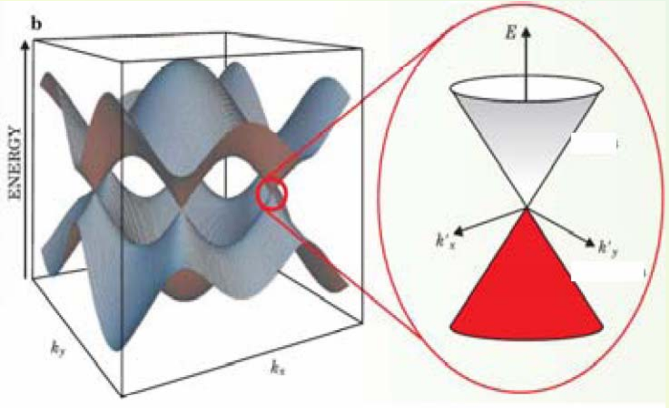
Caustics, focusing and Veselago lens for electrons in graphene *PN* junctions

Vladimir Falko

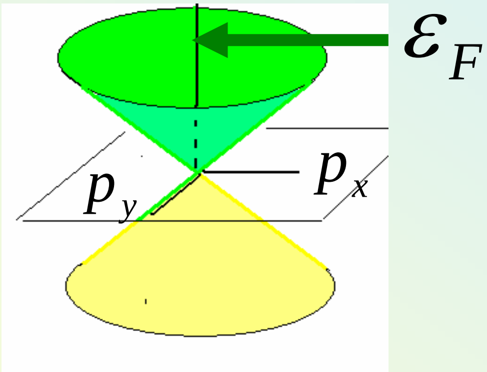
Vadim Cheianov

Centre for Nanoscale Dynamics
and Mathematical Physics

Boris Altshuler - Columbia U, NY



$$\varepsilon = vp$$



$$\hat{H} = v\vec{\sigma}\cdot\vec{p} = vp\,\vec{\sigma}\cdot\vec{n}$$

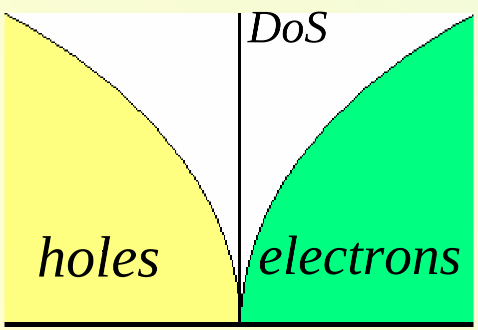
Bloch function amplitudes on the AB sites - 'isospin'

$$\psi = \begin{pmatrix} \varphi_A \\ \varphi_B \end{pmatrix}$$

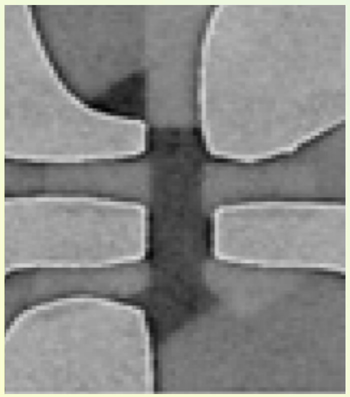
Chiral electrons:

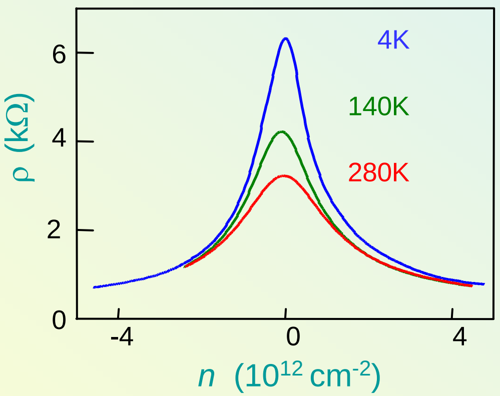
'isospin' direction is linked to the axis determined by the electron momentum:

$$\vec{\sigma}\cdot\vec{n} = 1$$



$$n_{carriers} \propto V_{gate}$$



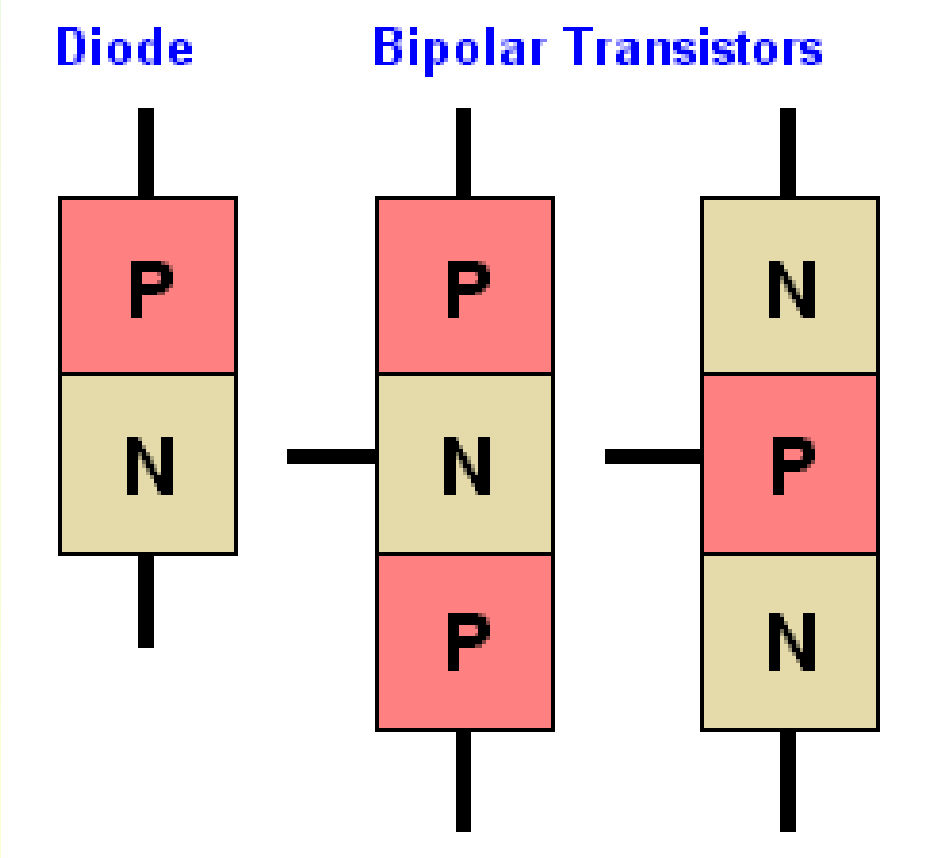


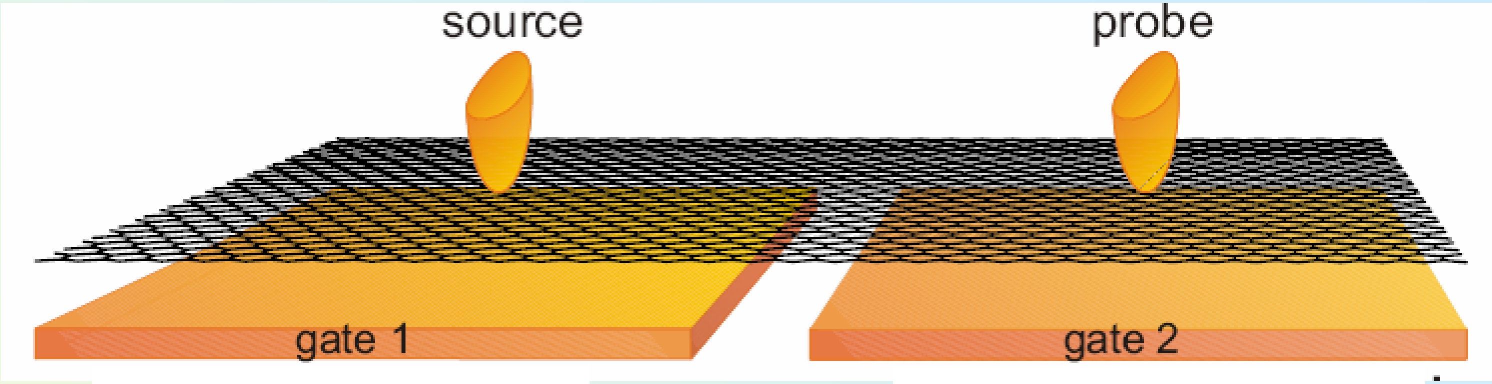
Novoselov *et al* - Science 306, 666 (2004)

Graphene:

2D zero-gap semiconductor

1939 - Jack Scaff and Henry Theurer discovered p-type and n-type regions in silicon at Bell Labs, Holmdel NJ



