Granular flows down inclines with and without side walls

Michel Louge

with Nicolas Taberlet, Alexandre Valance, Patrick Richard, Renaud Delannay, James Jenkins, Stephen Keast, and Dana Smith.

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Kavli Institute for Theoretical Physics

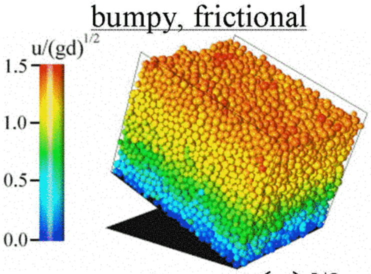
Cornell University

IFPRI
International Fine Particle Research Institute

NASA

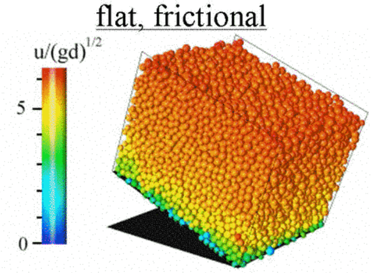
Effects of bottom boundary

bumpy, frictional



$$\dot{m}^\dagger \equiv \frac{\dot{m}/W}{\rho_s d\sqrt{gd}} \propto \left(\frac{h}{d}\right)^{5/2} \bar{\nu}$$

flat, frictional

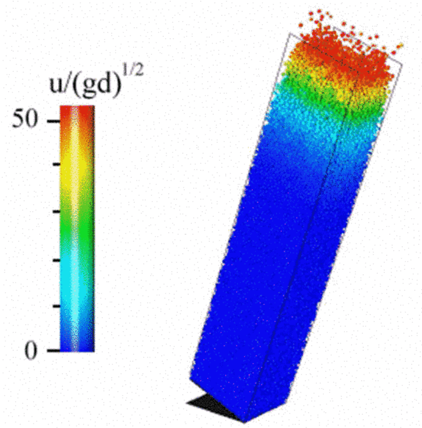


$$\dot{m}^\dagger \equiv \frac{\dot{m}/W}{\rho_s d\sqrt{gd}} \propto \left(\frac{h}{d}\right)^{3/2} \bar{\nu}^{3/2}$$

material density ρ_s, diameter d, volume fraction ν, flow thickness h

Taberlet, et al.: J. of Phys. Cond. Mat. 2005

Effects of frictional sidewalls



sidewall stabilized heap

Taberlet, et al. PRL 91, 264301 (2003)

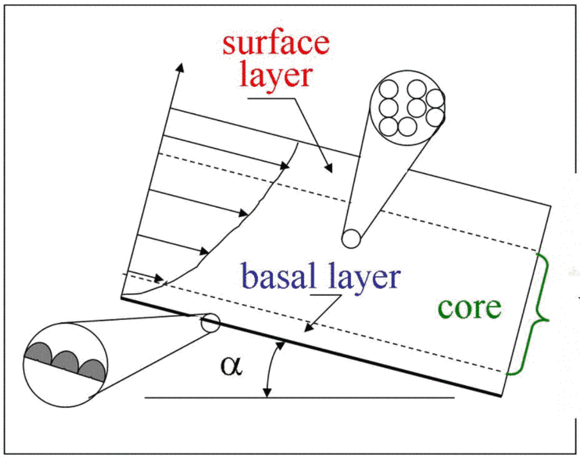
$$\dot{m}' \equiv \frac{\dot{m}}{\rho_s W^2 \sqrt{gW}} \propto \left(\frac{h}{W} - \frac{h_{stop}}{W}\right)$$

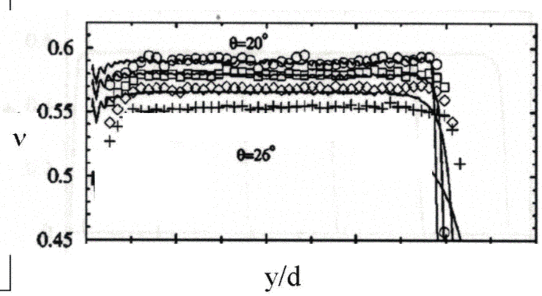
sidewall separation W

Taberlet, et al.: J. of Phys. Cond. Mat. 2005

Slow, shallow, steady, fully-developed flows down a bumpy incline without side walls

