Mixing and Segregation in Dense Flows: Rotating Cylinders

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Kavli Institute, Santa Barbara, June 20, 2005

Rotating Cylinders

Cement rotary kiln

Applications:

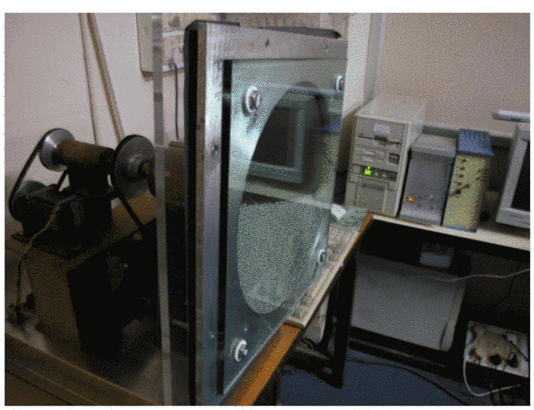
Drying, calcination, mineral processing, granulation, mixing, spray coating, incineration, …

Cement, fertilizer, seeds, grains, ores, pharmaceuticals, …

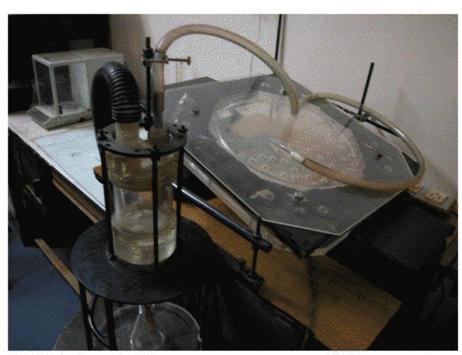
- Vast amounts of material are processed
- Mixing and segregation are very important

Experiments

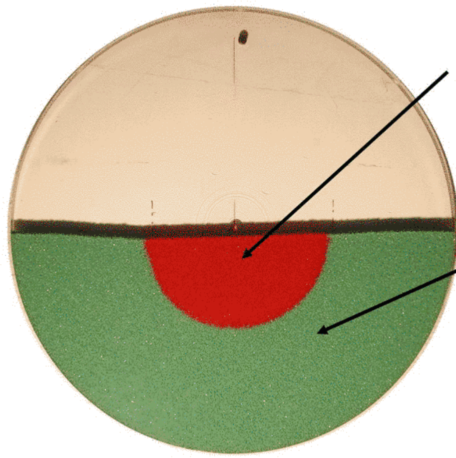
Quasi-2D Rotating Cylinder



Sampling setup

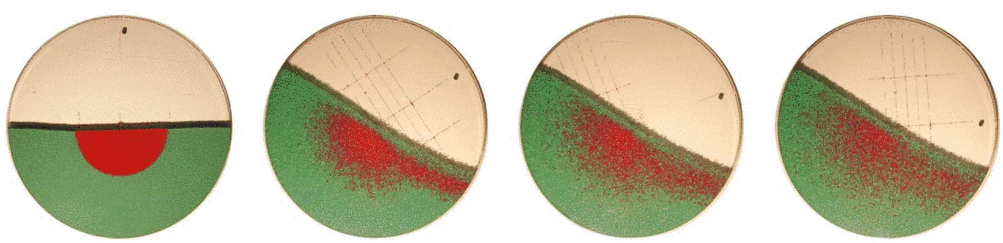


Tracer Mixing



Role of diffusion, model

Mixing of Tracers



Initial 0.5 rev. 1.0 rev. 1.5 rev.

Governing equation:

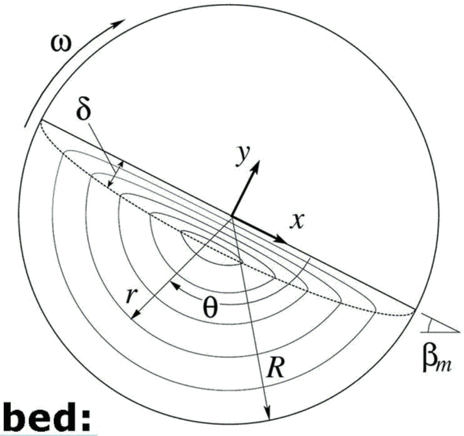
$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = D\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)$$

f: Number fraction of red beads

D: Self-diffusivity (constant, fitting parameter) $D \propto d^2$

(Hajra & Khakhar, *Phys. Fluids*, 2005)

