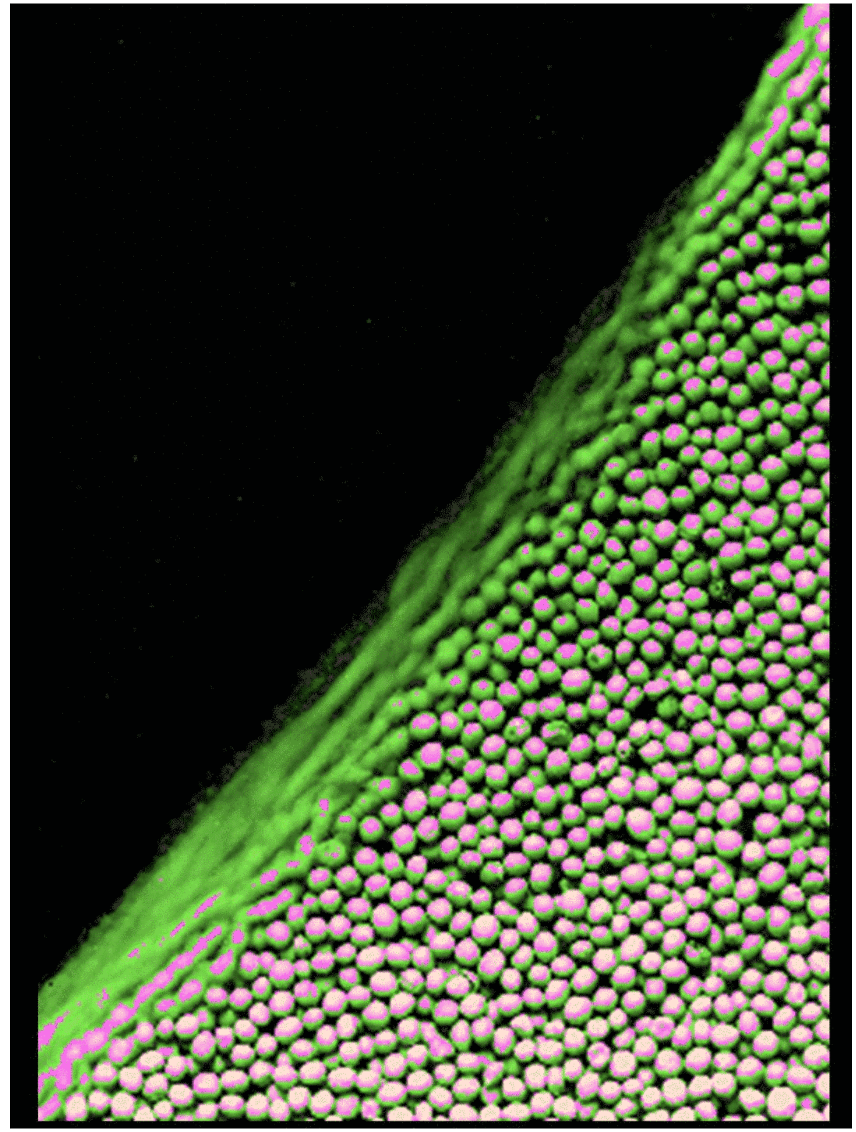
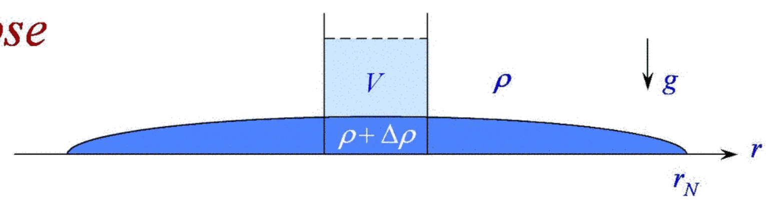

Institute of
Theoretical Geophysics
University of Cambridge
Kavli Institute for Theoretical Physics
Granular Physics Conference
Granular Column Collapses
Herbert E. Huppert
23 June 2005

