Granular Flow in a Modified Split Bottom Couette Cell

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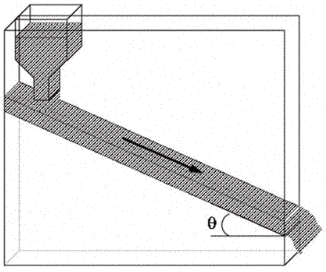
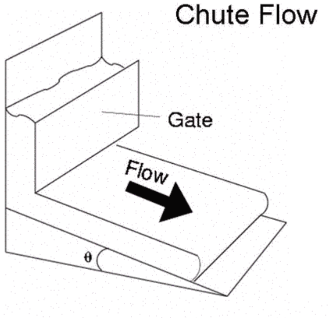
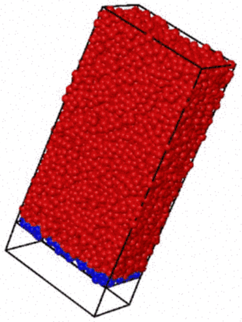
Dense Gravity Driven Flow



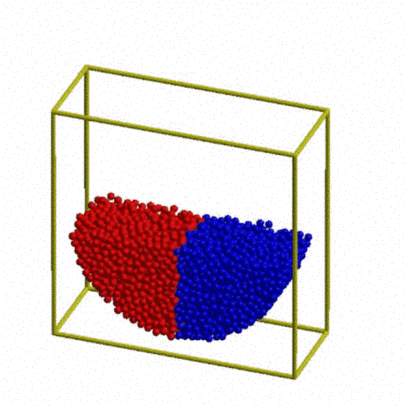
Heap Flow

Chute Flow





Dense Granular Flows

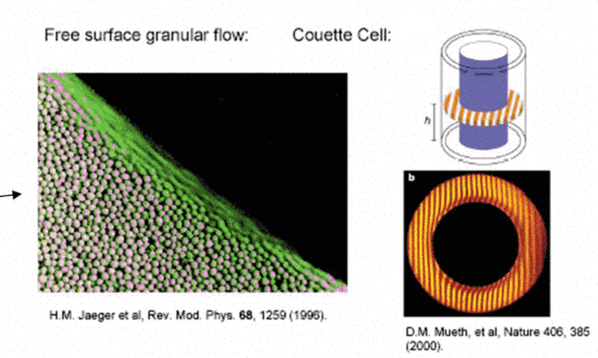


Rotating Drum

- Boundary Driven Flows
  - – Couette Flow
  - – Shear Flow

Shear Bands in Dense Granular Flow

- Shear bands: narrow and distinct bands of high rates of shear deformation (localization of energy dissipation)
  - – Phenomenon plays an important role in many applications
    - ballistic impact
    - explosive fragmentation
    - high speed machining
    - metal forming
    - interfacial friction
    - powder compaction
    - soil failure
    - seismic events
    - **granular flow**
- What is the role of micro-structure?
- Difficult to access range of a flowing states to test flow theories
- Non-universality



Free surface granular flow:

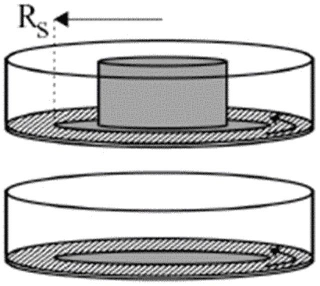
Couette Cell:

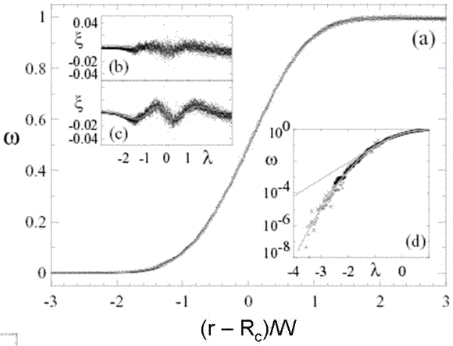
H.M. Jaeger et al, Rev. Mod. Phys. **68**, 1259 (1996).