Flow Model



In bed:

$v_x = \omega y$

$v_y = -\omega x$

In layer:

$v_x = \dot{\gamma}(\delta + y)$

$v_y = \omega x (y/\delta)$

$\delta = \delta_0 \left[1-(x/L)^2\right]^{1/2}$

$\delta_0 = (\omega/\dot{\gamma})^{1/2} L$

$\dot{\gamma} = \left[\frac{g \sin(\beta_m - \beta_s)}{cd \cos\beta_s}\right]^{1/2}$

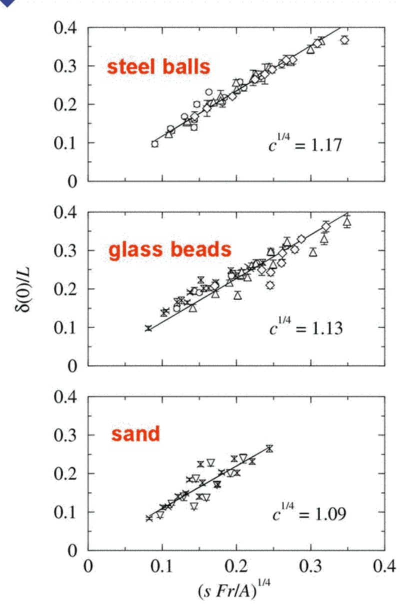
Parameters: c, β_s, β_m

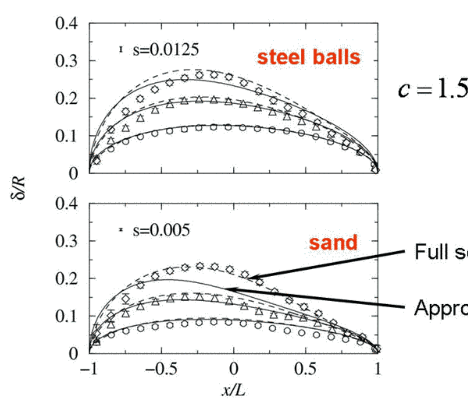
Low rotational speeds
$\dot{\gamma} \sim$ const.

D is const.

(Khakhar, Orpe and Ottino, *Adv. Complex Systems*, 2001)

Model Predictions

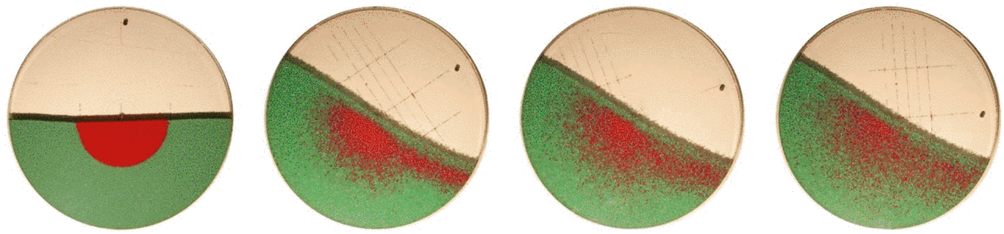




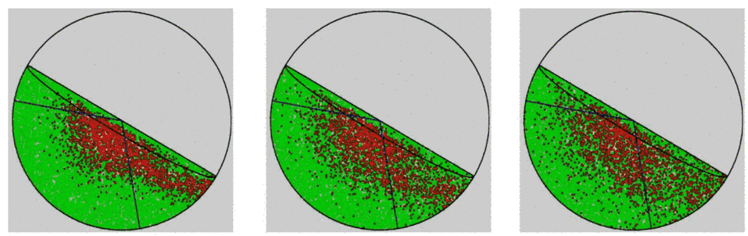
Ranges of validity:

- Froude numbers $Fr \in (0.002, 0.064)$
- Size ratios $s \in (0.05, 0.005)$
- Materials (shape)
  - Steel (spherical)
  - Glass (near spherical)
  - Sand (irregular)

Comparison



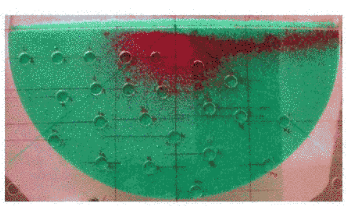
Initial 0.5 rev. 1.0 rev. 1.5 rev.