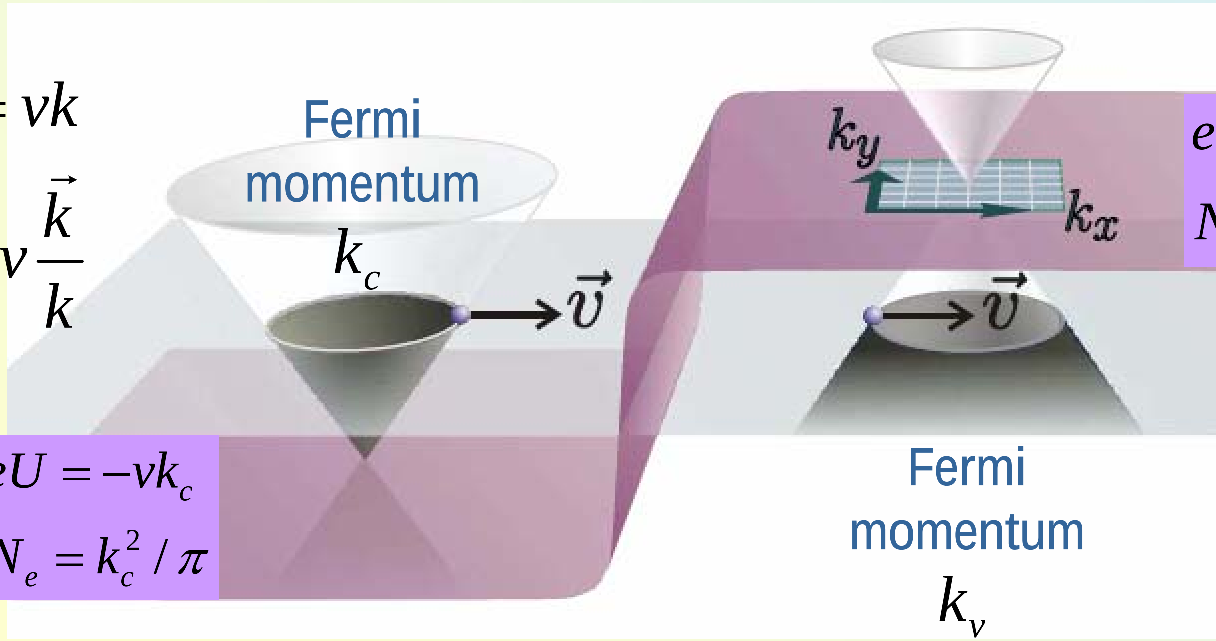


positive U, $n-type$ $\qquad$ *negative* U', $p-type$

The *PN* junction in graphene

$$\varepsilon_{cond}(\vec{k}) = vk$$

$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{k}} = v\frac{\vec{k}}{k}$$



$eU = -vk_c$

$N_e = k_c^2 / \pi$

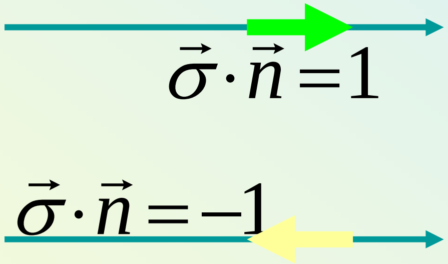
$eU' = vk_v$

$N_h = k_v^2 / \pi$

$$\varepsilon_{val}(\vec{k}) = -vk$$

$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{k}} = -v\frac{\vec{k}}{k}$$

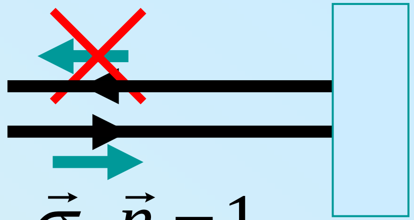
conduction band electrons



$\vec{\sigma}\cdot\vec{n}=1$ $\vec{p}$

$\vec{\sigma}\cdot\vec{n}=-1$ $\vec{p}$

valence band



$\vec{\sigma}\cdot\vec{n}=1$

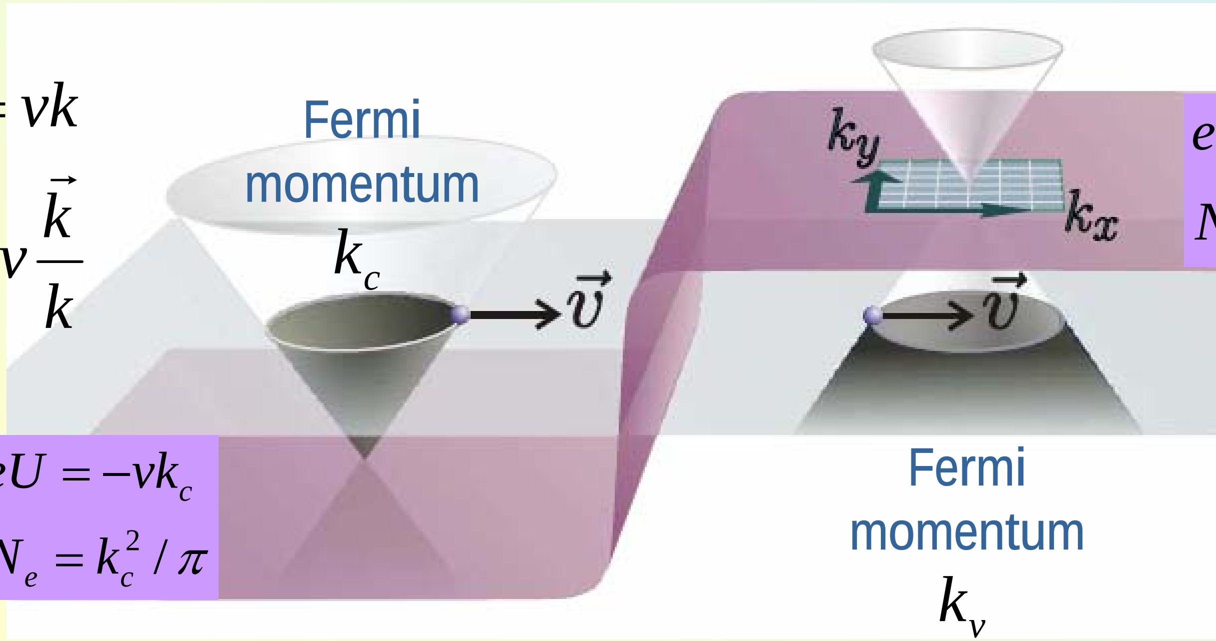
Due to the isospin conservation, A-B symmetric potential of *PN* junction cannot backward scatter chiral electrons,

Ando, Nakanishi, Saito, J. Phys. Soc. Jpn 67, 2857 (1998)

Cheianov, VF - PR B 74, 041403 (2006)

