Kinetic Theory of a Non-spherical Granular Intruder

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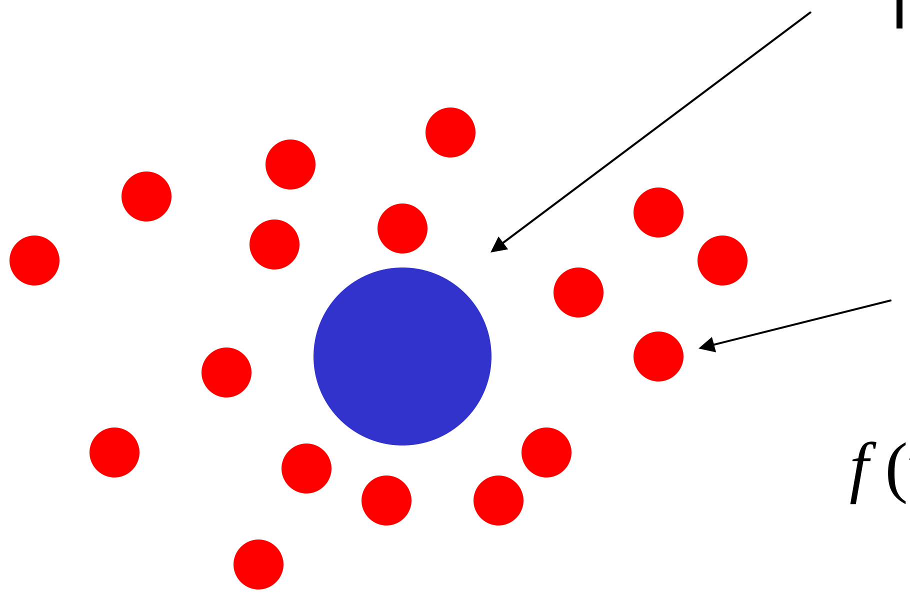
Non-spherical Granular Systems

Stationary granular gas



Intruder particle

$$F(V)\,?$$

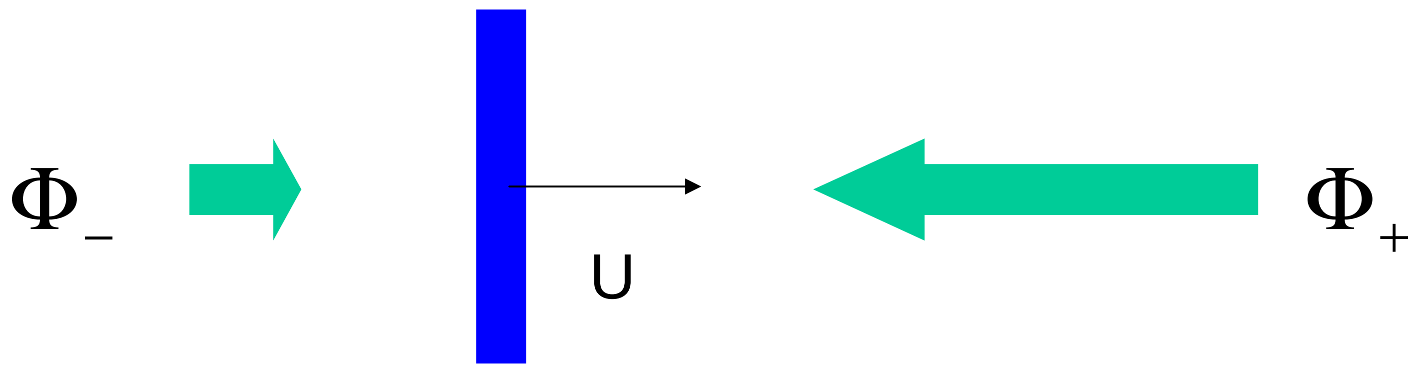
Bath particle

$$f(v, T_B) = A\exp\left(-\frac{mv^2}{2T_B}\right)$$

Intruder-bath particle collisions are inelastic with COR $\alpha < 1$

Martin and Piasecki, 1999; Garzo and Dufty, 1999; Barrat and Trizac, 2002

Particle flux depends on surface velocity



$$\Phi(U) = \Phi_+ + \Phi_- = \sqrt{\frac{m}{2\pi T}} \exp\left(-\frac{mU^2}{2T}\right) + U\mathrm{erf}\left(U\sqrt{\frac{\mathrm{m}}{2T}}\right)$$

$$P_+(U) = \frac{\Phi_+}{\Phi} = \frac{1}{2}\left(1 + \frac{U}{\Phi(U)}\right)$$

Kinetic Equation

$$\frac{\partial F(U,t)}{\partial t} = -\Phi(U)F(U,t) + \int_{-\infty}^{\infty} \Psi(V \to U)F(V,t)dV$$

Transition rate

Intruder particle velocity distribution

$$\Psi_{+}(V \to U) = \rho \int_{-\infty}^{V} dv(V-v) f(v,a)\delta(V - U + \frac{1+\alpha}{1+M}(v-V))$$

$$-\Phi(U)F(U) + \rho\left(\frac{1+M}{1+\alpha}\right)^2 \int_{-\infty}^{\infty} dV\,|U-V|\, f(V + \frac{1+M}{1+\alpha}(U-V), a)F(V) = 0$$