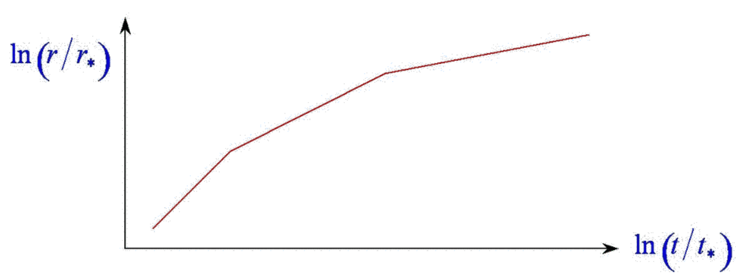


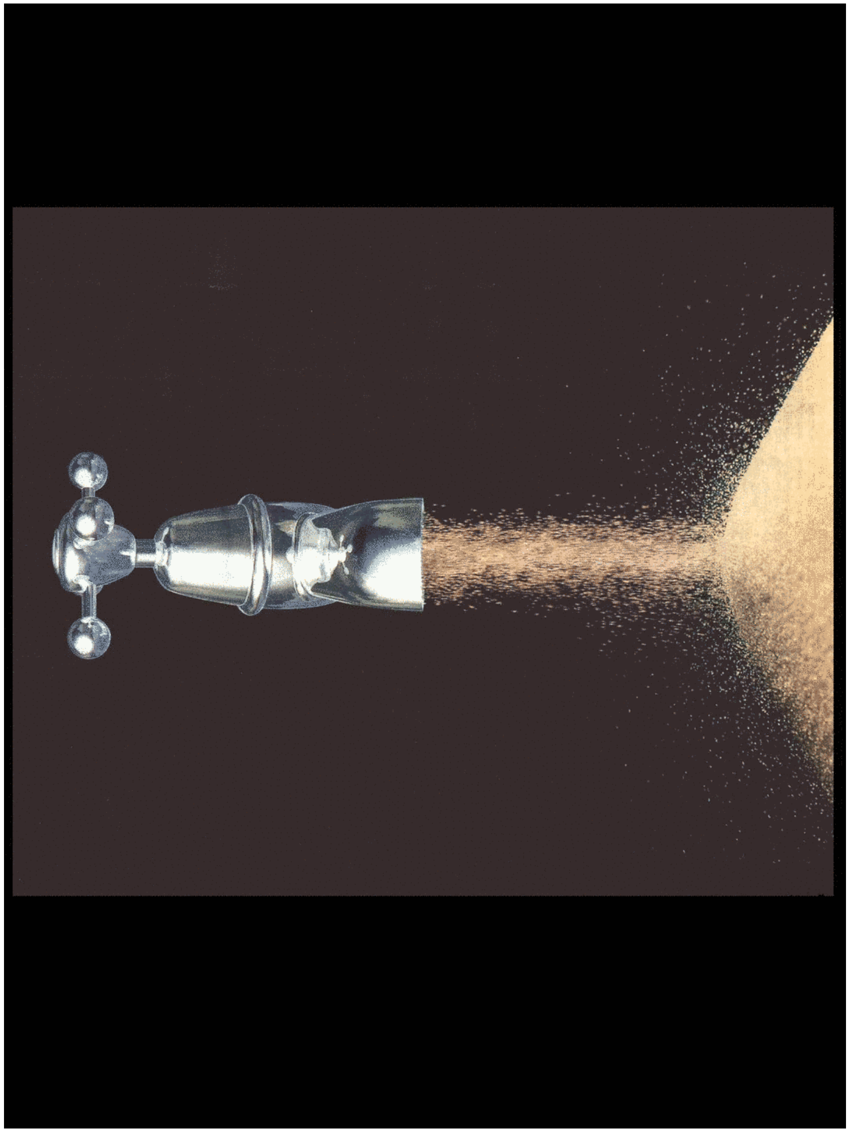
Fluid collapse

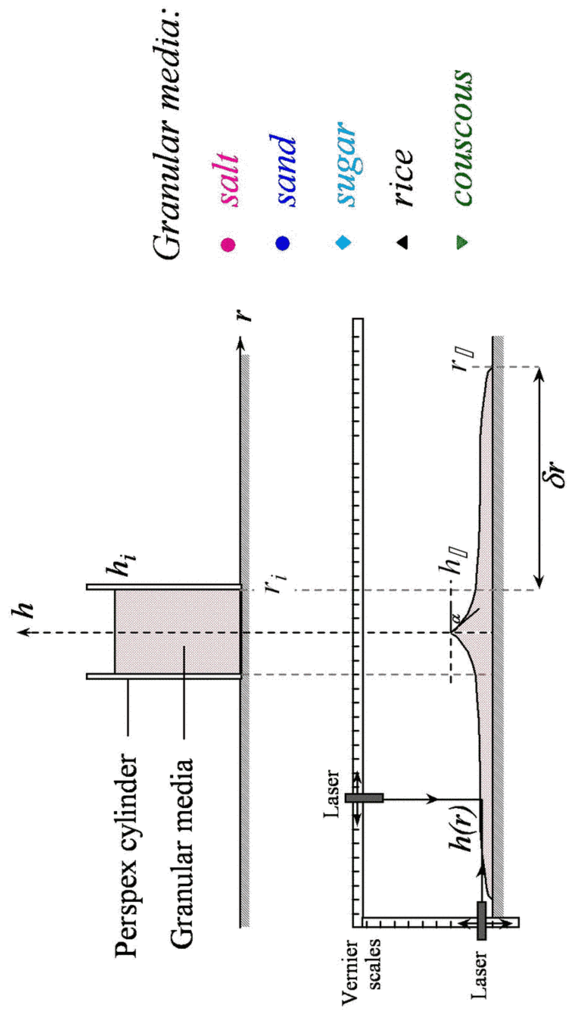
$g' = g\Delta\rho/\rho$

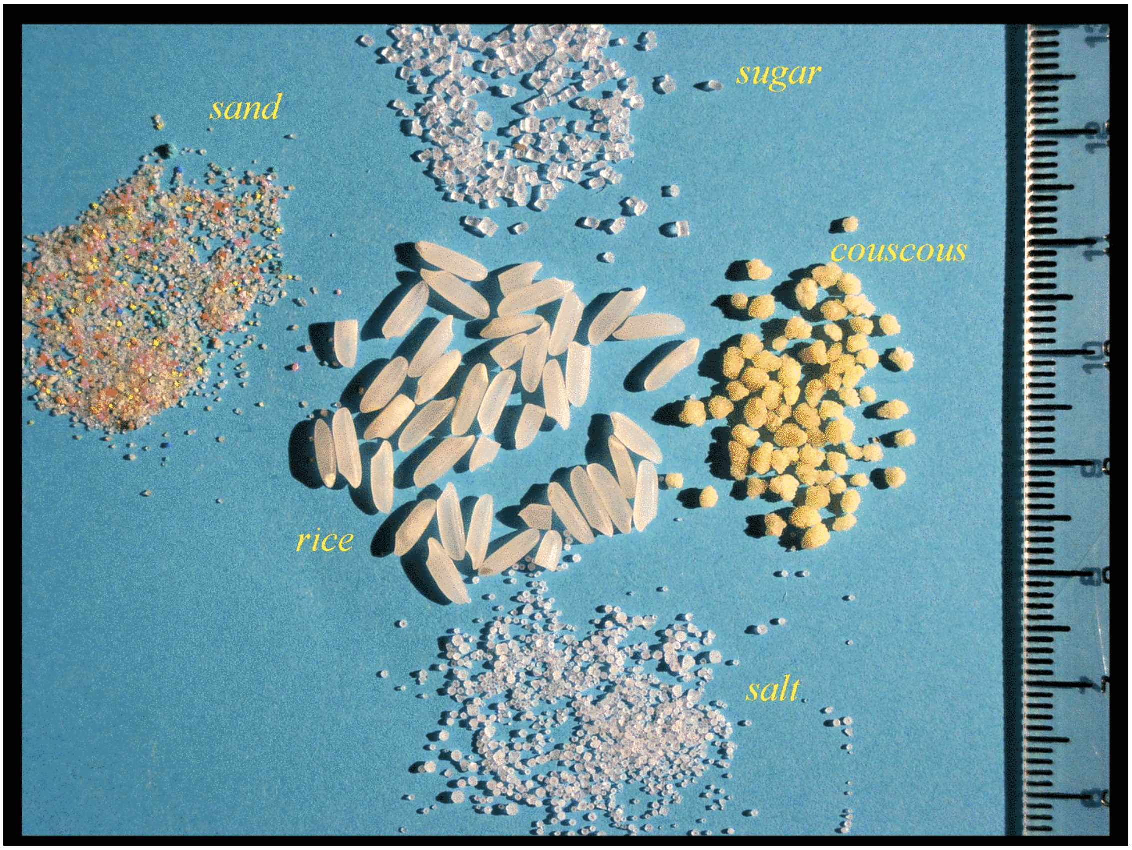


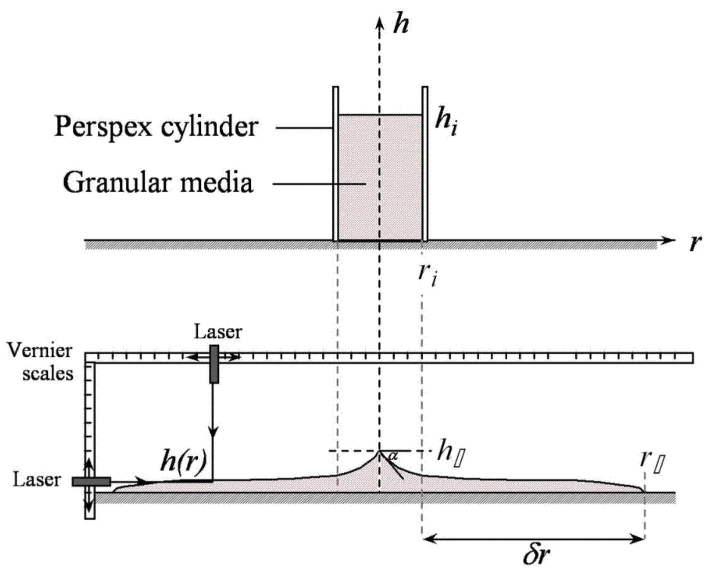
I Acceleration phase		(nobody)
II Slumping phase	$r_N \propto t$	(H[2] & JES, *JFM* 1980)
III Inertia/buoyancy phase	$r_N \propto (g'V)^{1/4} t^{1/2}$	(Hoult, 1972)
IV Viscosity/buoyancy phase	$r_N \propto (g'V/\nu)^{1/8} t^{1/8}$	(H[2] , *JFM* 1982)






Granular media:
salt
sand
sugar
rice
couscous
Perspex cylinder
Granular media
Vernier scales
Laser
Laser