$$\varepsilon_{cond}(\vec{k})=vk$$

$$\vec{v}=\frac{\partial\varepsilon}{\partial\vec{k}}=v\frac{\vec{k}}{k}$$



$$eU=-vk_c$$

$$N_e=k_c^2/\pi$$

$$eU'=vk_v$$

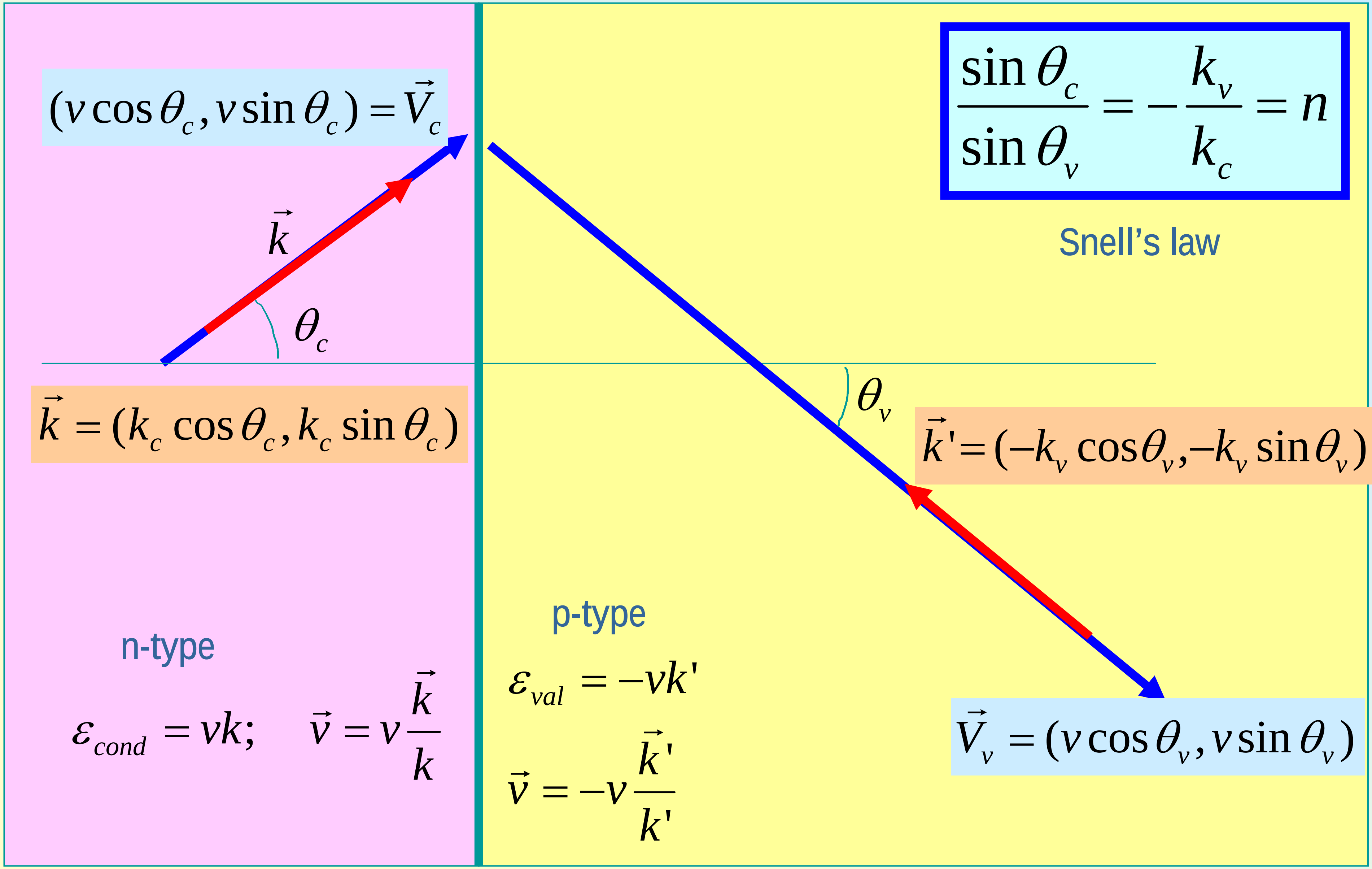
$$N_h=k_v^2/\pi$$

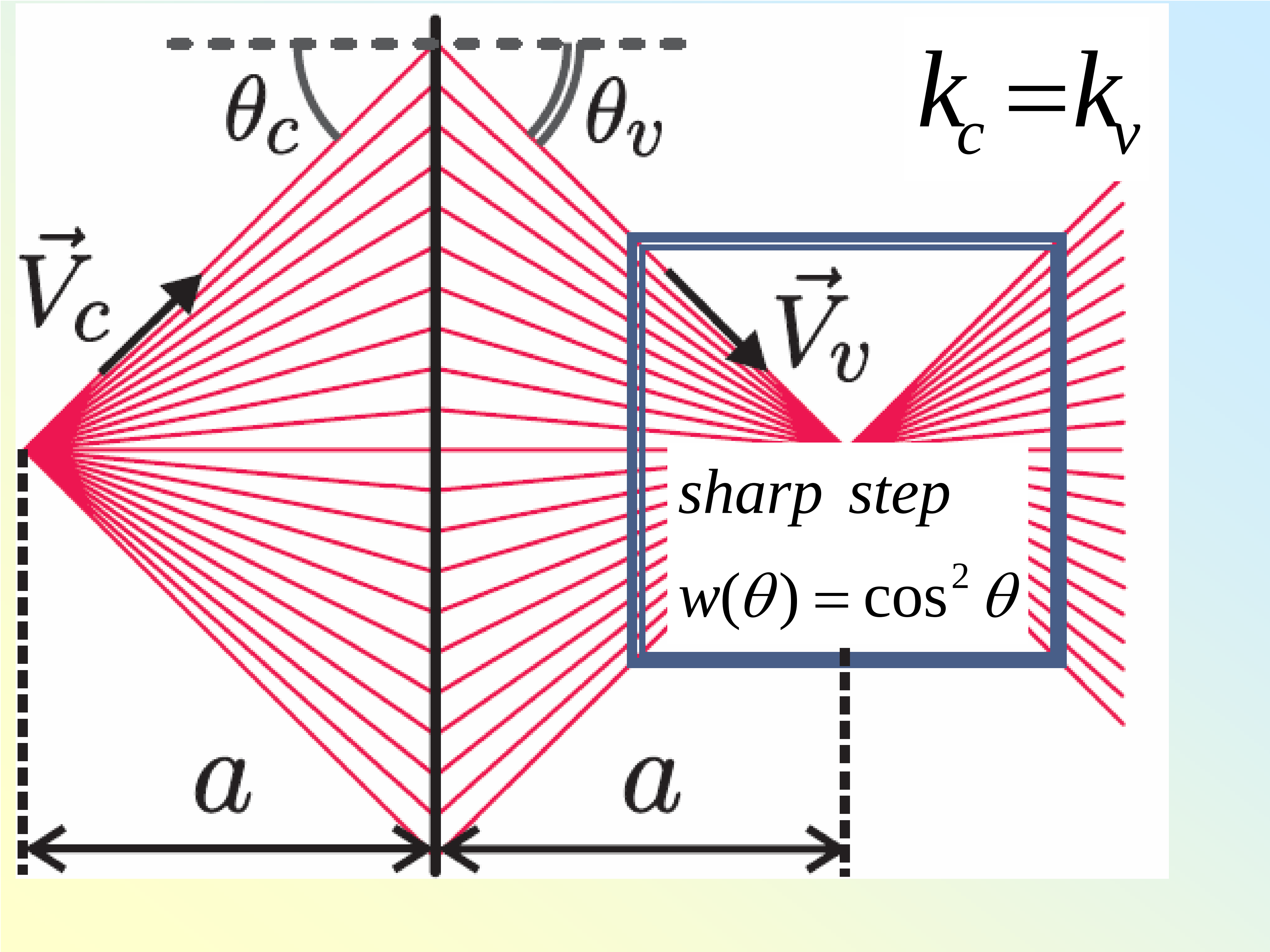
$$\varepsilon_{val}(\vec{k})=-vk$$

$$\vec{v}=\frac{\partial\varepsilon}{\partial\vec{k}}=-v\frac{\vec{k}}{k}$$

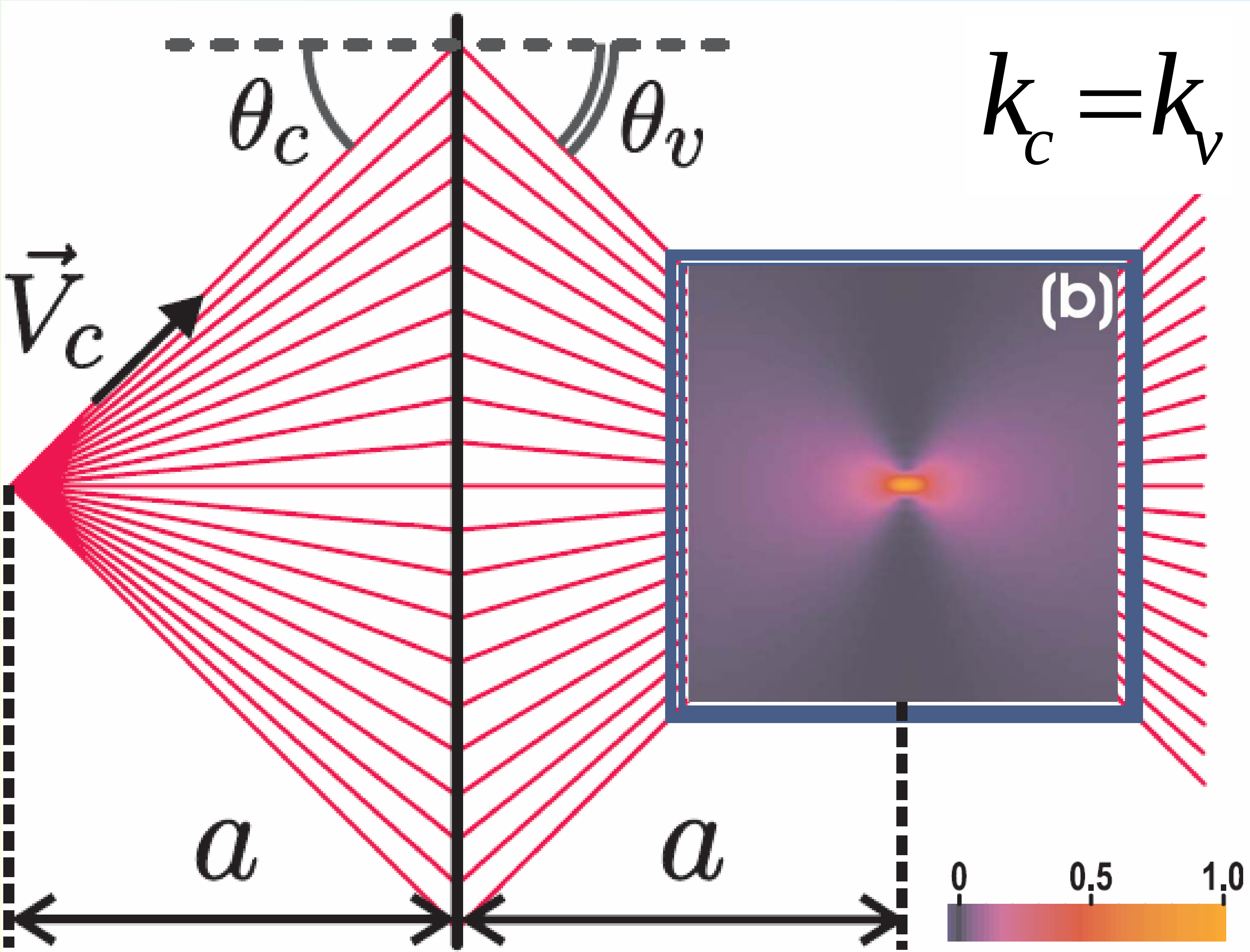
PN junction

$$k_y = k_y' \Rightarrow k_c \sin\theta_c = -k_v \sin\theta_v$$





θ_c
θ_v
$k_c = k_v$
$\vec{V}_c$
$\vec{V}_v$
sharp step
$w(\theta) = \cos^2\theta$
a
a



