The ac conductivity of monolayer graphene

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Talk is based on:
V.P. Gusynin, S.G. Sh., J.P. Carbotte, PRL **96**, 256802 (06), J. Phys.: Condens. Matter **19**, 026222 (07), cond-mat/0607727, cond-mat/0701053;
V.P. Gusynin, V.A. Miransky, S.G. Sh., I.A. Shovkovy, PRB **74**, 195429 (06); cond-mat/0612488

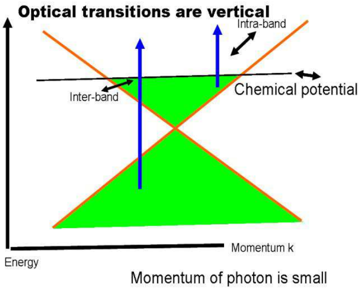
January 10, 2007

Outline

1. AC conductivity: introduction for experts
2. Microwave conductivity
3. Optical probe of the Dirac quasiparticles
4. Optical conductivity sum rule
5. New quantum Hall states, $\sigma_{xy} = 0, \pm e^2/h$ for $B > 20\text{T}$
6. Summary

AC probe of the Dirac quasiparticles:

zero magnetic field case, $B = 0$



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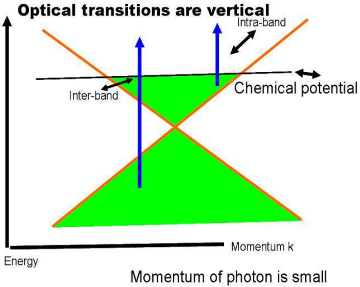

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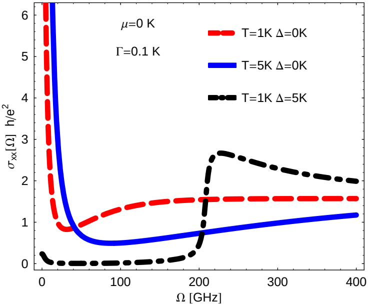


The microwave conductivity $\sigma_{xx}(\Omega, T)$ in units e^2/h vs frequency Ω in GHz. In addition to Drude peak (intraband) there is a constant background (interband).

V.P. Gusynin, S.G. Sh., J.P. Carbotte, PRL **96**, 256802 (2006).

Microwave conductivity: Drude term

In weak scattering limit for energy dependent scattering rate $\Gamma(\omega)$ conductivity in microwave range is given by

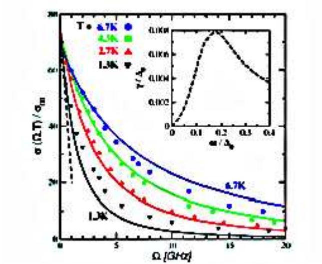
$$\sigma_{xx}(\Omega, T) = \sigma_{00} \int_{-\infty}^{\infty} d\omega \left(-\frac{\partial\, n_F(\omega)}{\partial \omega} \right) \frac{2\pi|\omega|\Gamma(\omega)}{\Omega^2 + 4\Gamma^2(\omega)},$$

with Ω photon frequency, $n_F(\omega) = 1/[e^{(\omega-\mu)/T} + 1]$ and $\sigma_{00} = e^2/(\pi^2\hbar)$.
Take $\mu = 0$ (Dirac point) and assume the weak impurity scattering limit in Born approximation: $\Gamma(\omega) = \gamma_{00} + \alpha|\omega|$, ($\gamma_{00}$ small), we get a cusp-like dependence

$$\sigma_{xx}(\Omega, T) \simeq \frac{\pi\sigma_{00}}{2\alpha}\left[1 - \frac{\pi}{8\alpha}\frac{\Omega}{T}\right], \qquad \gamma_{00} < \Omega \ll T.$$

W. Kim, F. Marsiglio, and J. P. Carbotte, PRB **70**, 060505(R) (04).

Microwave conductivity in HTSC



This cusp behavior of the microwave conductivity $\sigma(\Omega)$ is observed in a very pure $YBCO_{6.5}$ orthoII phase:

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For graphene found that for $\mu = 0$ or $V_g = 0$ the same equation $\sigma_{xx}(\Omega, T) \simeq \frac{\pi\sigma_{00}}{2\alpha}\left[1 - \frac{\pi}{8\alpha}\frac{\Omega}{T}\right]$, holds and predict in Born limit the same cusp like behavior.