(Hajra & Khakhar, *Phys. Fluids*, 2005)

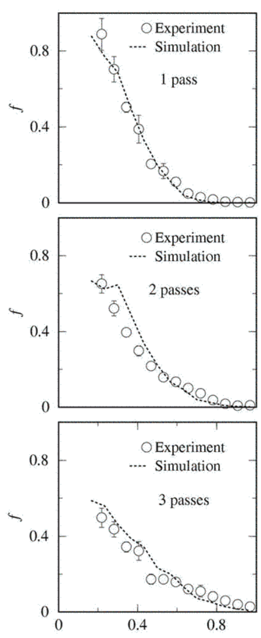
Comparison

Radial concentration profiles by sampling:

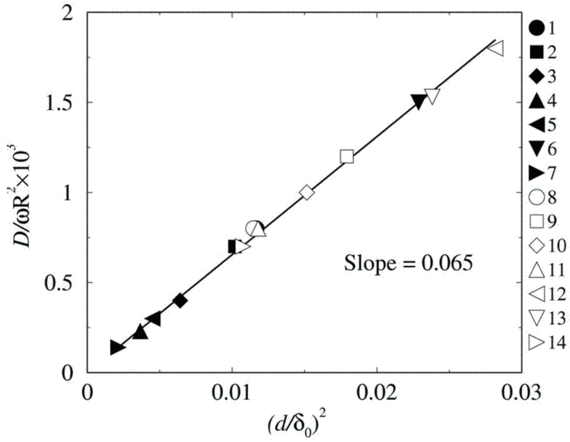


- 23 sampling points.
- Sampling volume constant.
- Precise location.
- Volume fraction by image analysis.

Error bars: Average over 3 experiments



Fitted Self-diffusivity

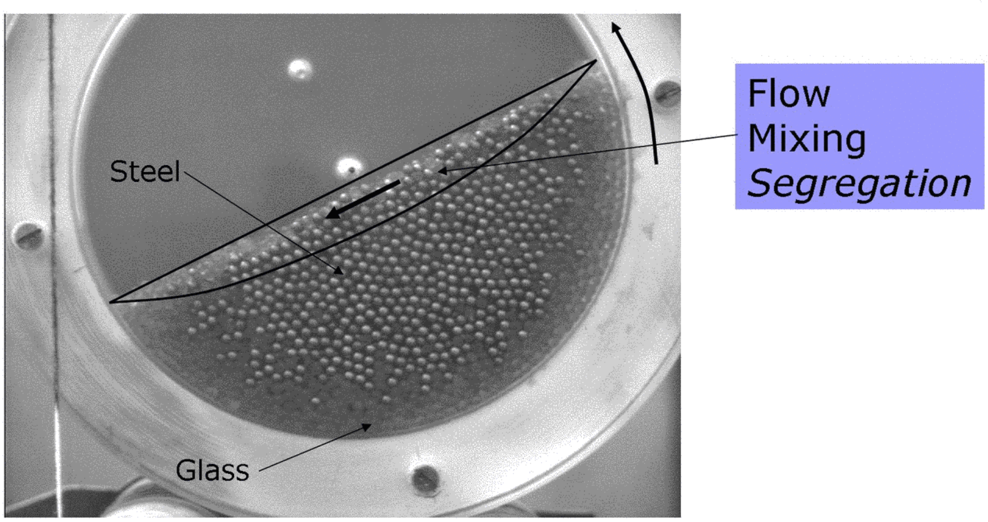


Correlation for diffusivity (Savage, 1993)

$$D = C_1 d^2 \dot{\gamma} = C_1 \omega R^2 \left(\frac{d}{\delta_0} \right)^2$$

(Hajra & Khakhar, *Phys. Fluids*, 2005)

Segregation: Density driven



Segregation Flux

Effective medium approach:

Segregation velocity

$$\mathbf{v}_H \propto \left(\frac{m_H}{V} - \langle\rho\rangle\right)\mathbf{g}$$