θ_c
θ_v
$k_c = k_v$
$\vec{V}_c$
(b)
a
a
0
0.5
1.0

$$E = \varepsilon + U(x) = 0$$

Electrons at electro-chemical potential, *E=0*

$$\hat{H} = -iv\vec{\sigma}\nabla + U(x)$$

$$[-iv\vec{\sigma}\nabla + U]G(\vec{r},\vec{r}_0) = \delta(\vec{r}-\vec{r}_0)$$

$$G(\vec{r},\vec{r}_0) = \int dk_y e^{ik_y y} G(x,k_y)$$

$$\left[-i\sigma_x\partial_x + \sigma_y k_y + \begin{Bmatrix} -k_c, x<0 \\ k_v, x>0 \end{Bmatrix}\right] G(x,k_y) = \delta(x+a)$$

$$E = \varepsilon + U(x) = 0$$

Electrons at electro-chemical potential, *E=0*

$$\left[\partial_x - \sigma_z k_y + i\sigma_x \begin{Bmatrix} -k_c, x<0 \\ k_v, x>0 \end{Bmatrix} \right] G(x,k_y) = i\sigma_x \delta(x+a)$$

$$G(x,y) = \int dk_y M(k_y) e^{i\phi(x,y;k_y)}$$

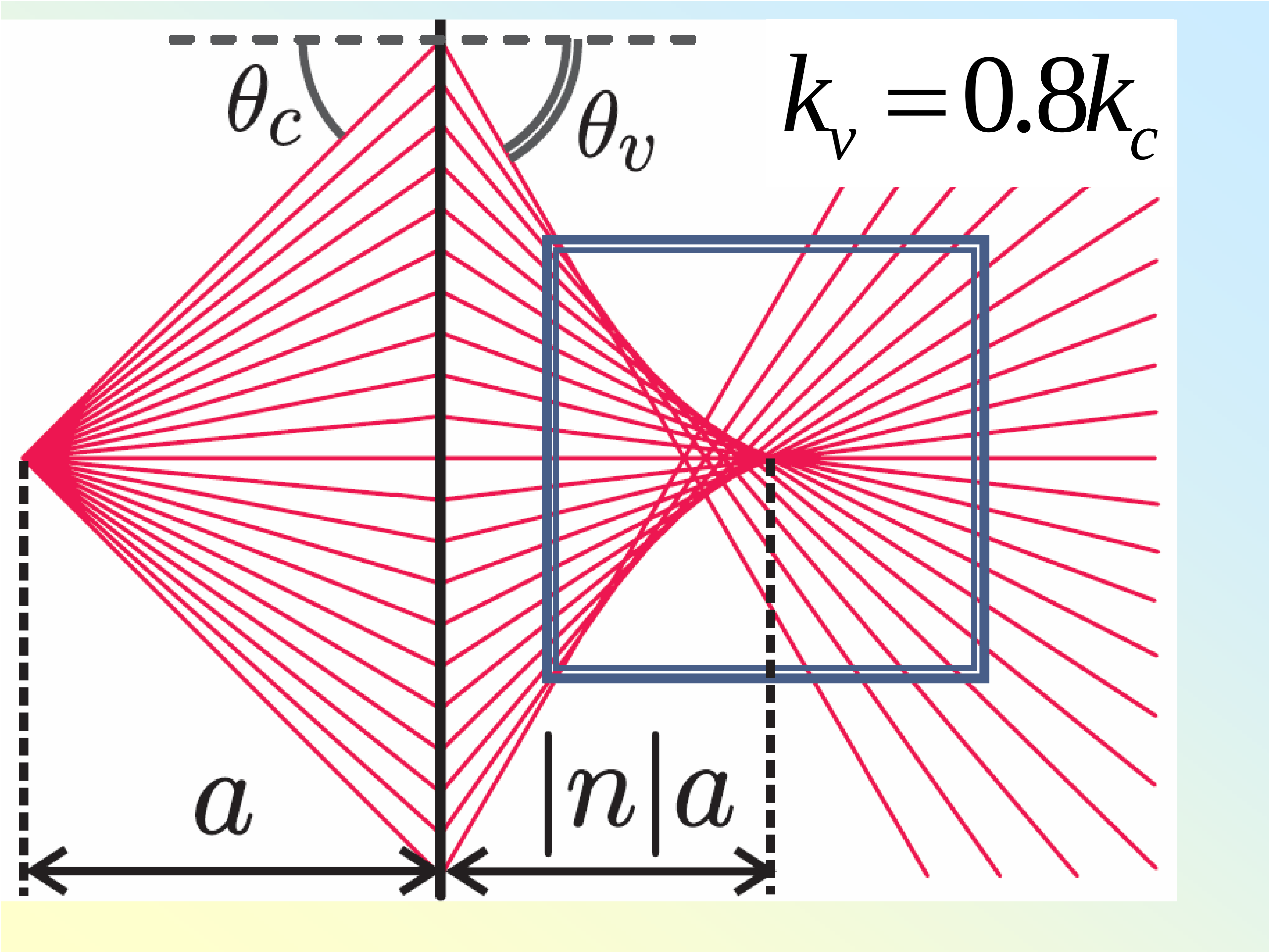
$$\phi(x,y;k_y) = a\sqrt{k_c^2 - k_y^2} - x\sqrt{k_v^2 - k_y^2} + yk_y$$

$$\frac{d\phi}{dk_y} = 0$$

Classical trajectory

$$y = a\tan\theta_c + x\tan\theta_v$$

$$\frac{\sin\theta_c}{\sin\theta_v} = -\frac{k_v}{k_c} = -\sqrt{\frac{N_h}{N_e}} = n$$


θ_c
θ_v
$k_v = 0.8k_c$
a
$|n|a$

$$y = a\tan\theta_c + x\tan\theta_v$$

$$\frac{\sin\theta_c}{\sin\theta_v} = -\frac{k_v}{k_c} = n$$

Snell's law

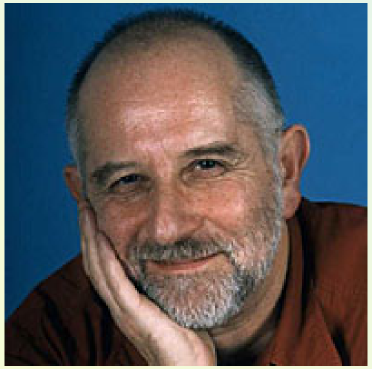
$$\left(\frac{d\phi}{dk_y} = 0\right)$$

$$\frac{dy}{d\theta_c} = 0$$ caustic $$\left(\frac{d\phi}{dk_y} = \frac{d^2\phi}{dk_y^2} = 0\right)$$

$$\frac{dy}{d\theta_c} = \frac{d^2y}{d\theta_c^2} = 0$$ cusp $$\left(\frac{d\phi}{dk_y} = \frac{d^2\phi}{dk_y^2} = \frac{d^3\phi}{dk_y^3} = 0\right)$$

general catastrophe theory


Rene Thom


Michael Berry


Vladimir Arnold

$$y = a\tan\theta_c + x\tan\theta_v$$

$$\frac{\sin\theta_c}{\sin\theta_v} = -\frac{k_v}{k_c} = n$$

Snell's law

$$\left(\frac{d\phi}{dk_y} = 0\right)$$

$$\frac{dy}{d\theta_c} = 0$$

caustic

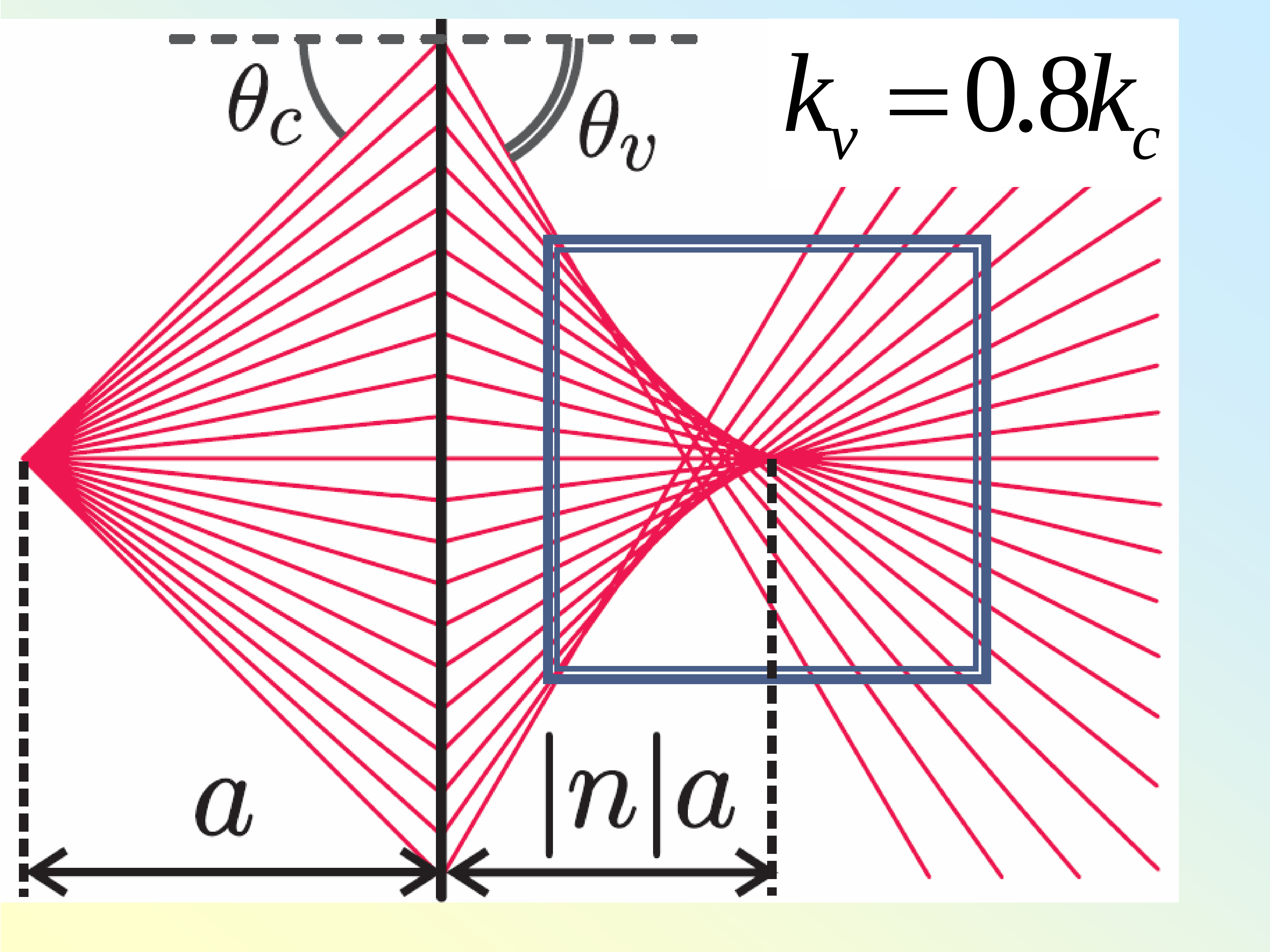
$$\left(\frac{d\phi}{dk_y} = \frac{d^2\phi}{dk_y^2} = 0\right)$$