Bumpy base, no side walls





Silbert, et al.: *Physical Review E* **64**, 051302-1 (2001)
Olivier Pouliquen: *Physics of Fluids* **11**, 542-548 (1999)

Energy balance in the core - kinetic theory

$$0 = -\frac{dq}{dy} + S_I \frac{du}{dy} - \gamma$$

heat flux shear work dissipation

$$\gamma = f_3(\nu)\rho_s T^{3/2}/d$$

$$S_I = f_1(\nu)\rho_s d\sqrt{T}\frac{du}{dy}$$

$$N_I = f_4(\nu)\rho_s T$$

$$q = -\kappa\frac{dT}{dy}$$

$$\kappa_I = f_2(\nu)\rho_s d\sqrt{T}$$

Incorporate momentum balance

$$N = N_I \approx \rho_s \bar{\nu} g \cos\alpha (h-y) \qquad S = S_I \approx \rho_s \bar{\nu} g \sin\alpha (h-y)$$

$$T \approx \frac{\bar{\nu}}{f_4} g \cos\alpha (h-y) \qquad s^* \equiv \frac{h-y}{d}$$

$$\frac{f_1}{f_4^{1/2}} \sqrt{s^*} \frac{d}{ds^*} \left\{ \frac{f_2}{f_4^{3/2}} \sqrt{s^*} \left[1 - \left(\frac{d \ln f_4}{d \ln \nu} \right) \left(\frac{d \ln \nu}{d \ln s^*} \right) \right] \right\} + s^{*2} \left[\tan^2\alpha - \frac{f_1 f_3}{f_4^2} \right] = 0$$

$$\left[\tan^2\alpha - \frac{f_1 f_3}{f_4^2} \right] = 0$$

$$0 = -\frac{dq}{dy} + S_I \frac{du}{dy} - \gamma$$

Conductivity invariance in the core

$$\left. \begin{aligned} &\frac{dq}{dy} \equiv 0 \\ &T \approx \frac{\bar{\nu}}{f_4} g \cos\alpha (h-y) \\ &q = -\kappa \frac{dT}{dy} \end{aligned} \right\} \quad \frac{\mathrm{d}\kappa}{\mathrm{d}y} \equiv 0$$

<u>invariant</u>

$S/N,\ \nu,\ q,\ \kappa$

$$I \equiv \frac{(du/dy)d}{\sqrt{N/\rho_s \nu}} \qquad \eta \equiv \frac{S_E}{S}$$

GDR Midi: Euro. Phys. J. E 14 (2004)

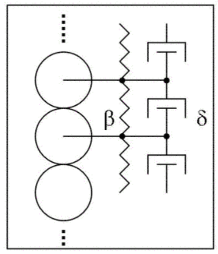
Louge, PRE 67, 061303 (2003)

<u>not invariant</u>

$du/dy,\ T,\ S,\ N,\ \gamma$

$$\kappa_I = f_2(\nu) \rho_s d \sqrt{T}$$

"Phonon" conduction



$$\frac{\partial^2 \xi}{\partial t^2} = c^2 \frac{\partial^2 \xi}{\partial x^2} + D \frac{\partial^3 \xi}{\partial t \partial x^2}$$

$$c^2 = \frac{\beta d^2}{m} \qquad D = \frac{d^2 \delta}{m}$$

$$\tilde{\kappa}_E = \frac{3}{2} \rho_s \nu D \left[1 - \frac{j}{\sqrt{\varepsilon}} \right] \qquad \varepsilon \equiv \frac{D^2 \omega^2}{c^4} \qquad \kappa_E \approx \sqrt{\frac{54}{\pi^3}} \nu \left(\frac{\ell_E}{d} \right) \sqrt{\rho_s \beta d} >> \kappa_I$$

where $$\left(\frac{\ell_E}{d} \right) \sim \frac{\sqrt{N / \rho_s \nu}}{d(du/dy)} = I^{-1} = \frac{a_1}{8 (S_I / N_I)^{3/2}} \sqrt{\frac{a_1 a_3 \nu g_{12}}{\sin \alpha}}$$

Halsey and Ertas, arXiv:cond-mat/0506170 (2005)

