Anisotropy in dense granular flows

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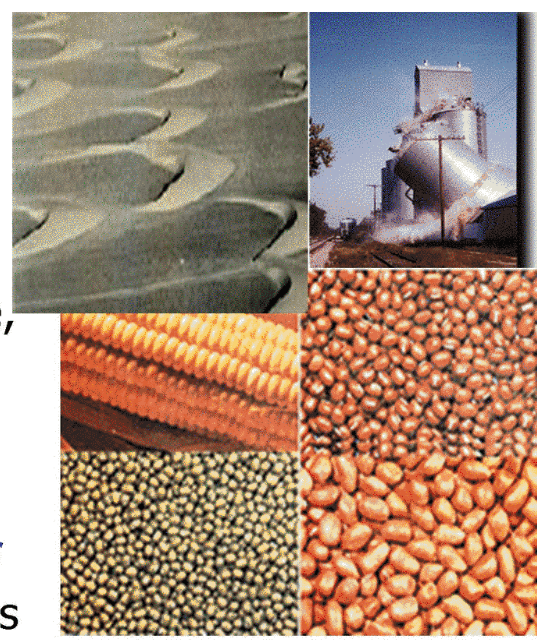
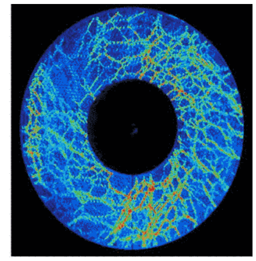
Granular Materials



Real:
- grain, rice, lentils,
- powder, pills, granulate,
- sand, soil, rock,
- cement, concrete, …

Model Granular Materials
- steel/aluminum spheres
- spheres(disks) with dissipation/friction (cohesion)

Why Granular Materials



Numberless applications:

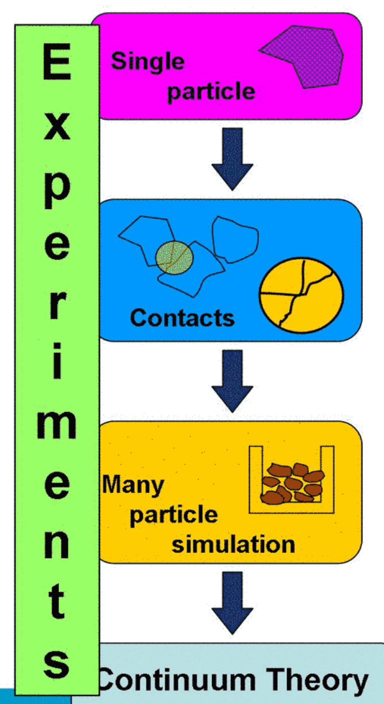
- constructions, industry(silos), agriculture
- everyday life (e.g. coffee powder, sugar, salt, …)

Challenges for Statistical Pysics and Materials- & Comput. Science and Engineering

- many particle systems, non-linear, non-equilibrium
  - localization (shearbands)
  - force chains (wide distributions)
  - damage, segregation, …

Contents

- Introduction
- Particle contacts/interactions
- Many particle simulation
- Continuum theory
- Sintering/Cementation
- Conclusion



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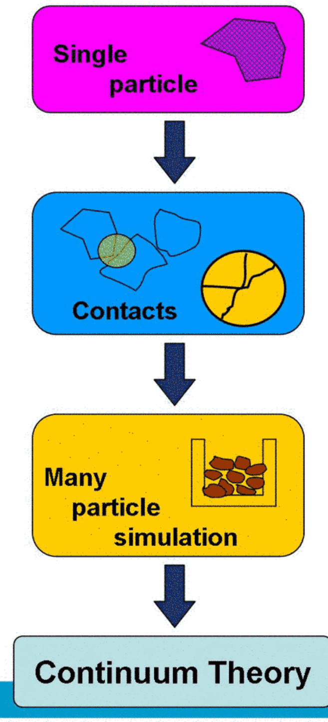
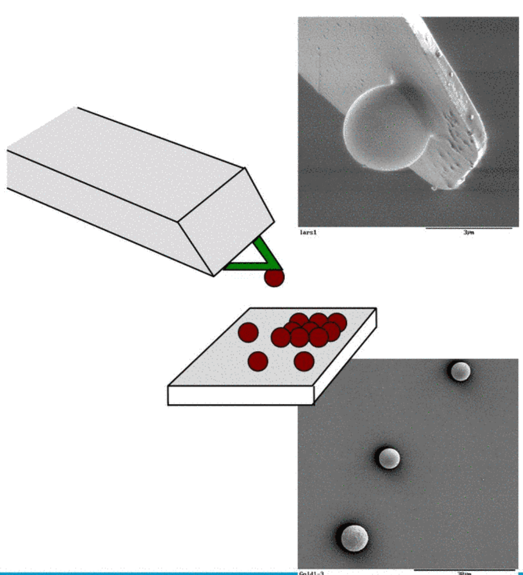
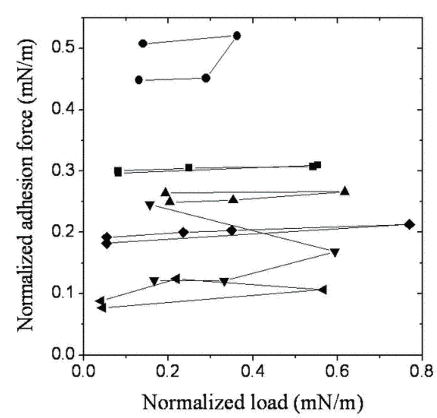
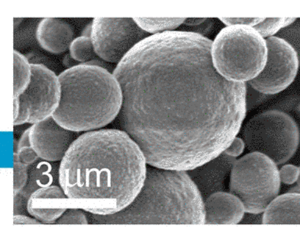


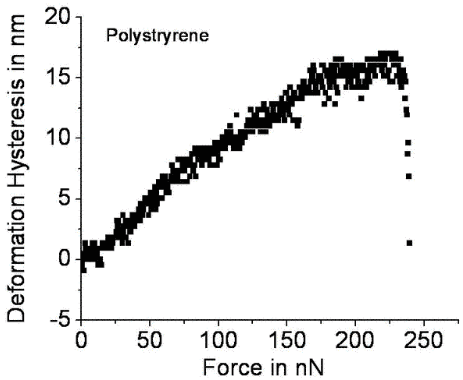
Contact force measurement (PIA)

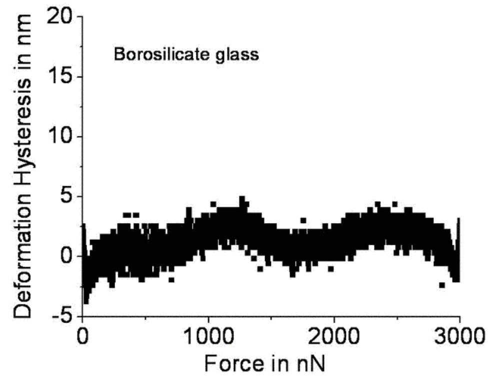






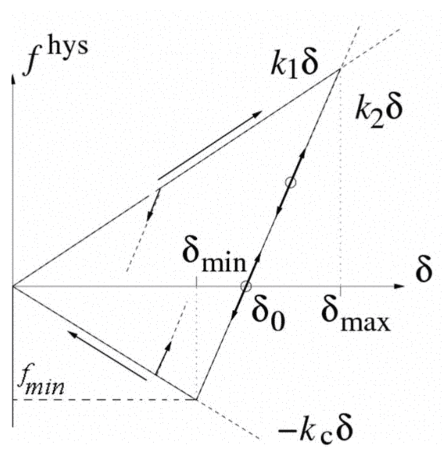
Hysteresis (plastic deformation)





MPI-Polymer Science (Butt)
Contact properties via AFM

Contact model



- (too) simple ☺
- piecewise linear
- easy to implement

$$f_i^{hys} = \begin{cases} k_1\delta & \text{for loading} \\ k_2(\delta-\delta_0) & \text{for un-/reloading} \\ -k_c\delta & \text{for unloading} \end{cases}$$

Maximum overlap δ_{max}

stress-free overlap $\delta_0 = (1 - k_1/k_2)\delta_{max}$

strongest attraction at: $\delta_{min} = \frac{k_2 - k_1}{k_2 + k_c}\delta_{max}$

the max. attractive force: $f_{min} = -k_c\delta_{min}$

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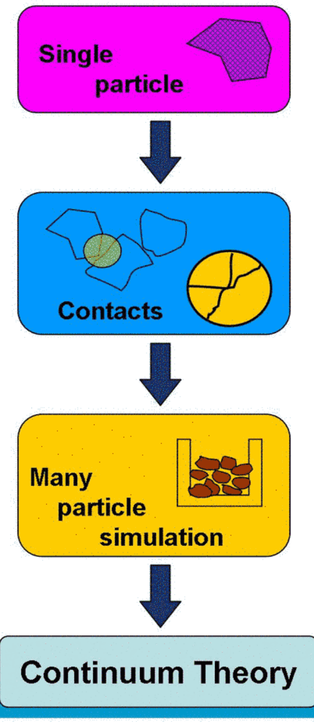
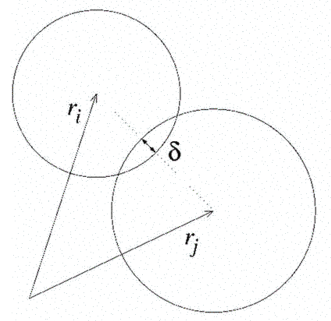


Method and Why ?