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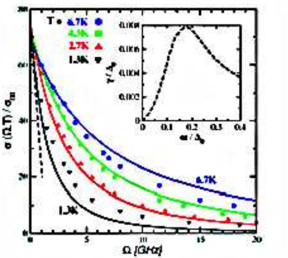

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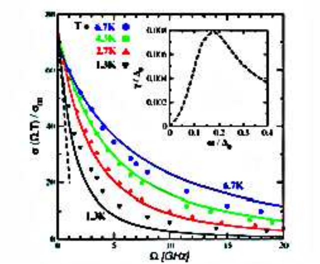
Unitary limit

[see e.g. N.M.R. Peres, F. Guinea, A.H. Castro Neto, PRB **73**, 125411 (06)] is, of course, different, but HTSC shows that the cusp is not impossible...

Microwave conductivity in HTSC



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What happens if we vary the gate voltage V_g?

Tunable Drude peak

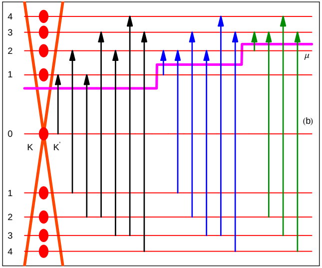
Away from the Dirac point $\mu = 0$
$\mu \propto \text{sgn}\, V_g \sqrt{V_g}$ is tunable by the gate voltage as well as the width of the Drude peak:

$$\sigma_{xx}(\Omega, T) = \sigma_{00} 2\pi |\mu| \frac{\Gamma(\mu)}{\Omega^2 + 4\Gamma^2(\mu)}, \quad |\mu| \gg T.$$

A possibility to measure the dependence $\Gamma(\omega)$
by changing μ and extracting $\Gamma(\mu)$.

Optical probe of the Dirac quasiparticles:

finite magnetic field case, $B \neq 0$



The allowed transitions between LLs $n = 0, \ldots 4$.

The pair of cones at **K** and **K**$'$ are combined.

Left — $E_0 < \mu < E_1$;

middle — $E_1 < \mu < E_2$;

right — $E_2 < \mu < E_3$.

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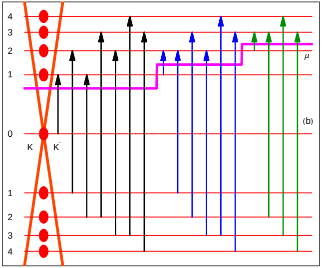

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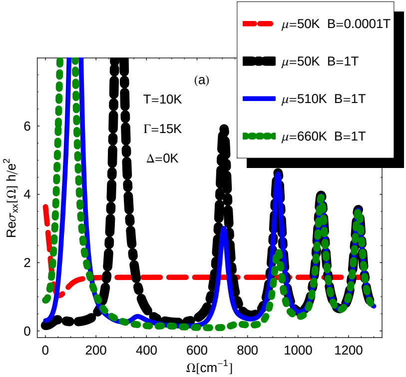


$\mathrm{Re}\,\sigma_{xx}(\Omega)$ in units of e^2/h vs Ω.
red – $\mu = 50$K and $B = 10^{-4}$T,
black – $\mu = 50$K, blue – $\mu = 510$K,
green – $\mu = 660$K all three for $B = 1$T.

Optical probe: filling sweep

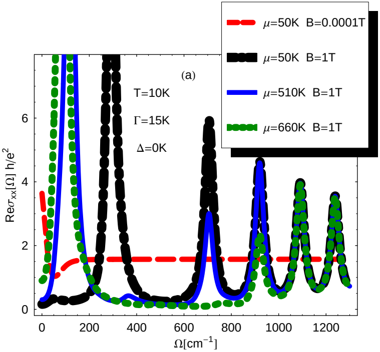
$\Leftarrow$ Dependence of Hall conductivity, $\sigma_{xy}(\nu) = 2e^2/h(1 + 2[\nu/4])$ at $T = 0$ ([] denotes integer part) on the filling factor ν which varies linearly with the time.

The first absorption line $\Omega = 294\,\mathrm{cm}^{-1}$ always appears with full intensity or is entirely missing, while all other lines disappear in two steps. This indicates the Dirac nature of quasiparticles in graphene.

$\Leftarrow$ Dependence of diagonal ac conductivity, $\sigma_{xx}(\Omega)$ at $T \neq 0$ on optical frequency Ω as the function of ν.