Solution

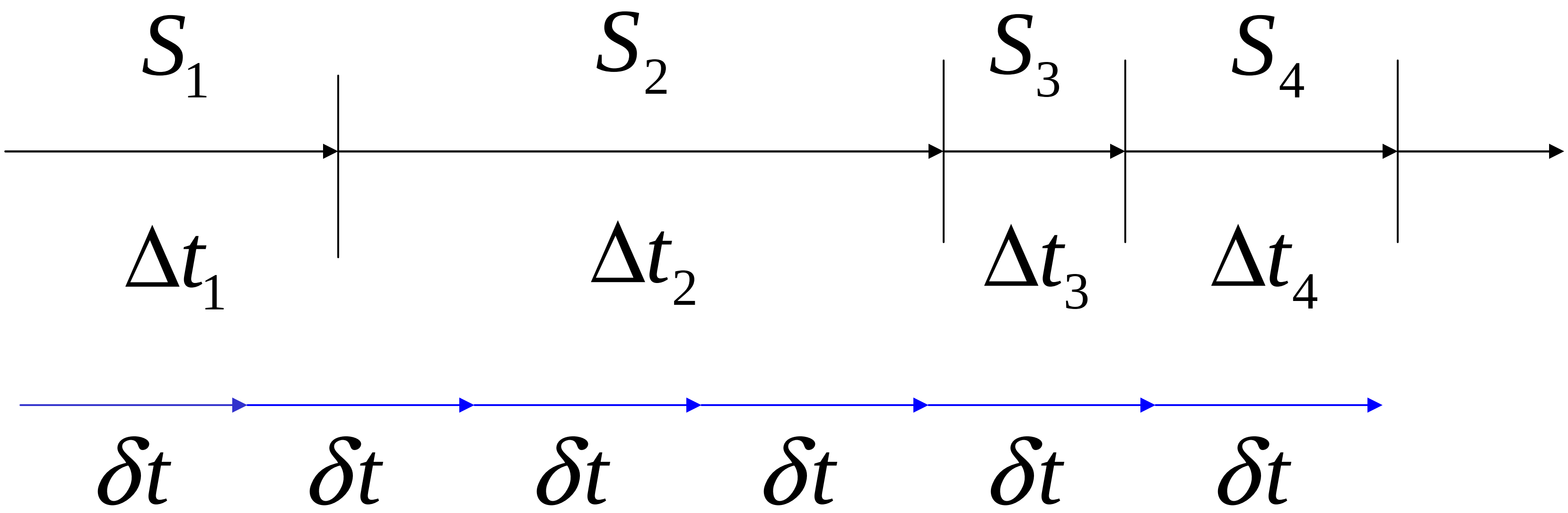
$$F(U) = \exp\left(-\frac{mU^2}{2\gamma T_B} \right)$$

The intruder particle possesses a stationary Maxwellian velocity distribution with an effective granular temperature $T_{eff} = \gamma T_B$