$$\frac{dy}{d\theta_c} = \frac{d^2y}{d\theta_c^2} = 0$$

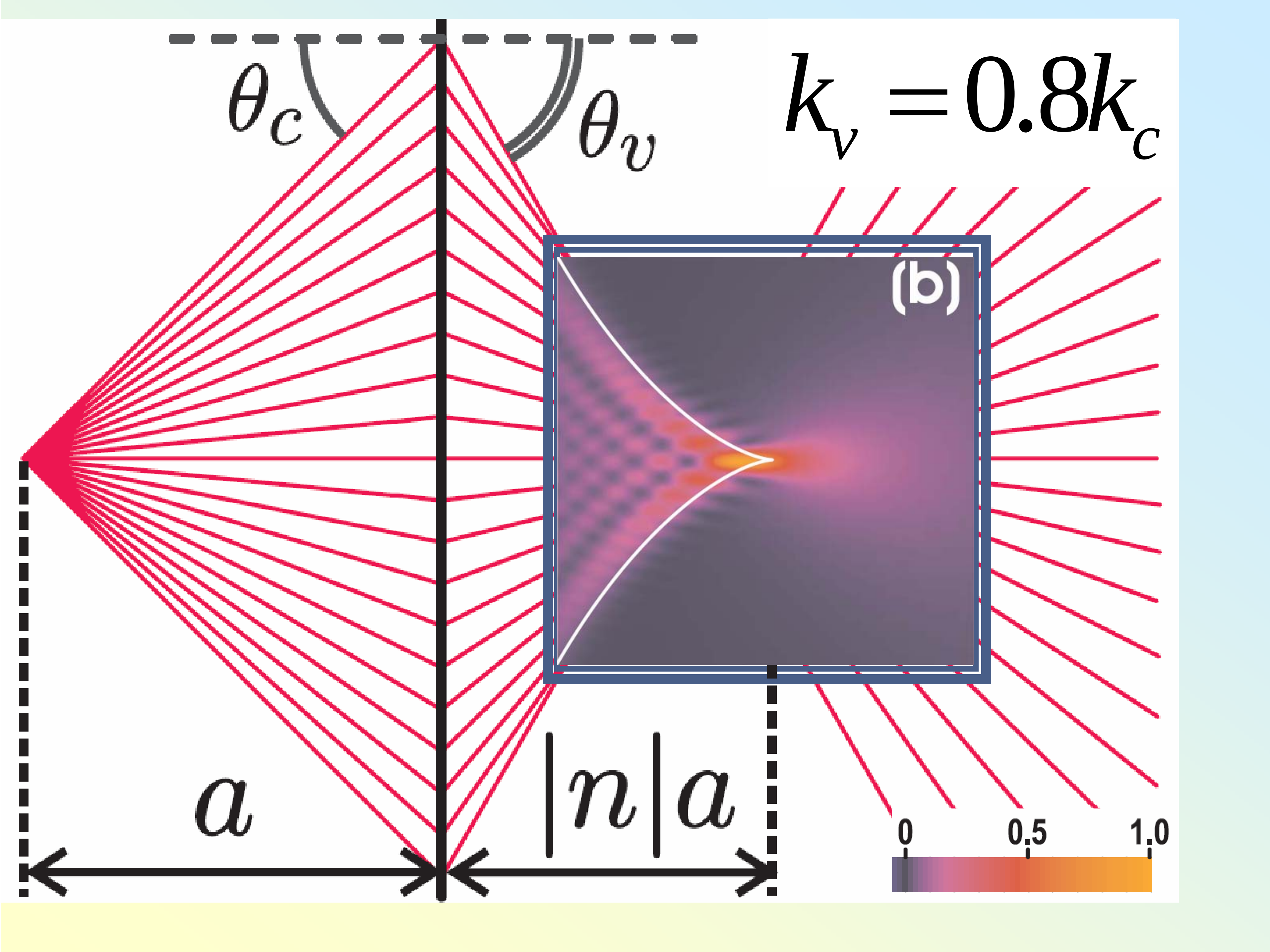
cusp

$$\left(\frac{d\phi}{dk_y} = \frac{d^2\phi}{dk_y^2} = \frac{d^3\phi}{dk_y^3} = 0\right)$$

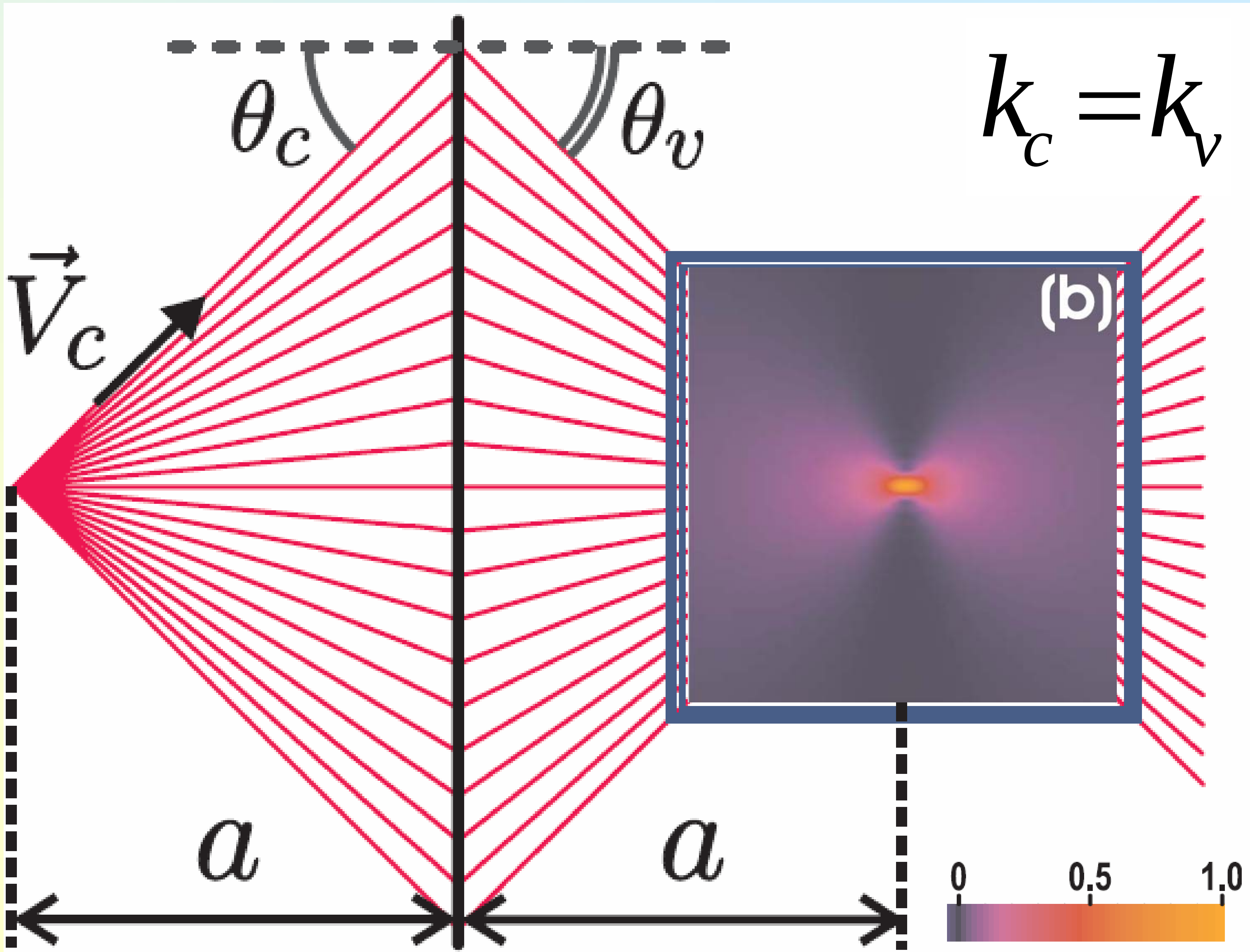
$$y_{\text{caust}}(x) = \pm\sqrt{\frac{[x^{2/3} - x_{\text{cusp}}^{2/3}]^3}{n^2 - 1}}, \quad x_{\text{cusp}} = |n|a$$