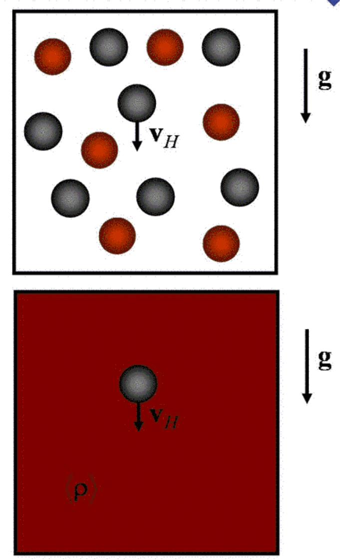
Average density

$$\langle\rho\rangle = f\frac{m_H}{V} + (1-f)\frac{m_L}{V}$$

Segregation flux

$$\mathbf{j}_H = nf\mathbf{v}_H \propto nf(1-f)(m_H - m_L)\mathbf{g}/V$$

m: mass, V: volume of particles
f: number fraction of H, n: number density



$$\mathbf{j}_H = -\mathbf{j}_L = Kf(1-f)(m_H - m_L)\nabla p$$ Pressure diffusion flux

Kinetic Theory: Ideal Gas

(Hirschfelder, Curtiss and Bird, 1954)

Ordinary diffusion: $\mathbf{j}_H^f = -\frac{m_H m_L n^2}{\rho} D\nabla f$
Mixing

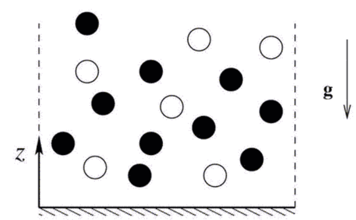
Pressure diffusion: $\mathbf{j}_H^p = \frac{m_H m_L D n^2}{\rho^2 T} f(1-f)(m_H - m_L)\nabla p$
Segregation: higher mass to high *p*

Temperature diffusion: $\mathbf{j}_H^T = -\frac{m_H m_L D n^4}{\rho^2 T} K_T f(1-f)(m_H - m_L)\nabla T$
Segregation: higher mass to low *T*

Valid for slightly inelastic spheres: Jenkins and Mancini (1989)

Equilibrium Profile

- Ideal gas
- Equal size particles
- Elastic particles
- Constant temperature



Balance of fluxes at equilibrium:

$$\frac{m_H m_L D}{\rho^2 T} f(1-f)(m_H - m_L)\frac{dp}{dz} - \frac{m_H m_L n^2 D}{\rho}\frac{df}{dz} = 0$$

pressure ordinary

Hydrostatic pressure:

$$\frac{dp}{dz} = -\rho g$$

Result:

$$\ln\frac{f}{1-f} = \ln\frac{f_0}{1-f_0} - \frac{(m_H - m_L)gz}{T}$$

(Khakhar, McCarthy & Ottino, CHAOS, 1999)

Dense Gas: Equilibrium profile

(Jenkins and Mancini 1989, Arnarson and Willits 1998)

Conditions for equilibrium:

$$m_H g + \left(\frac{\partial \mu_H}{\partial n_H}\right)\frac{dn_H}{dz} + \left(\frac{\partial \mu_H}{\partial n_L}\right)\frac{dn_L}{dz} = 0$$

$$m_L g + \left(\frac{\partial \mu_L}{\partial n_H}\right)\frac{dn_H}{dz} + \left(\frac{\partial \mu_L}{\partial n_L}\right)\frac{dn_L}{dz} = 0$$

Assumption:
Negligible temperature diffusion

Chemical potential for equal size particles:

$$\mu_i = T\ln n_i + TF(\phi), \quad i = H, L$$

Simplified equations

$$m_H g + \frac{T}{n_H}\frac{dn_H}{dz} + \left(\frac{dn_H}{dz} + \frac{dn_L}{dz}\right)V\frac{dF}{d\phi} = 0$$

$$m_L g + \frac{T}{n_L}\frac{dn_L}{dz} + \left(\frac{dn_H}{dz} + \frac{dn_L}{dz}\right)V\frac{dF}{d\phi} = 0$$

Ideal gas Non-ideal

Solution

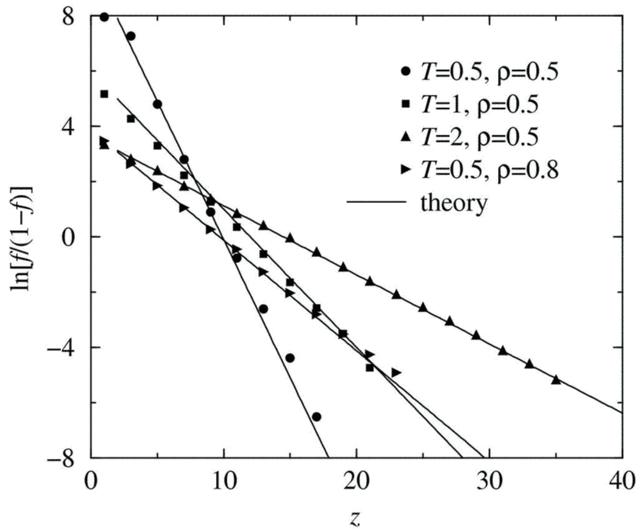
$$\ln\frac{n_H}{n_L} = \ln\frac{n_{H0}}{n_{L0}} - \frac{(m_H - m_L)gz}{T}$$

