Vibrofluidization

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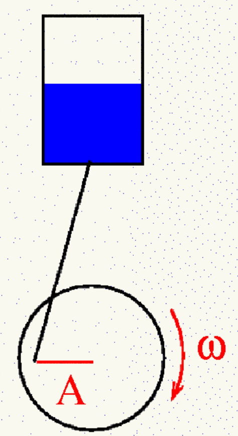
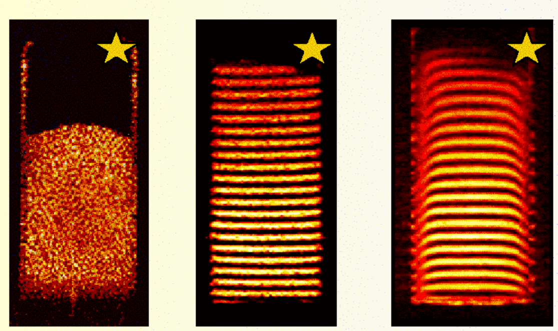


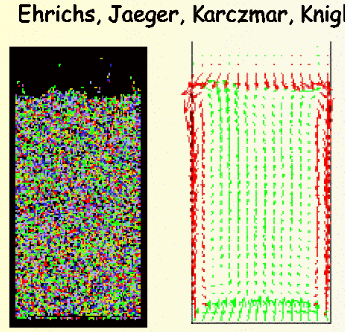
1. Vertically vibrated granular material, some effects
2. Conditions for convection: Theory
3. Experiment

1. Vibrated granular material: effects

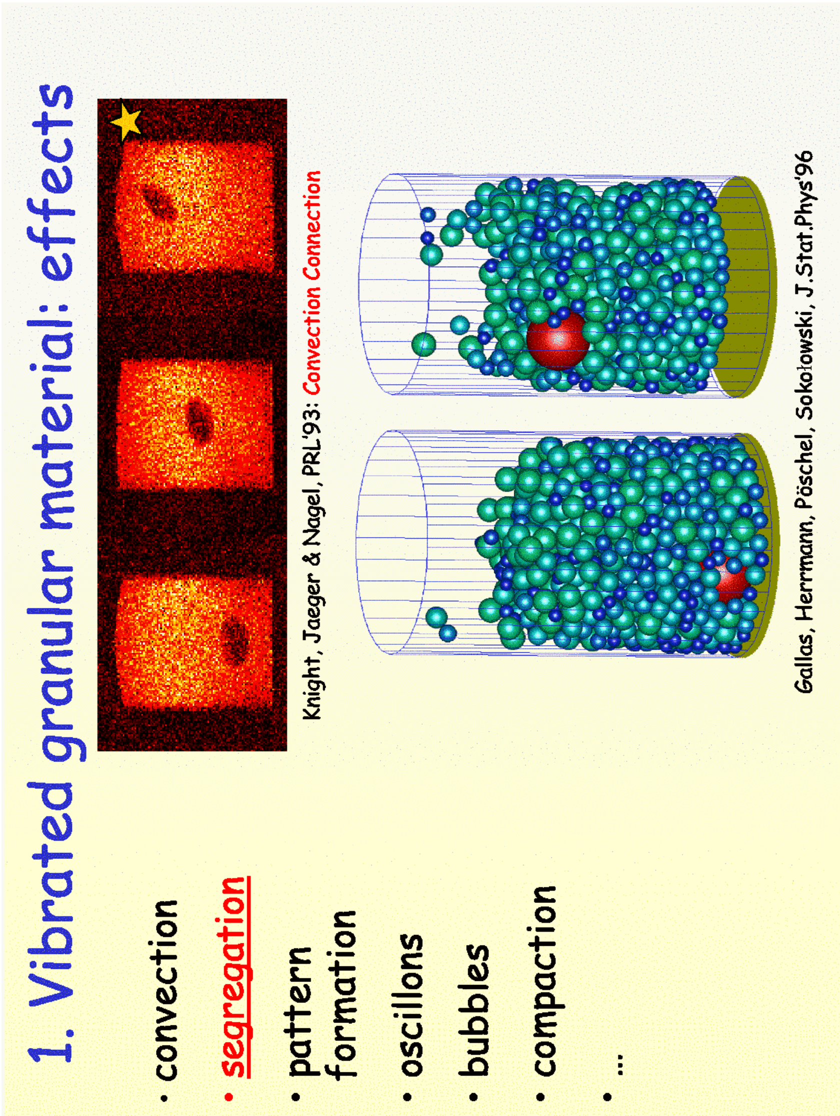
- convection
- segregation
- pattern formation
- oscillons
- bubbles
- compaction
- ...