- `Many particle' simulations work for small systems only ($10^4 – 10^6$)
- Industrial scale applications rely on FEM
- FEM relies on continuum mechanics + constitutive relations
- `Micro-macro' can provide those !
- Homogenization/Averaging

Discrete particle model



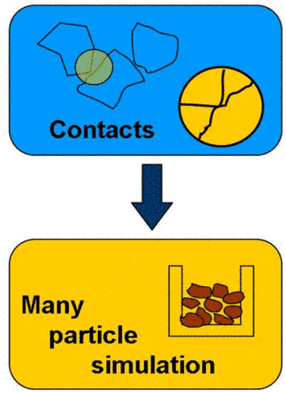
Equations of motion

$$m_i \frac{d^2\vec{r}_i}{dt^2} = \vec{f}_i \qquad I_i \frac{d\vec{\omega}_i}{dt} = \vec{t}_i$$

Forces and torques:

$$\vec{f}_i = \sum_c \vec{f}_i^{\,c} + \sum_w \vec{f}_i^{\,w} + m_i g$$

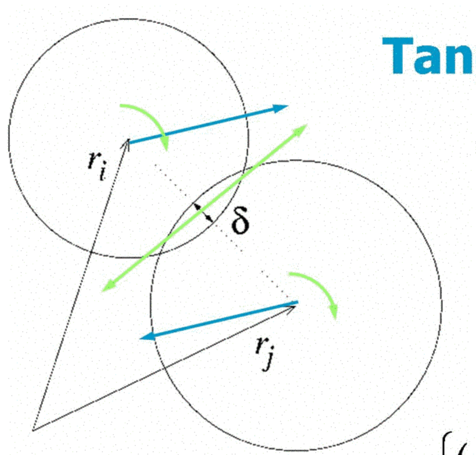
$$\vec{t}_i = \sum_c \vec{r}_i^{\,c} \times \vec{f}_i^{\,c}$$



Overlap $\delta = \frac{1}{2}(d_i + d_j) - (\vec{r}_i - \vec{r}_j)\cdot\vec{n}$

Normal $\hat{n} = \vec{n}_{ij} = \frac{(\vec{r}_i - \vec{r}_j)}{|\vec{r}_i - \vec{r}_j|}$

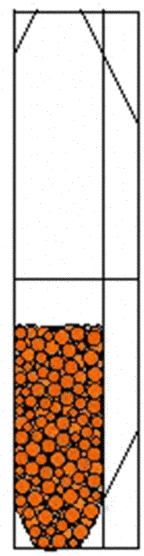
Tangential contact model



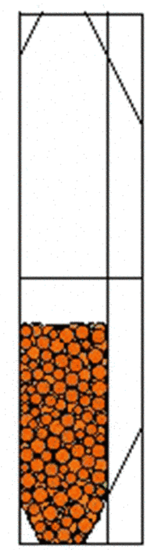
Sliding contact points:
- Static Coulomb friction
- Dynamic Coulomb friction

$$v_t = \begin{cases} (v_i - v_j)^t + \hat{n}\times(a_i\omega_i + a_j\omega_j) & \text{sliding} \\ \hat{n}\times(b_i\omega_i - b_j\omega_j) & \text{rolling} \\ \hat{n}\hat{n}\cdot(c_i\omega_i - c_j\omega_j) & \text{torsion} \end{cases}$$

Silo Flow with friction



$\mu = 0.5$



$\mu = 0.5$

$\mu_r = 0.2$

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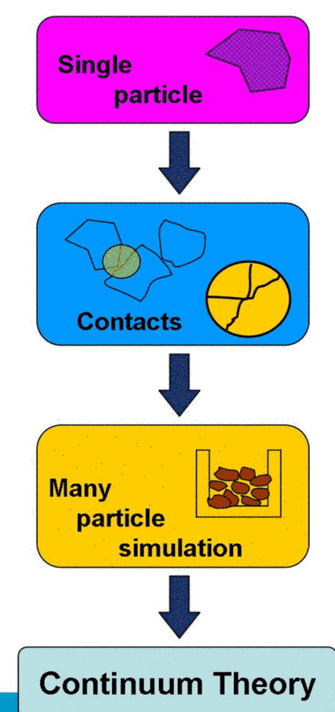
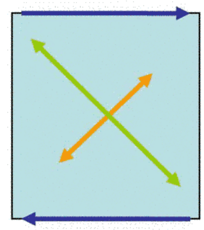


Anisotropy ↔ Shear ?

- Simple shear

$$\varepsilon = \begin{pmatrix} 0 & 2\varepsilon_s \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & \varepsilon_s \\ -\varepsilon_s & 0 \end{pmatrix} + \begin{pmatrix} 0 & \varepsilon_s \\ \varepsilon_s & 0 \end{pmatrix}$$

Rotation + symmetric shear



Anisotropy ↔ Shear ?

- Simple shear

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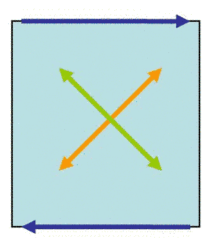
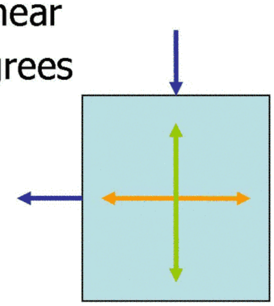
Rotation + symmetric shear



- Rotate symmetric shear tensor by 45 degrees

$$R_{45} \cdot \begin{pmatrix} 0 & \varepsilon_s \\ \varepsilon_s & 0 \end{pmatrix} \cdot R_{45}^T = \begin{pmatrix} \varepsilon_s & 0 \\ 0 & -\varepsilon_s \end{pmatrix}$$



- Biaxial "shear": compression+extension

Biaxial box set-up

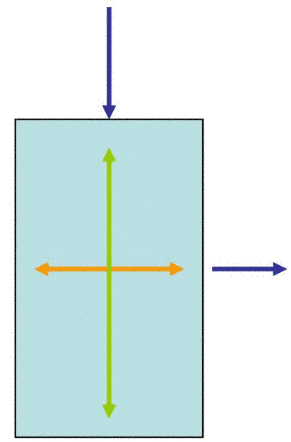
- Top wall: strain controlled

$$z(t) = z_f + \frac{z_0 - z_f}{2}\left(1 + \cos \omega t\right)$$

- Right wall: stress controlled

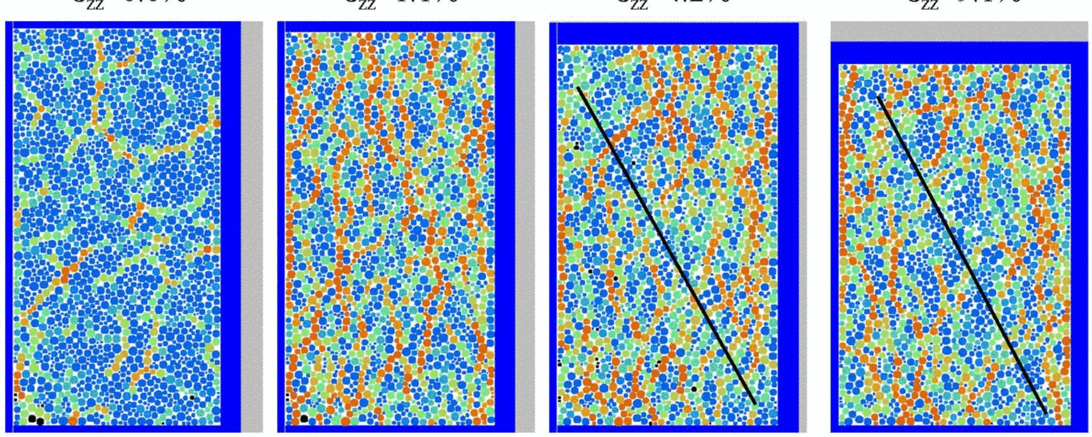
$$p = \text{const.}$$

- Evolution with time … ?