D.M. Mueth, et al, Nature 406, 385 (2000).

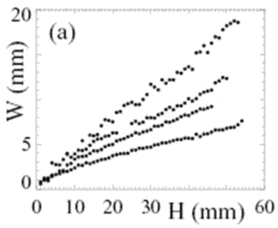
exponential velocity profiles

Split Bottom Couette Cell





Rescaled surface velocity profiles for various heights collapse onto a universal curve regardless of particle properties



Shear zones are detached from the walls and become wide with increasing total height H.

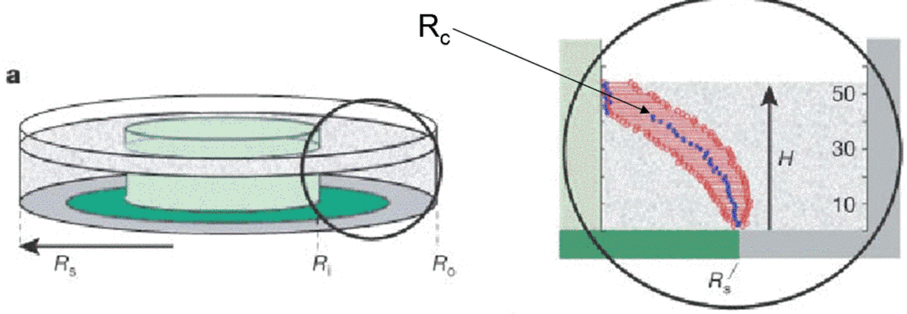
Fenistein *et al.* (Nature **425**, 256 (2003) ; PRL **92**, 094301 (2004))

- Address questions of velocity fluctuations, particle diffusion, and granular temperature?
- Relationship between micro and macro scales?

focus attention on ‘shallow’ packs

Shape of the Shear Zones

- Parameters involved in rescaling center $\mathbf{R_c = f(R_s, H)}$ and width $W = f$(H,particle) of shear zone appear to have separate length scales
- Theoretical description/predictions for shape of shear zone $R_c(r,h)$



Fenistein *et al.* (Nature **425**, 256 (2003); PRL **92**, 094301 (2004))

Discrete Element Simulations

- Allows observation of bulk behavior away from influence of side walls
- Allows detailed measurements of microscopic quantities (e.g., inter-particle forces)
- Observe bulk behavior, go beyond "shallow" regime, test theory

Integrate Newton's equations-translational and rotational d.o.f.

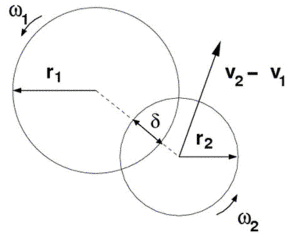
$$\mathbf{F}_n = f(\delta/d)(k_n \delta \mathbf{n}_{ij} - \frac{m}{2}\gamma_n \mathbf{v}_n)$$

$$\mathbf{F}_t = f(\delta/d)(-k_t \Delta \mathbf{s}_t - \frac{m}{2}\gamma_t \mathbf{v}_t)$$