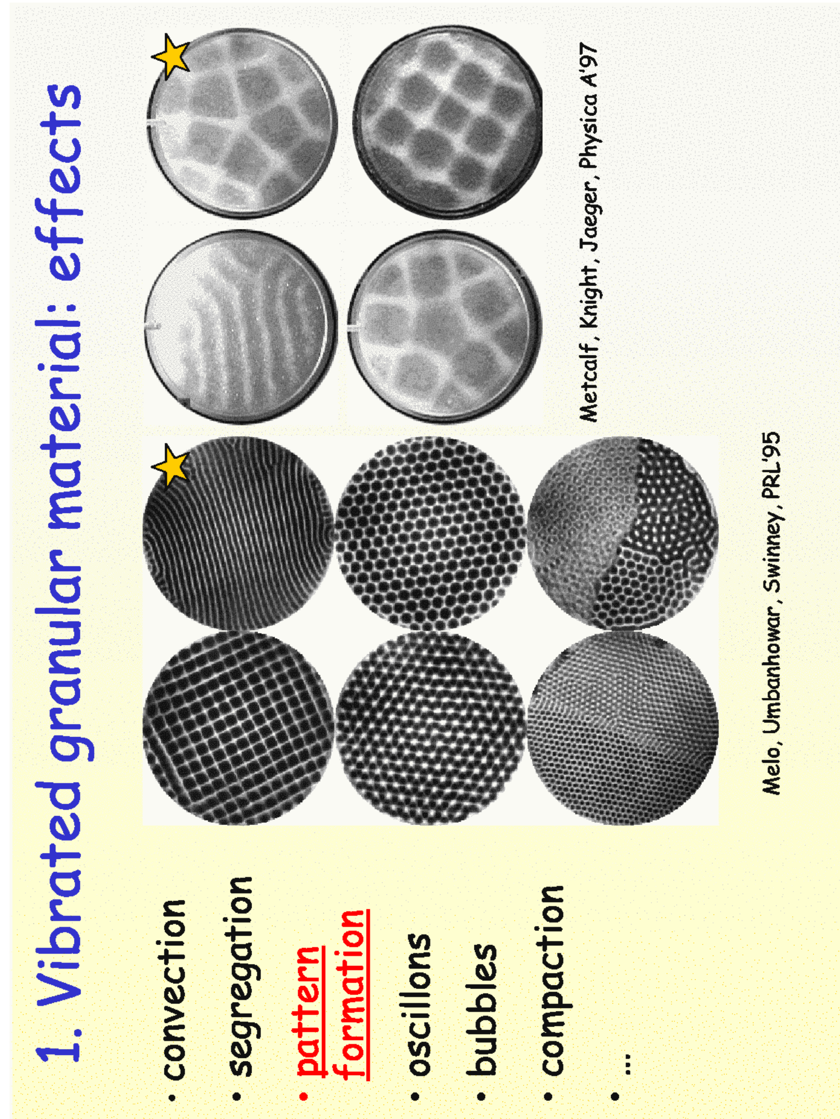


Ehrichs, Jaeger, Karczmar, Knight, Kuperman, Nagel, Science'95



Salueña, Pöschel, Esipov'97


1. Vibrated granular material: effects
• convection
• segregation
• pattern formation
• oscillons
• bubbles
• compaction
• ...
Knight, Jaeger & Nagel, PRL'93: Convection Connection
Gallas, Herrmann, Pöschel, Sokolowski, J.Stat.Phys'96


1. Vibrated granular material: effects
• convection
• segregation
• pattern formation
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• bubbles
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• ...
Metcalf, Knight, Jaeger, Physica A'97
Melo, Umbanhowar, Swinney, PRL'95

1. Vibrated granular material: effects

- convection
- segregation
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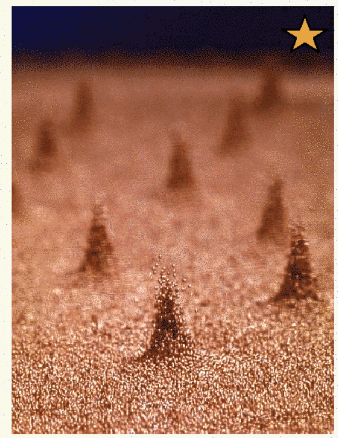
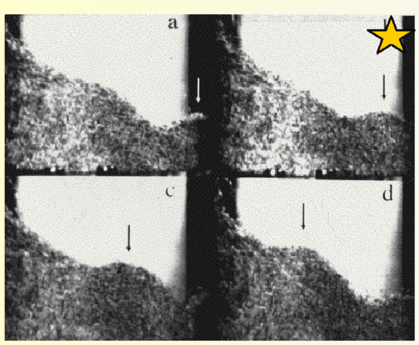


Umbanhowar, Melo, Swinney, Nature'96

1. Vibrated granular material: effects

- convection
- segregation
- pattern formation
- oscillons
- bubbles
- compaction
- ...

vibrated annular container



travelling waves

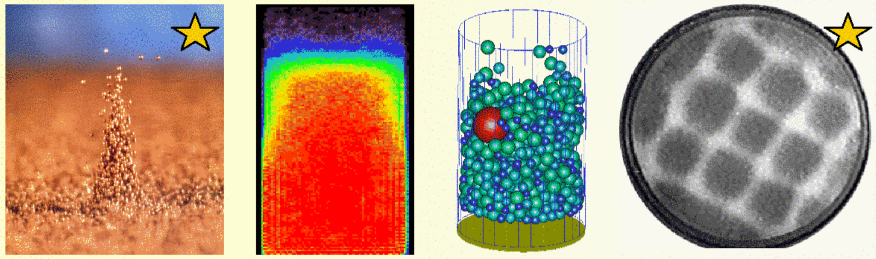
Pak & Behringer PRL'93

bubbles

Pak & Behringer, Nature'94

1. Vibrated granular material: effects

What have all these effects in common?



For all these effects the particles have to lose contact for part of the period. → Vibrofluidization

motion of the vibrating table: $z_0(t) = A_0 \cos \omega_0 t$

necessary condition:

$$\left(\frac{d^2 z_0}{dt^2}\right)_{\max} = A_0 \omega_0^2 > g$$

$$\Gamma \equiv A_0 \omega_0^2 / g > 1$$

Froude number

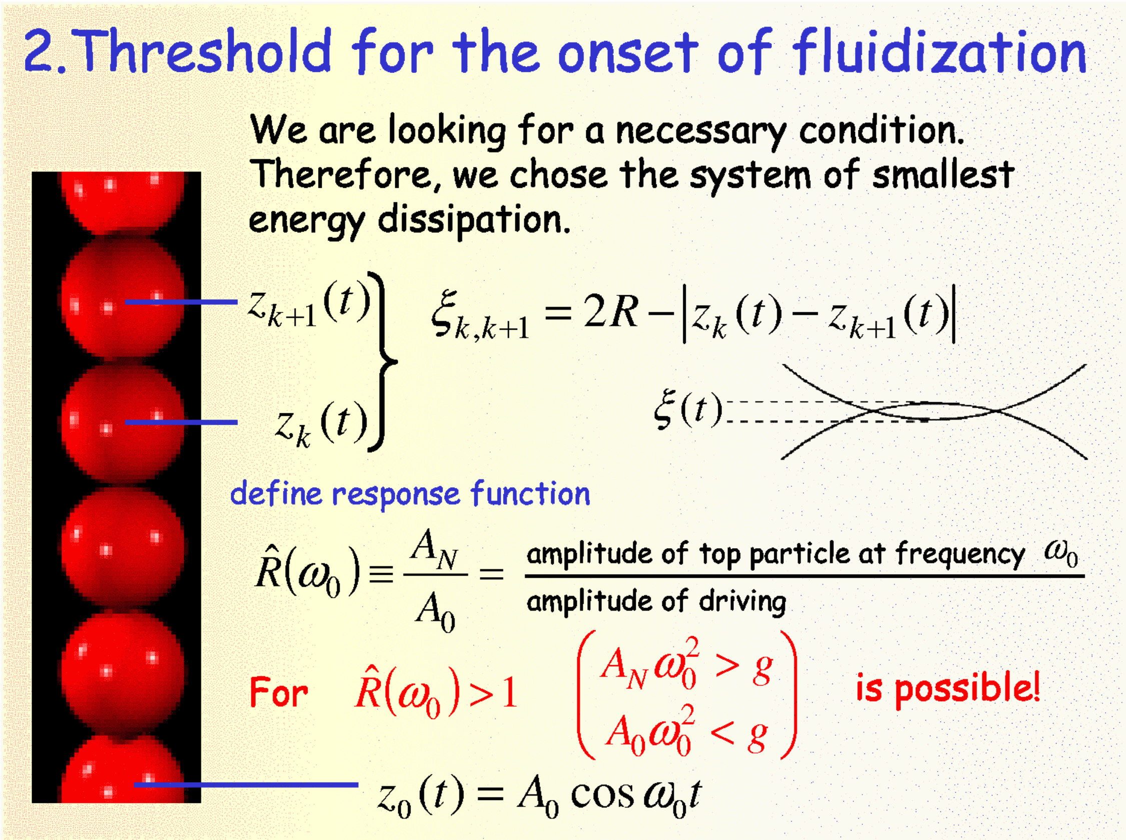
???