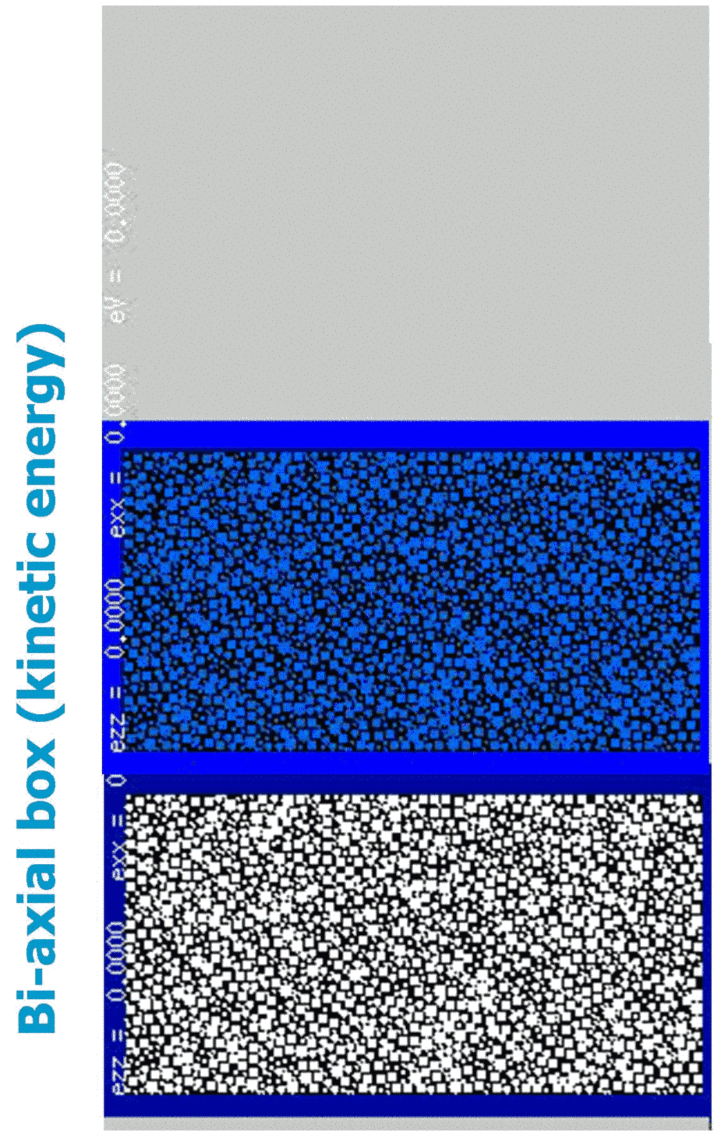
Simulation results (closer look)

ε_{zz}=0.0% ε_{zz}=1.1% ε_{zz}=4.2% ε_{zz}=9.1%

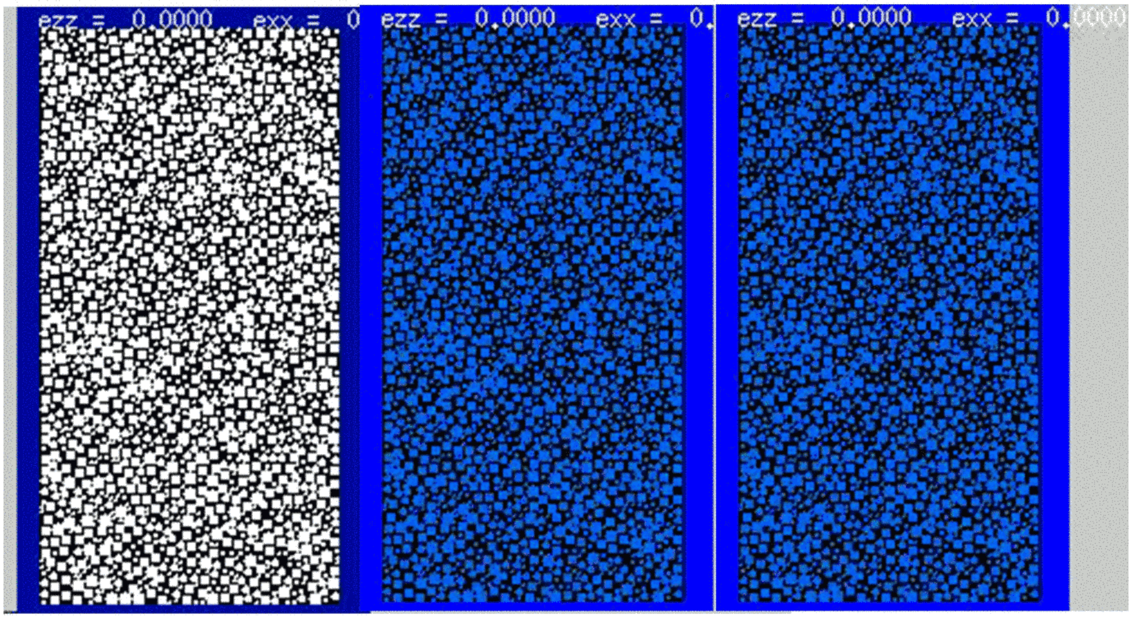


shear band localization

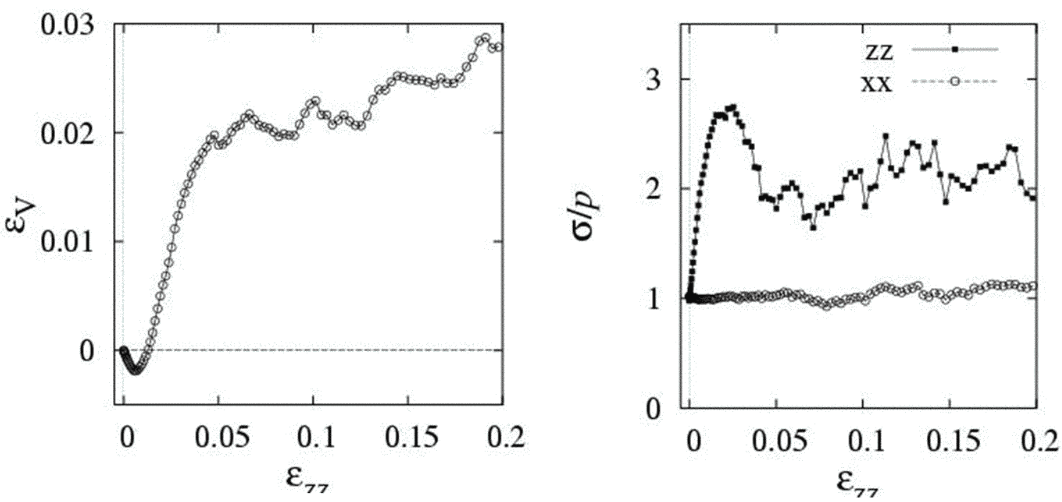

Bi-axial box (stress chains)


Bi-axial box (kinetic energy)

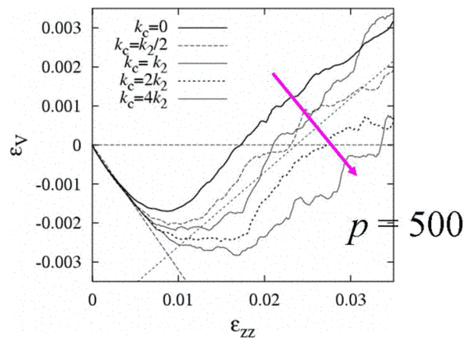
Bi-axial box (rotations)

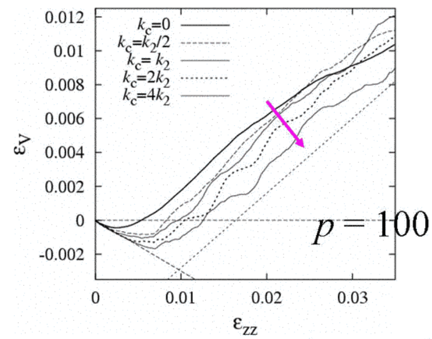


Bi-axial compression with p_x=const.