$$\frac{n_H}{n_L} = \frac{f}{1-f}$$

Profile same as the ideal gas case

Monte Carlo Computations

Equilibrium profiles



Elastic particles

Density segregation under gravity

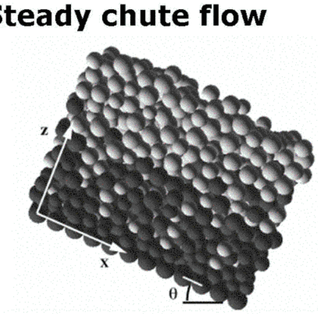
Constant temperature

Predictions of theory good at all number densities

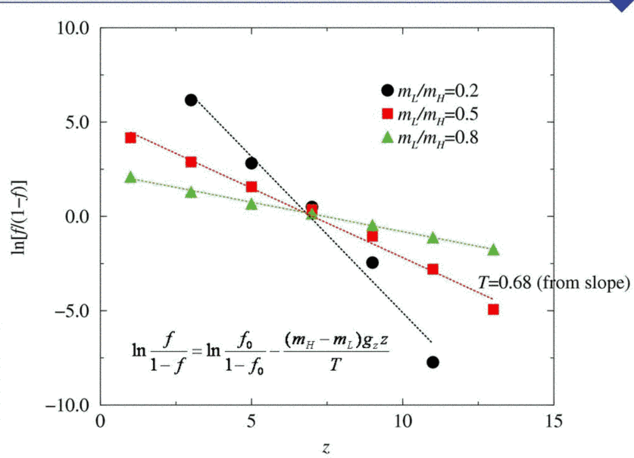
(Khakhar, McCarthy & Ottino, *CHAOS*, 1999)

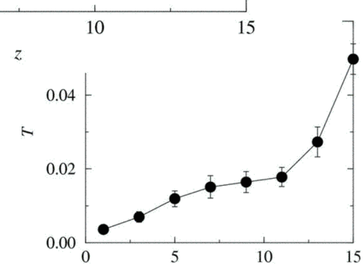
Particle Dynamics

Steady chute flow



* Soft particle simulations
* Inelastic and frictional particles



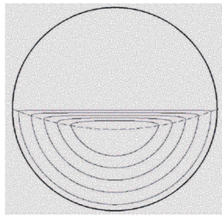


- Good agreement with theory
- Effective *T* (from slope) is constant and much larger than *T* from computations

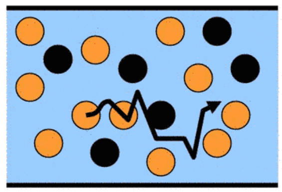
(Khakhar, McCarthy & Ottino, *CHAOS*, 1999)

Rotating Cylinder

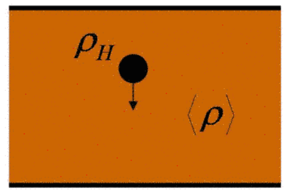
Equal size, different density particles



Flow
(unchanged)



Diffusion
(unchanged)



