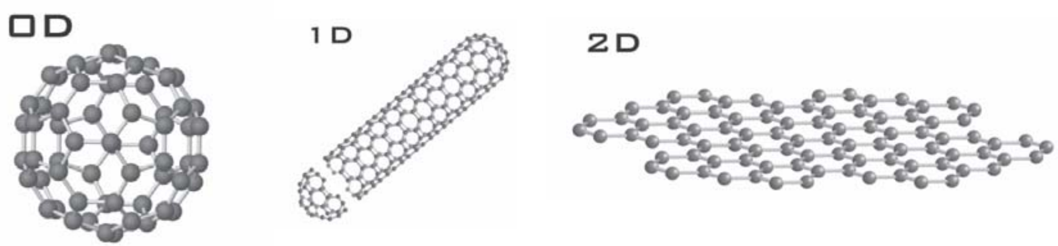
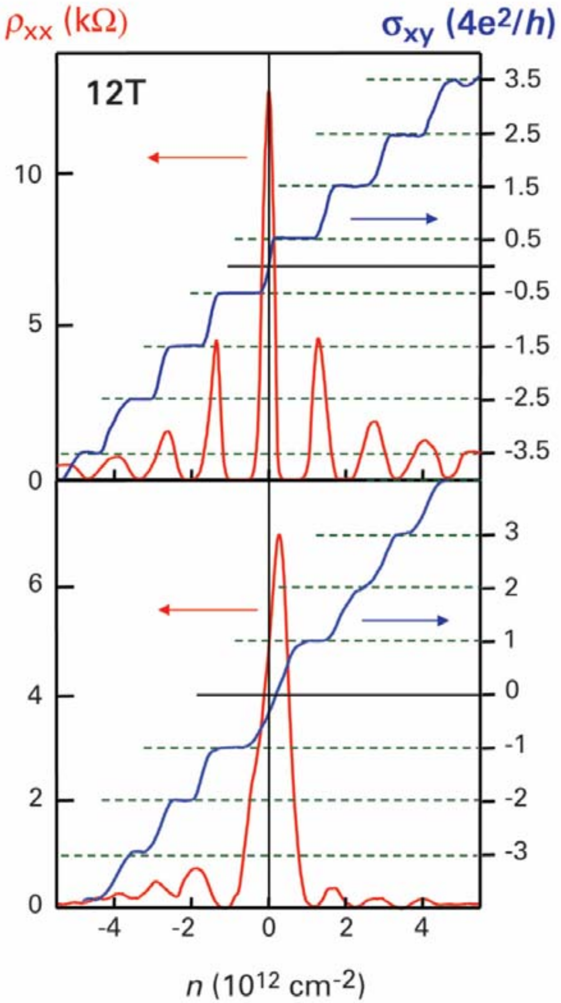
Evolution