Material parameters cohesion





Initial Compression:

$$\frac{\varepsilon_v}{\varepsilon_{zz}} = \tan^{-1}(1-2\nu)$$

Poisson-ratio: $\nu \approx 0.66$

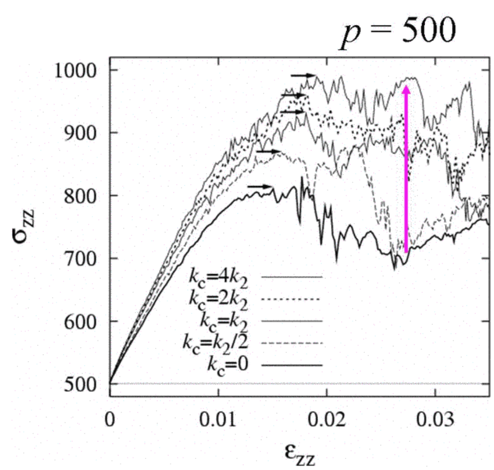
Dilatancy: $d' = \tan^{-1}\left(\frac{2\sin\psi}{1-\sin\psi}\right)$

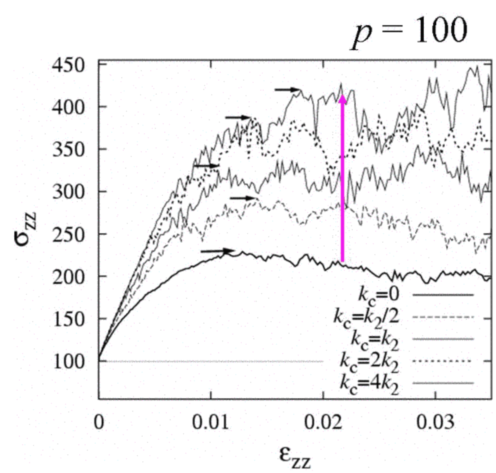
Dilatancy Angle:

$\psi \approx 0.088$ **for** $p = 500$

$\psi \approx 0.190$ **for** $p = 100$

Modulus and yield stress cohesion

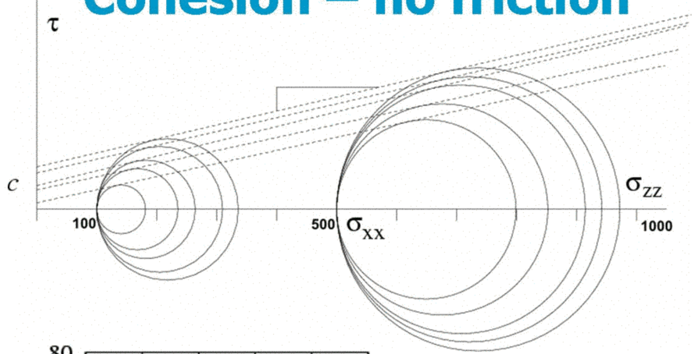




Modulus
(initial slope)

Yield Stress ….
(peak value)

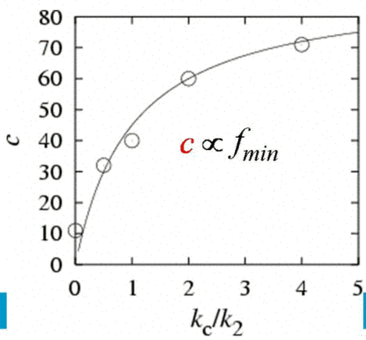
Cohesion – no friction



$k_c / k_2 = 0, 1/2, 1, 2,$ and 4

geometrical friction angle

$\phi \approx 13^{\circ}$



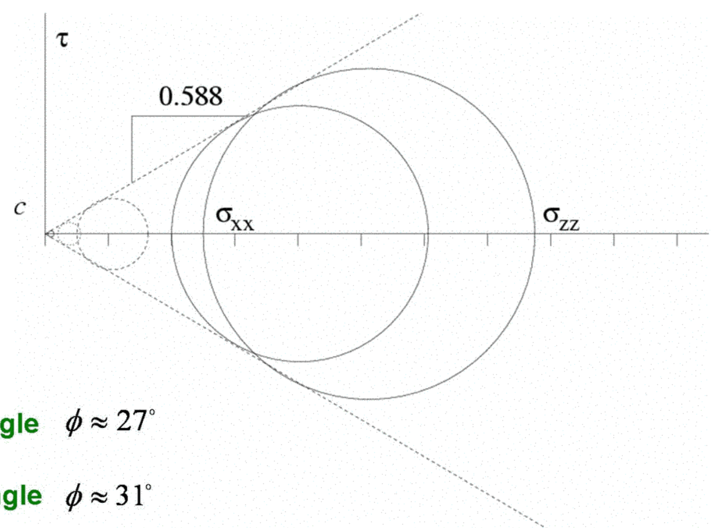
macro cohesion

$c = 1.8 \times 10^{-3} \frac{1 - k_1/k_2}{1 + k_2/k_c}$

k_c/k_2	p_x	σ_{zz}	p_x	σ_{zz}	c
0	100	183	500	798	11
½	100	234	500	853	32
1	100	264	500	915	40
2	100	310	500	941	60
4	100	336	500	972	71