$$\gamma = \frac{1+\alpha}{2+(1-\alpha)\frac{m}{M}} \leq 1$$

Martin and Piasecki

Event driven simulation

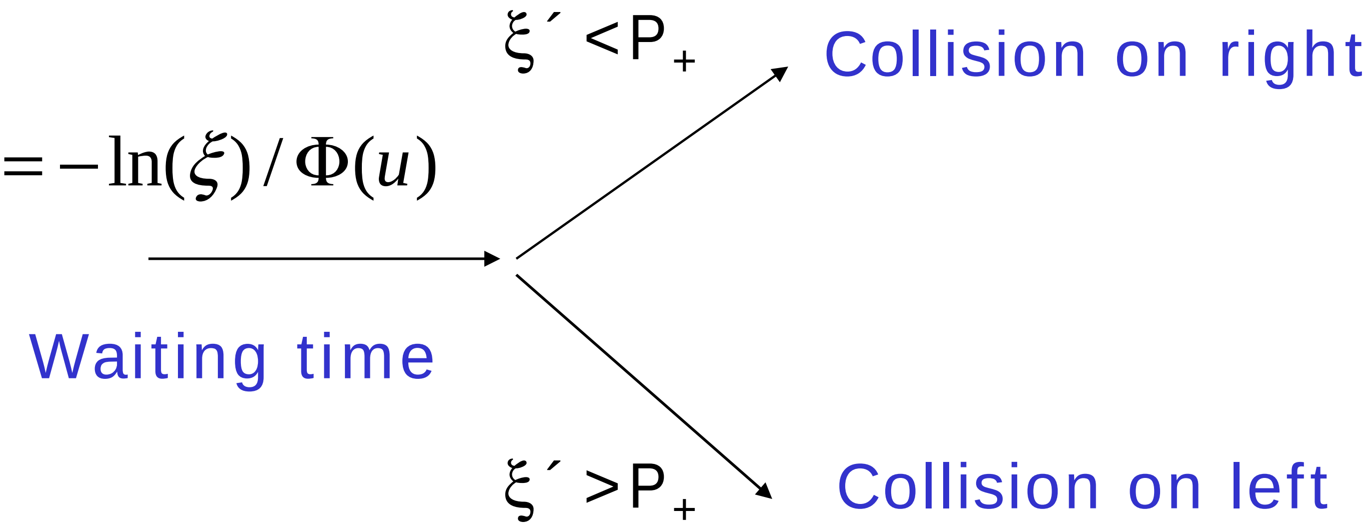


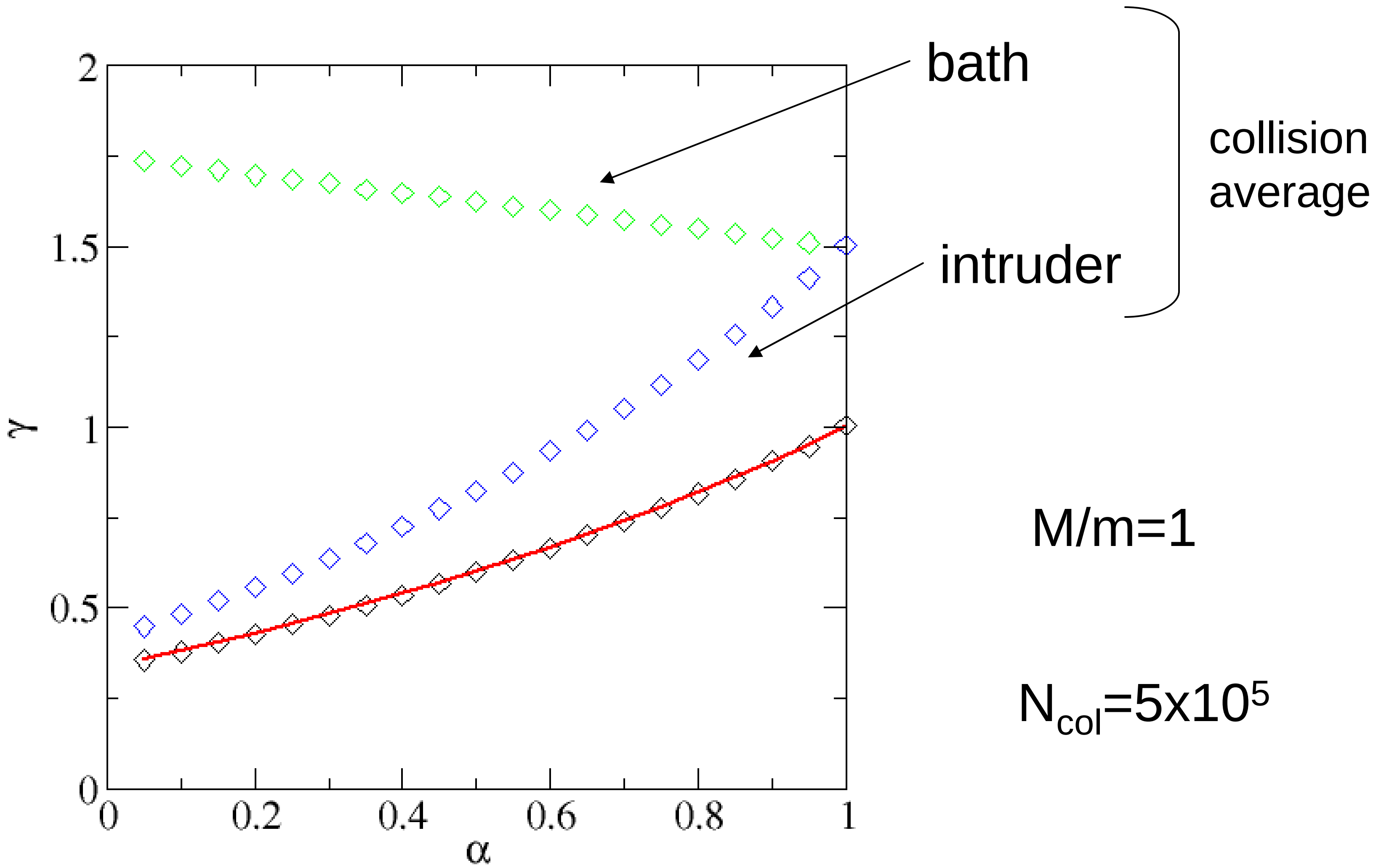
$$P(\Delta t) = \exp(-R(S_i)\Delta t)$$

Mean event rate

ED simulation

$$\Delta t = -\ln(\xi)/\Phi(u)$$

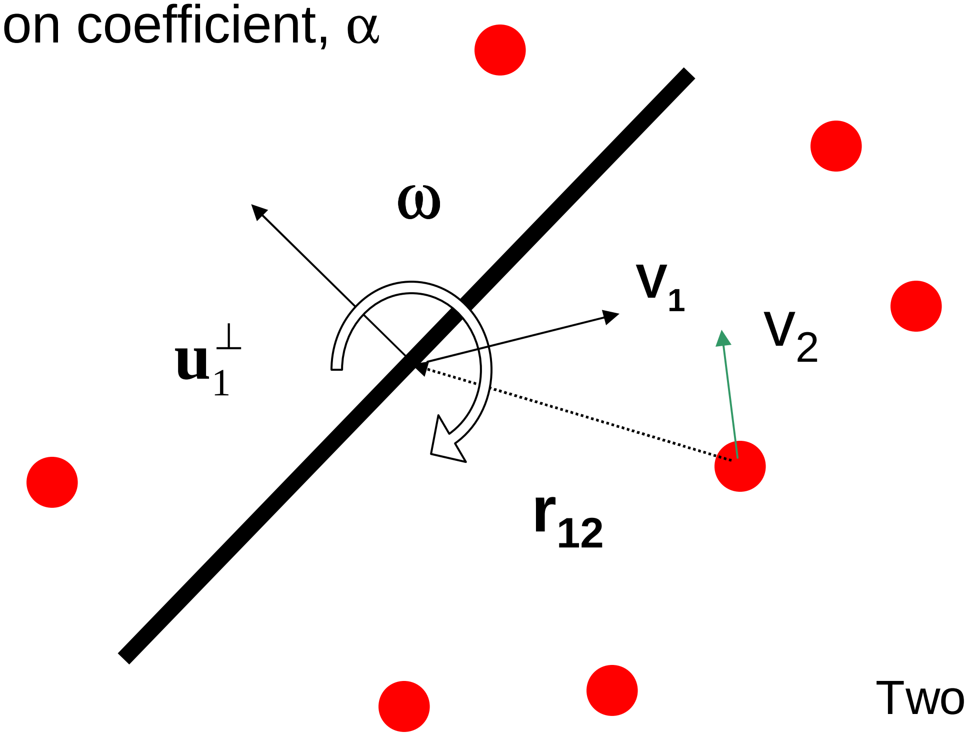




bath
collision
average
intruder
2
1.5
1
0.5
0
γ
0
0.2
0.4
0.6
0.8
1
α
M/m=1
N_col=5x10^5

Needle System

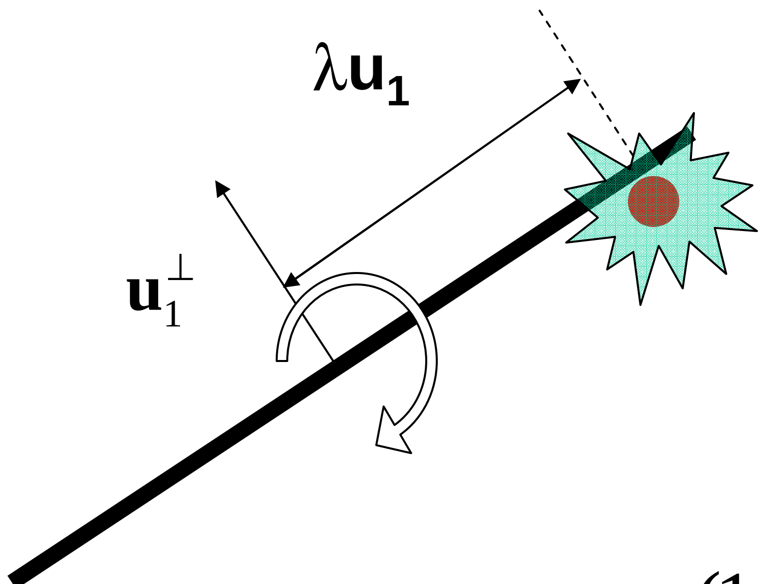
Needle length, L
Moment of inertia, I
Normal restitution coefficient, α