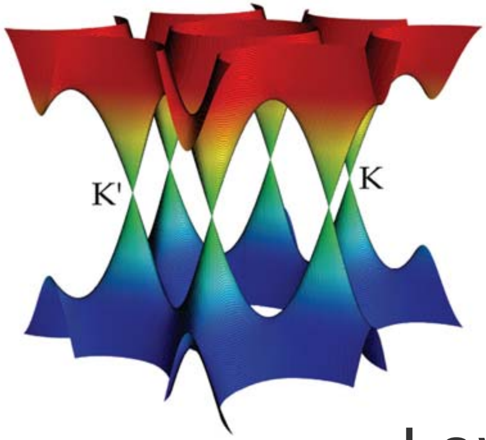


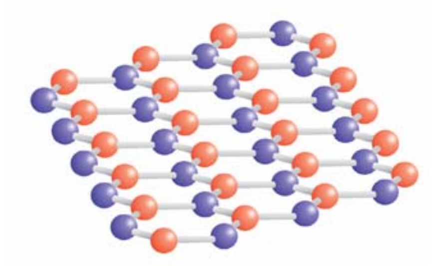


Novosolov *et al*., Nature 2005
Zhang *et al*., Nature 2005

Band structure

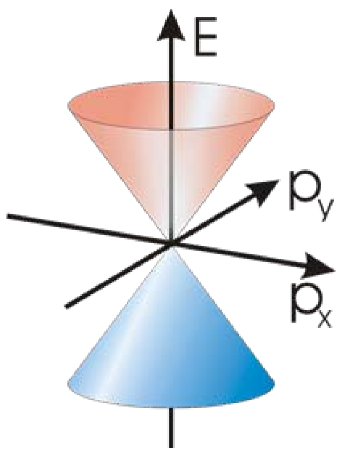
Tight-binding description





$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j$$

Low-energy Dirac Hamiltonian



Dirac points

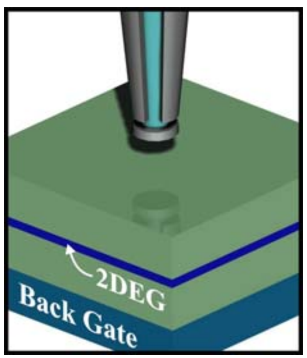
$$H = \mathrm{v}\,\boldsymbol{\Sigma}\cdot\mathbf{p}$$

$$H = \mathrm{v}\begin{bmatrix} 0 & p_x - ip_y & 0 & 0 \\ p_x + ip_y & 0 & 0 & 0 \\ 0 & 0 & 0 & -(p_x - ip_y) \\ 0 & 0 & -(p_x + ip_y) & 0 \end{bmatrix} \begin{matrix} A_1 \\ B_1 \\ B_2 \\ A_2 \end{matrix}$$

Motivation

Graphene is exposed at surface:

- directly accessible by local probes
- enhanced spatial resolution

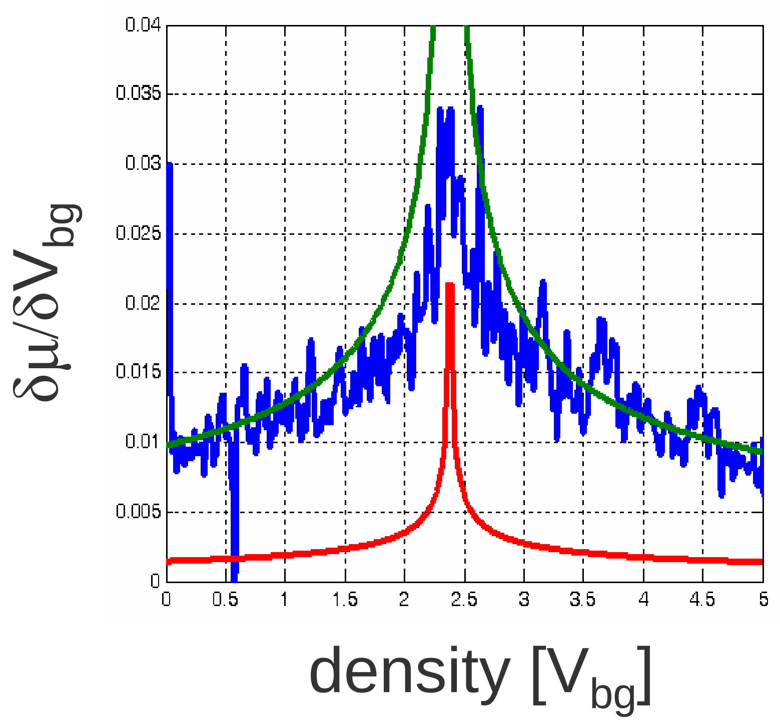


Scanning SET
(A. Yacoby et al.)

STM
(e.g. talk by E. Andrei next week)

Motivation

Scanning SET: (A. Yacoby et al.)



(local) compressibility

$$\kappa^{-1} \sim \left[\frac{d\mu}{dn}\right]$$

Hartree-Fock:

$$\mu = \hbar \mathrm{v}(2\pi n_0)^{1/2} - \sqrt{\frac{2}{\pi}}\frac{e^2}{2\pi\varepsilon_{\mathrm{eff}}} n^{1/2} + E_{\mathrm{corr}}$$

measured from Dirac pt

all electrons

see MacDonald et al. (2007)

Motivation

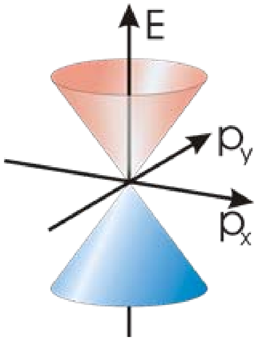
STM: (local) tunneling density of states