GDR Midi: Euro. Phys. J. E 14 (2004)

Louge, PRE 67, 061303 (2003)

See also: Kondic & Behringer: Europhys. Lett. (2004)

Weighing theories

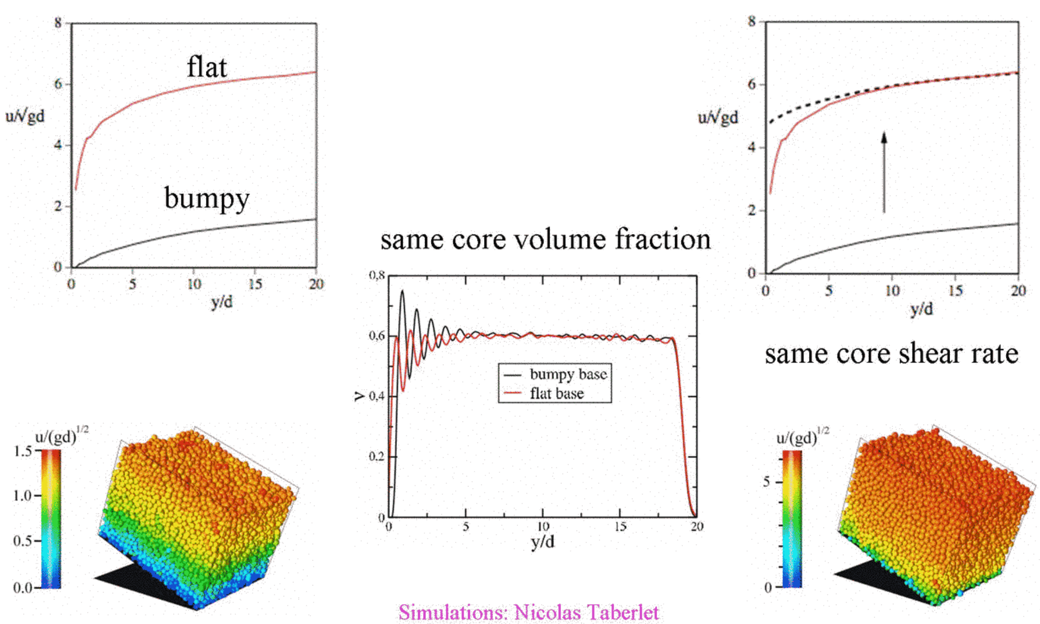
Slow flows on a bumpy base

- are mobilized through the depth
- have invariant ν in the core
- $h_{stop}/d = f(\alpha)$
- $\dot{m}^{\dagger} \propto f(\alpha)(h/d)^{5/2}$
- are insensitive to restitution
- $\tan \alpha_{\min} < \tan \alpha < \tan \alpha_{\max}$

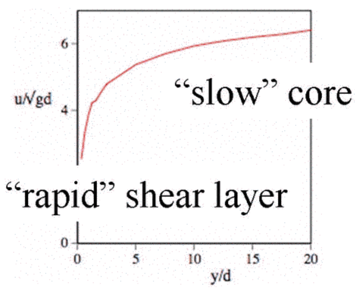
QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

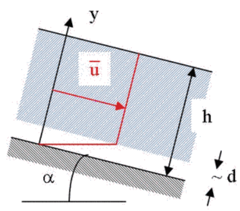
Slow, shallow, steady, fully-developed flows down a frictional plane without side walls

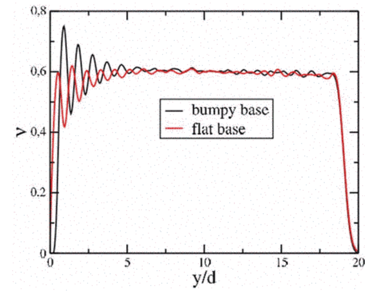
“Slow” core on a flat boundary



Velocity on a flat boundary







$$S \sim \rho_s g h \bar{\nu} \sin\alpha \sim f_1(\nu_0)\rho_s d\sqrt{T_0}\,\frac{\bar{u}}{d}$$

$$T_0 \sim \frac{\bar{\nu}}{f_4(\nu_0)} g h \cos\alpha$$

$$\frac{\bar{u}}{\sqrt{gd\cos\alpha}} \sim \left(\frac{\bar{\nu}^{1/2} f_4(\nu_0)^{1/2}}{f_1(\nu_0)}\right)\left(\frac{h}{d}\right)^{1/2} \tan\alpha$$

Louge & Keast: Phys. Fluids 13 (2001).