Two-dimensional

Collision

$$\mathbf{V} = \mathbf{v}_{12} + \lambda \dot{\mathbf{u}}_1$$



$$\mathbf{V}'.\mathbf{u}_1^{\perp} = -\alpha \mathbf{V}.\mathbf{u}_1^{\perp}$$

$$\mathbf{V}'.\mathbf{u}_1 = \mathbf{V}.\mathbf{u}_1$$

$$\Delta p = -\frac{(1+\alpha)\mathbf{V}.\mathbf{u}_1^{\perp}}{\frac{1}{M} + \frac{1}{m} + \frac{\lambda^2}{I}}$$

Kinetic Theory

$$\frac{\partial f^{(1)}(v_1,\omega)}{\partial t} = \int dv_2 \int dr_2 \overline{T_{12}} f^{(2)}(v_1,\omega,v_2)$$

$$f^{(2)}(v_1,\omega,v_2) = f^{(1)}(v_1,\omega)\Phi(v_2)$$

$$\Phi(v_2) \propto \exp\left(-\frac{mv_2^2}{2T}\right)$$

Analysis

$$\frac{dT_T}{dt} \propto \int dr_2 \int dv_2 \int d\omega \int d\theta \overline{T_{12}} f^{(2)}(v_1, \omega, v_2) E_1^T = 0$$

$$\frac{dT_R}{dt} \propto \int dr_2 \int dv_2 \int d\omega \int d\theta \overline{T_{12}} f^{(2)}(v_1, \omega, v_2) E_1^R = 0$$

Maxwell distribution of needle angular and translational velocities is not a stationary state of the Enskog-Boltzmann Equation

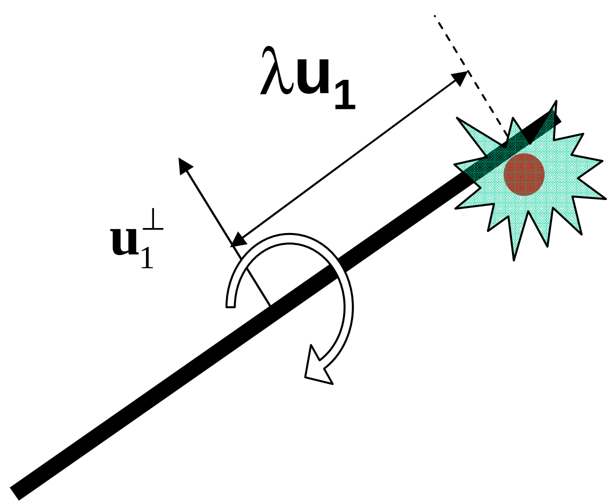
But we use it as a trial function…

$$f^{(1)}(v_1, \omega_1) \propto \exp\left(-\frac{Mv_1^2}{2\gamma_T T} - \frac{I\omega_1^2}{2\gamma_R T} \right)$$

Collision Operator

$$T_{12} = \Theta(L/2 - |\lambda|)\delta(|\mathbf{r}_{12}.\mathbf{u}_1^{\perp}| - 0^+)\left|\frac{d\,|\mathbf{r}_{12}.\mathbf{u}_1^{\perp}|}{dt}\right|\Theta\left(-\left|\frac{d\,|\mathbf{r}_{12}.\mathbf{u}_1^{\perp}|}{dt}\right|\right)(b_{12} - 1)$$

contact **flux** **approach** **difference**



Solution

Let $k = \dfrac{L^2}{4I(1/m + 1/M)}$

$$b\int_0^1 dx \frac{\sqrt{1+akx^2}}{1+kx^2} = \frac{1+\alpha}{2}\int_0^1 dx \frac{(1+akx^2)^{3/2}}{(1+kx^2)^2} \qquad (1)$$

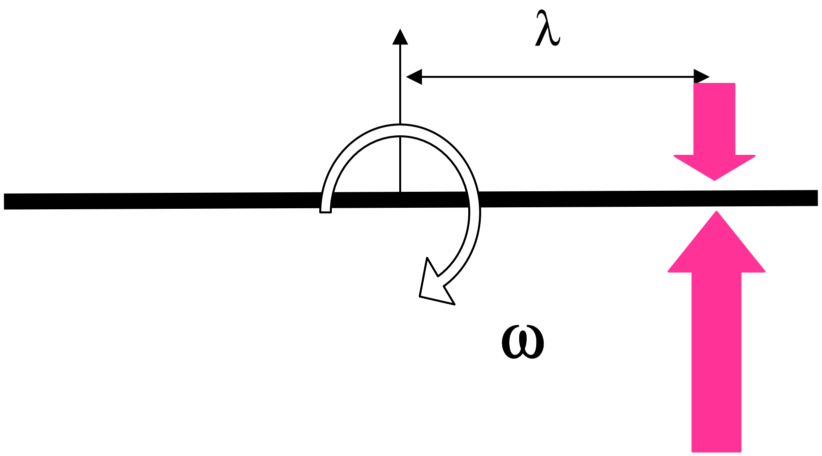
$$a\int_0^1 dx x^2 \frac{\sqrt{1+akx^2}}{1+kx^2} = \frac{1+\alpha}{2}\int_0^1 dx \frac{x^2(1+akx^2)^{3/2}}{(1+kx^2)^2} \qquad (2)$$

$$a = \gamma_R \frac{M+m}{M+m\gamma_T} \quad (3) \qquad\qquad b = \gamma_T \frac{M+m}{M+m\gamma_T} \quad (4)$$

Solve (2) for *a*. Then find *b* from (1). Then find γ_R and γ_T from (3) and (4)