Optical conductivity sum rule

Partial optical spectral weight up to Ω_m



Area under conductivity gives total optical spectral weight to energy Ω_m:
$W(\Omega_m) = \int_0^{\Omega_m} d\Omega \, \mathrm{Re}\, \sigma_{xx}(\Omega)$

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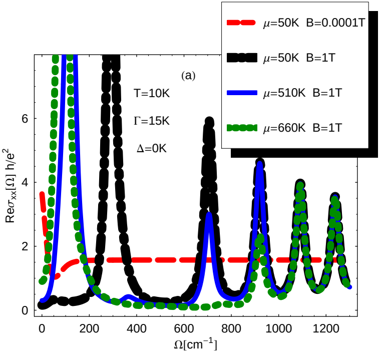

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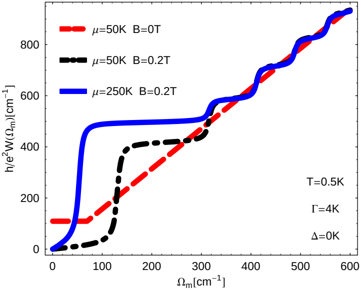

Plateaux with the steps corresponding to the various peaks in $\mathrm{Re}\, \sigma_{xx}(\Omega)$.

Full optical conductivity sum rule

Due to two atoms per unit cell and inter- as well as intra-band optical transitions sum rule is different from sum over bands:

$$\frac{2}{\pi}\int_0^\infty d\Omega \,\mathrm{Re}\,\sigma_{xx}(\Omega) = \frac{2e^2}{\hbar^2}\int_{BZ}\frac{d^2\mathbf{k}}{(2\pi)^2}\left[n_F\left(\epsilon(\mathbf{k})\right) - n_F(-\epsilon(\mathbf{k}))\right]$$

$$\times\left[\frac{\partial^2}{\partial k_\alpha^2} - \left(\frac{\partial\varphi(\mathbf{k})}{\partial k_\alpha}\right)^2\right]\epsilon(\mathbf{k}), \quad \text{here} \quad \varphi : t\sum_{\boldsymbol{\delta}} e^{i\mathbf{k}\boldsymbol{\delta}} \equiv \epsilon(\mathbf{k})e^{i\varphi(\mathbf{k})}$$

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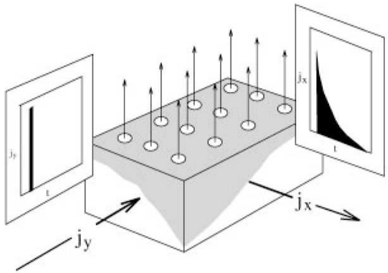
or in the linearized approximation

$$\frac{2}{\pi}\int_0^\infty d\Omega \, \mathrm{Re}\,\sigma_{xx}(\Omega) = \alpha\frac{e^2 t}{\hbar^2} - \frac{e^2 a^2}{9\pi\hbar^4 v_F^2}\left(|\mu|^3 + \pi^2|\mu|T^2\right), \quad |\mu| \gg T$$

where t is the hopping parameter, three vectors $\boldsymbol{\delta}$ connect nearest neighbors, $\alpha \approx 0.61$, v_F is the Fermi velocity, a is the lattice constant.

Optical Hall angle sum rule



The Hall response
$j_x(\Omega) = \sigma_{xy}(\Omega) E_y(\Omega)$,
where, because an injected current is $j_y(\Omega) = \sigma_{xx}(\Omega) E_y(\Omega)$ the response function $t_H(\Omega)$:

$$j_x(\Omega) = t_H(\Omega) j_y(\Omega)$$

Optical Hall angle:

$$t_H(\Omega) \equiv \tan \theta_H(\Omega) = \frac{\sigma_{xy}(\Omega)}{\sigma_{xx}(\Omega)}$$

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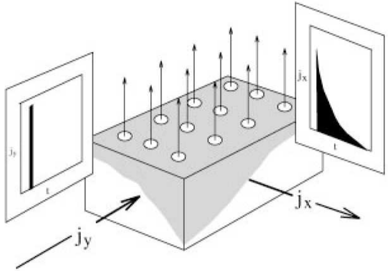

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Optical Hall angle:

$$t_H(\Omega) \equiv \tan\theta_H(\Omega) = \frac{\sigma_{xy}(\Omega)}{\sigma_{xx}(\Omega)}$$