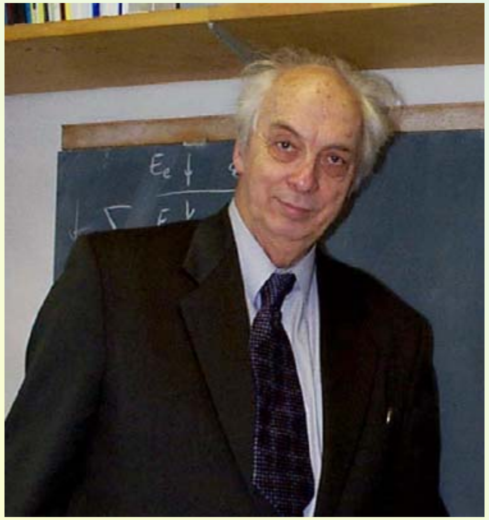
θ_c
θ_v
$k_v = 0.8k_c$
a
$|n|a$

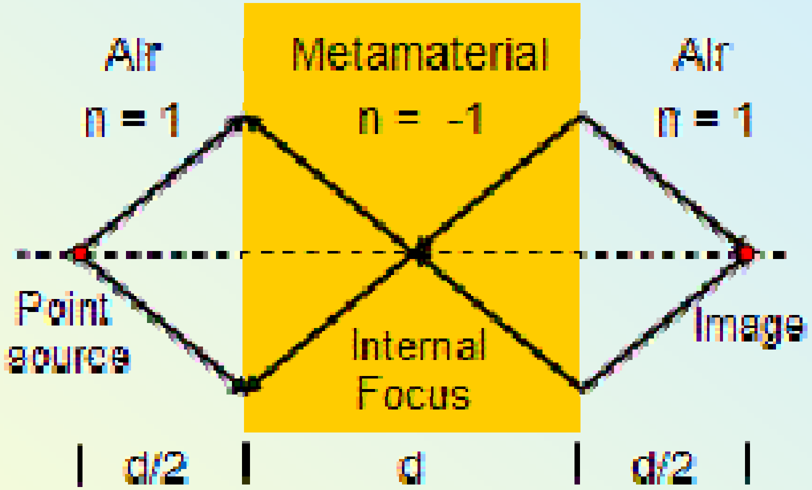


θ_c
θ_v
$k_v = 0.8k_c$
(b)
a
$|n|a$
0
0.5
1.0


θ_c
θ_v
$k_c = k_v$
$\vec{V}_c$
(b)
a
a
0
0.5
1.0

Veselago Lens

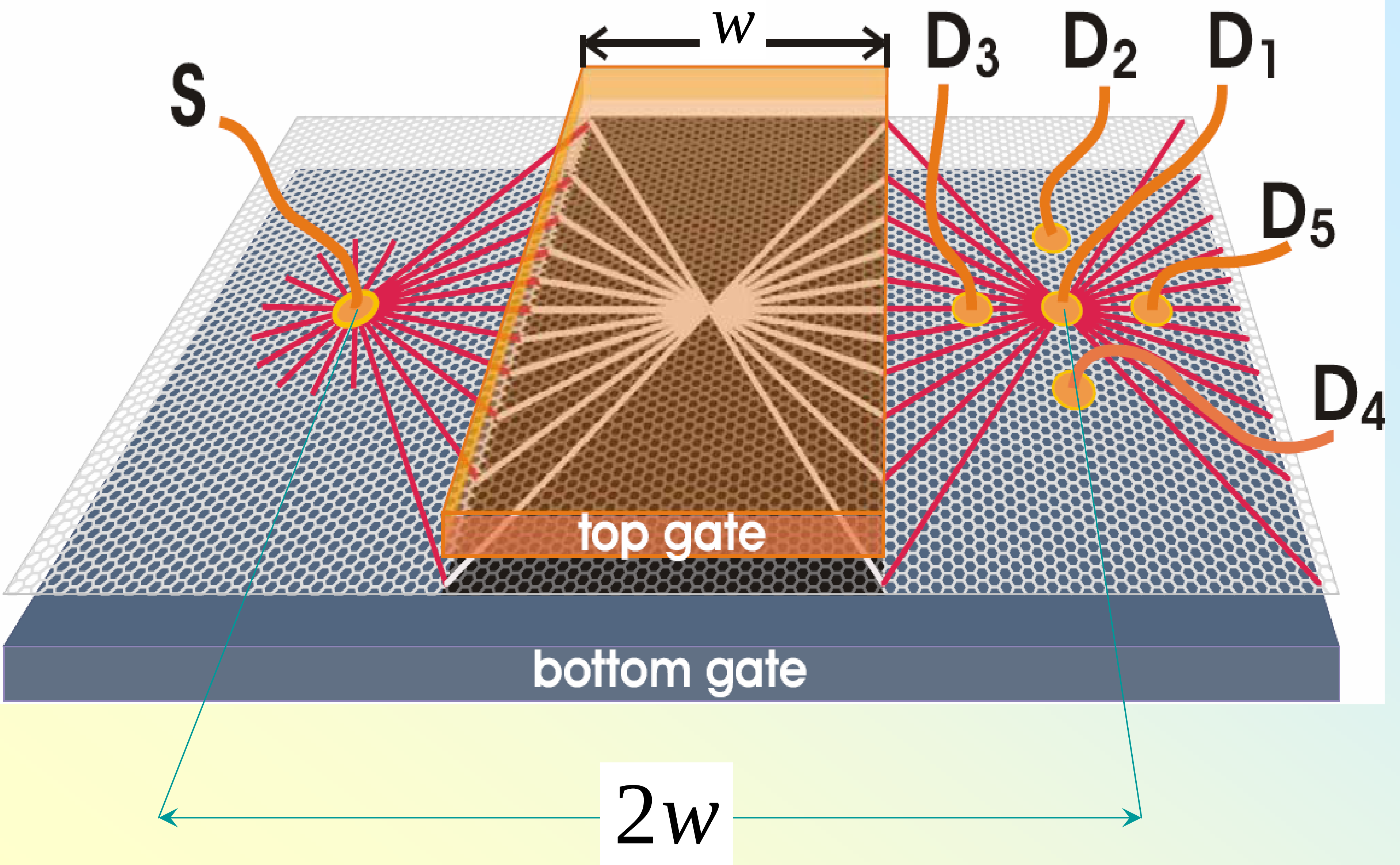




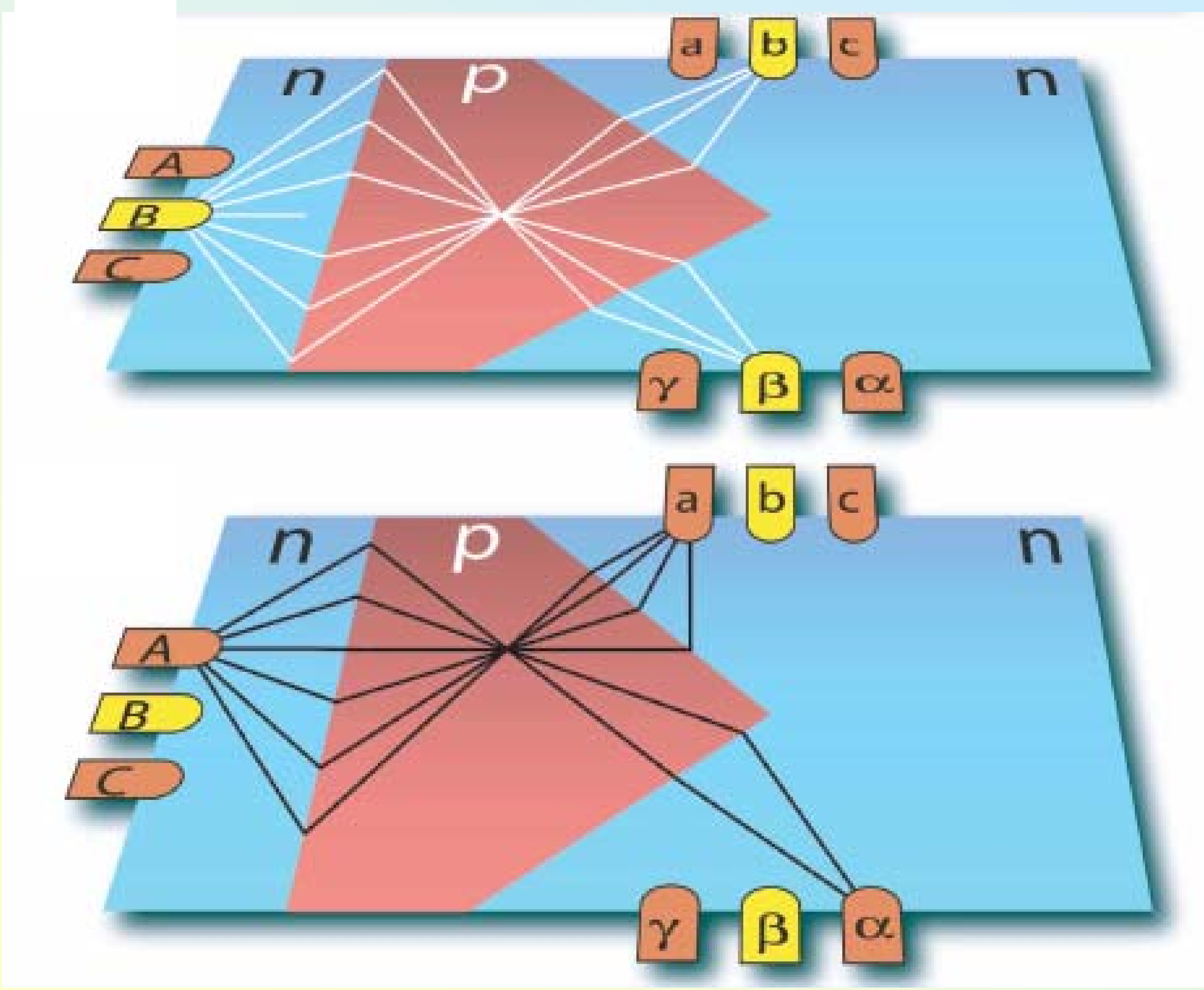
Veselago, Sov. Phys.-Usp., 10, 509 (1968)

Pendry, Phys. Rev. Lett., 85, 3966 (2000)

Graphene bipolar transistor: Veselago lens for electrons



Focusing beam-splitter for electrons



Wishful thinking about graphene microstructures:

Focusing/caustics and Veselago lens in graphene *PN* junctions.