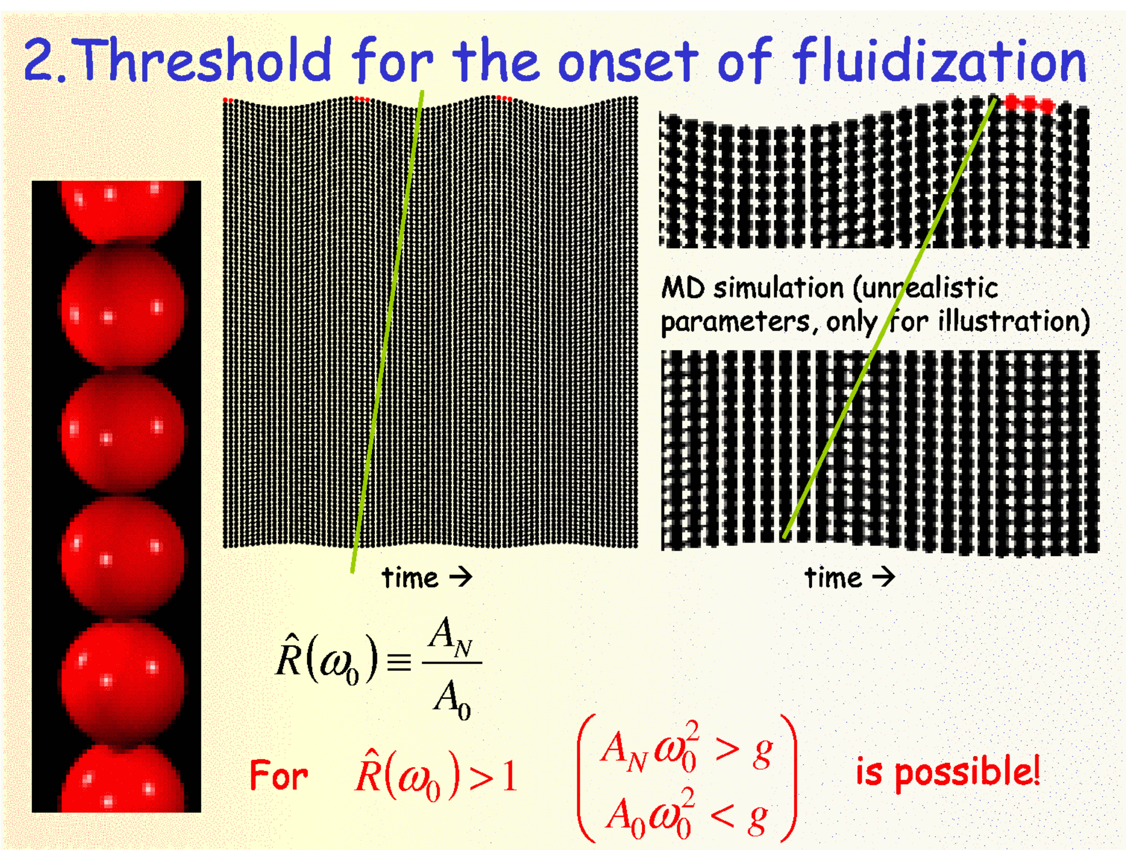
2. Threshold for the onset of fluidization

Kitchen table experiment: shake sand with tracers $f = 8Hz, \quad \Gamma = 0.96g$

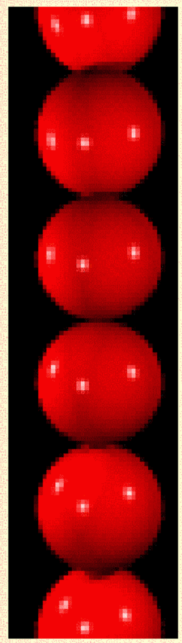
1 frame / min

(details later)


2. Threshold for the onset of fluidization
We are looking for a necessary condition. Therefore, we chose the system of smallest energy dissipation.
$z_{k+1}(t)$
$z_k(t)$
$\xi_{k,k+1} = 2R - \left|z_k(t) - z_{k+1}(t)\right|$
$\xi(t)$
define response function
$\hat{R}(\omega_0) \equiv \frac{A_N}{A_0} = \frac{\text{amplitude of top particle at frequency } \omega_0}{\text{amplitude of driving}}$
For $\hat{R}(\omega_0) > 1$ $\begin{pmatrix} A_N\omega_0^2 > g \\ A_0\omega_0^2 < g \end{pmatrix}$ is possible!
$z_0(t) = A_0 \cos \omega_0 t$


2. Threshold for the onset of fluidization
MD simulation (unrealistic parameters, only for illustration)
time →
time →
$\hat{R}(\omega_0) \equiv \frac{A_N}{A_0}$
For $\hat{R}(\omega_0) > 1$ $\begin{pmatrix} A_N\omega_0^2 > g \\ A_0\omega_0^2 < g \end{pmatrix}$ is possible!

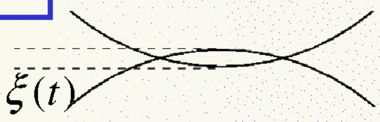
2. Threshold for the onset of fluidization



$$\frac{d^2 z_k}{dt^2} = \frac{1}{m}\left(F_{k,k+1} - F_{k-1,k}\right) - g$$

Newton's equation of motion

particle model: viscoelastic spheres



$$F(t) = F^{el}(t) + F^{dis}(t)$$

$$F^{el} = -\mu\sqrt{R}\,\xi^{3/2} \qquad \mu = \frac{1}{3}\frac{Y}{1-\nu^2}$$

H. Hertz'1881

$$F^{dis} = -\alpha\sqrt{R}\sqrt{\xi}\,\frac{d\xi}{dt} \qquad \alpha = \frac{\sqrt{2}}{3}\frac{YA}{1-\nu^2}$$

Brilliantov, Spahn, Hertzsch, Pöschel, PRE'96

define $u_k = z_k - 2Rk \qquad (k = 0,..,N)$

$$\xi_{k,k+1} = u_k - u_{k+1}$$

top particle loses contact if $u_N > u_{N-1}$

2. Threshold for the onset of fluidization

$$\frac{\partial^2 u}{\partial t^2} = -g - \frac{\partial}{\partial z}\left[\kappa\left(-\frac{\partial u}{\partial z}\right)^{3/2} - \beta\frac{\partial^2 u}{\partial t\,\partial z}\sqrt{-\frac{\partial u}{\partial z}}\right] \qquad \left.\frac{\partial u}{\partial z}\right|_{z=L} = 0$$

$\kappa = 2Y/\pi\rho\left(1-\nu^2\right) \qquad \beta = A\kappa \qquad \rho$-density