STM

Dirac equation describes
coarse grained local TDOS

Dirac dispersion

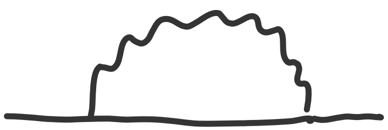
$$E_p = \mathrm{v}\, p$$



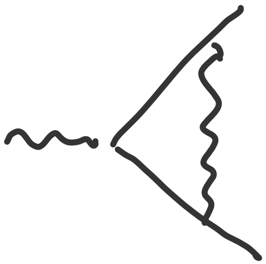
$$\nu(E) = \frac{|E|}{2\pi\hbar^2\,\mathrm{v}^2}$$

Coulomb interactions

"Fine structure constant" $$g = \frac{e^2}{\hbar \mathrm{v}} \sim 1$$



$$E(p) = [\mathrm{v} + (e^2/4)\ln(D/\mathrm{v}\,p)]p$$



$$\delta e = 0$$

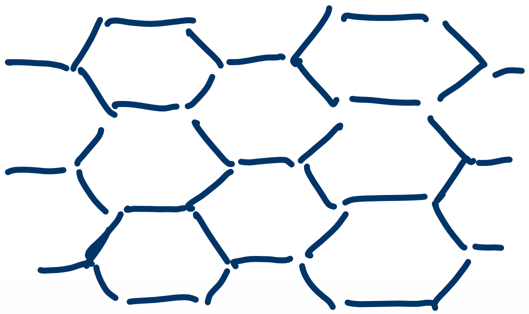
TDOS

$$\nu(E) = \frac{|E|}{2\pi\hbar^2[\mathrm{v} + (e^2/4)\ln(D/\mathrm{v}\,p)]^2}$$

J. González et al., Nucl. Phys. B 424, 595 (1994)

Disorder

Tight binding



$$H = -t \sum_{\langle ij \rangle} c_i^\dagger c_j$$

- site energies
- random hopping
- lattice defects

Peres et al. PRB 2006

Dirac theory

general T-invariant local operator:

$$\delta H = u_0(\mathbf{r})\mathbf{1} + \sum_{ij} u_{ij}(\mathbf{r})\Lambda_i \Sigma_j$$

where

$$\Lambda_{x,y} = \pi_{x,y} \otimes \sigma_z \qquad \Lambda_z = \pi_z \otimes \sigma_0$$

and

Λ_i, Σ_j odd under T-reversal

McCann et al. PRL 2006

Disorder

	scalar potential	(ficticious) gauge field
intra-valley	u_0, u_{zz}	u_{zx}, u_{zy} (abelian)
inter-valley	u_{xz}, u_{yz}	$u_{xx}, u_{xy}, u_{yx}, u_{yy}$ (nonabelian)

McCann et al. PRL 2006

Smooth disorder

Neglecting intervalley scattering

$$H = \mathrm{v}\,\boldsymbol{\Sigma}\cdot(\mathbf{p}+\mathbf{a}(\mathbf{r}))+V(\mathbf{r})$$

ficticious gauge field due to random hopping

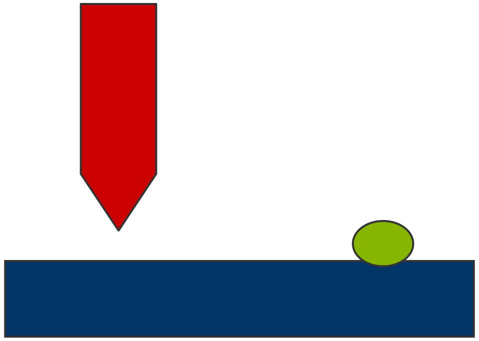
random site energies

Time-reversal symmetry:

$$\mathbf{a}(\mathbf{r}) = \Pi_z \otimes \mathbf{1}\begin{pmatrix} u_{zx}(r) \\ u_{zy}(r) \end{pmatrix}$$

Disorder

Weak disorder (ballistic regime):



