Pushing through the sand: local jamming, penetration, and drag in granular media

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Part 1: Pushing an object through grains

When a stress is applied to a dense collection of grains, the grains form a rigid "jammed" structure to resist the stress.

What is the nature of the jammed state resulting from a locally applied stress?

How strong are jammed states? How do they fail?

Behringer group
Study the granular drag force

Drag is force required to reorganize grains to allow motion
Principles of granular drag at low velocities

Grains jam, and then jammed state breaks

\[ F_{\text{drag}} \equiv \text{avg. force to reorganize static grains} \]

\[ F_{\text{drag}} \text{ should be velocity independent -- akin to friction} \]

Simple mean-field or detailed calculation suggests:

\[ F_{\text{drag}} = \eta g \rho d_c H^2 \text{ for vertical cylinder} \]

- \( \eta \) = dimensionless constant (grain surface/morphology/packing)
- \( \rho \) = density of grain material
- \( d_c \) = cylinder diameter
- \( H \) = depth of insertion
Measure Drag Force at Low Velocities

Rotating Bucket of Glass Spheres, Cylinder Dipped In

Measure Force to Keep Cylinder Fixed

Vary grain size, velocity, depth, cylinder diameter
Drag Force: Vertical cylinder moving horizontally

\[ F_{\text{drag}} = \eta \rho d_c H^2 \] in agreement with theoretical expectations independent of velocity and grain size

\[ F = \eta \rho g H A \] ~ Phys. Rev. Lett. 82, 205 (1999) and 84 5122 (2000)

- Cylinder diameter \( d_c = 10 \) mm. Grain diameter \( d_g = 0.9 \) mm
Drag does not depend on cylinder surface

Phys. Rev. E 64, 031307 (2001) and 64, 061303 (2001)

Drag determined by the force needed to collapse the bulk jammed state
Look at finite size effect with penetrometer


- Probe effects of boundaries on strength of jammed state by measuring resistance to penetration

- Vary:
  - bead diameter
  - bucket size
  - diameter of plate
  - velocity,
  - texture of bottom surface
Careful filling procedure required

Slowly lower bucket so grains fill without free fall

Get reproducible and homogeneous packing fraction $\sim 59\%$
Height dependence of penetration force

- Initial linear force distribution with subsequent rollover
- Rapid increase as penetrometer approaches bottom
- Work in a regime of no bucket size or velocity dependence
Obtain the effect of the bottom by subtracting off data taken with deeply filled bucket

Obtain:

“Bulk” force as a function of depth, \( F_{\text{bulk}} \)

Measure of stress at bucket bottom, \( F_0 \)

0.9 mm beads
25.4 mm plate
How close to the bottom boundary does the penetration force reflect that a bottom exists?

\[ \Delta F \propto e^{-\frac{z}{\lambda}} \]

For all grain sizes and real sand

Implies the existence of an intrinsic length scale

Length scale determined by....

Pressure? Plate Diameter? Grain Diameter? Something else?
Grain diameter appears not to affect length scale
Plate diameter dependence of length scale

• $\lambda$ increases with penetrating plate size

• Larger penetrating object detects bottom earlier

$\Delta F \propto e^{-z/\lambda}$
Fill height dependence of length scale

Fill height affects $\lambda$ through ambient stress

Get measure of stress through $F_0 = F_{\text{bulk}}(z = 0)$
Dependence of length scale on system parameters

\[ \lambda \propto \sqrt{\frac{F_0}{r}} \]

Grain Diameter
- 0.92mm (square plates)
- 1.89 mm (square plates)
- 0.38 mm
- 0.92 mm
- 2.27 mm

\[ \sqrt{\frac{F_0}{r}} \text{ (N/mm)}^{1/2} \]
Scaling of length scale

Plate size dependence

\[ \Delta F \Rightarrow \frac{\Delta F}{F_0} \]

\[ z \Rightarrow \frac{z}{\sqrt{F_0 / r}} \]

Fill height/pressure dependence

Plate size & Fill height/pressure dependence

0.9 mm grains Smooth bottom
Where does dependence of length scale come from?

\[ \lambda \propto \sqrt{\frac{F_0}{r}} \]

If we take \( F_0 = P_{\text{eff}}(\pi r^2) \) where \( P_{\text{eff}} \) is an effective granular pressure we get:

\[ \lambda \propto \sqrt{P_{\text{eff}} r} \]

Since \( \lambda \) is the effective size of the jammed state caused by penetration, it would be interesting to understand its origins...theory needed!!!
What does length scale mean?

Image inside 3 dimensional bead pack: MRI on mustard seeds: preliminary data only (Igor Veretennikov, Notre Dame)
What’s next: the effect of a free boundary

How much force is needed to lift the coffin lid?

Of great interest to “taphephobics”!
New apparatus (preliminary data): penetration from below

What is initial force needed to start motion?

How does free boundary affect resistance?

• Minimal friction through bearing
• Careful and reproducible filling
• Plate ($d_{\text{plate}} >> d_{\text{grain}}$) which starts flush with the bottom
• Controlled elasticity through spring
Raw Data: Force vs. Height

Rich set of phenomena to investigate....
Point where grains flow under the plate depends on grains size

Grains flow under rising plate

Makes sense, since need a crack at least one diameter wide!
Part 2: Temperature effects on granular materials

In studies of granular media, we usually only consider temperature as a statistical measure of grain kinetic energy.

But grains are made of materials which change with changing temperature....
Thermal cycling can effect granular samples....

Difference in thermal contraction between container and medium will cause the grains to settle each time there is a thermal cycle.

Change in packing changes granular properties, can have more drastic effects...
Simple thermal cycling experiment

Glass beads and plastic container
Thermal cycling results do not depend strongly on filling or cylinder diameter (preliminary data)

105C, 2hr annealing, 0.5mm beads
1000ml cylinder filled to different height

1.5mm beads, 58.3% initial PF, 105C annealing temp
Multiple cycles result in increased packing

Can be fit well by double exponential

\[
y = A_1 \exp(-x/t_1) + A_2 \exp(-x/t_2) + y_0
\]

- \( y_0 = 61.65564 \pm 0.76245 \)
- \( A_1 = -1.37035 \pm 0.65612 \)
- \( t_1 = 1.11197 \pm 0.82305 \)
- \( A_2 = -1.91976 \pm 0.54095 \)
- \( t_2 = 14.56763 \pm 16.12865 \)
Conclusions/Questions

Interesting physics in local perturbations

What defines the length scale for the jammed state?

What are the microscopic dynamics of the collapse process?

Temperature can be an important parameter in granular media

New way to study packing

What will happen as we change thermal expansion coefficients/initial packing/etc.?