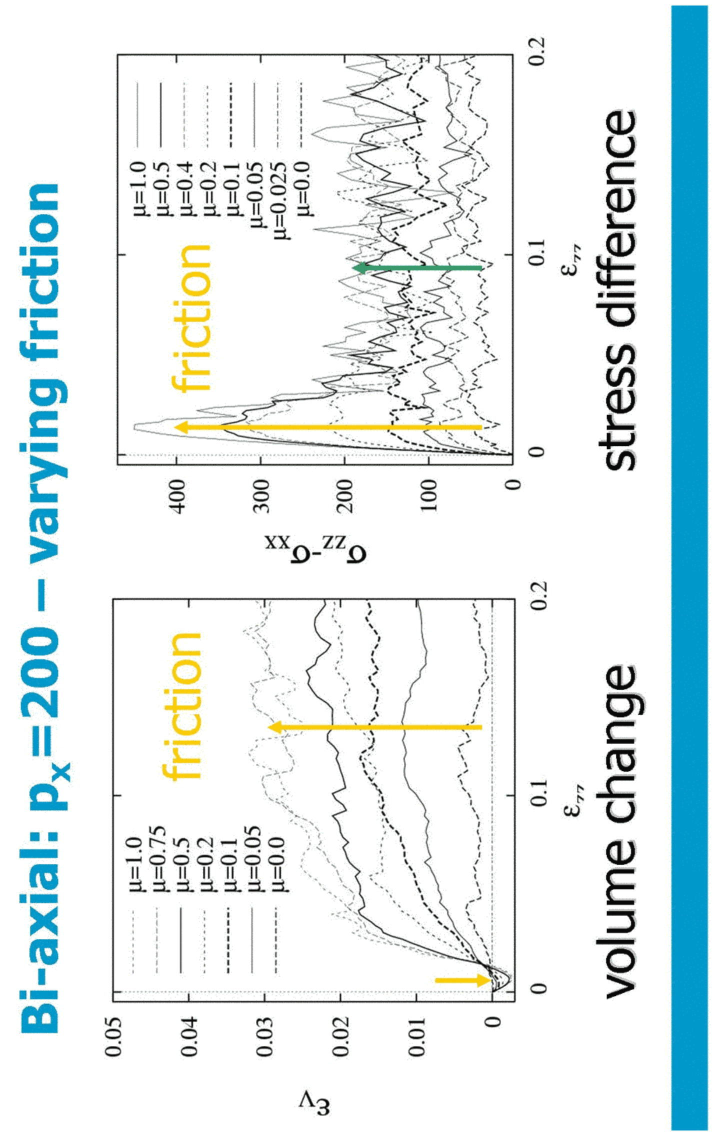
Friction – no cohesion

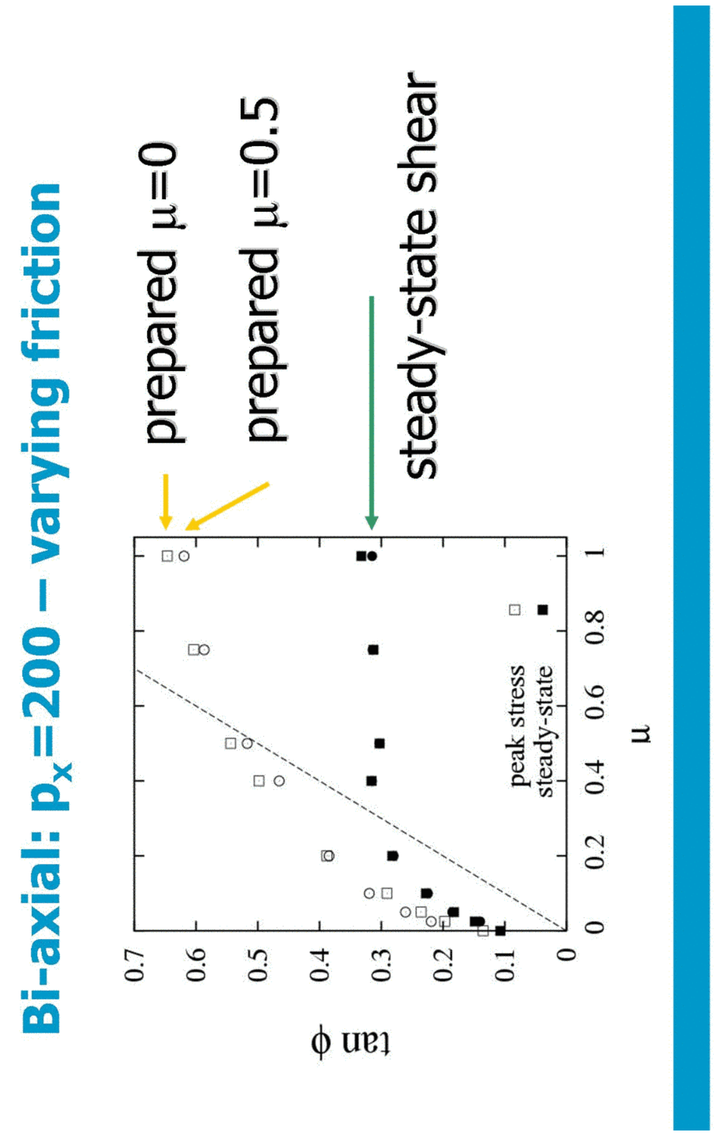
$k_c = 0$ and $\mu = 0.5$



Internal friction angle $\phi \approx 27^{\circ}$

Total friction angle $\phi \approx 31^{\circ}$


Bi-axial: p_x=200 – varying friction
friction
μ=1.0
μ=0.75
μ=0.5
μ=0.2
μ=0.1
μ=0.05
μ=0.0
ε_V
ε_zz
volume change
friction
μ=1.0
μ=0.5
μ=0.4
μ=0.2
μ=0.1
μ=0.05
μ=0.025
μ=0.0
σ_zz-σ_xx
ε_zz
stress difference


Bi-axial: p_x=200 – varying friction
prepared μ=0
prepared μ=0.5
steady-state shear
peak stress
steady-state
tan φ
μ

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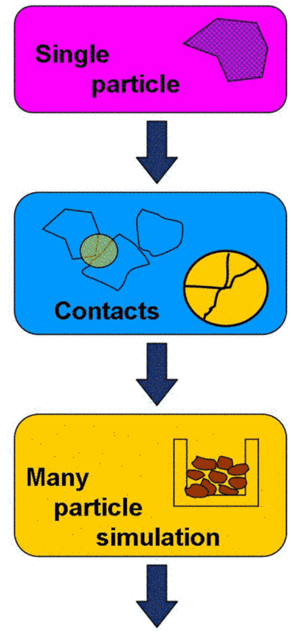
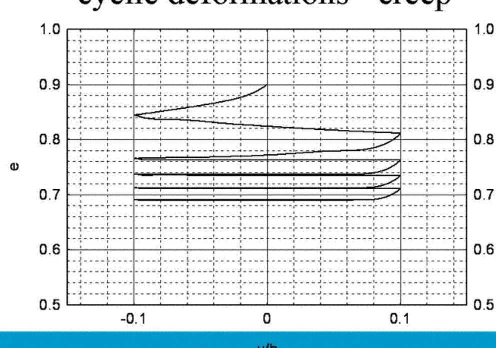


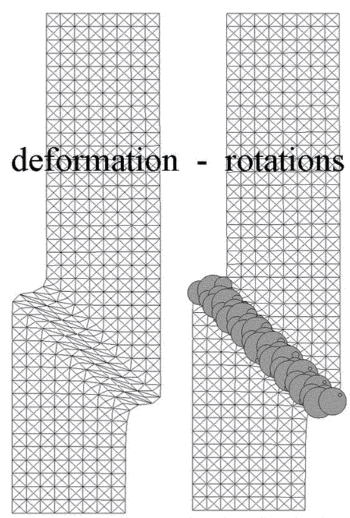
Continuum Theory

Hypoplastic FEM model

J. Tejchman et al. (2003)
+ successful tool – few parameters
- microscopic foundations ?
- extensions & parameter identification

cyclic deformations - creep





Continuum Theory

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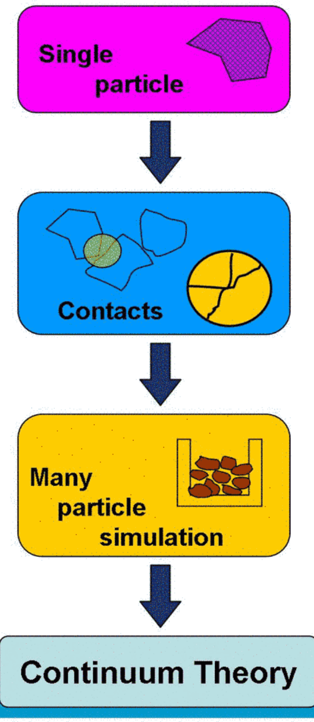
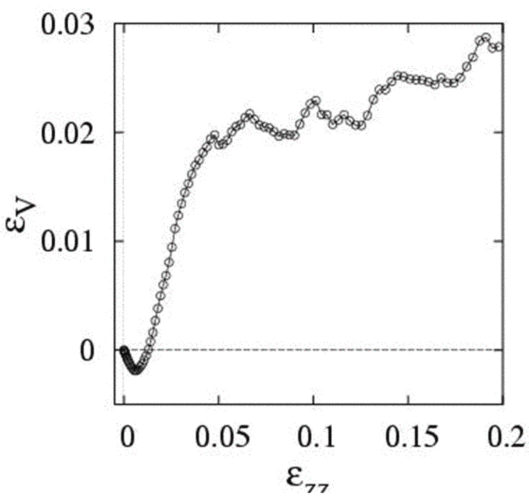
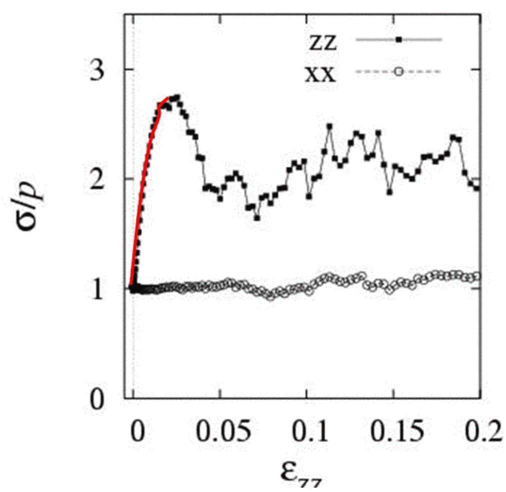


An-isotropy



in stress



An-isotropy (Stress)

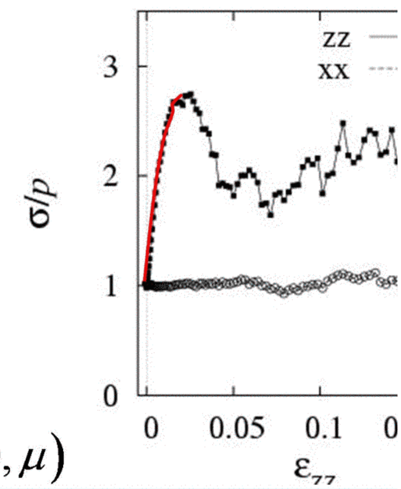
- Stress: Isotropic: $\mathrm{tr}\,\boldsymbol{\sigma}$, and deviatoric: $\mathrm{dev}\,\boldsymbol{\sigma} = \sigma_{zz}-\sigma_{xx}$
  - Minimal eigenvalue: σ_{xx}
  - Maximal eigenvalue: σ_{zz}
- Dev. Stress fraction $s_D = \mathrm{dev}\,\boldsymbol{\sigma} / \mathrm{tr}\,\boldsymbol{\sigma}$

$$\frac{\partial}{\partial \varepsilon_D} s_D = \beta_s \left(s_{\max} - s_D \right)$$