h(r)
δr
h
r





Granular media:

- ● *salt*
- ● *sand*
- ◆ *sugar*
- ▲ *rice*
- ▼ *couscous*

External parameters: r_i , h_i, $\left(V_i = \pi r_i^2 h_i\right)$, g , ~~μ~~

$\underbrace{\qquad}$ r_i , $a = h_i / r_i$

In experiments: 2.25 cm < r_i < 9.7 cm 150 cc < V < 13,700 cc

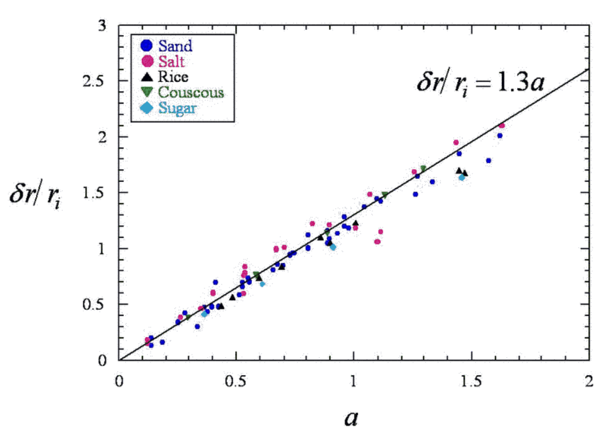
$$\delta r = r_i f_r(a) \qquad h_\infty = r_i f_h(a) \qquad t_\infty = (r_i/g)^{1/2} f_t(a)$$

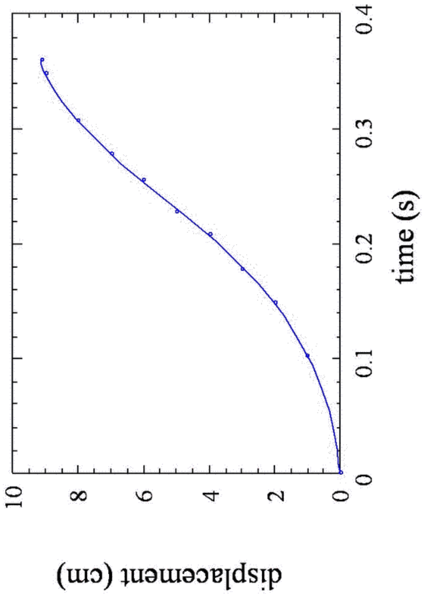
Small a results

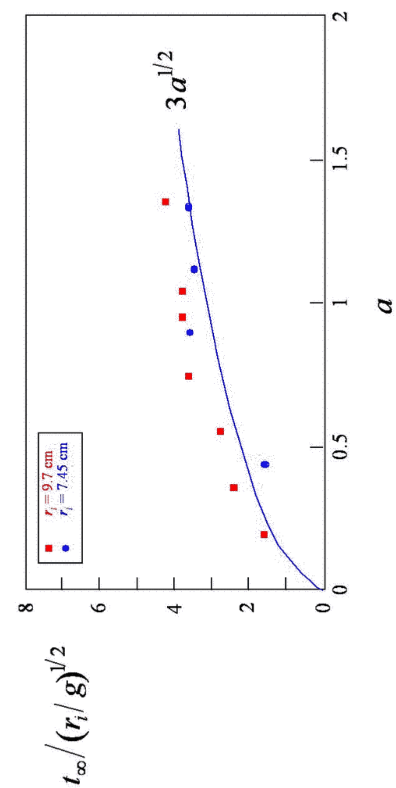
$$\delta r = r_i f_r(a) \qquad h_\infty = r_i f_h(a) \qquad t_\infty = (r_i/g)^{1/2} f_t(a)$$