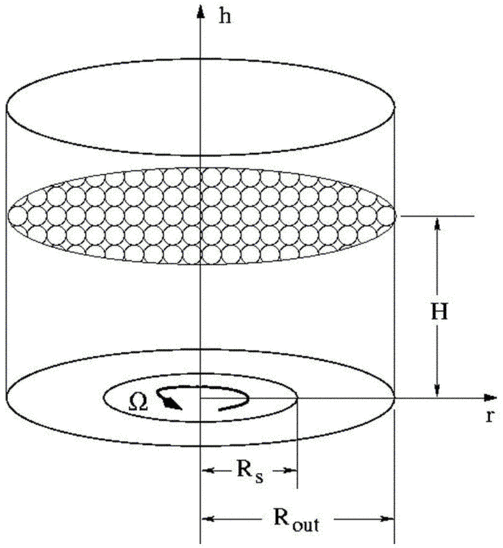
$f(x) = \sqrt{x}$ **Hertzian springs**

$\Delta \mathbf{s}_t$ **Elastic tangential displacement**

$F_t \le \mu F_n$ **Coulomb Failure Criterion**



DEM System Parameters



$R_s = 30.0d$
$R_{out} = 37.8d$

$\Omega = 0.014$ **rad/**τ **where** $\tau = \sqrt{d/g}$

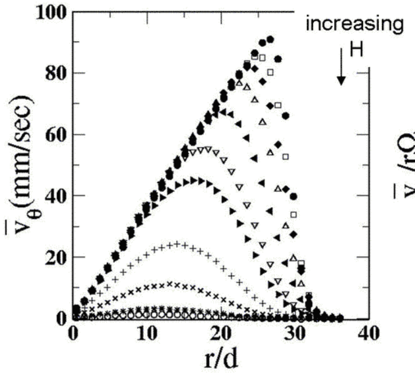
$5.4d \le H \le 34.2d$
20,000 – 180,000 particles

rough bottom composed of layer of glued particles

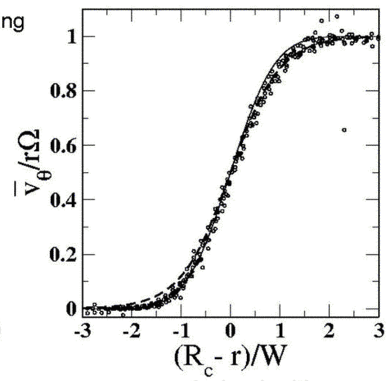
- All experiments, simulations are in hydrostatic regime ($H<6R_{out}$)

- Values picked to exactly match experimental system for poppy seeds (d=0.8mm) - particle tracking at surface
- MRI – poppy and rajagara seeds – R_{out}=51.6d, R_s=40.6d

Experimental Surface Velocity Profiles – Poppy Seeds



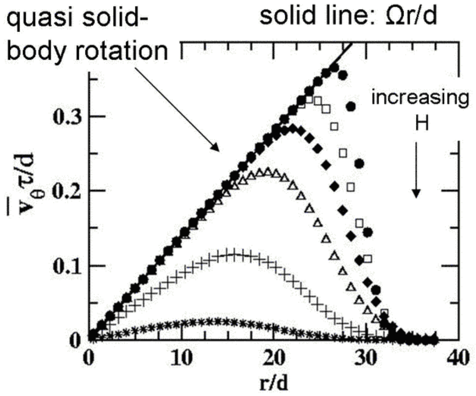
azimuthal velocities



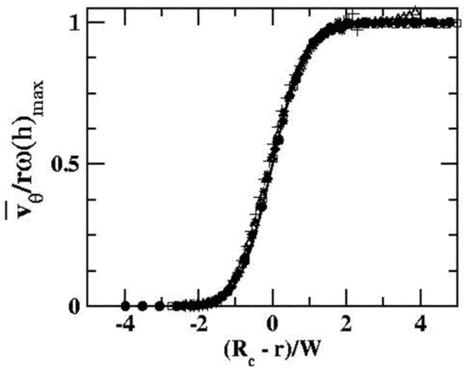
rescaled velocities

- Linear azimuthal velocity profile near center for shallow packs (regime of previous work)
- Slight asymmetry in the rescaled velocities