Segregation

Convective-diffusion in layer

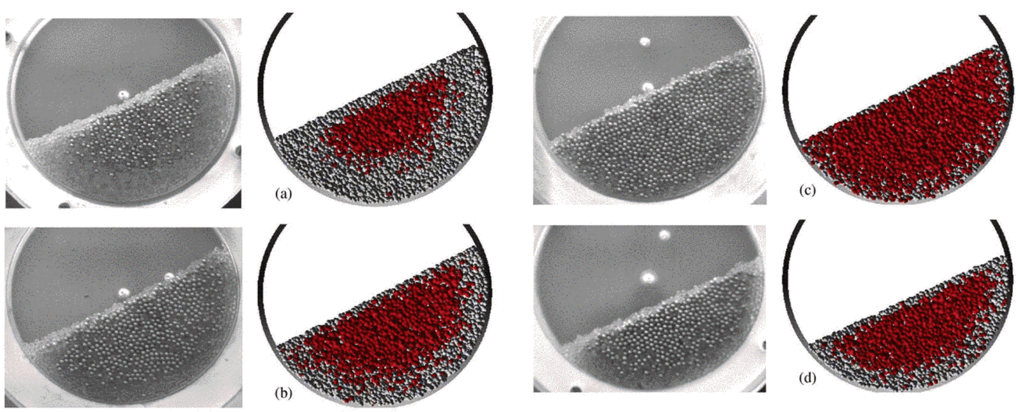
$$\frac{\partial f}{\partial t} + v_x \frac{\partial f}{\partial x} + v_y \frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left(\underbrace{D \frac{\partial f}{\partial y}}_{\text{diffusion}} - \underbrace{\frac{BD}{d}\left[1 - \frac{m_L}{m_H}\right] f(1-f)}_{\text{segregation}} \right)$$

Solid body rotation in bed *Kinetic theory form*

Parameters: B, D

(Khakhar, McCarthy & Ottino, *Phys. Fluids*, 1997)

Comparison



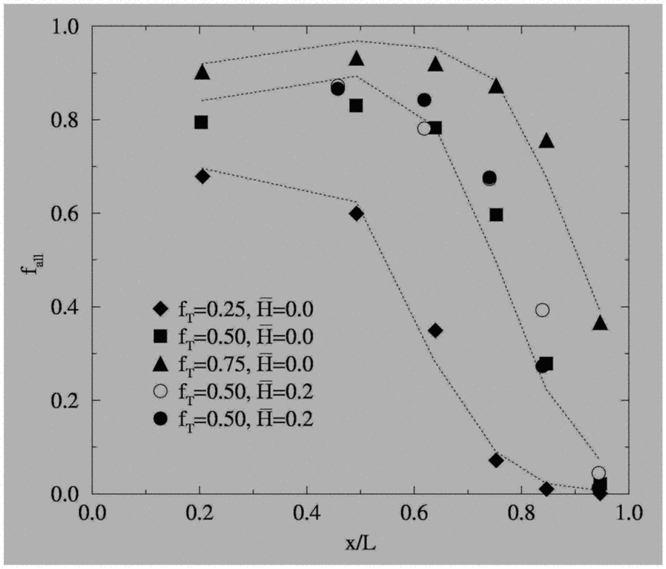
Computations: Lagrangian simulations

Parameters: $B = 2$ (fitted)

D: estimated from mixing expts

(Khakhar, McCarthy & Ottino, *Phys. Fluids*, 1997)

Comparison

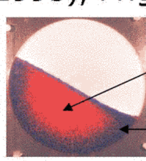


Experiments: Setting with gel and slicing at equilibrium.

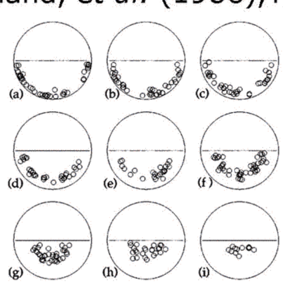
(Khakhar, McCarthy & Ottino, *Phys. Fluids*, 1997)

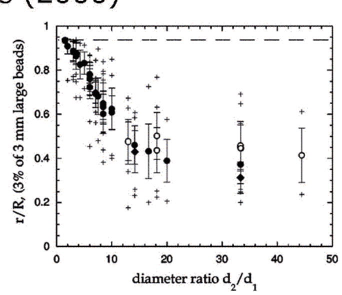
Size Segregation

- **Regular segregation:**
  Donald and Roseman (1962), Alonzos *et al.* (1991), Cantelaube and Bideau (1995), Prigozhin and Kalman (1998)



- **Reverse segregation:**
  Nityanand, *et al.* (1986),Thomas (2000)

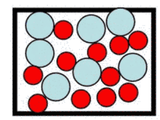




Mechanisms

Regular segregation:
Savage and Lun (1988)