Cheianov, VF, Althsuler (2006)

Mirage of a perturbation *u* on the other side of a sharp *PN* junction represented by the local density of states image (STM).

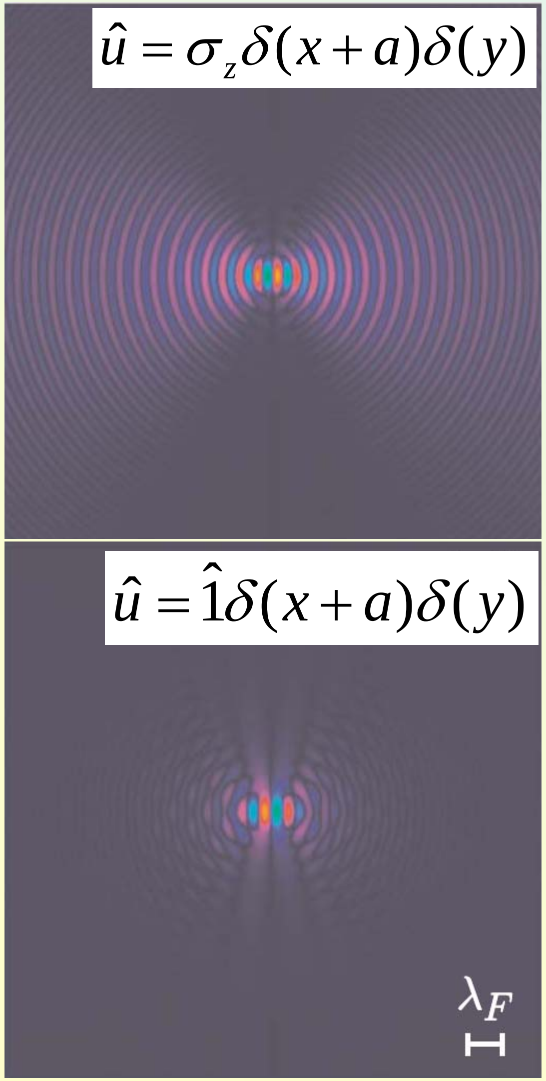
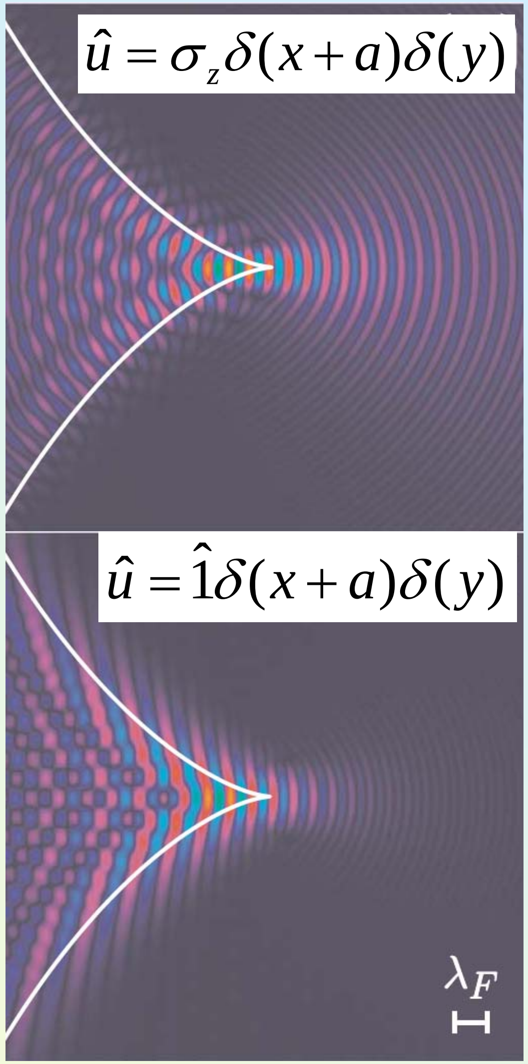
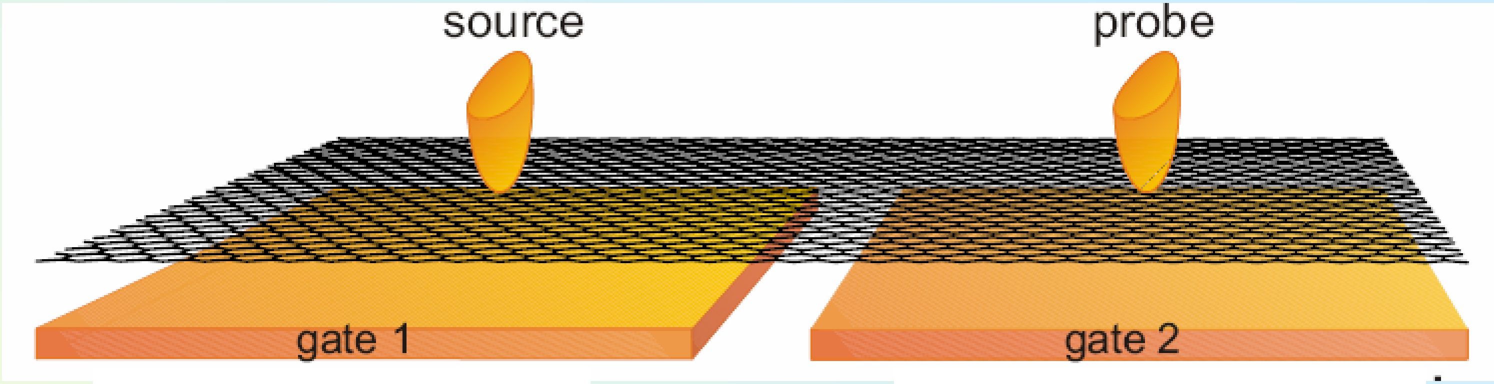


Image of a bilayer island ('dot') embedded in monolayer graphene: images show the modulation of the difference between tunnelling currents to the A and B sites.

Image of a charged tip on the other side of the junction: 'teleported' Friedel oscillations.



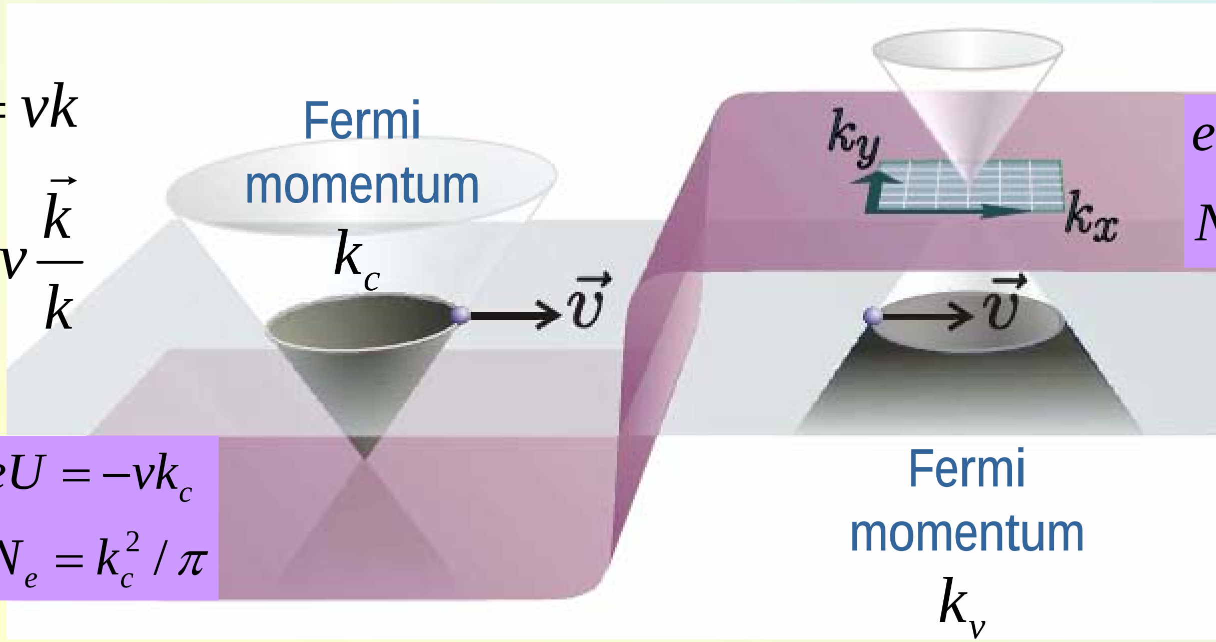


$positive\ U,\ \ n-type$ $negative\ U',\ \ p-type$

A smooth ‘high density’ *PN* junction graphene, $d \gg \lambda_F$

$$\varepsilon_{cond}(\vec{k}) = vk$$

$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{k}} = v\frac{\vec{k}}{k}$$