Simulation Surface Velocity Profiles

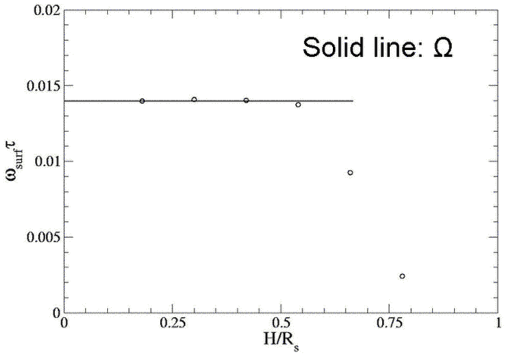


Azimuthal velocity profiles
for varying pack heights



Rescaled velocity profiles

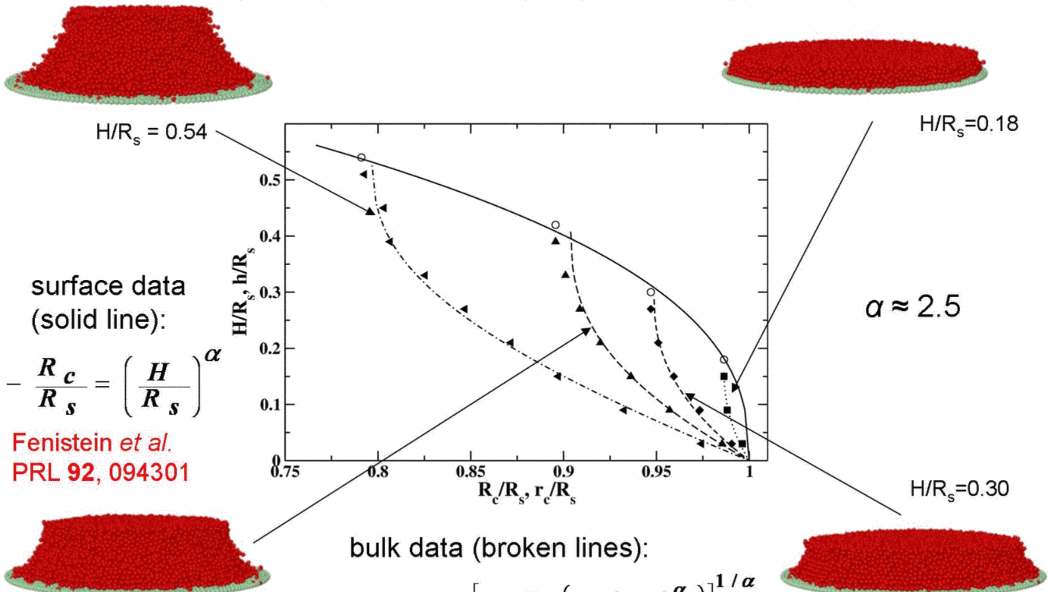
Shallow vs. Deep Packs



- As observed from the surface: qualitative change at $H/R_s \approx 0.5$ in agreement with previous work

What happens in the bulk?

Shape of Shear Zone and Inner Core for Shallow Packs



$\alpha \approx 2.5$

surface data
(solid line):

$$1 - \frac{R_c}{R_s} = \left(\frac{H}{R_s}\right)^{\alpha}$$

Fenistein *et al.*
PRL **92**, 094301

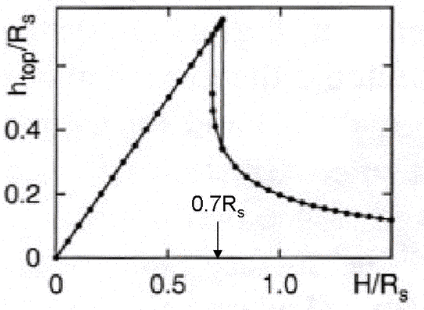
bulk data (broken lines):

$$\frac{h}{R_s} = \frac{H}{R_s} - \frac{r}{R_s}\left[1 - \frac{R_s}{r}\left(1 - \left(\frac{H}{R_s}\right)^{\alpha}\right)\right]^{1/\alpha}$$