Fluctuating sieve

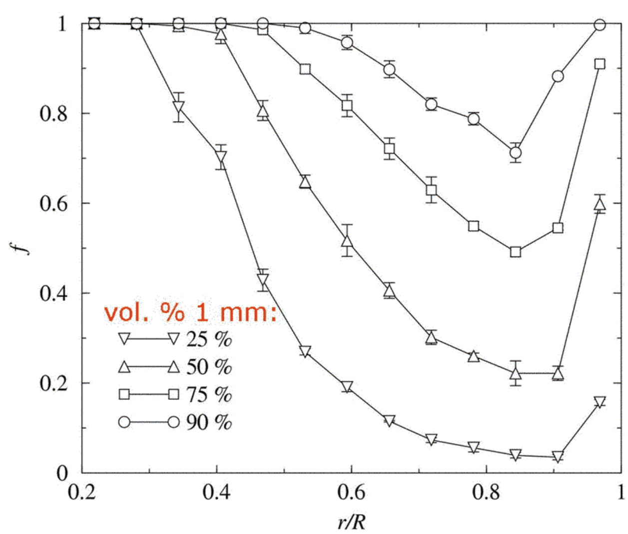


Small particles drop into voids with greater probability than large particles in dense flows
– net flux of small particles down : small particles in core

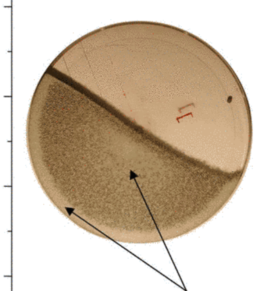
Reverse segregation:
Thomas (2000)

Massive large particles move down by pushing other particles aside
– net flux of large particles down : layer of small particles at periphery

Radial Size Segregation



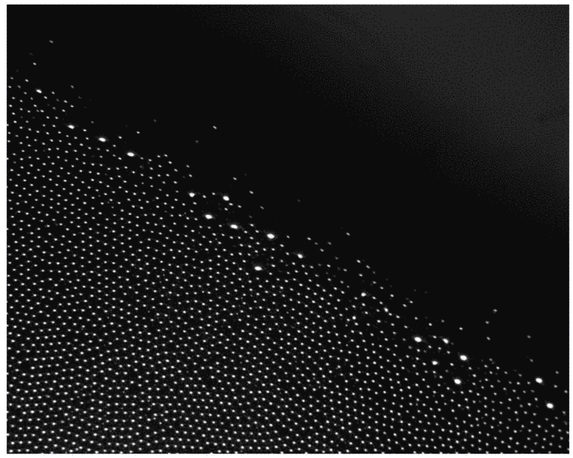
1 & 2 mm stainless steel balls



Double segregation for all cases

(Hajra and Khakhar, *PRE*, 2004)

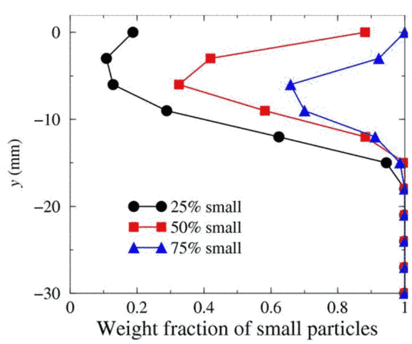
Measurements in the Layer

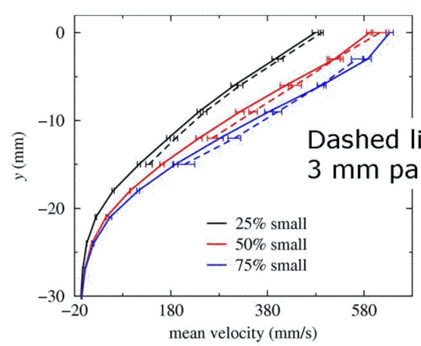


Tracking of big and small particles:
Mean and rms velocities, composition

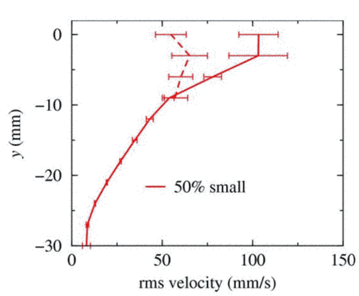
(T. A. V. Prasada Rao, in progress)