ED Simulation

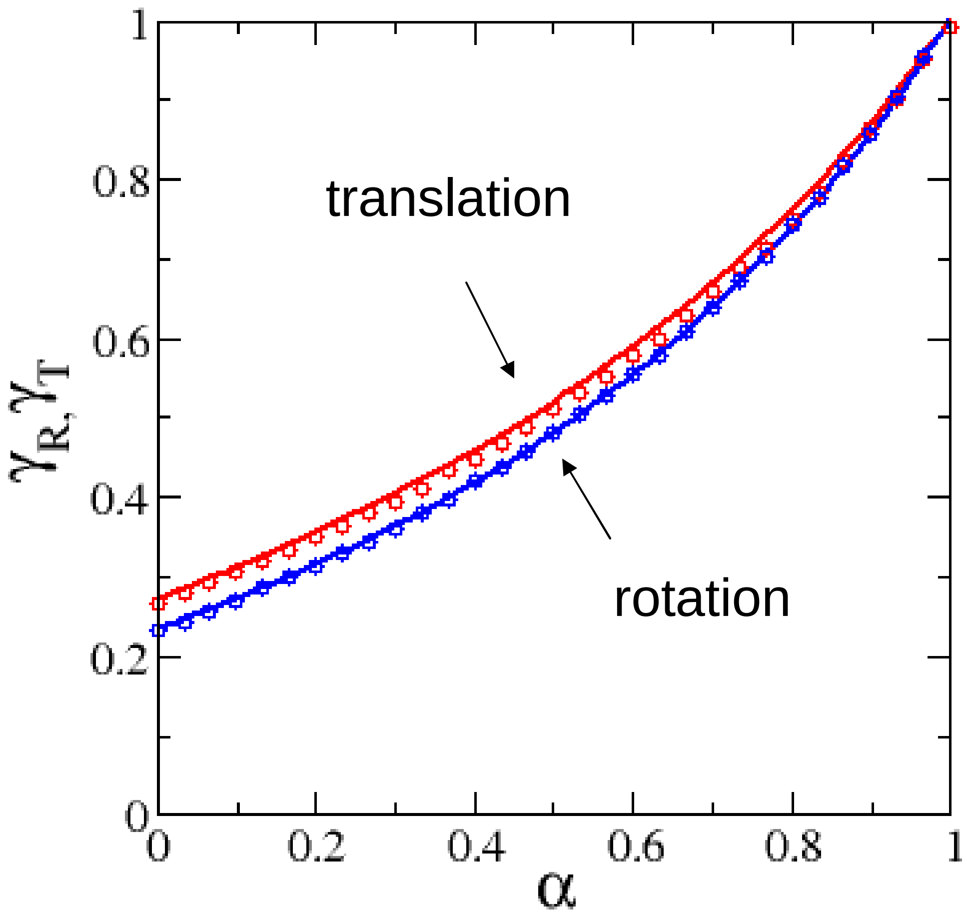
$V^{\perp}$ $\Phi(V^{\perp} - \lambda\omega)$