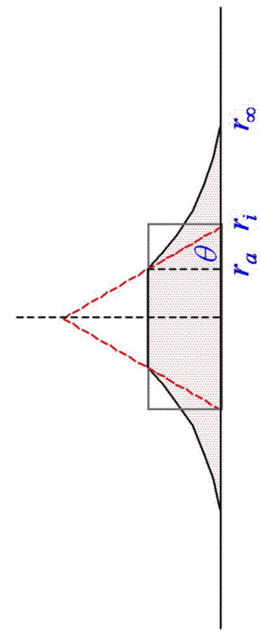
For $a = h/r_i << 1$ results must be independent of r_i

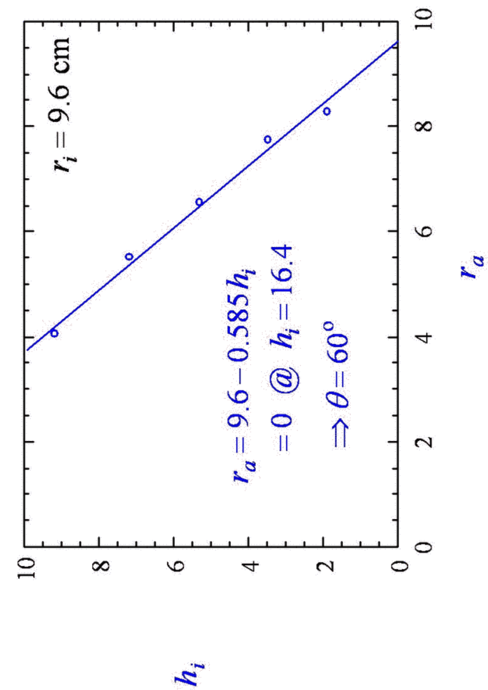
$$\delta r/r_i \propto a \qquad h_\infty = h_o \qquad t_\infty/(r_i/g)^{1/2} \propto a^{1/2}$$

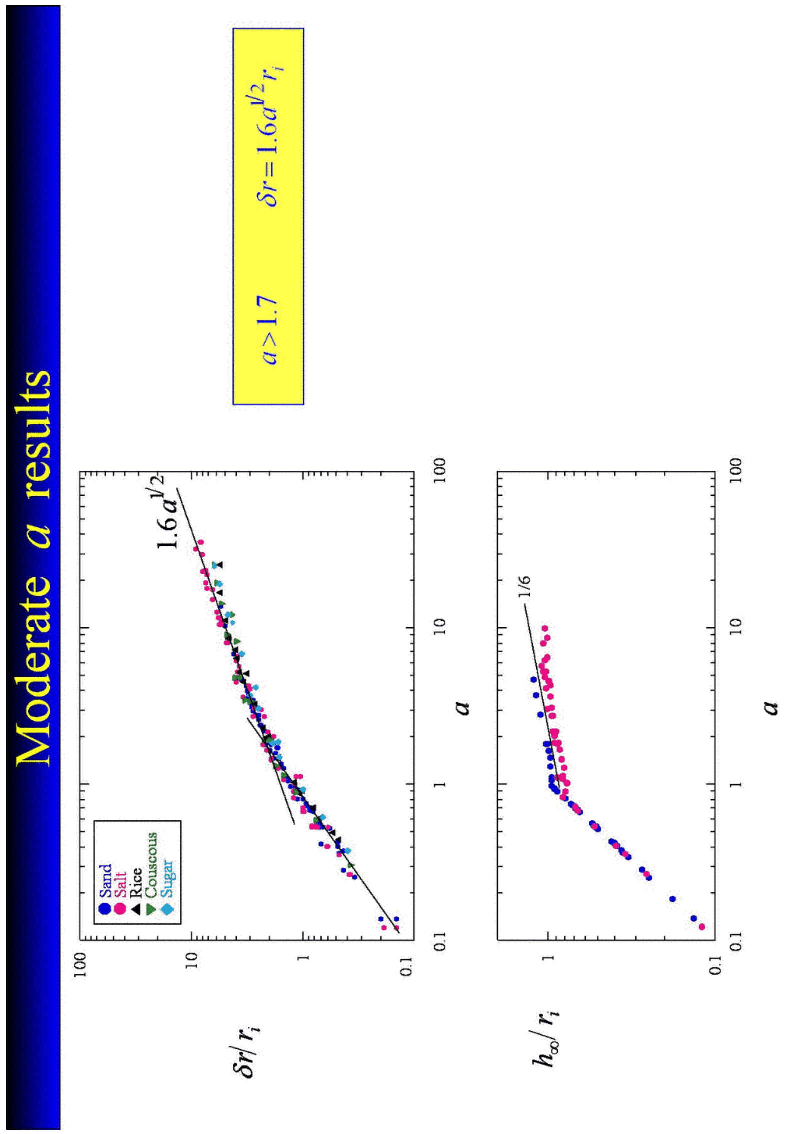


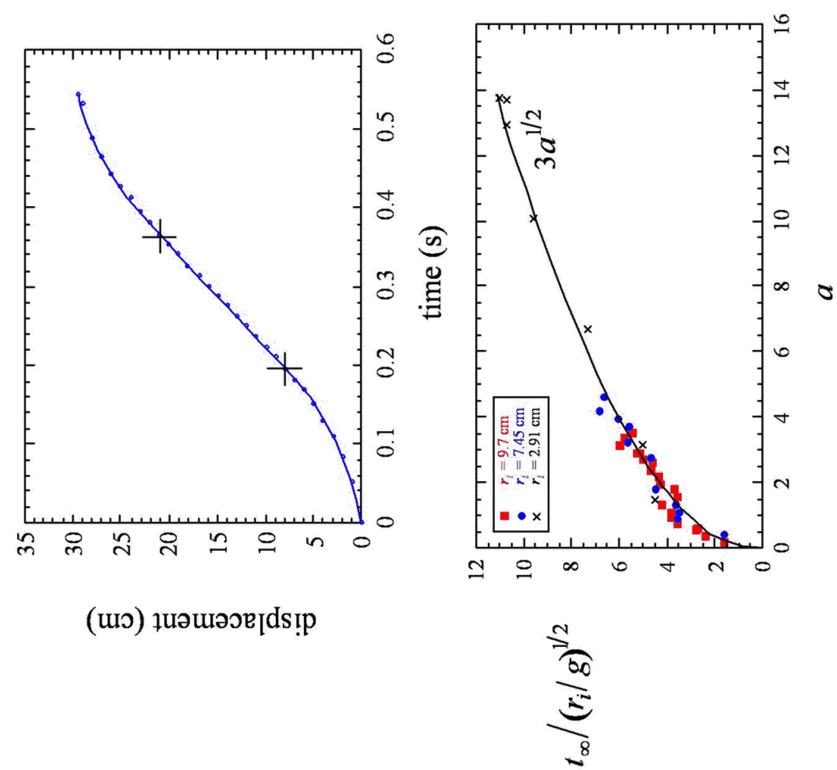

displacement (cm)
time (s)