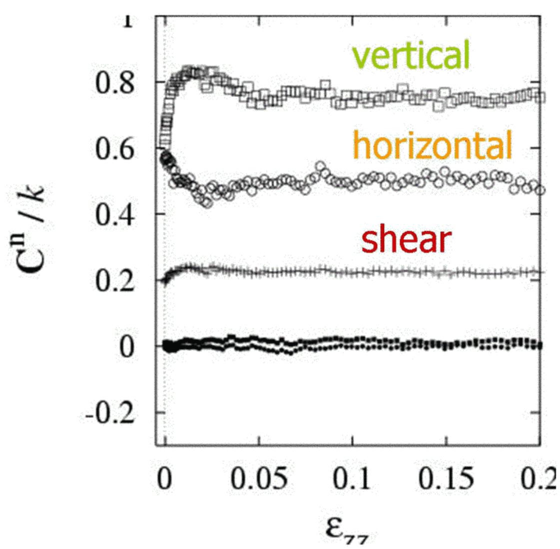
- Exponential approach to peak

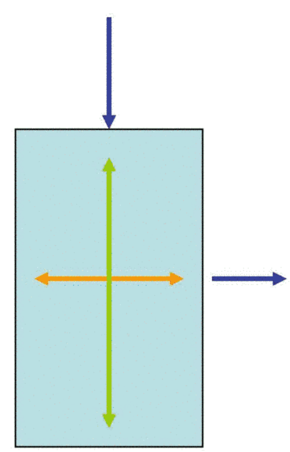
$$1 - s_D / s_{\max} = \exp\left(-\beta_s \varepsilon_D\right)$$

$\beta_s\left(\rho, p, \mu\right)$



Stiffness tensor



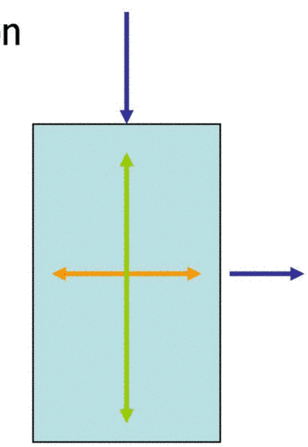


Different moduli:
- against shear C_2
- perpendicular C_1
- *one* shear modulus

An-isotropy (Structure)

- Structure changes with deformation
- Different stiffness:
  - More stiffness against shear C_2
  - Less stiffness perpendicular C_1
- One (only?) shear modulus
- Anisotropy $A = C_2 - C_1$ evolution

$$\frac{\partial}{\partial \varepsilon_D} A = \beta_F \left(A_{\max} - A \right)$$



- Exponential approach to maximal anisotropy

... see Calvetti et al. 1997

An-isotropy

- Three different stiffness entries:
  - Vertical stiffness $C_{zzzz} =: C_2$
  - Horizontal stiffness $C_{xxxx} =: C_1$
  - Shear modulus $C_{xxzz} =: G$
- Isotropic compression modulus: $2E := C_2 + C_1 + 2G$
- Anisotropic modulus $2A := C_2 - C_1$
- Deviatoric modulus $2B := C_2 + C_1 - 2G$

Constitutive model – scalar

(in the biaxial box eigen-system)

$$\delta\sigma_V = E\varepsilon_V + A\varepsilon_D$$

$$\delta\sigma_D = A\varepsilon_V + B\varepsilon_D$$

Constitutive model – isotropic, scalar

(in the biaxial box eigen-system)

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Constitutive model – scalar

(in the biaxial box eigen-system)

$$\delta\sigma_V = E\varepsilon_V + A\varepsilon_D$$

$$\delta\sigma_D = A\varepsilon_V + B\varepsilon_D$$

Constitutive model – tensorial

(arbitrary eigen-system)

$$\delta\sigma_V = E\varepsilon_V + A\varepsilon_D \cos\left(2\phi_\varepsilon - 2\phi_C\right)$$

$$\delta\boldsymbol{\sigma}_D = A\varepsilon_V \hat{\mathbf{D}}\left(\phi_C\right) + \frac{E}{2}\varepsilon_D \hat{\mathbf{D}}\left(\phi_\varepsilon\right) + B_2\varepsilon_D \hat{\mathbf{D}}\left(\phi_\varepsilon - 2\phi_C\right)$$

Constitutive model – isotropic, scalar

(in the biaxial box eigen-system)

$$\delta\sigma_V = E\varepsilon_V + A\varepsilon_D$$

$$\delta\sigma_D = A\varepsilon_V + B\varepsilon_D$$

Constitutive model – isotropic, tensorial

(arbitrary eigen-system)

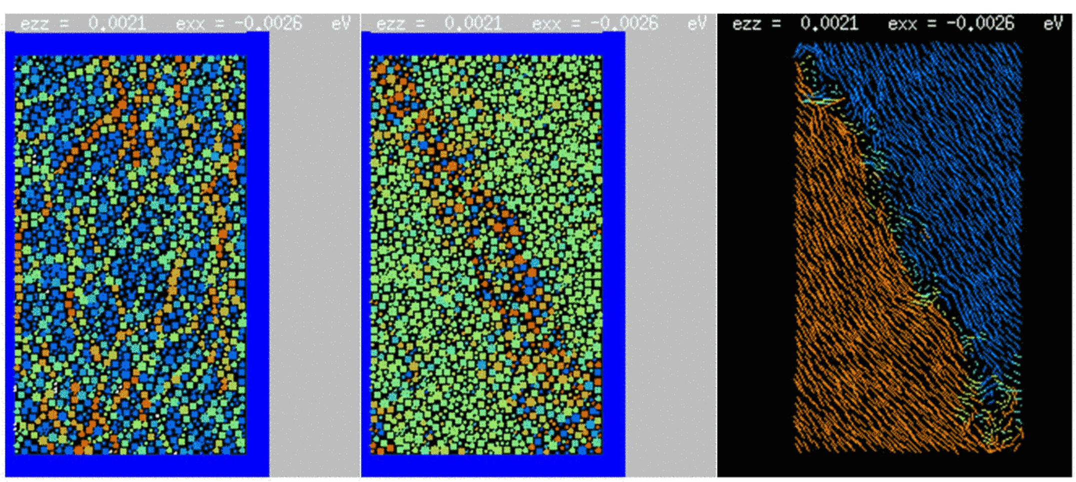
$$\delta\sigma_V = E\varepsilon_V + A\varepsilon_D \cos\left(2\phi_\varepsilon - 2\phi_C\right)$$

$$\delta\boldsymbol{\sigma}_D = A\varepsilon_V \hat{\mathbf{D}}\left(\phi_C\right) + B\varepsilon_D \hat{\mathbf{D}}\left(\phi_\varepsilon\right) + B_3\varepsilon_D \hat{\mathbf{D}}\left(\phi_\varepsilon - 2\phi_C\right)$$

Open questions

- Quantitative experimental verification
- Main challenge
  - micro-macro transition & constitutive modeling
- Collective flow behavior (rheology)
  - Anisotropy
  - Size distributions / Composition
  - Friction/Cohesion/Shape effects
  - Micro-polar (rotations) continuum theory

Micro informations: shear bands



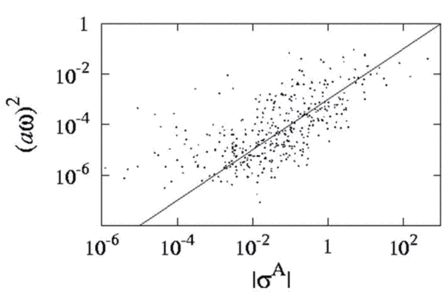
potential energy rotations displacements

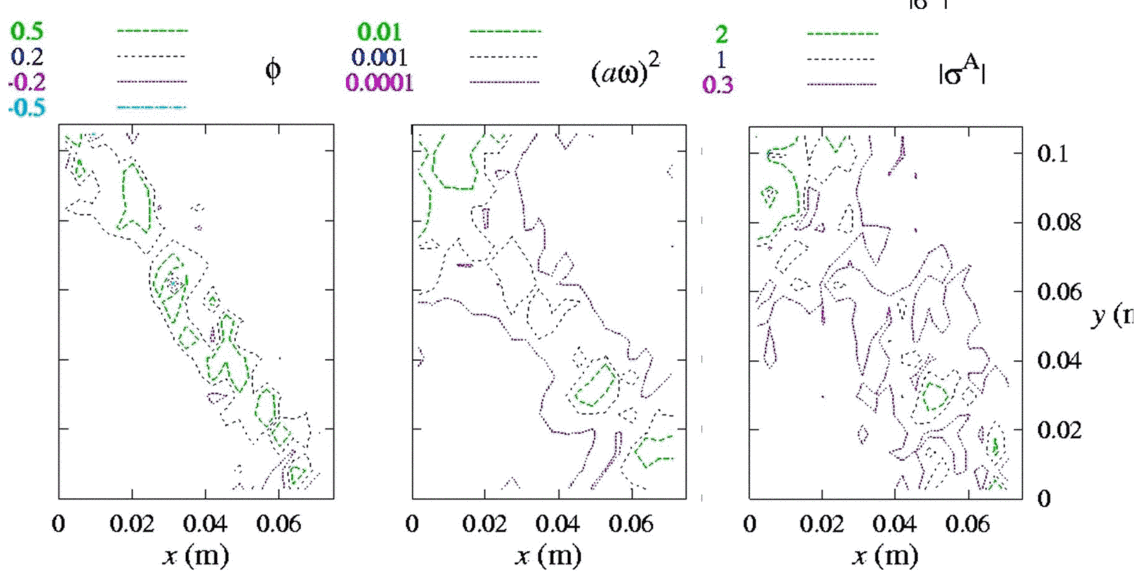
Global average vs. Local average

- Global
  - + Experimentally accessible data
  - - Wall effects
  - - Averaging over inhomogeneities
- Local
  - - Difficult to compare to experiment
  - + Averages away from the walls
  - + Average over `similar' volume elements