STM

chiral symmetry

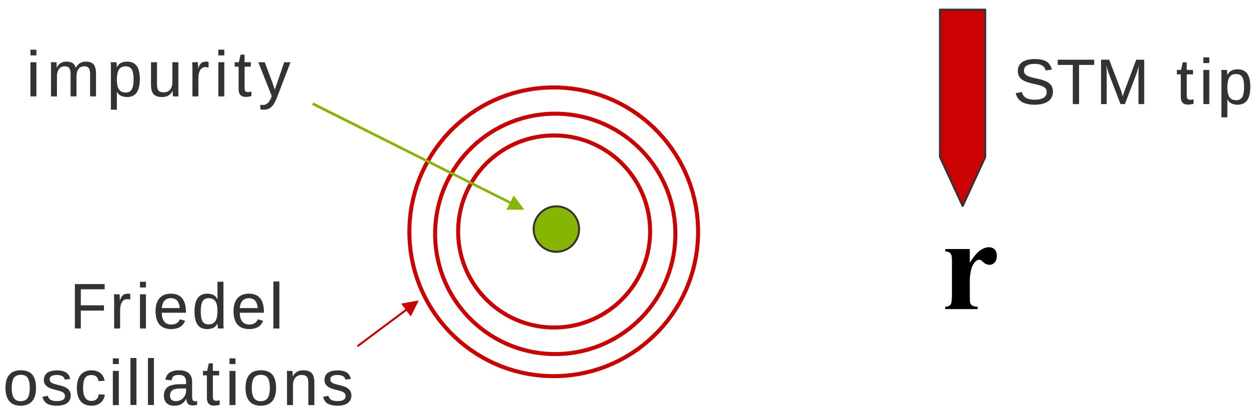
↓

no backscattering

$$G(\mathbf{r},\mathbf{r};E) = G_0(\mathbf{r},\mathbf{r};E) + G_0(\mathbf{r},\mathbf{0};E)\hat{u}G_0(\mathbf{0},\mathbf{r};E) + \ldots$$

$$\delta\nu(E) \sim \mathrm{tr}\,\delta\,\mathrm{G}(\mathbf{r},\mathbf{r};E) \sim \mathrm{tr}\,[\mathrm{s}_{\mathbf{r}}\hat{u}s_{-\mathbf{r}}] \sim 0$$

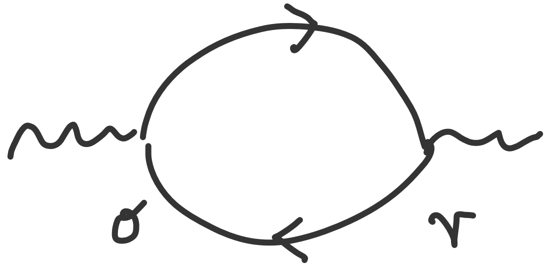
Zero-bias anomaly



- Fate of zero-bias anomaly as E_F approaches Dirac pt ?
  - wavelength of Friedel oscillations diverge
  - strength of electron-electron interaction increases

- Probe of ficticious gauge field ?

Friedel oscillations

$\delta\rho(\mathbf{r}) \sim$ 

$\sim \int d\varepsilon \, \mathrm{tr}\, G(\mathbf{r},\mathbf{0};\varepsilon)\hat{u}G(\mathbf{0},\mathbf{r};\varepsilon)$

Green function:

$$\mathbf{G}^{R}_{\epsilon}(\mathbf{p}) = \frac{\hat{s}_{-\mathbf{p}}}{\epsilon + vp + i\eta} + \frac{\hat{s}_{\mathbf{p}}}{\epsilon - vp + i\eta}$$

$$\mathbf{G}^{R}_{\epsilon}(\mathbf{r},0) \simeq -\frac{e^{i\pi/4}p_\epsilon}{\sqrt{2\pi}v}\frac{e^{ip_\epsilon r}}{\sqrt{p_\epsilon r}}\left[\hat{s}_{\mathbf{r}} + \frac{i}{4p_\epsilon r}\boldsymbol{\Sigma}\cdot\frac{\mathbf{r}}{r}\right]$$

extra 1/r

$\delta\rho(\mathbf{r}) \sim \frac{1}{r^3}$ as opposed to $1/r^2$ in usual 2d electron gas

Cheianov *et al.* PRL 2006

Fock contribution

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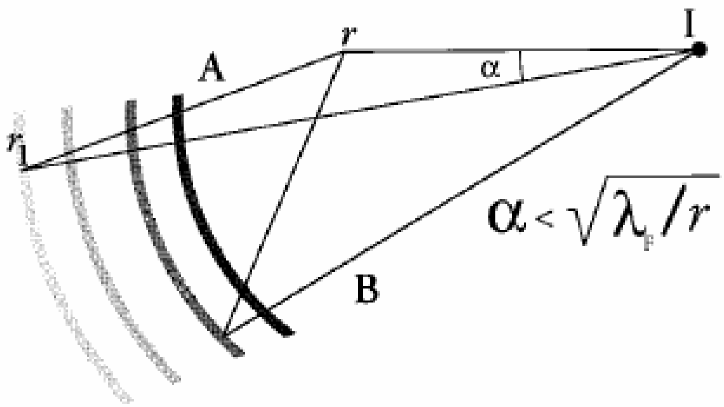
Ballistic regime:

perturbation theory in disorder and interactions

Rudin et al. PRB 1997



$$\hat{V}_F(\mathbf{r}_1, \mathbf{r}_2) = \frac{1}{2} V(\mathbf{r}_1 - \mathbf{r}_2)\delta\rho(\mathbf{r}_1, \mathbf{r}_2)$$

$$\hat{H}_{HF}(\mathbf{r}_1, \mathbf{r}_2) = \hat{V}_H(\mathbf{r}_1)\delta(\mathbf{r}_1 - \mathbf{r}_2) - \hat{V}_F(\mathbf{r}_1, \mathbf{r}_2)$$

$$\delta\mathbf{G}^R_\epsilon(\mathbf{r}, \mathbf{r}) = \int d\mathbf{r}_1 d\mathbf{r}_2\, \mathbf{G}^R_\epsilon(\mathbf{r}, 0)\hat{u}\mathbf{G}^R_\epsilon(0, \mathbf{r}_1)\, \hat{H}_{HF}(\mathbf{r}_1, \mathbf{r}_2)\mathbf{G}^R_\epsilon(\mathbf{r}_2, \mathbf{r})$$

Results

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Global TDOS (finite density n_i of impurities):

$$\frac{\delta\nu_\omega}{\nu_F} \simeq \frac{4\nu_F^2 n_i}{\sqrt{\pi}k_F^2}\mathrm{Tr}\left[2\hat{u}^2 - \sum_{\alpha=x,y}\hat{\Sigma}_\alpha\hat{u}\hat{\Sigma}_\alpha\hat{u}\right]\ln\left(\frac{\hbar\omega}{E_F}\right)$$

(valid for $1/\tau < \omega < \mathrm{v} k_F$)

- logarithmic singularity tied to Fermi energy (zero-bias anomaly)
- relative strength of anomaly independent of Fermi energy (since $v_F \sim k_F$)

Results

Potential disorder
(e.g. u_{zz})

$$\frac{\delta\nu_\omega(\mathbf{r})}{\nu_F} \simeq -\frac{4\nu_F^2}{\pi^{3/2}}\frac{u_{zz}^2}{(k_F r)^2}$$

(valid for $\lambda_F < r < \mathrm{v}/\omega$)

Ficticious gauge field
(e.g. u_{zx})

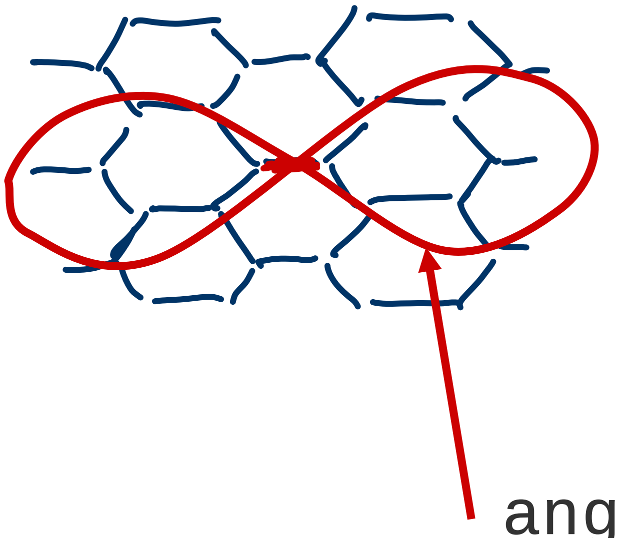
$$\frac{\delta\nu_\omega(\mathbf{r})}{\nu_F} \simeq -\frac{4\nu_F^2}{\pi^{3/2}}\frac{u_{zx}^2\sin^2\phi}{(k_F r)^2}$$

angular dependence!

- u_{zy} gives $\cos^2\phi$ dependence
- analogous for nonabelian gauge field

Angular dependence

e.g. random hopping



contributes to u_{zy} (abelian), u_{yz} and u_{xz} (nonabelian)

angular dependence for intravalley contribution

Conclusions

- Logarithmic zero-bias anomaly in tunneling density of states due to interactions and disorder
- Originates from Fock contribution alone
- Relative strength of zero-bias anomaly is independent of doping
- Angular dependence of tunneling density of states around impurity is signature of ficticious gauge field

E. Mariani, L. Glazman, A. Kamenev, FvO, unpublished