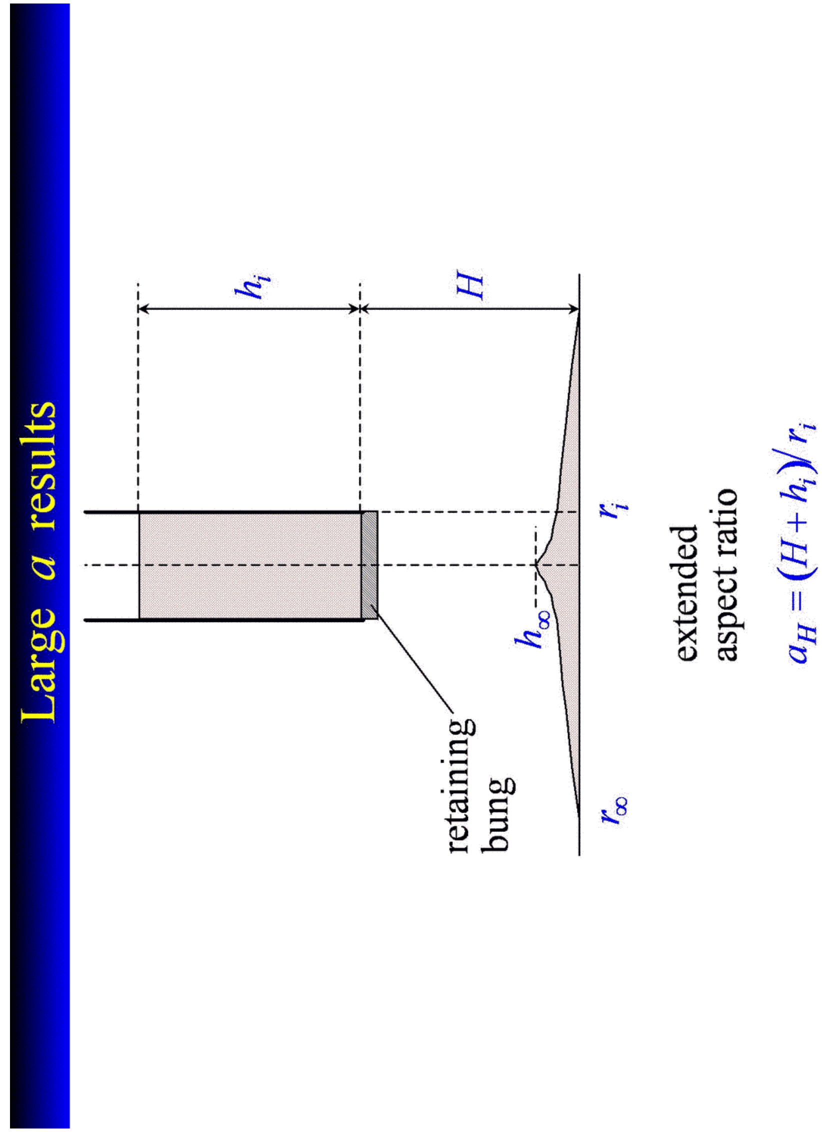

$t_\infty / (r_i / g)^{1/2}$
a
$3a^{1/2}$
$r_i = 9.7$ cm
$r_i = 7.45$ cm


θ
r_a
r_i
r_∞

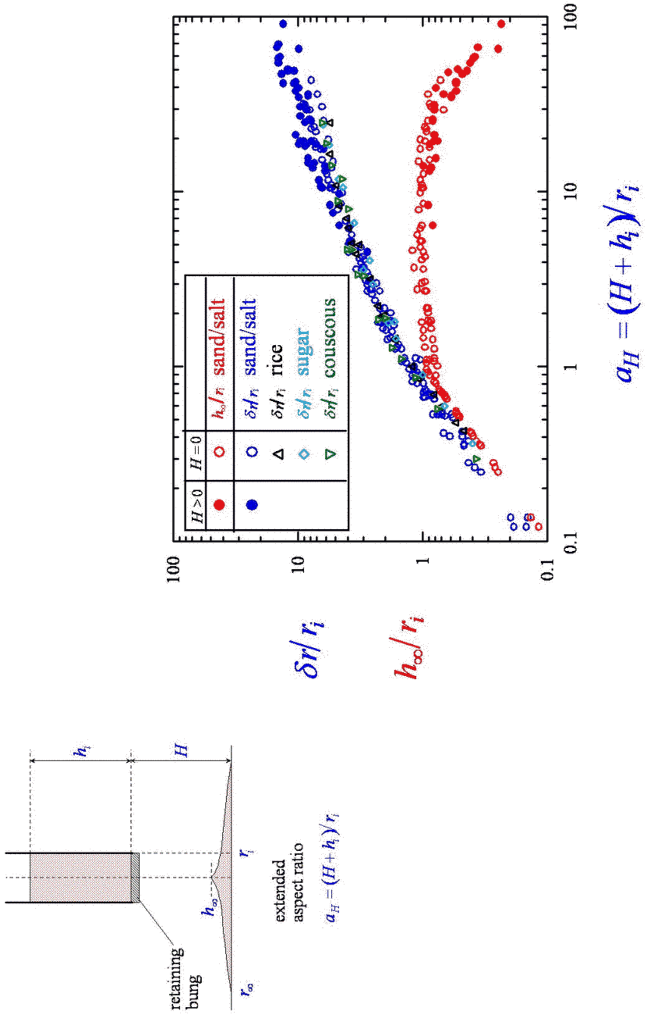

h_i
r_a
$r_i = 9.6$ cm
$r_a = 9.6 - 0.585 h_i$
$= 0$ @ $h_i = 16.4$
$\Rightarrow \theta = 60°$