Complication: flux is not constant

$$P(t) = \exp\left(-\int_0^t \Phi(t')dt' \right)$$

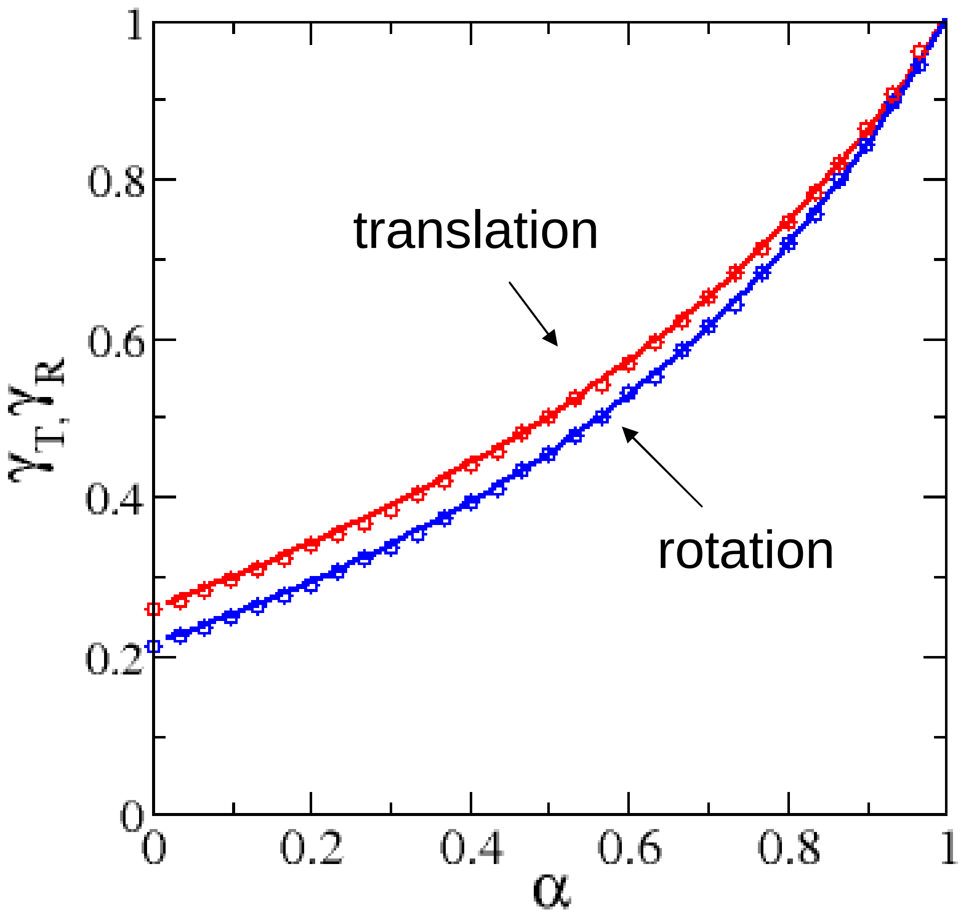
Granular temperatures



M=m

$$I = \frac{Ml^2}{12}$$

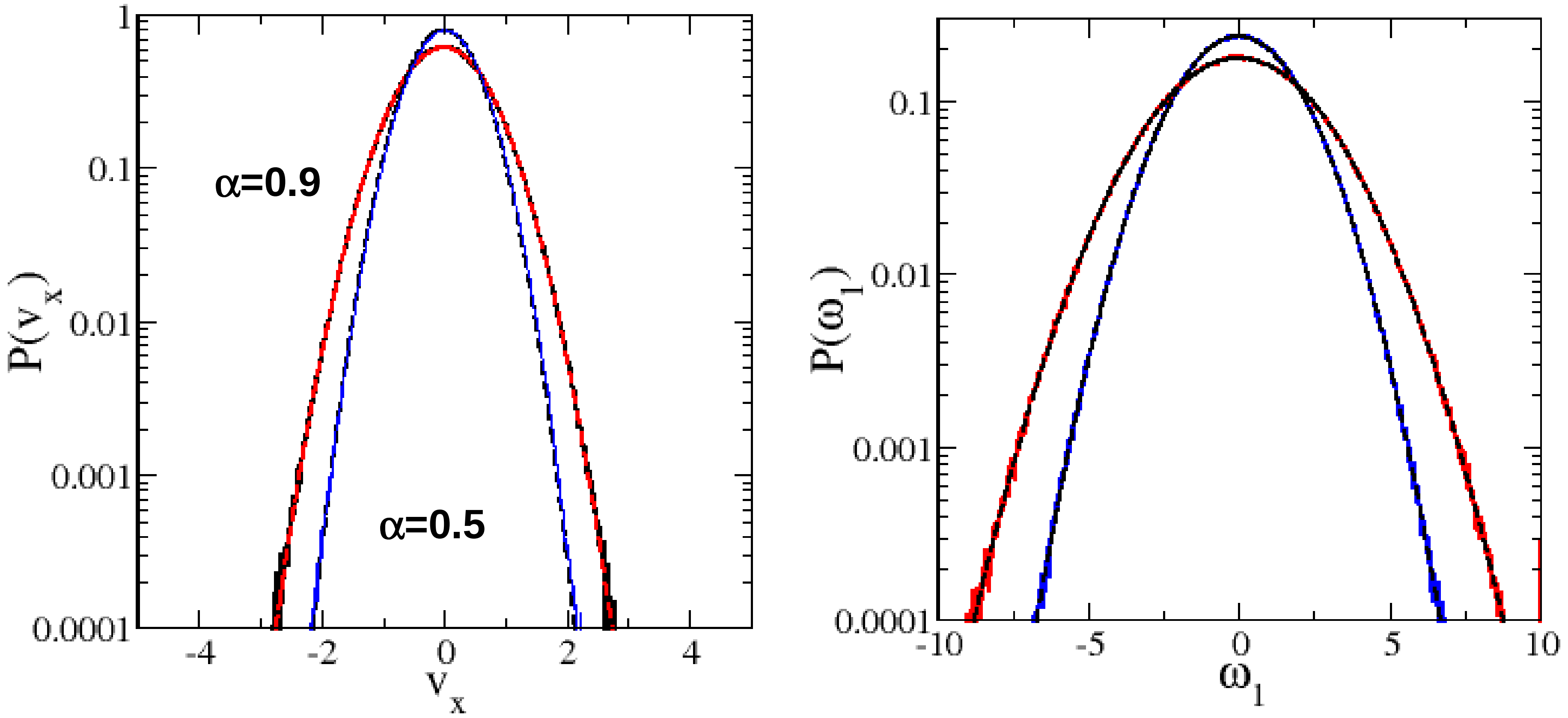
Inhomogeneous Needle



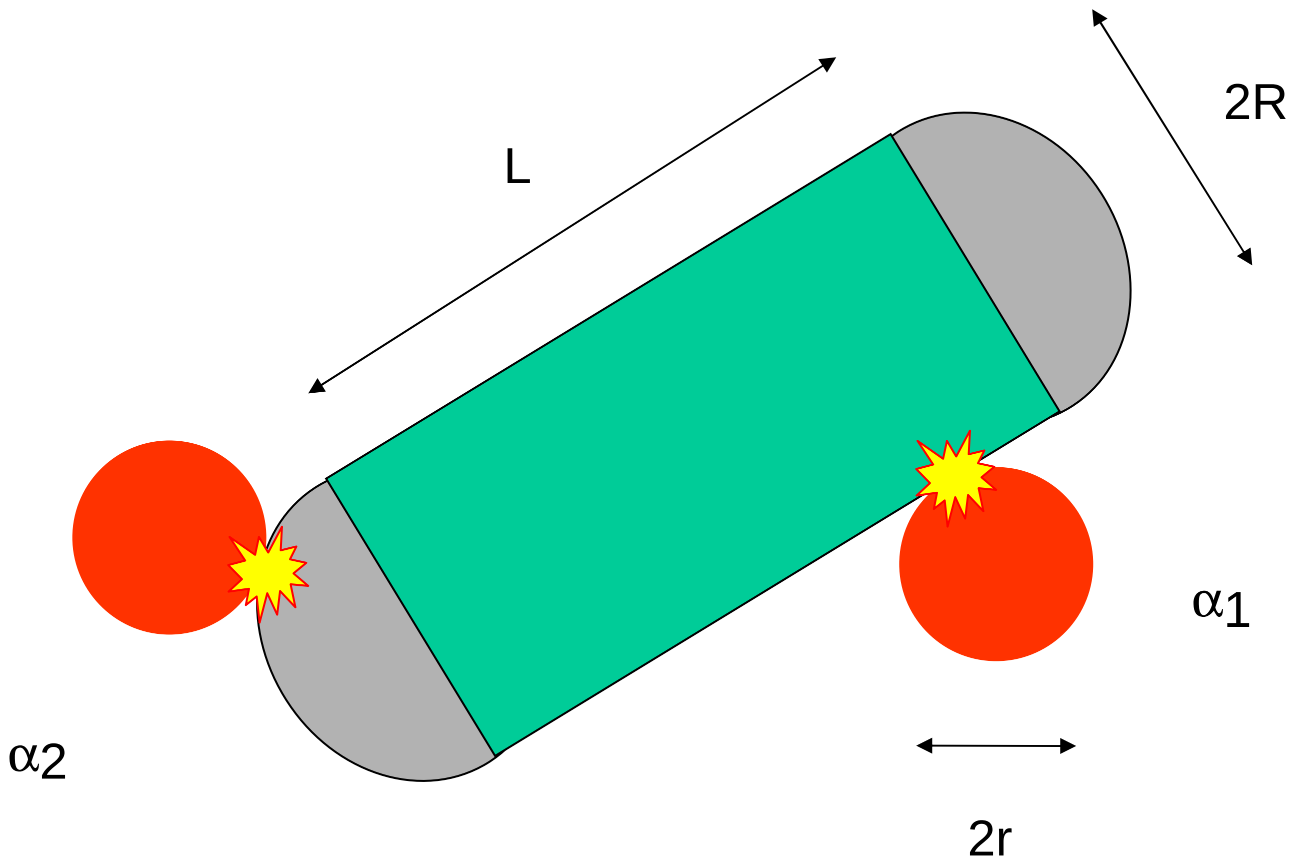
M = m

$$I = \frac{Ml^2}{16}$$

Velocity distributions



Discorectangle