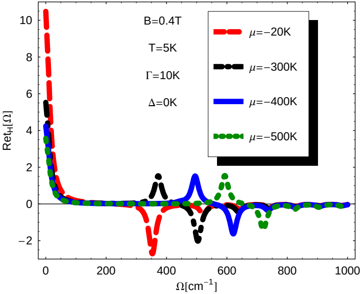
obeys the sum rule

$$\frac{2}{\pi}\int_0^\infty d\Omega \mathrm{Re}\, t_H(\Omega) = \omega_H,$$

where in 2D electron gas the Hall frequency ω_H coincides with a bare cyclotron frequency,
$\omega_H = \omega_c = \frac{eB}{mc}$.

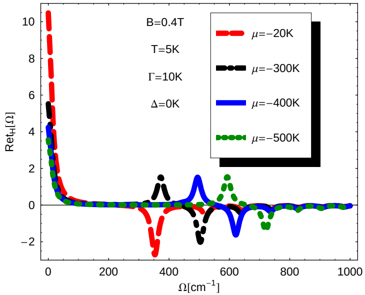
H.D. Drew and P. Coleman, PRL **78**, 1572 (97).

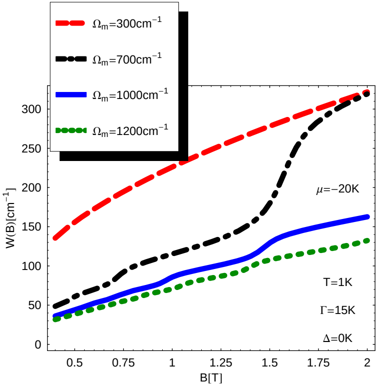
Partial Hall angle sum rule



Optical Hall angle, $\mathrm{Re}\, t_H(\Omega)$
Notice that while $\mathrm{Re}\, \sigma_{xx}(\Omega)$ and $\mathrm{Im}\, \sigma_{xy}(\Omega)$ peaked near $\Omega =$ Landau level energy difference, Hall angle $\mathrm{Re}\, t_H(\Omega)$ has a "Drude"-like peak.

Partial Hall angle sum rule





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Area under $\mathrm{Re}\, t_H(\Omega)$ gives total weight to energy Ω_m:
$W(\Omega_m) = \int_0^{\Omega_m} d\Omega\, \mathrm{Re}\, t_H(\Omega)$
Observe crossover from $W \propto \sqrt{B}$ for low Ω_m to $W \propto B$ for large Ω_m

Hall angle sum rule for graphene

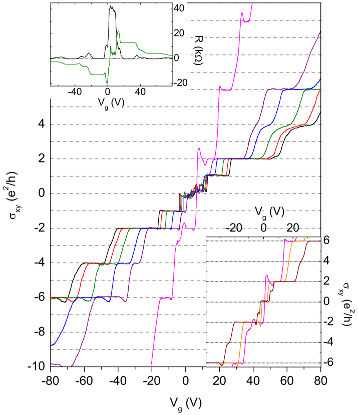
$$\frac{2}{\pi}\int_0^\infty d\Omega\,\mathrm{Re}\,t_H(\Omega) = \omega_H = -\frac{1}{4\alpha}\frac{eB}{c}\frac{ta^2}{\hbar^2}\rho a^2$$
$$= -\frac{4}{9\pi\alpha}\frac{eB\hbar v_F^2}{c}\frac{\mu^2 \mathrm{sgn}\mu}{\hbar t^3},$$

where t is the hopping parameter, $\alpha \approx 0.61$, v_F is the Fermi velocity, a is the lattice constant, ρ is the carrier imbalance.
Full spectral weight linear in B and $\propto \rho \propto V_G$.
Interestingly, when the spectrum is gapped (as considered by many groups): $E = \sqrt{\hbar^2 v_F^2(p_1^2 + p_2^2) + \Delta^2}$

$$\rho = \frac{1}{\pi\hbar^2 v_F^2}(\mu^2 - \Delta^2)\theta(\mu^2 - \Delta^2)\mathrm{sgn}\mu$$

The gap Δ can be extracted from the change in ω_H obtained from magneto-optical measurements as done by Drew in HTSC.