Newton's equation of motion for the displacement of the particles

Limit of elastic particles

new variables $x = 1 - \frac{z}{L} \qquad \tau = \left(\frac{g\kappa^2}{L^5}\right)^{1/6} t \qquad \gamma = \left(\frac{g^2 L^5}{\kappa^2}\right)^{1/6}$

$$\frac{\partial^2 u}{\partial \tau^2} = -\gamma^2 + \frac{1}{\gamma}\frac{\partial}{\partial x}\left[\left(\frac{\partial u}{\partial x}\right)^{3/2}\right] \qquad \left.\frac{\partial u}{\partial x}\right|_{x=0} = 0$$

amplification factor   Pöschel, Schwager, Salueña, PRE'2000

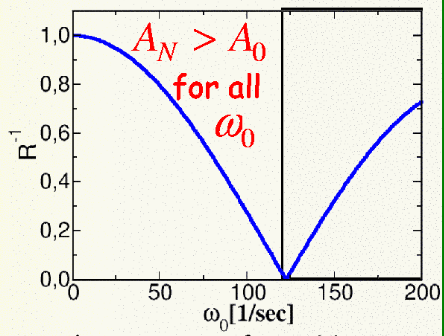
$$\hat{R}^{-1} = \left(\frac{6}{25}\right)^{1/5}\Gamma\left(\frac{3}{5}\right)\Omega_0^{2/5}\left|J_{-2/5}\left(\frac{2}{5}\right)\sqrt{6}\,\Omega_0\right| \qquad \Omega = \left(\frac{L^5}{g\kappa^2}\right)^{1/6}\omega$$

2. Threshold for the onset of fluidization

Results for the limit of elastic particles

amplification factor

$$\hat{R}^{-1} = \left(\frac{6}{25}\right)^{1/5} \Gamma\left(\frac{3}{5}\right)\Omega_0^{2/5} \left| J_{-2/5}\left(\frac{2}{5}\right)\sqrt{6}\Omega_0 \right|$$



(parameters for rubber, $Y = 4\times10^7$ Pa, L=0.6m)

Taylor expansion:

$$\hat{R}^{-1} = 1 - \frac{2}{5}\left(\frac{L^5}{g\kappa^2}\right)^{1/3} \omega_0^2 + \ldots$$

one particle: $A_0 = \frac{g}{\omega_0^2}$

a column: $A_0 = \frac{g}{\omega_0^2} - \frac{2}{5}\left(\frac{L^5}{g\kappa^2}\right)^{1/3}$

$$\kappa = 2Y / \pi\rho\left(1-\nu^2\right)$$

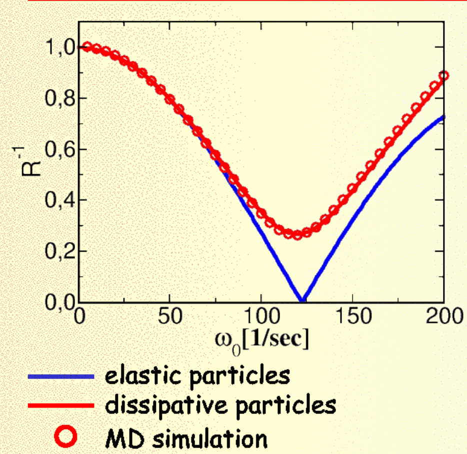
2. Threshold for the onset of fluidization

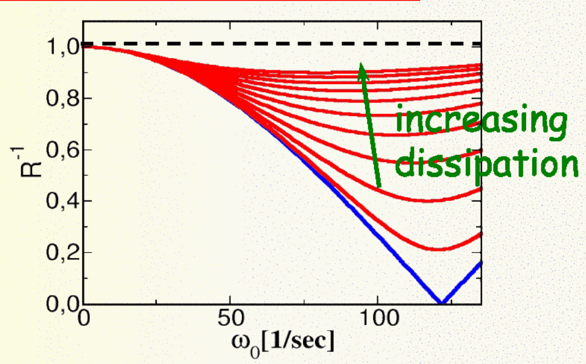
$$\frac{\partial^2 u}{\partial t^2} = -g - \frac{\partial}{\partial z}\left[\kappa\left(-\frac{\partial u}{\partial z}\right)^{3/2} + \beta \frac{\partial^2 u}{\partial t\,\partial z}\sqrt{-\frac{\partial u}{\partial z}}\right] \qquad \left.\frac{\partial u}{\partial z}\right|_{z=L} = 0$$

Newton's equation of motion for the displacement of the particles

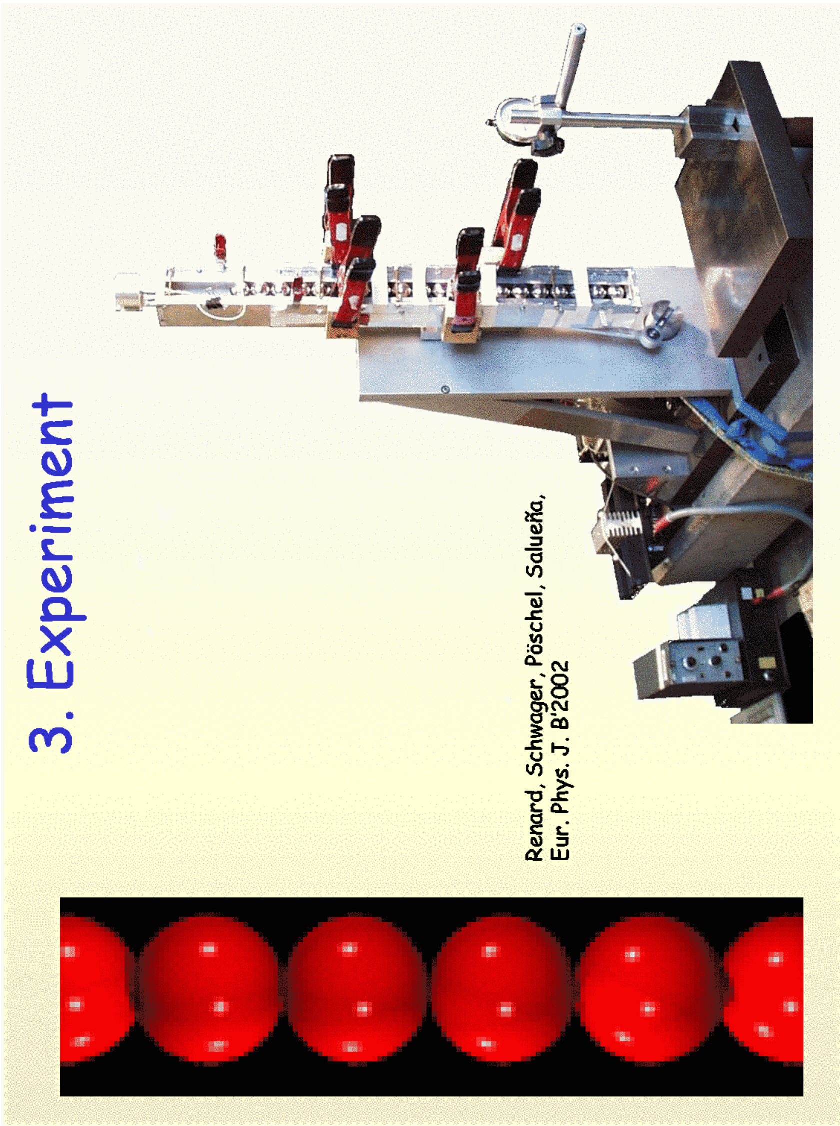
$$\hat{R}^{-1} = 1 - \omega_0^2 \frac{2}{5}\left(\frac{L^5}{g\kappa^2}\right)^{1/3} + \omega_0^4\left(\frac{L^5}{g\kappa^2}\right)^{2/3}\left[\frac{3}{100} + \frac{8}{45}\beta^2\left(\frac{g}{\kappa^4 L^5}\right)^{1/3}\right]$$

