Granular gases:
a window into (simple?) far from equilibrium systems

- Background and motivation
  very (deceptively) simple non-equilibrium system
  testing ground: kin. theory, simulation & expt.
  model system
  non-Maxwell-Boltzmann velocity distributions

- Questions (for us)
  how to inject energy (boundary/uniform)?
  role of spatial correlations?
  how distinct, what controls behavior?

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Experiments Velocity correlations

*Non-Gaussian (Boltzmann)*

\[ P(v) \approx \exp(-Av^\beta), \text{ where } \beta \approx 1.55 \]

not

\[ P(v) \approx e^{-mv^2/2kT} \]

Rouyer & Menon '01

See also

Olafsen & Urbach '98, '99
Losert *et al.* '99 (also $\beta \approx 1.5$)
Kudrolli *et al.* '97,'00, '01
(not universal?)
Approaches

Kinetic theories
e.g., van Noije & Ernst ’98

Simulations
e.g., Moon et al. ’01
Barrat & Trizac ’02
Brey & Ruiz-Montero ’03

Questions:
role/method of energy input?
role of spatial correlations?
why $\beta < 2$, what controls this?

The model

event-driven simulation,

$$v_i' = v_i - \frac{1}{2} \eta (v_i \cdot \hat{r}_{ij} - v_j \cdot \hat{r}_{ij}) \hat{r}_{ij},$$

$\eta < 1$,
Coefficient of restitution
Clustering vs. Collapse

Inelastic collisions lead to clustering.

Collapse $= \text{infinite number of collisions in a finite time}$

(McNamara & Young, 1994)

Radial dependence

"Heat"
Uniform heating

Little significant deviation from Gaussian
- see also Barrat and Trizac, 2003
- Barrat, Trizac, & Ernst cond-mat/0411435
Boundary heating

FIG. 2. Velocity distributions for $N = 350$ and $\phi = 0.02$. Shown are both results for uniform heating with $\eta = 0.8$ (○), $\eta = 0.1$ (□) and results for boundary heating with $\eta = 0.9$ (◇), $\eta = 0.6$ (▽), and $\eta = 0.4$ (★). (a) $P(v_x/\sigma_x)$; (b) $-\ln[-\ln[P(v_x/\sigma_x)]]$ versus $\ln(v_x/\sigma_x)$. A Gaussian is shown as a solid line.

Comparison with experiments

FIG. 10: Boundary heating. (a) $-\ln[-\ln[P(v_x/\sigma_x)]]$ versus $\ln(v_x/\sigma_x)$ for $N = 350$, $\phi = 0.05$ and $\eta = 0.9$ (○), $N = 500$, $\phi = 0.05$ and $\eta = 0.9$ (□), $N = 350$, $\phi = 0.05$ and $\eta = 0.8$ (○), $N = 350$, $\phi = 0.25$ and $\eta = 0.9$ (△). The solid lines correspond to the fit as made by Rouyer and Menon and has an exponent $\alpha = 1.52$. The range of the solid lines corresponds to half the range used by Rouyer and Menon in their fit, but contains about 80% of their data points.
What controls non-Gaussians?

Data collapse for various $\phi$, $N$, & $\eta$

\[ q = \frac{N \eta}{N_c} \]

heating

collisions

Estimates:

\[ \ell_c \sim \frac{1}{\phi} \]

\[ \ell_h \sim (N/\phi)^{1/2} \]

\[ q \sim 1/(N \phi)^{1/2} \]

Spatial correlations irrelevant

no spatial degrees of freedom, but same $q$

FIG. 5. $-\ln[-\ln(P(v_r/\sigma_r))]$ versus $\ln(v_r/\sigma_r)$. The symbols shown are velocity distributions acquired by simulation for $q = 120$ (uniform heating, ●), $0.08$ (boundary heating, □), $0.012$ (homogeneous two-point heating [4,13], ○). The lines show the velocity distributions found in the model for the same values of $q$ (solid, dotted, dashed, respectively).
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- non-Gaussian distributions
  NOT “overpopulated” tails of kinetic theory
  continuous range of effective exponents
- Boundary vs. uniform/homogeneous heating
- Importance of energy cascade
  energy input at rate $N_H$
  dissipated at rate $N_C$
  important parameter $q = N_H/N_C$
- Role of spatial correlations