Rotations (local)

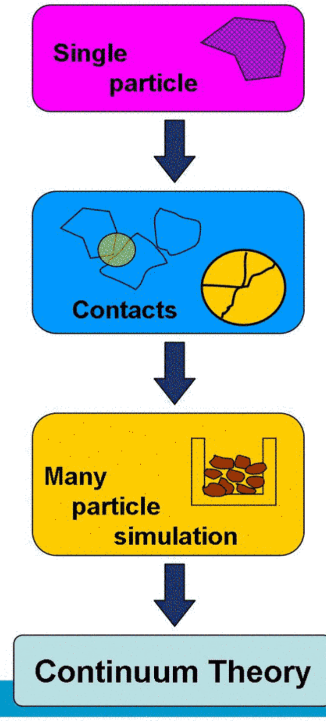
Direction, amplitude,
anti-symmetric (!) stress





Contents

- Introduction
- Particle contacts/interactions
- Many particle simulation
- Continuum theory
- Sintering/Cementation
- Conclusion



Summary

Micro-**macro** transition:

- **moduli** (bulk, shear, anisotropic)
- **cohesion** (max. adhesion force)
- **friction** f (contact friction)

Anisotropy … (arbitrary orientation)

- *exponential approach to max.*
- valid for stress & A (=deviator fabric)

… still to be understood:

What is the max. stress/fabric anisotropy?

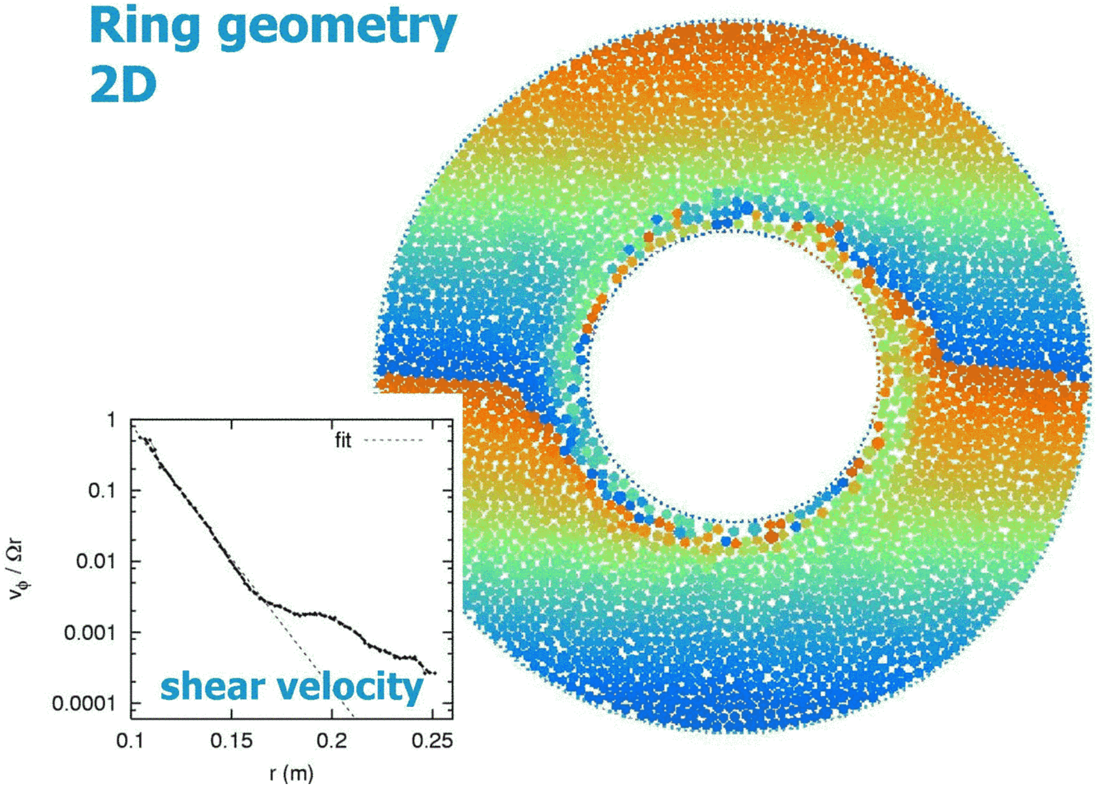
Summary

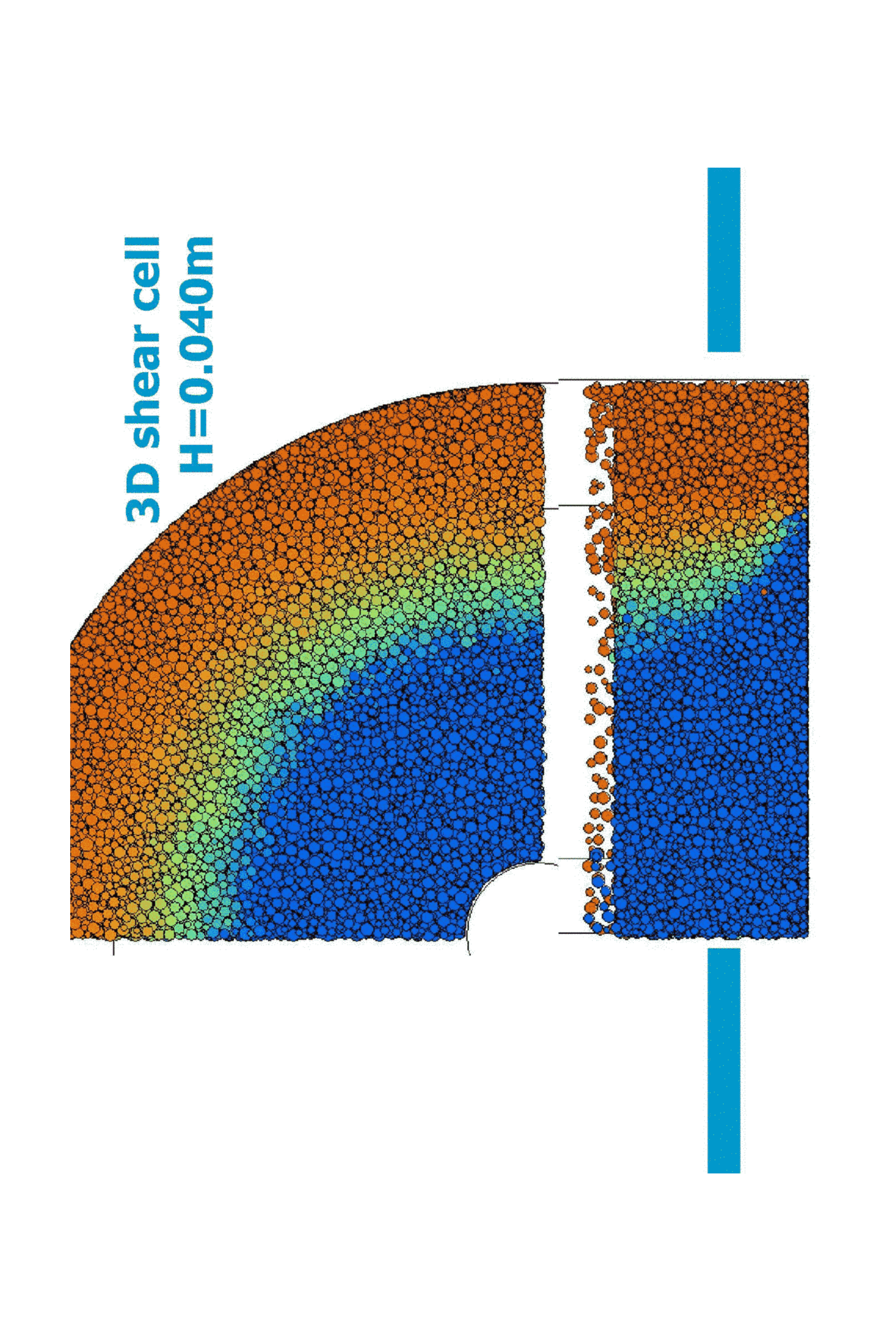
Granular media are:
- compressible + dilatant
- inhomogeneous (force-chains)
- (almost always) an-isotropic
- micro-polar (rotations)