Moderate a results
a > 1.7
δr = 1.6a^{1/2} r_i
Sand
Salt
Rice
Couscous
Sugar
1.6a^{1/2}
1/6
δr/r_i
h_∞/r_i
a


displacement (cm)
time (s)
r_i = 9.7 cm
r_i = 7.45 cm
r_i = 2.91 cm
3a^{1/2}
t_∞/(r_i/g)^{1/2}
a

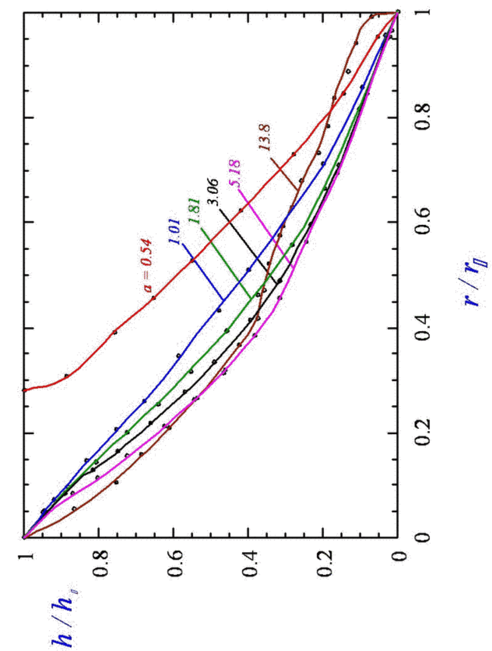

Large a results
h_i
H
r_i
h_∞
r_∞
retaining bung
extended aspect ratio
$a_H = (H + h_i)/r_i$