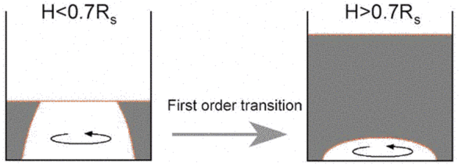
Unger *et al.* PRL **92**, 214301

Proposed Theory – Unger *et al.*

- Least dissipation (minimum torque)
  - Infinitely thin shear surface between two bulk solid regions
  - Hydrostatic pressure
  - Coulomb friction between solid regions
- Describes shape of shear zone for shallow packs based on bulk stress state
- Predicts for tall packs:
  - transition in shape of shear zone (open → closed)
  - first order accompanied by hysteresis
  - height of the shear zone, h_{top} is proportional to R_s/H

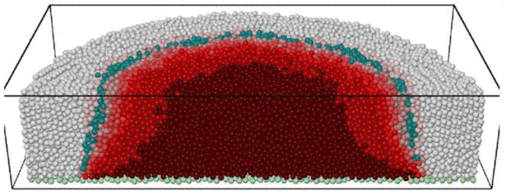


Unger *et al.* PRL **92**, 214301 (2004)



Normalized Angular Velocity

"open" shape

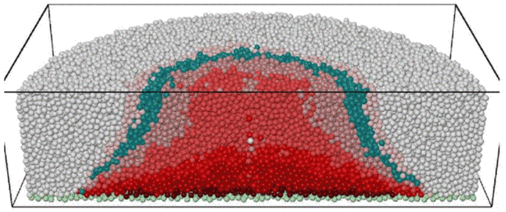


$H/R_s = 0.54$

$\frac{\omega}{\Omega} \geq 0.95$ **dark red**

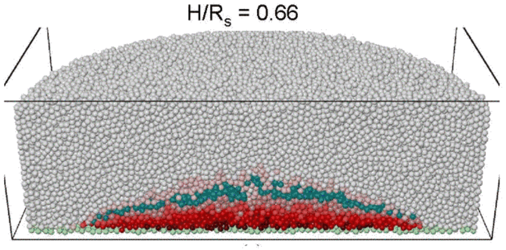
$0.45 \leq \frac{\omega}{\Omega} < 0.55$ **teal**