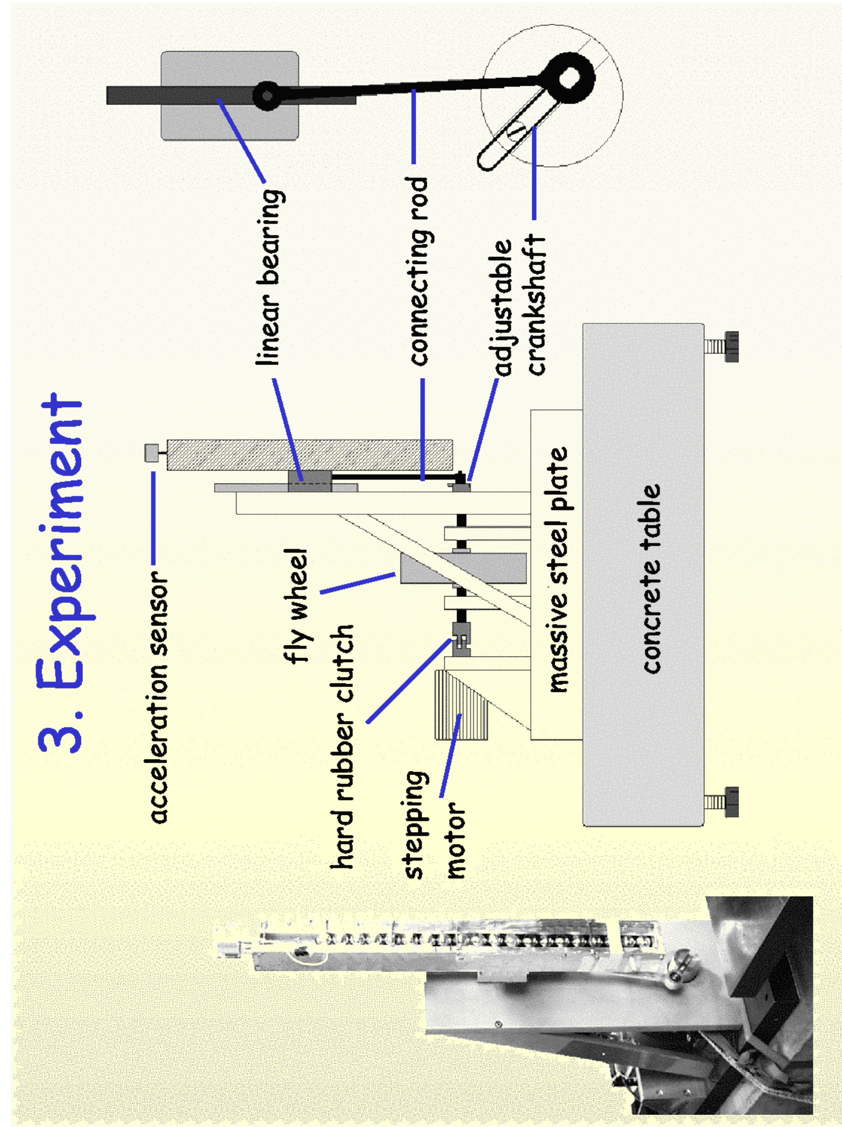
Pöschel, Schwager, Salueña, PRE'2000

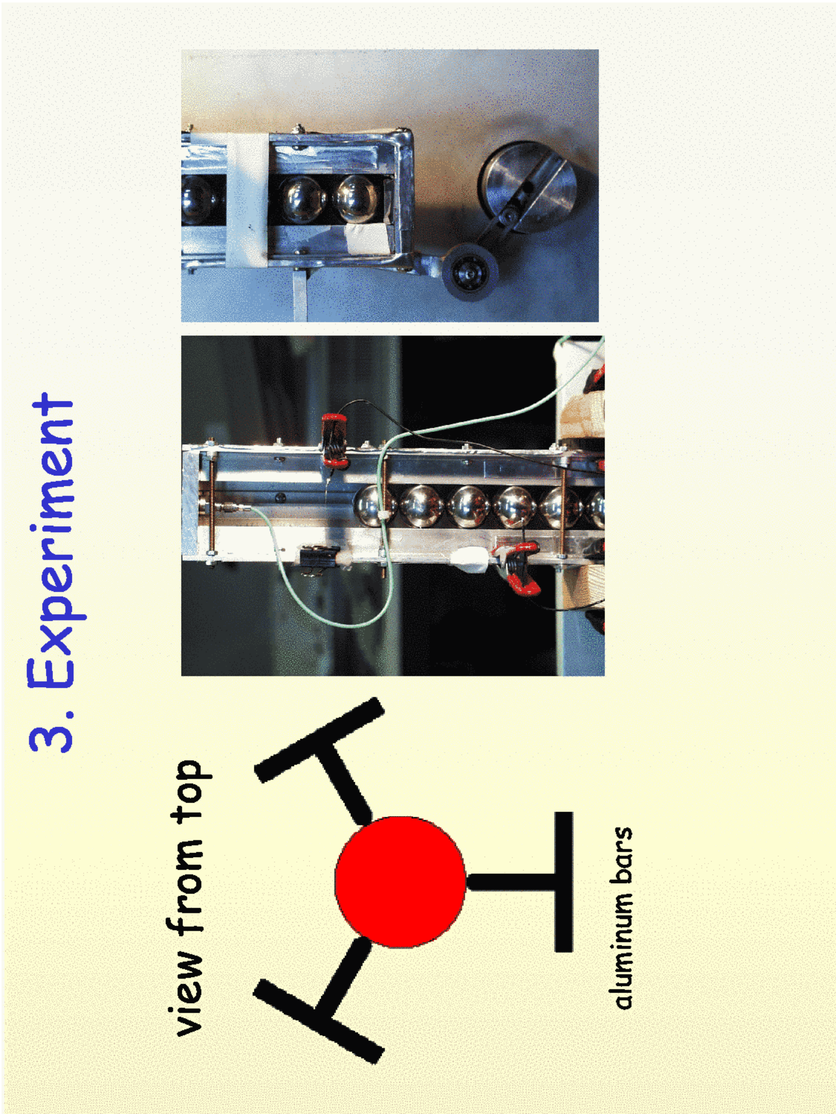