$$eU = -vk_c$$

$$N_e = k_c^2 / \pi$$

$$eU = vk_v$$

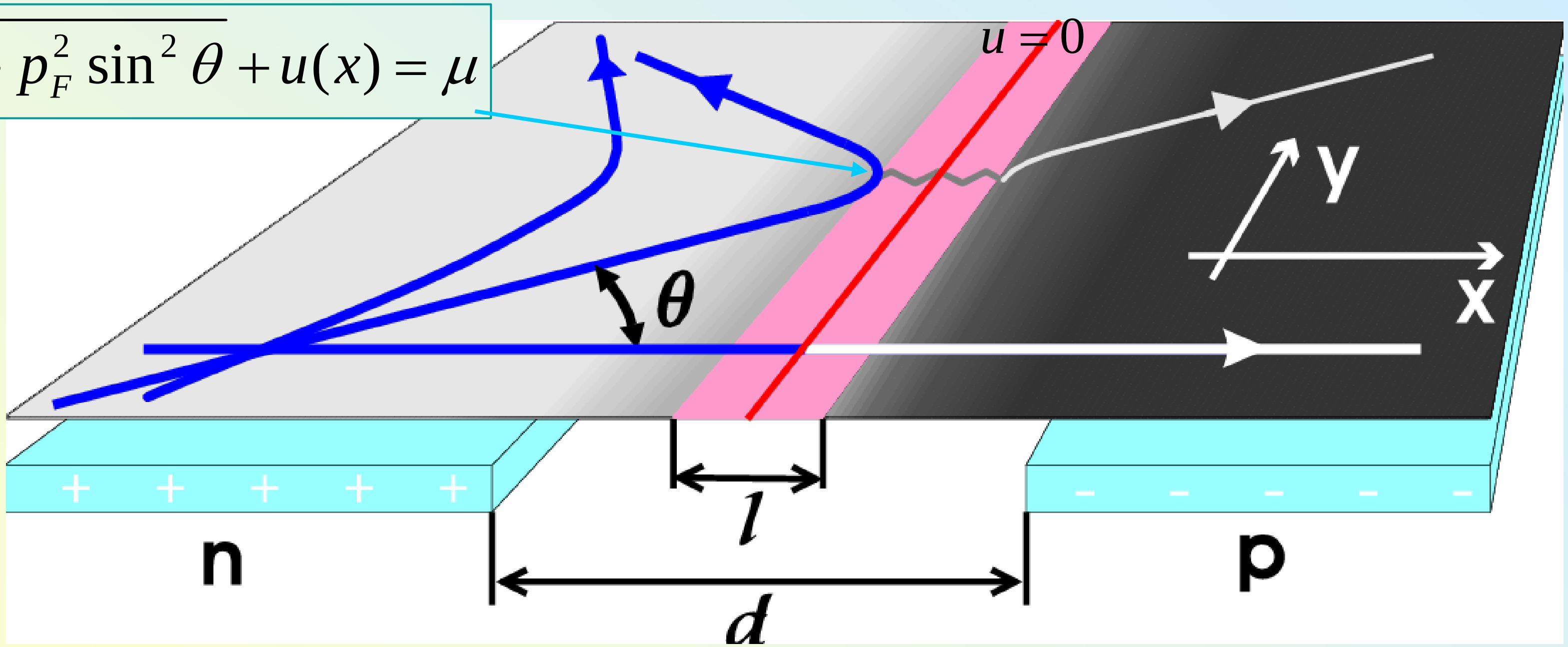
$$N_h = k_v^2 / \pi$$

$$\varepsilon_{val}(\vec{k}) = -vk$$

$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{k}} = -v\frac{\vec{k}}{k}$$

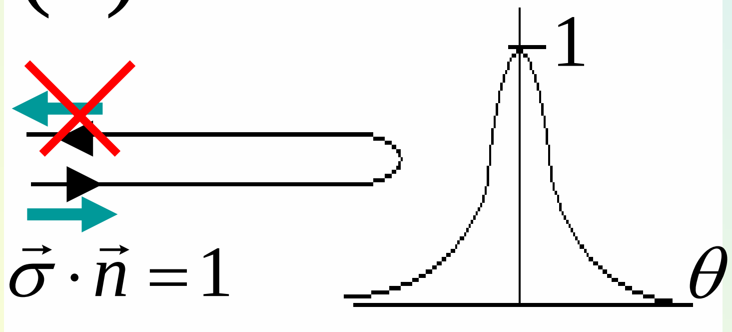
Transmission of chiral electrons through the *PN* junction in graphene

$$\sqrt{p_x^2 + p_F^2 \sin^2\theta} + u(x) = \mu$$



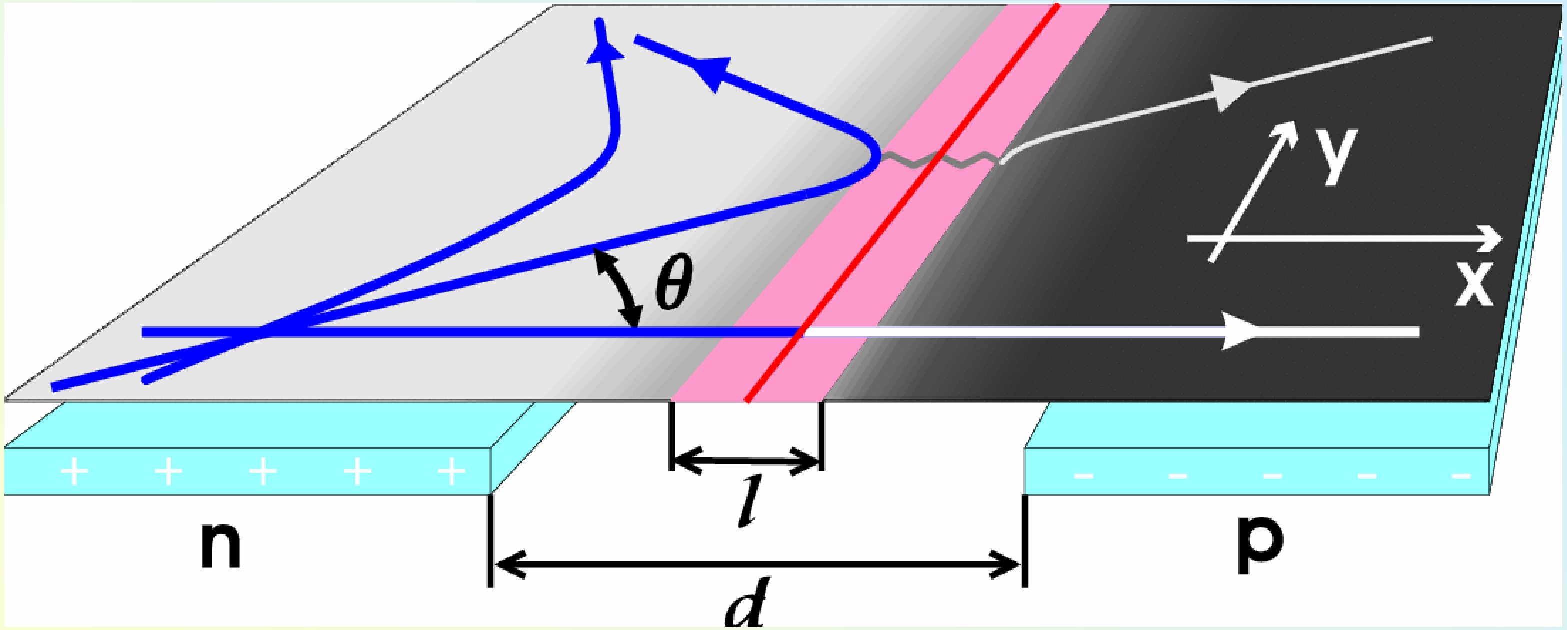
$$w(\theta) = e^{-\pi k_F d \sin^2\theta} \cos^2\theta$$

Due to the isospin conservation, electrostatic potential cannot scatter chiral fermions in backward direction.



Cheianov, VF - PR B 74, 041403 (2006)

Transmission of chiral electrons through the *PN* junction in graphene



Due to selective transmission of electrons with a small incidence angle, an *PN* junction in graphene should display a finite conductance per unit length (no pinch-off) and a characteristic Fano factor in the shot noise.

Cheianov, VF - PR B 74, 041403 (2006)

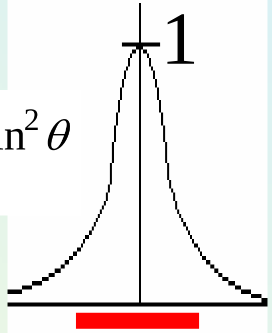
$$g_{np} = \frac{2e^2}{\pi h}\sqrt{\frac{k_F}{d}}$$

$$\langle I \cdot I \rangle = (1-\sqrt{\tfrac{1}{2}})\ eI$$

Smooth junction, $d >> \lambda_F$

Selective transmission in a smooth junction, $d \gg \lambda_F$

$$w(\theta) = e^{-\pi k_F d \sin^2 \theta}$$



$$|\theta| < \theta_0 = \sqrt{\pi k_F d} \ll 1$$

$$\theta' \approx \frac{r}{r_c(B_*)} = \theta_0$$

weak-field magnetoresitance of ballistic *P-N* junctions

$$R(B) = R_{cont} + \frac{2\pi a}{g_{np}} f\left(\frac{B}{B_*}\right)$$

$$g_{npn} = \frac{g_{np}}{\sqrt{\pi}} \int_{-\infty}^{\infty} \frac{dz}{e^{z^2} + e^{(z+B/B_*)^2} - 1}$$