Layer Profiles



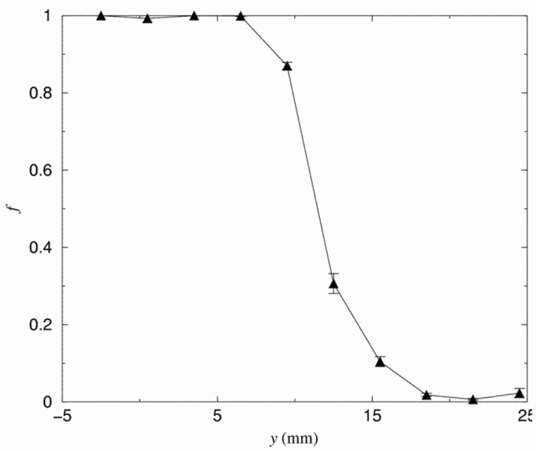


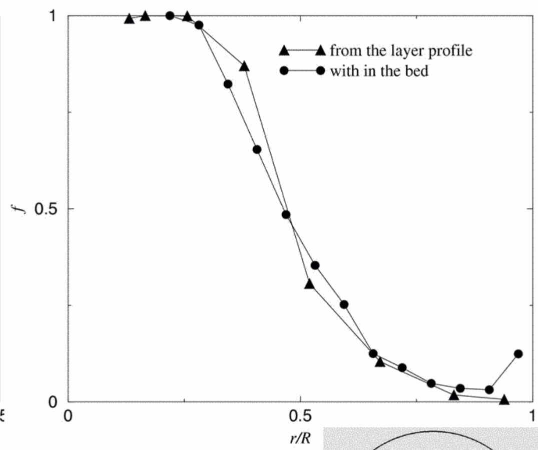
Dashed lines:
3 mm particles

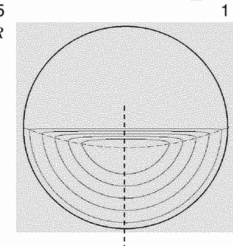


2 mm and 3 mm
Stainless steel

Radial to Layer Profile



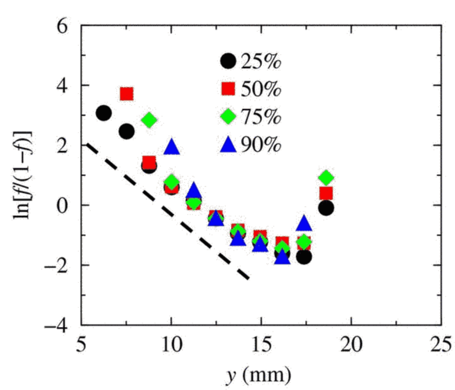




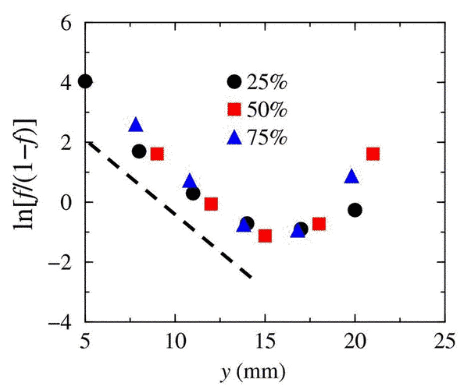
Relation between *y* and *r* by a mass balance: $\int_0^y v_x dy = \int_\delta^r \omega r dr$

Assume no change in *f* along a streamline

Concentration Scaling

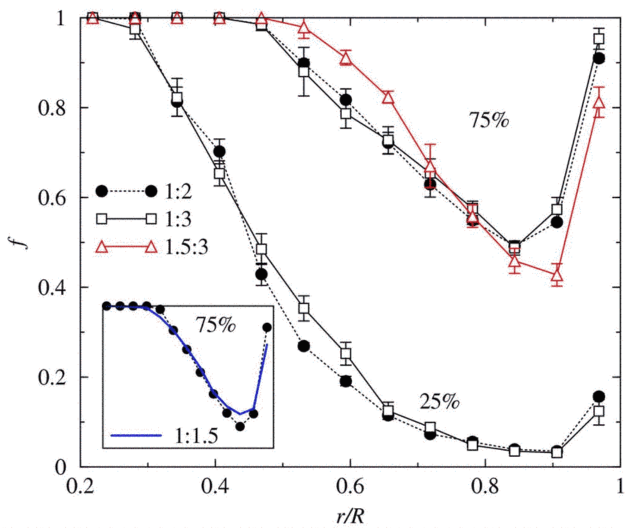


1 mm & 2 mm particles
Converted radial profiles



2 mm & 3 mm particles
Measured profiles

Particle Size Ratio



Segregation determined by the size of small particles – Size ratio not so important.