$\frac{\omega}{\Omega} < 0.35$ **white**



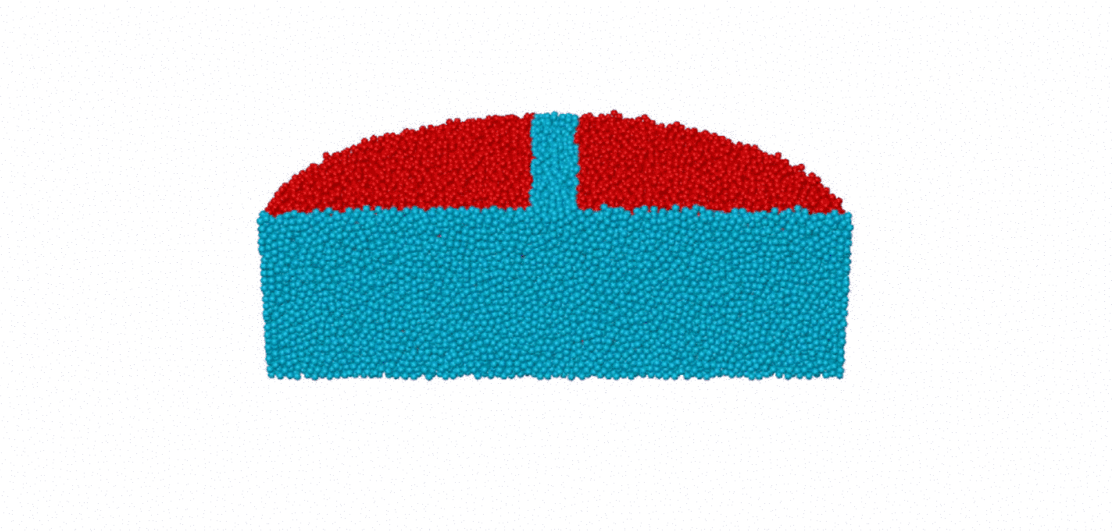
$H/R_s = 0.66$

"closed" shape



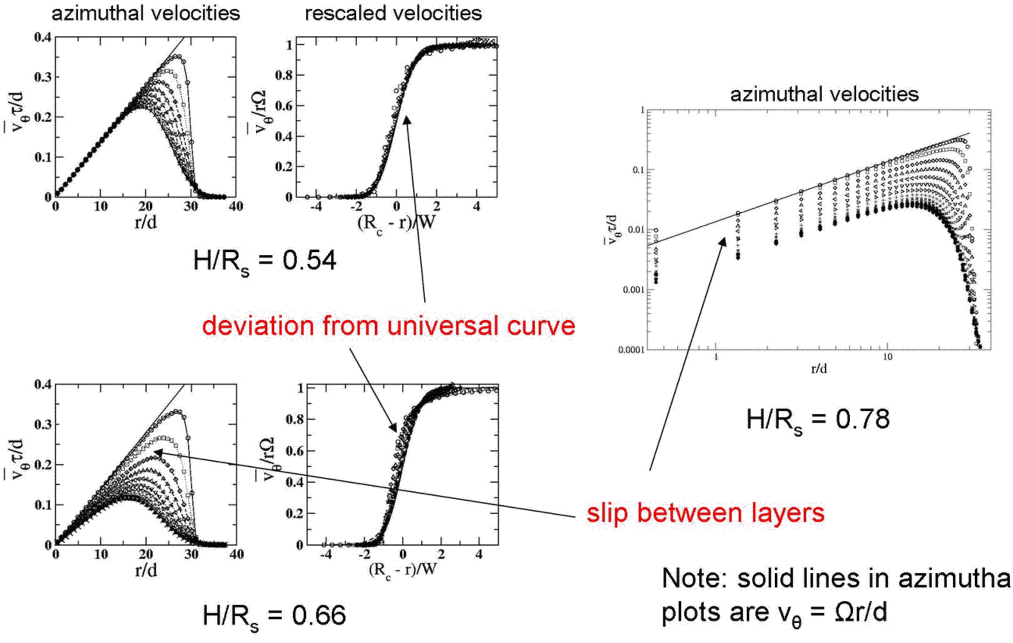
$H/R_s = 0.78$

Shear in Deep Pack



$H/R_s = 0.78$

Bulk Velocity Profiles



$H/R_s = 0.54$

deviation from universal curve

$H/R_s = 0.78$

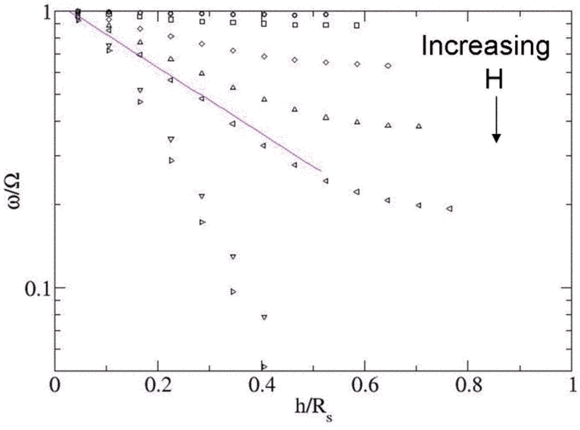
slip between layers

$H/R_s = 0.66$

Note: solid lines in azimuthal plots are $v_\theta = \Omega r/d$

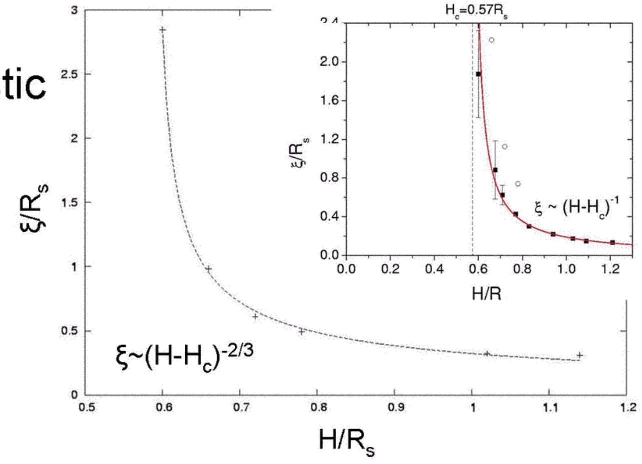
Slip Between Layers

- For $H/R_s > 0.5$ slip between layers increases with H
- MRI give good data for deep packs and longer times