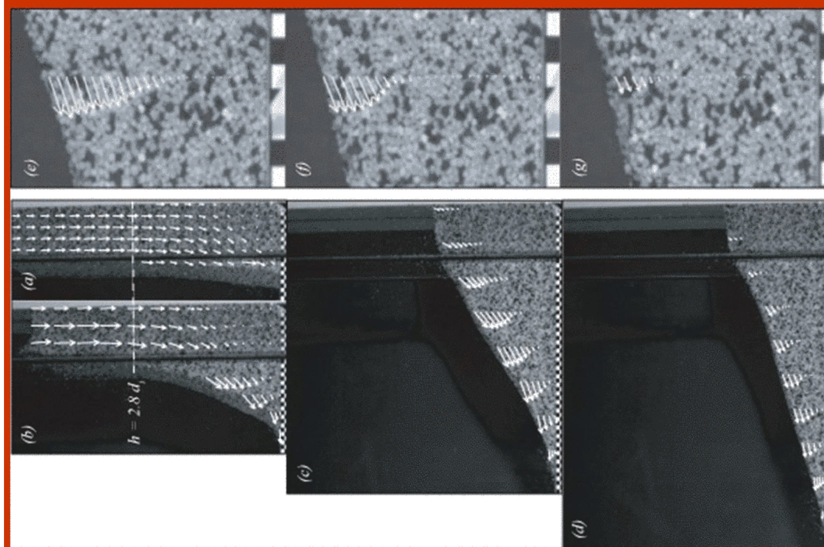
Large a results

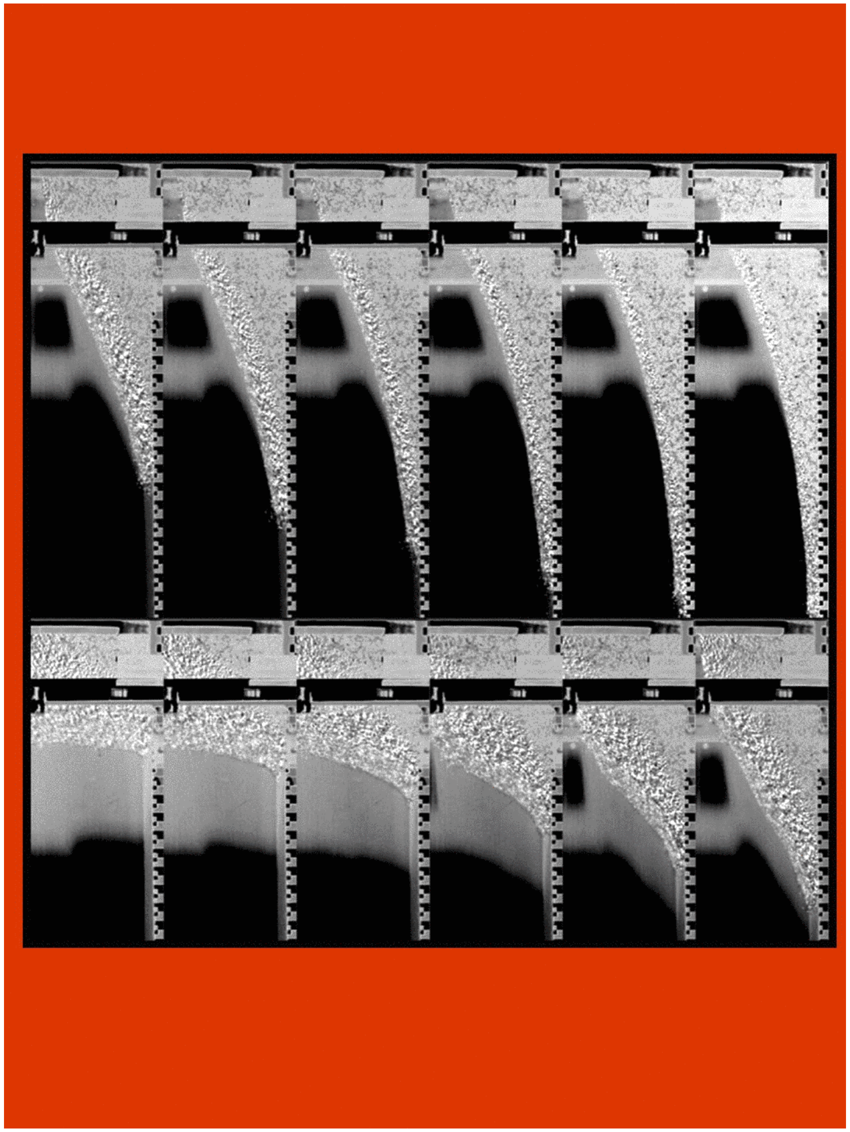


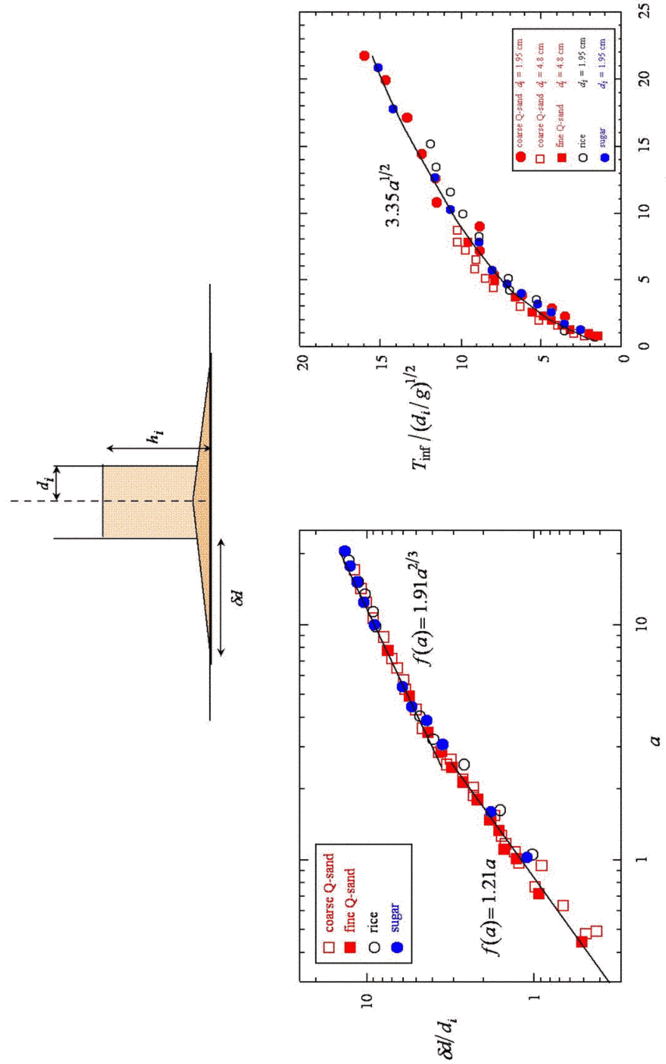
Final radius dependent on maximum height only

$h/h_\infty = f(r/r_\infty;\, a)$

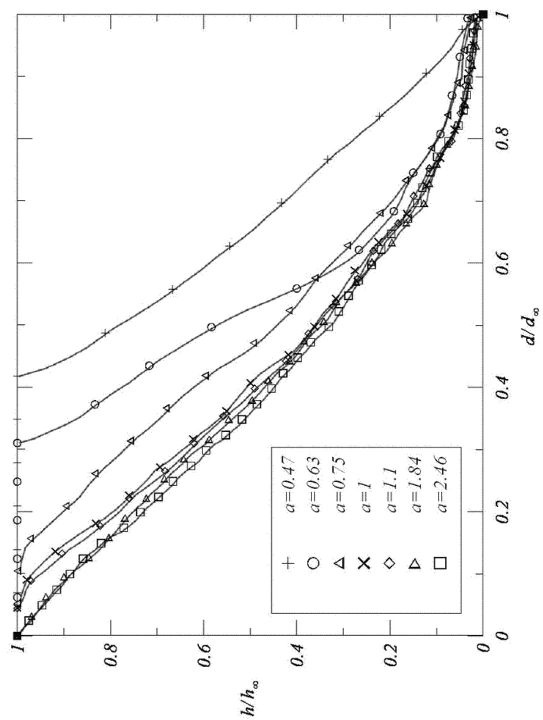



(a)
(b)
(c)
(d)
(e)
(f)
(g)
h = 2.8 d

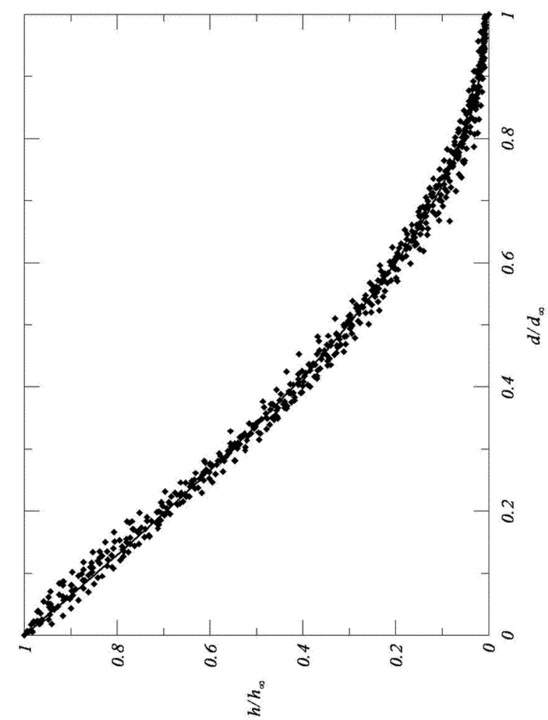




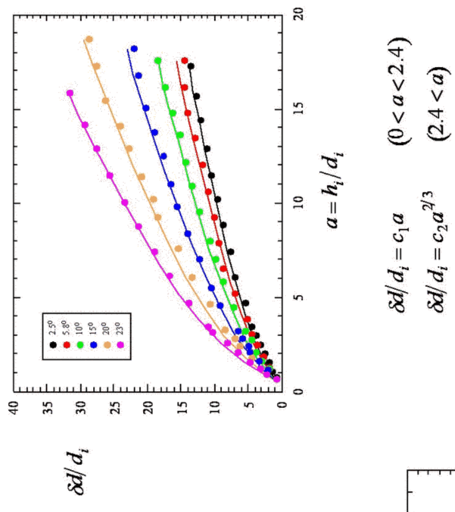
coarse Q-sand
fine Q-sand
rice
sugar
f(a)= 1.21a
f(a)= 1.91a^2/3
3.35a^1/2
δd/d_i
a
h_i
d_i
δd