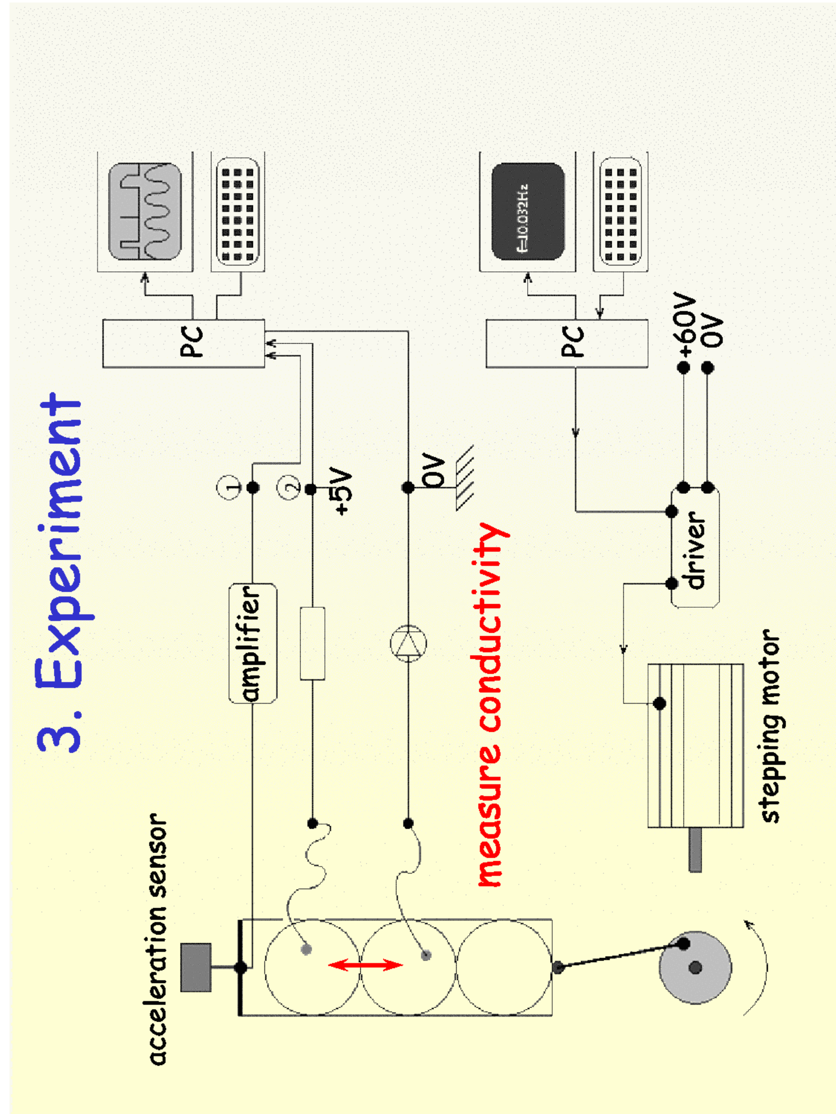


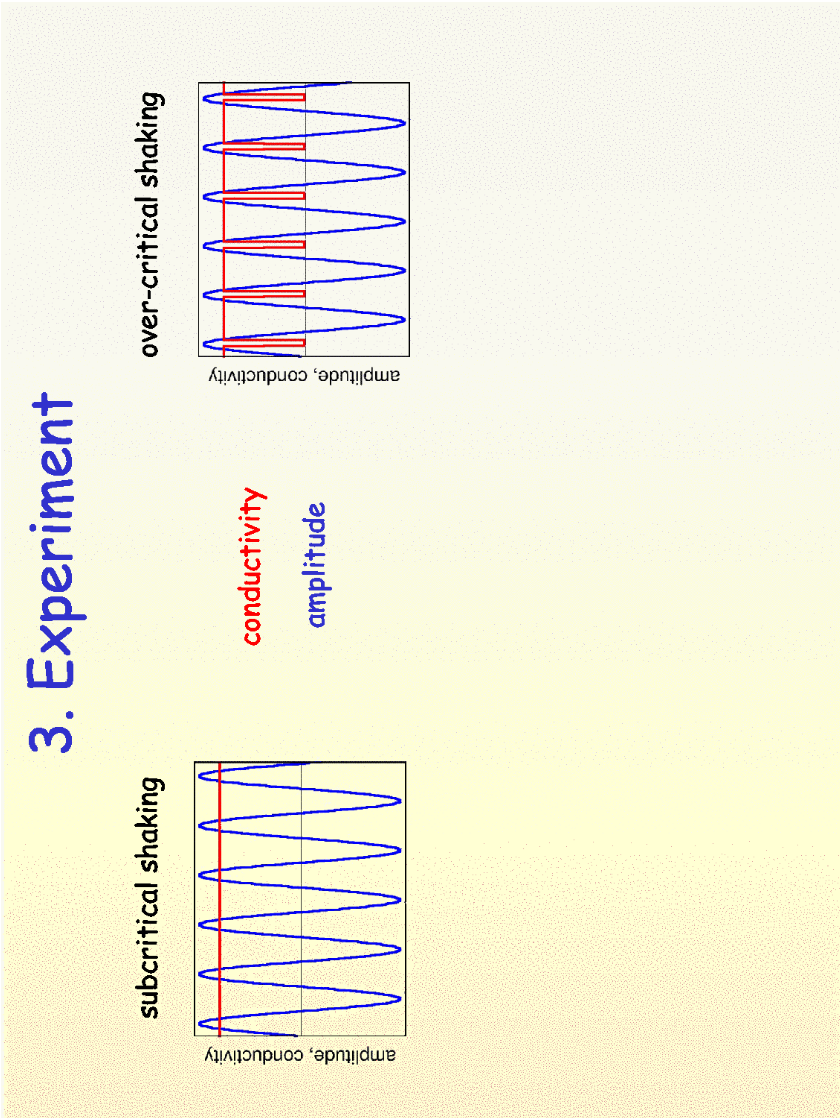
Even for large dissipation there is always a region of amplification.