New quantum Hall states, $\sigma_{xy} = 0, \pm e^2/h$



Y. Zhang, *et al.*, PRL **96**, 136806 (2006).

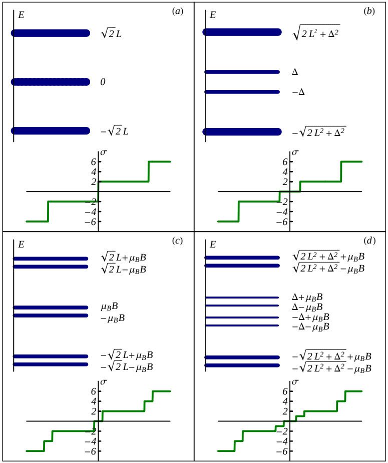
—— 9 **T**, —— 11.5 **T**, —— 17.5 **T**,
—— 25 **T**, —— 30 **T**, —— 37 **T**,
—— 37 **T**, —— 42 **T**, —— 45 **T**.

The observed filling factor sequence:
$\nu = 0$ for $B > 11$Tesla,
$\nu = \pm 1$ for $B > 17$Tesla.

Thus the four fold (sublattice-spin) degeneracy of $n = 0$ Landau level is totally resolved for $B > 17$Tesla.

The four fold degeneracy of $n = 1$ level is partially resolved into $\nu = \pm 4$ which originates from spin splitting leaving two fold degeneracy.

Theoretical predictions



V.P. Gusynin, *et al.*, PRB **74**, 195429 (2006).

Illustration of the spectrum and the Hall conductivity in the $n = 0$ and $n = 1$ Landau levels:

(a) $\Delta = 0$ and $E_Z = 0$ (no Zeeman term).

(b) $\Delta \neq 0$ and $E_Z = 0$.

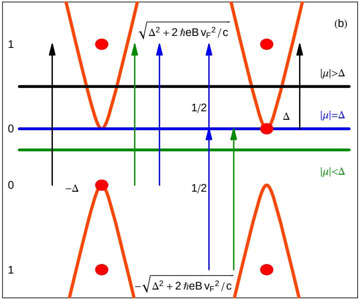
(c) $\Delta = 0$ and $E_Z \neq 0$.

(d) $\Delta \neq 0$ and $E_Z \neq 0$.

Thickness of the lines represents the degeneracy $\times 4$, $\times 2$, and $\times 1$ of the energy states; $L = \sqrt{\hbar v_F^2 |eB|/c}$.

Both gap, $\Delta \neq 0$ and Zeeman term, $E_Z \neq 0$ are necessary to explain $\sigma_{xy} = 0, \pm e^2/h$ plateaux.

Manifestation of the gap in optics



Transitions between LLs with an excitonic gap Δ.
green – $|\mu| < \Delta$,
blue – $\mu = \Delta$,
black – $|\mu| > \Delta$.
For $|\mu| > \Delta$ two different lengths for $\uparrow$ transitions $\Longrightarrow$ peak splits

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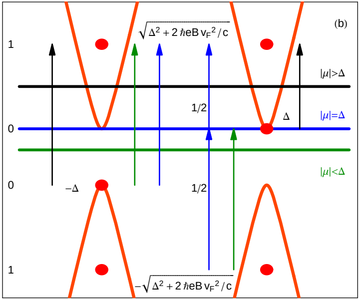

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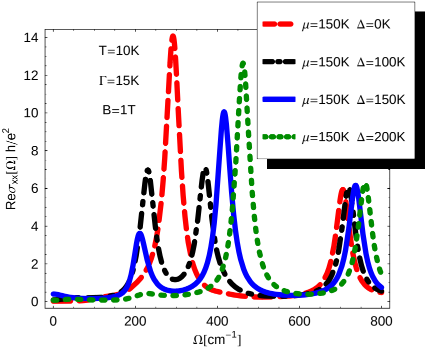

$\mathrm{Re}\,\sigma_{xx}(\Omega)$ in units of e^2/h vs Ω.
red – $\Delta = 0$K,
black – $\Delta = 100$K,
blue – $\Delta = 150$K,
green – $\Delta = 200$K.

Summary

- Optical measurements can be used to establish the presence of the Dirac quasiparticles, anomaly of the first peak
- Optical signatures of the Dirac quasiparticles already observed in epitaxial graphite M.L. Sadowski, *et al.*, PRL **97**, 266405 (2006), but not quite, because the sample is multilayer.
  Observation in graphene APS 2007 abstract of P. Kim....
- Theoretical work on FIR in bilayer D.S.L. Abergel, V.I. Fal'ko, Preprint cond-mat/0610673.