a=0.47
a=0.63
a=0.75
a=1
a=1.1
a=1.84
a=2.46
d/d∞
h/h∞

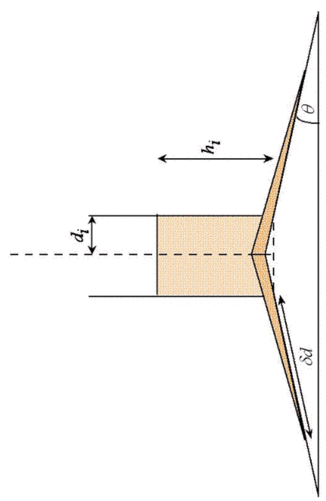


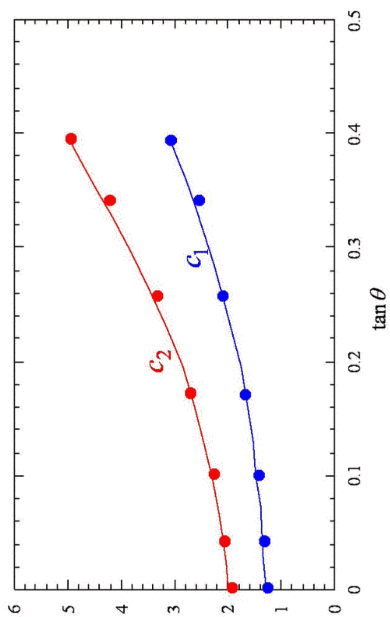
d/d∞
h/h∞

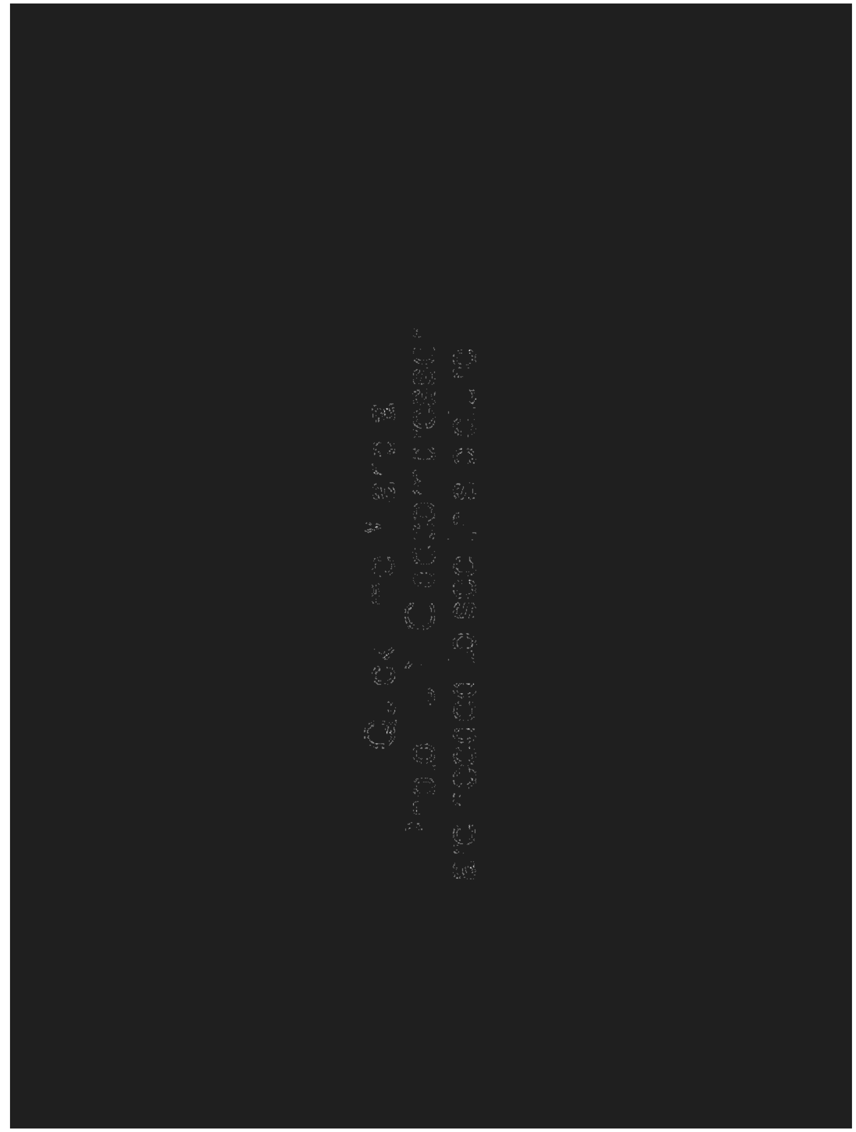


$$\delta d/d_i = c_1 a \quad (0 < a < 2.4)$$

$$\delta d/d_i = c_2 a^{2/3} \quad (2.4 < a)$$








Theoretical
explanation

Conclusions

- Granular collapses of different materials can be systematically described in terms of a, r_i and g, independent of material and frictional parameter values. This is consistent with the idea that internal friction is only important in stopping the flow quite abruptly in the last moment.
- An internal cone of material, of a slope of 60° to the horizontal, remains static throughout the motion.
- For $a < 1.7$, an initial acceleration phase is followed immediately by a deceleration phase, and, independent of r_i,

$$\delta r_i = 1.3 h_i , \quad h_\infty = h_i \quad \text{and} \quad t_\infty = 3(h_i / g)^{1/2}$$

- Beyond $a < 1.7$, there is a break in the mathematical forms and there is a phase of constant velocity between the acceleration and deceleration phases. Explicitly,

$$\delta r = 1.6 r_i a^{1/2}, \quad h_\infty = 0.88 r_i a^{1/6} \quad \text{and} \quad t_\infty = 3(r_i / g)^{1/2} a^{1/2}$$

- There is a further break around $a = 10$, where h_∞ appears to decrease monotonically with a.