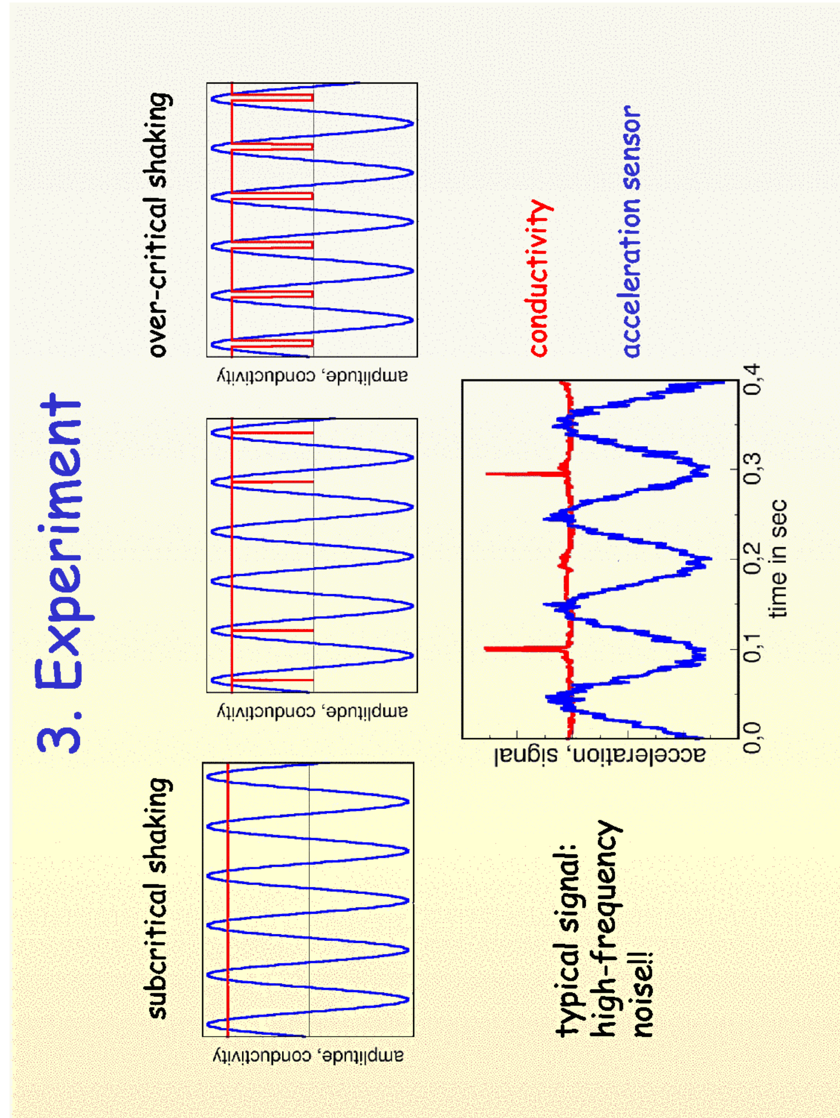

3. Experiment
Renard, Schwager, Pöschel, Salueña,
Eur. Phys. J. B'2002


3. Experiment
acceleration sensor
linear bearing
connecting rod
adjustable crankshaft
fly wheel
hard rubber clutch
stepping motor
massive steel plate
concrete table


3. Experiment
view from top
aluminum bars


3. Experiment
PC
PC
+60V
0V
driver
stepping motor
1
2
+5V
0V
amplifier
measure conductivity
acceleration sensor


3. Experiment
subcritical shaking
amplitude, conductivity
conductivity
amplitude
over-critical shaking
amplitude, conductivity


3. Experiment
subcritical shaking
amplitude, conductivity
amplitude, conductivity
over-critical shaking
amplitude, conductivity
conductivity
acceleration sensor
acceleration, signal
time in sec
0,0
0,1
0,2
0,3
0,4
typical signal:
high-frequency
noise!!

3. Experiment

What do we expect?

- 1 sphere: fluidization for $\Gamma \equiv A\omega^2 / g > 1$
- 20 spheres: fluidization for

$$\frac{A\omega^2}{g} > \Gamma\left(\frac{3}{5}\right)\left|M^{2/5} J_{-2/5}(2M)\right| \qquad M = \text{material and system constants}$$

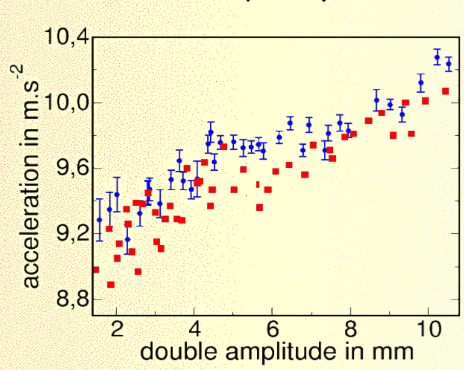
Taylor expansion: $$\frac{A\omega^2}{g} > 1 - B_2\omega^2 + B_4\omega^4$$

$$B_2 = \frac{18^{2/3}}{45}\left(\frac{\pi^2\rho^2L^5}{g\mu^2}\right)^{1/3}$$

$$B_4 = \frac{1}{18^{2/3}}\left(\frac{\pi^2\rho^2L^5}{g\mu^2}\right)^{2/3}\left(\frac{3}{100} + \frac{4\cdot 324^{2/3}}{405}\frac{\alpha^2}{\pi^2}\left(\frac{g\pi^4}{\mu^4\rho^2L^5}\right)^{1/3}\right)$$

3. Experiment

determine for different amplitudes the critical frequency (acceleration)

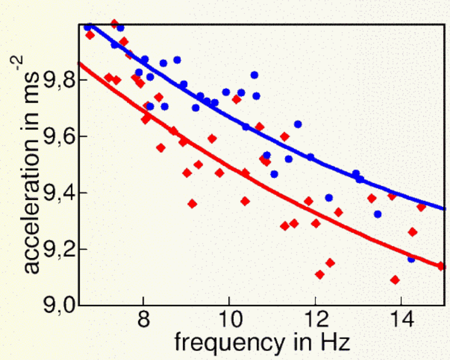


1 sphere → systematic errors

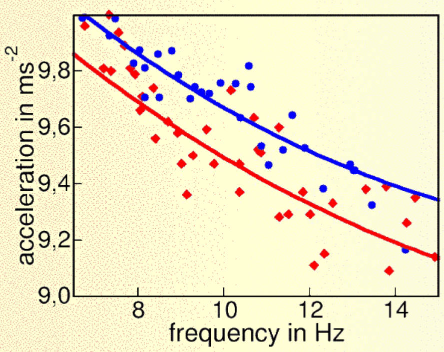
20 spheres

same data in different representation together with best fit

$$f(\omega) = \left(a_2\omega^2 + a_1\omega + a_0\right)^{-1}$$



3. Experiment



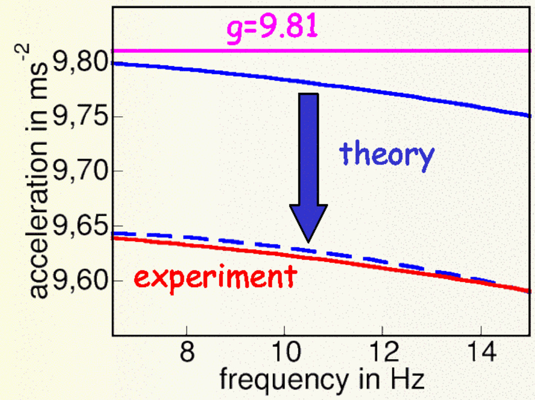
1 sphere

20 spheres

Eliminate systematic errors by dividing both curves.