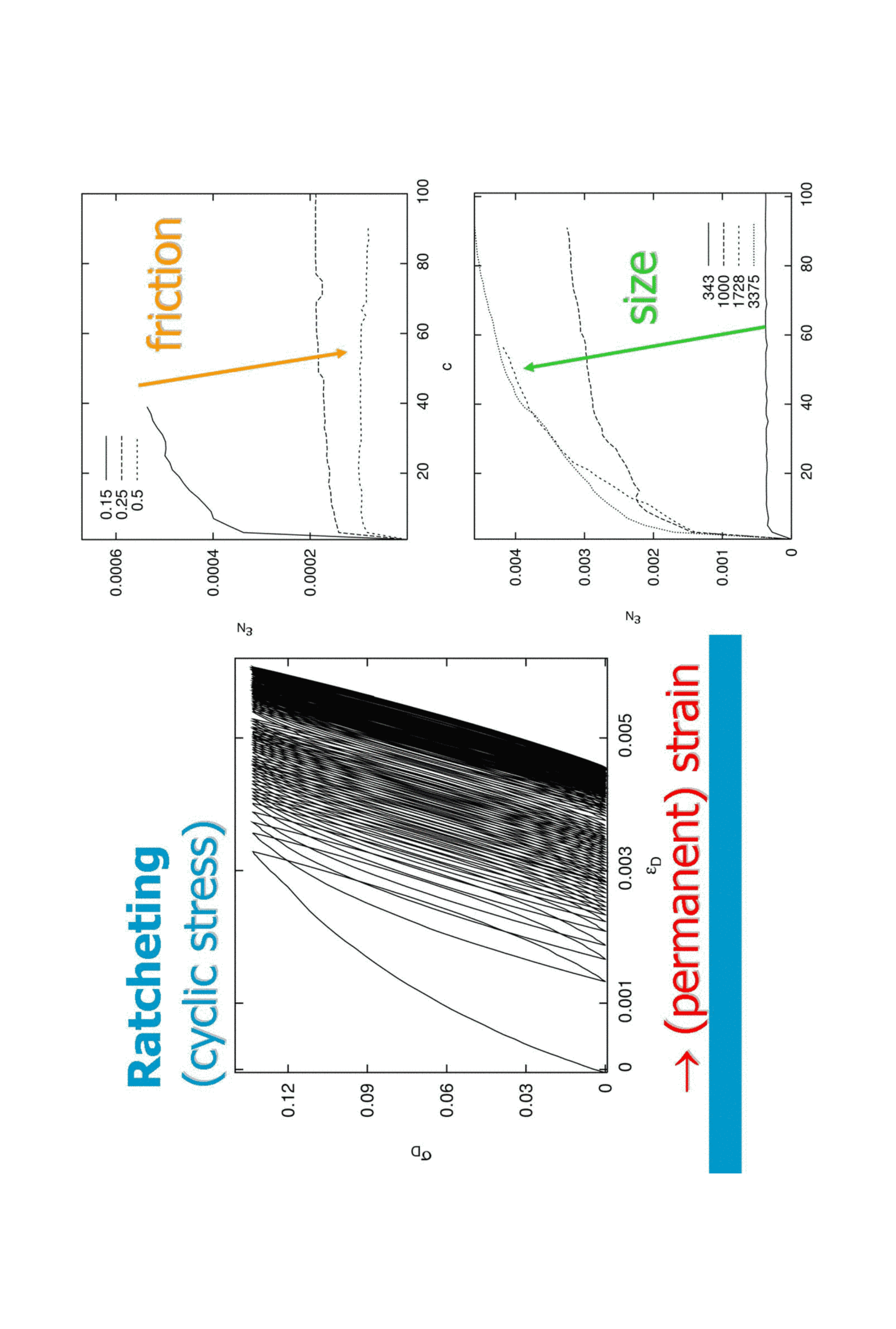
Granular media are …

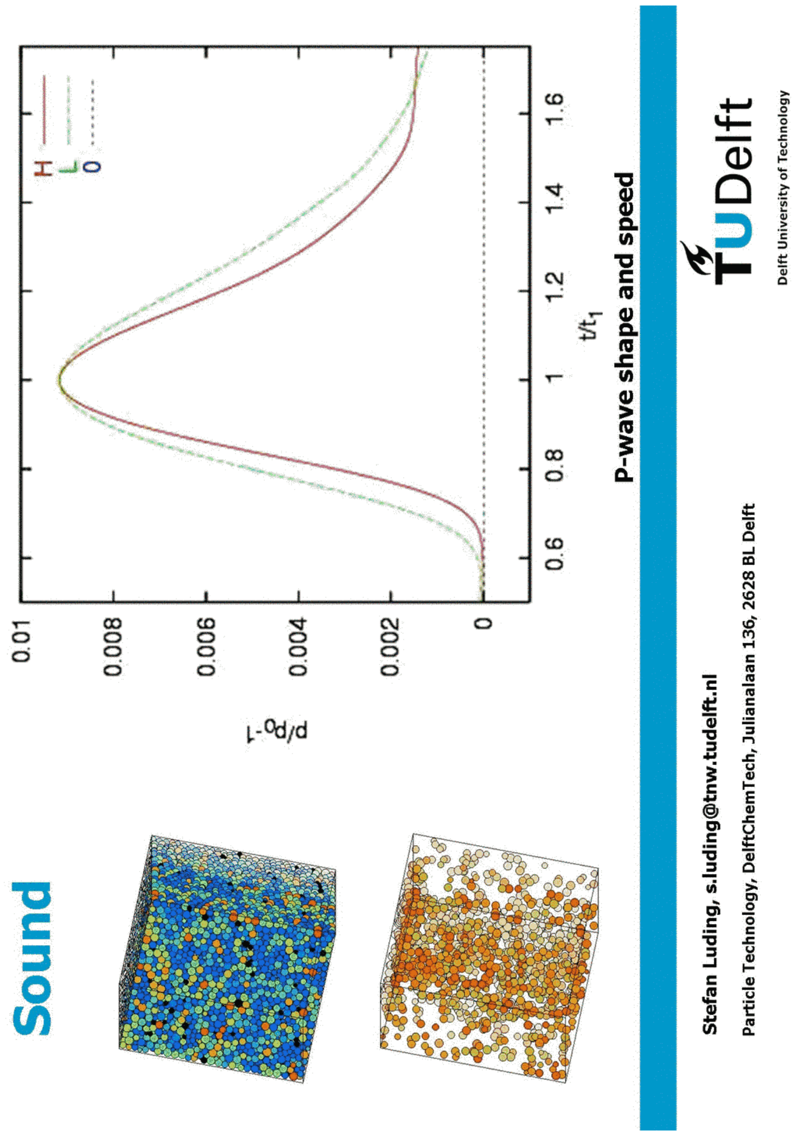
… still interesting

Ring geometry 2D




3D shear cell
H=0.040m


friction
0.15
0.25
0.5
size
343
1000
1728
3375
c
N3
Ratcheting
(cyclic stress)
→ (permanent) strain
σD
εD


Sound
P-wave shape and speed
t/t1
p/p0-1
H
L
0
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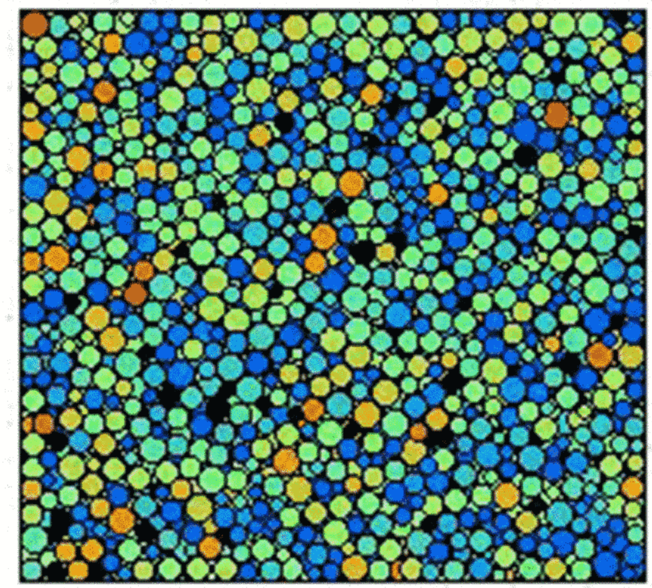

The End