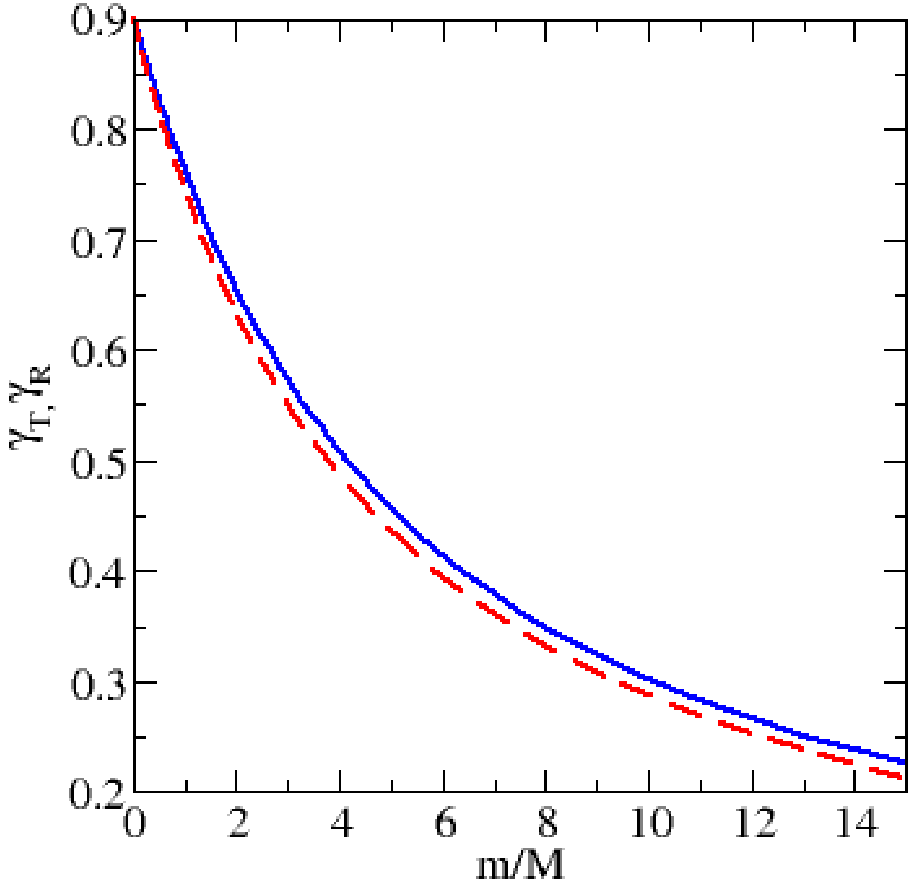
Steady State Solution

$$b[(1+\alpha_1)cI_1^{01}(a,k)+(1+\alpha_2)(1-c)J_1^{01}(a,k)]=\frac{(1+\alpha_1)^2}{2}cI_2^{03}(a,k)+\frac{(1+\alpha_2)^2}{2}(1-c)J_2^{03}(a,k)]$$

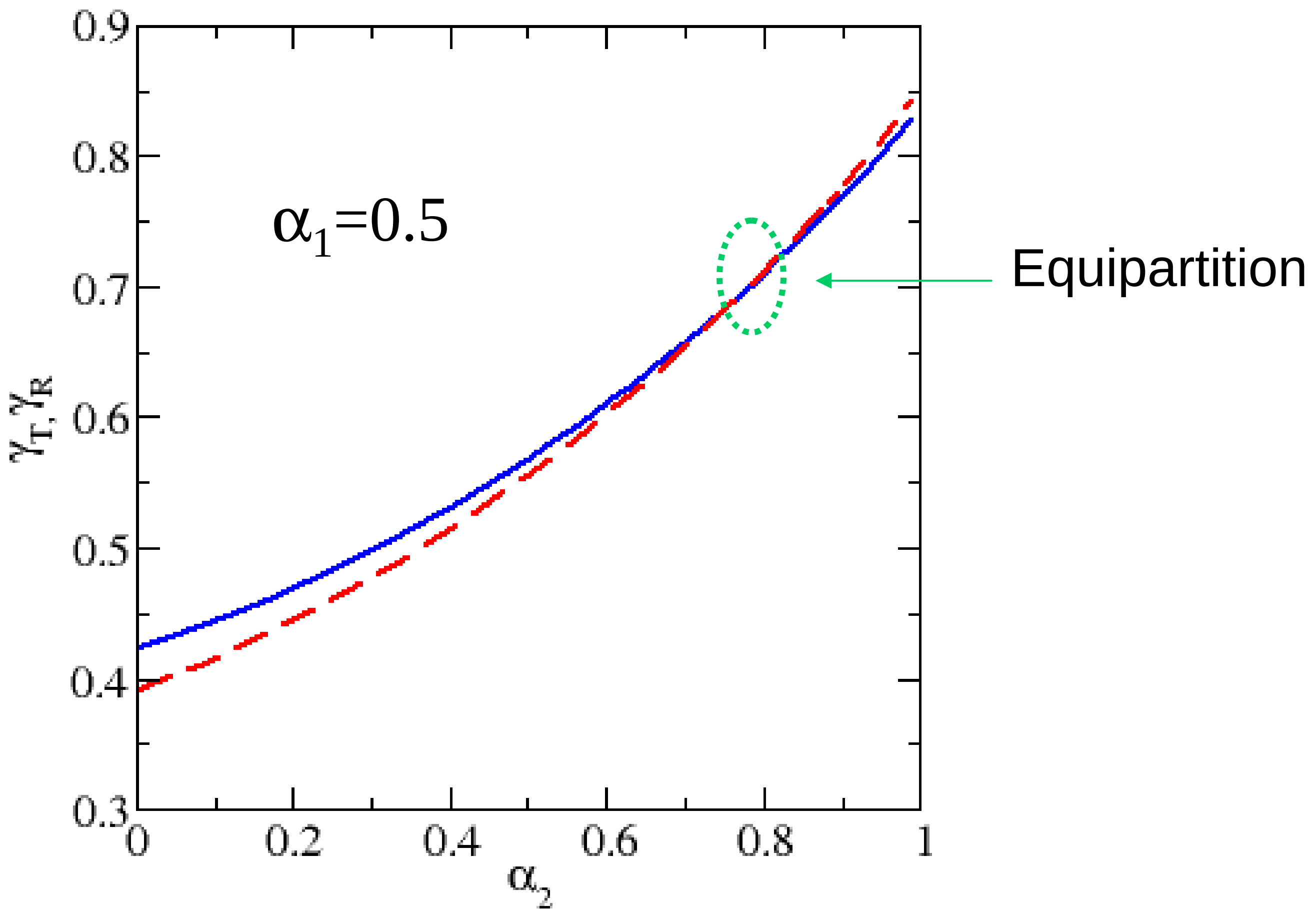
$$a[(1+\alpha_1)cI_1^{11}(a,k)+(1+\alpha_2)(1-c)J_1^{11}(a,k)]=\frac{(1+\alpha_1)^2}{2}cI_2^{13}(a,k)+\frac{(1+\alpha_2)^2}{2}(1-c)J_2^{13}(a,k)]$$