amplification factor:

$$\frac{\left(a_2\omega^2+a_1\omega+a_0\right)^{-1}}{\left(a_2\omega^2+a_1\omega+a_0\right)^{-1}}$$



Summary

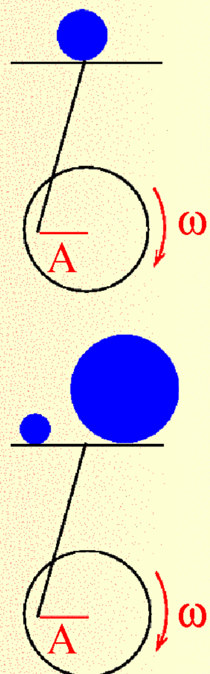
- fluidization implies separation of particles
- column of viscoelastic spheres:
  analytical: There is always a region of amplification for the driving frequency
  MD: agreement
  Experiment: agrees too
- The effect has not been confirmed for real (3D) granular systems
- There are MD simulations which describe fluidization for $A\omega^2 < g$

Taguchi, PRL 69, 1367 (93); EPL 24, 203 (93)
Luding et al., PRE 50, 3100 (94)
Barker, Mehta, PRA 45, 3435 (92)
Gallas, Herrmann, Sokolowski, J. Physique II 2, 1389 (92)

Claim: $\Gamma \equiv A\omega^2/g > 1$ is insufficient to describe the onset of fluidization (and possibly other effects too). Instead, we propose

$$\Gamma \equiv A\omega^2/g > R^{-1} \quad \text{with} \quad R^{-1} = R^{-1}(\omega)$$

Comment 1: Does Γ describe a single particle?



particle starts to jump at

$A\omega^2 < g \quad A\omega^2 = g \quad A\omega^2 > g$

Large particle jumps for smaller frequency.

Reason: Finite size body behaves like a non-linear oscillator, even if the material is linear.

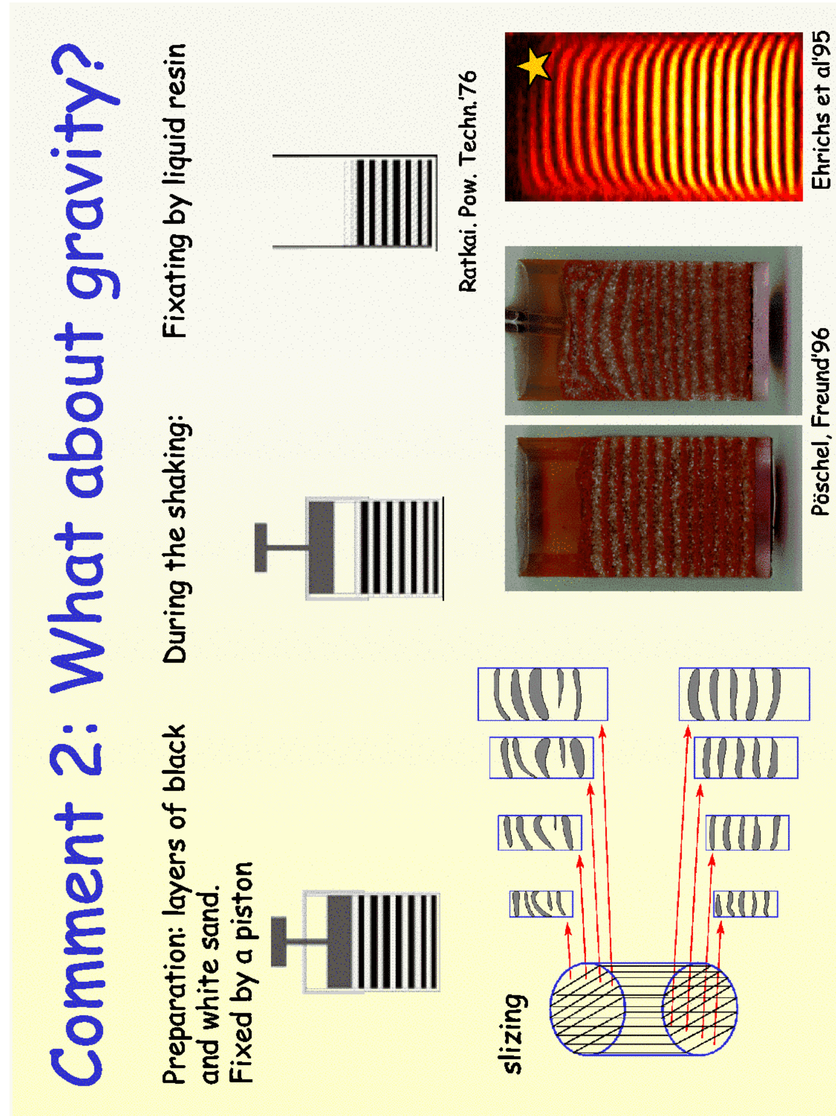
Comment 2: What about gravity?

Both condition for the onset of fluidization depend on the value of gravity:

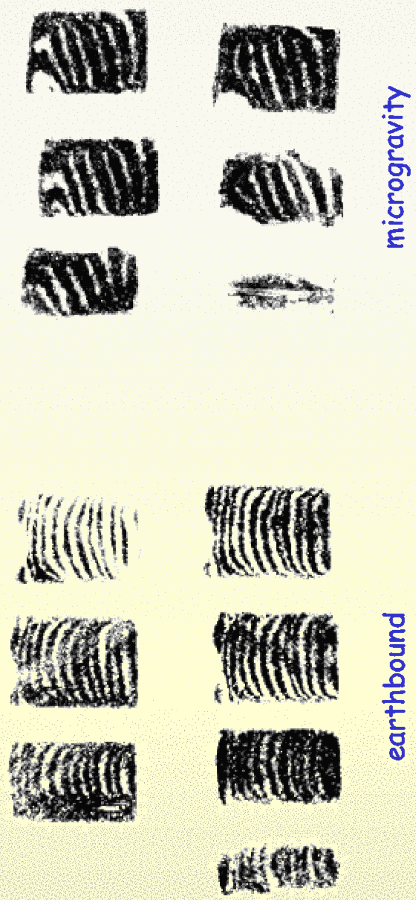
$$\frac{A\omega^2}{g} > 1 \qquad \frac{A\omega^2}{g} > 1 - B_2\omega^2 + B_4\omega^4$$

What do we expect in the absence of gravity ?


Comment 2: What about gravity?
20 SECONDS/PARABOLA
20 PARABOLA/HOUR
2 HOURS/FLIGHT
3 FLIGHTS/CAMPAIGN
ZERO G
PARABOLA
ZERO G


Comment 2: What about gravity?
Preparation: layers of black and white sand.
Fixed by a piston
During the shaking:
Fixating by liquid resin
Ratkai. Pow. Techn.'76
Pöschel, Freund'96
Ehrichs et al'95
slizing

Comment 2: What about gravity?



$$\frac{A\omega^2}{g} << 1$$

Pöschel, Freund'96

Comment 2: What about gravity?