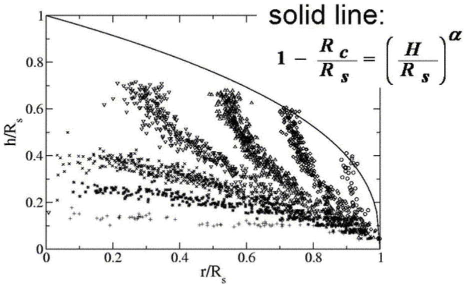
Slip Transition?

- From w(h) ~ exp(-H/ξ)
- ξ gives characteristic length scale
- ξ diverges with decreasing H



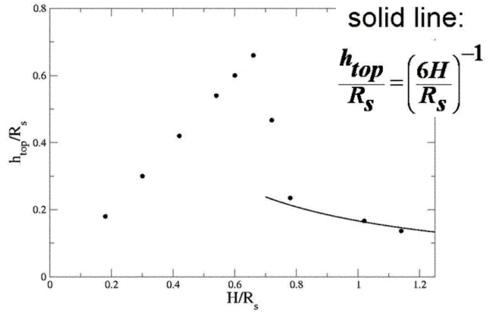
Nature of the Transition

- Basic assumptions violated
  - Shear zone has finite width
  - Smooth transition between moving and stationary regions
  - Slip between layers
- Direct test: How to define top of shear zone?
  - Choose $\frac{\omega}{\Omega} = 0.5$?



solid line:

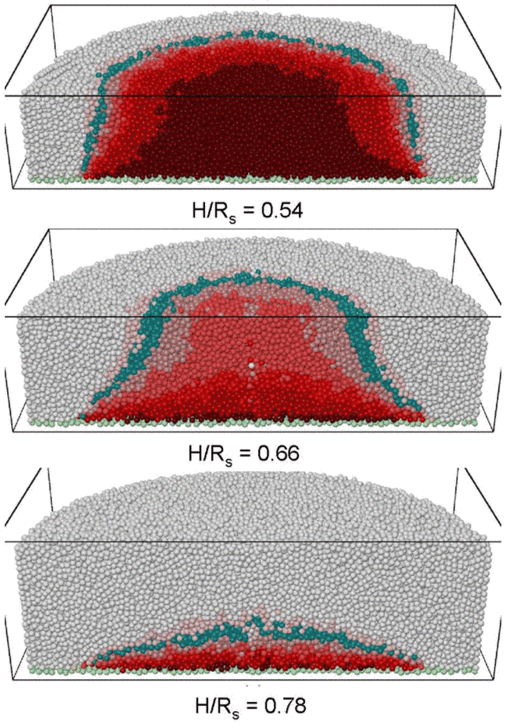
$$1 - \frac{R_c}{R_s} = \left(\frac{H}{R_s}\right)^{\alpha}$$



solid line:

$$\frac{h_{top}}{R_s} = \left(\frac{6H}{R_s}\right)^{-1}$$

Normalized Angular Velocity



"open" shape

$H/R_s \le 0.5$

$\frac{\omega}{\Omega} \ge 0.95$ **dark red**

$0.45 \le \frac{\omega}{\Omega} < 0.55$ **teal**

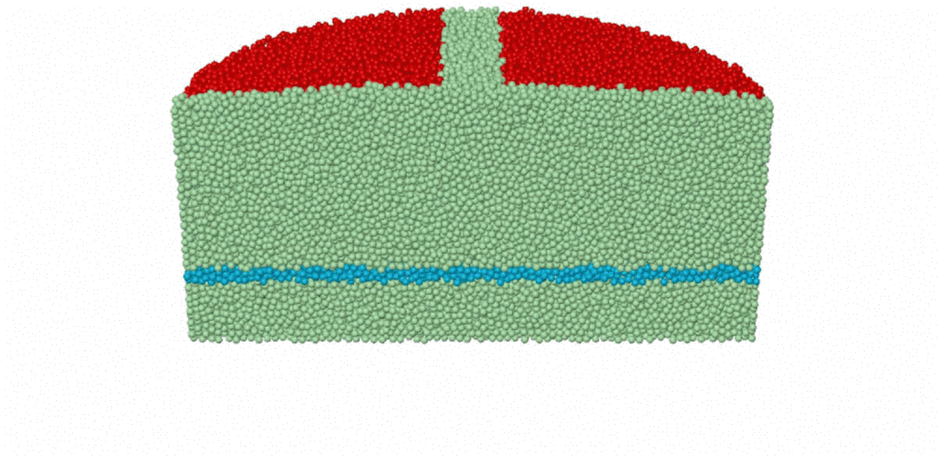
$\frac{\omega}{\Omega} < 0.35$ **white**

$\sim 0.5 < H/R_s < \sim 0.7$

"closed" shape

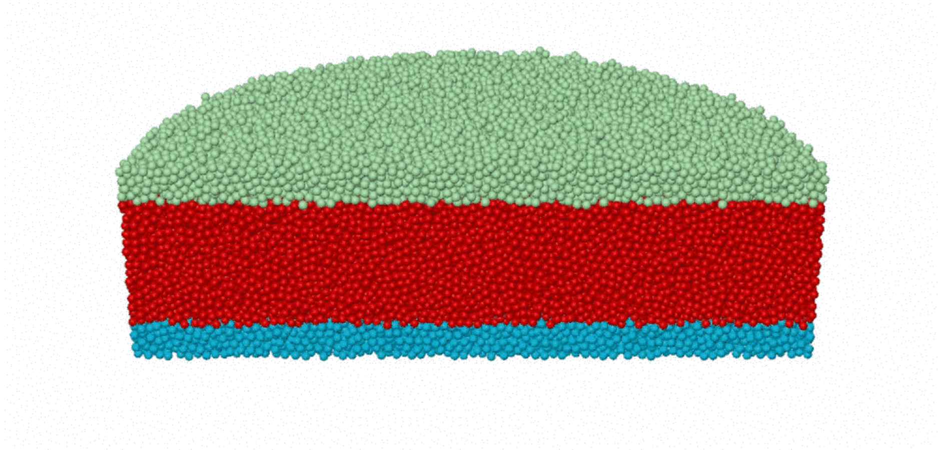
$H/R_s > 0.7$

Shear in Deepest Pack



$H/R_s = 1.14$

Convection in Deep Pack



$H/R_s = 0.78$

Conclusions

- Slip between layers is an increasingly significant mode of deformation for packs of $H > 0.5\ R_s$
  - apparently related to torsion failure of the inner core
- Results indicate a second order slip transition
- Theory captures bulk stress state, but misses details of the flow.
  - Finite width of the shear zone, torsion failure
  - A “shape transition” may still be present
  - For deepest packs, trend is $h_{top}\ \alpha\ R_s/H$
- Attraction interactions reduce the value of transition to lower H
- Convection observed for intermediate piles
- Can variational theory be extended to account for these or is there a more fundamental difference in the nature of the flow?