$$k=\frac{L^2}{4I\left(\frac{1}{m}+\frac{1}{M}\right)} \qquad c=\frac{L}{L+2(r+R)}$$

$\alpha_1 = \alpha_2$; c=4/7

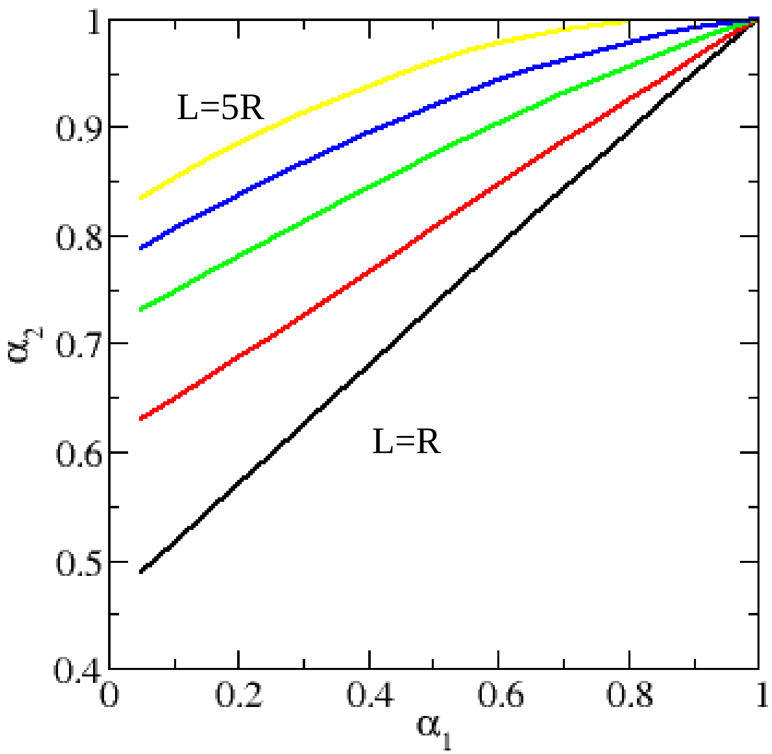


M = m, L = 2R, R = r

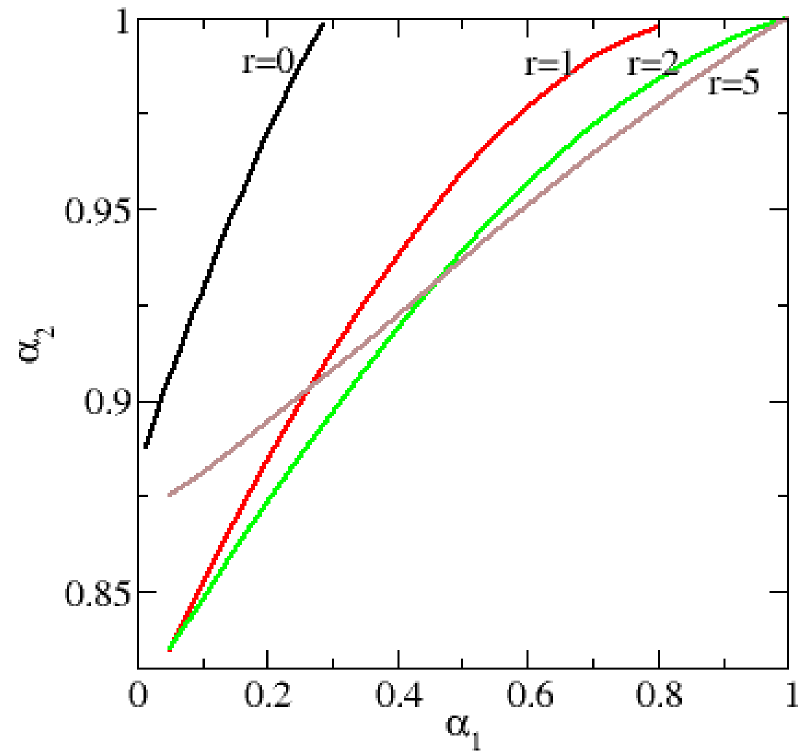


0.9
0.8
0.7
0.6
0.5
0.4
0.3
γ_T, γ_R
α_1=0.5
Equipartition
0
0.2
0.4
0.6
0.8
1
α_2

m = M, r = R


L=5R
L=R
α_2
α_1
0
0.2
0.4
0.6
0.8
1
0.4
0.5
0.6
0.7
0.8
0.9
1

m = M, L = 8R


r=0
r=1
r=2
r=5
α_2
α_1
0
0.2
0.4
0.6
0.8
1
0.85
0.9
0.95
1

Summary

- Theory gives results in excellent agreement with ED simulations
- Equipartition is generally not obtained except:
  - in the limit of light bath particles
  - If COR depends on position of impact
- For uniform COR, $T_R < T_T$
- Small deviations from gaussian
- Non equipartition also expected for higher dimensions and more complex shapes
- Experimental verification in 2D?