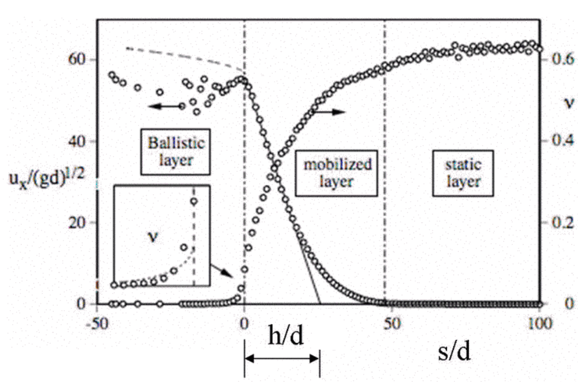
Bare theories

Flows on a frictional base

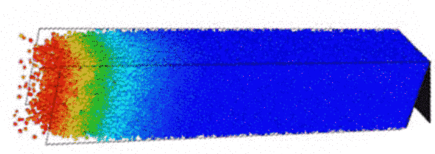
- have a rapid shear layer
- have invariant ν in a slower core
- $\dot{m} \propto f(\alpha)(h/d)^{3/2}$
- exhibit traffic waves
- $\mu_I < \tan\alpha_{\min} < \tan\alpha < \tan\alpha_{\max} < \mu_E$

Steady, fully-developed flows on
an erodible heap with side walls

Sidewall Stabilized Heap (SSH)



$$\nu = \nu_c[1 - \exp(-s/\ell)]$$

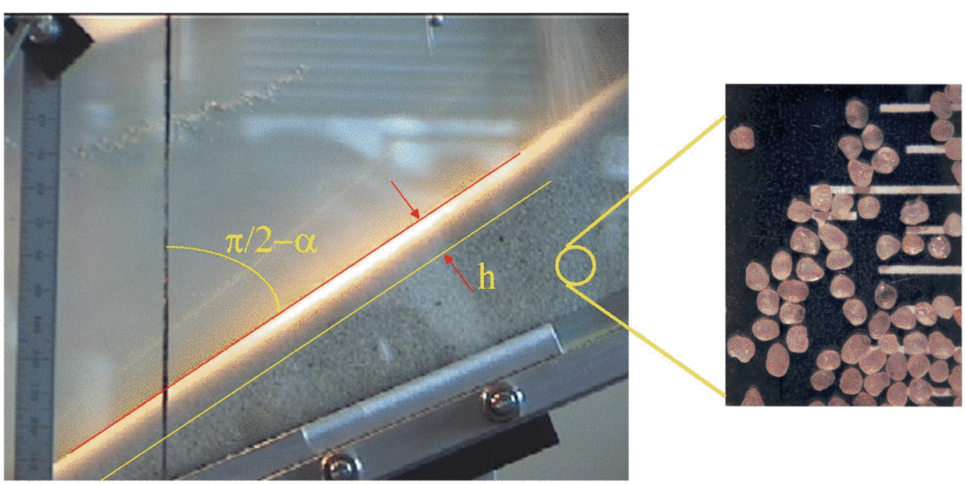


Taberlet, et al.: J. of Phys. Cond. Mat. 2005

Sidewall Stabilized Heap

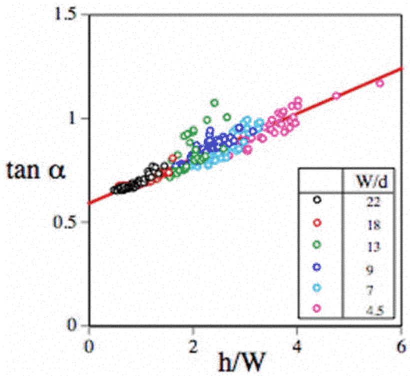
QuickTime™ and a
DV/DVCPRO - NTSC decompressor
are needed to see this picture.

SSH Experiments



rounded Ottawa sand, nearly monodisperse, 0.71mm

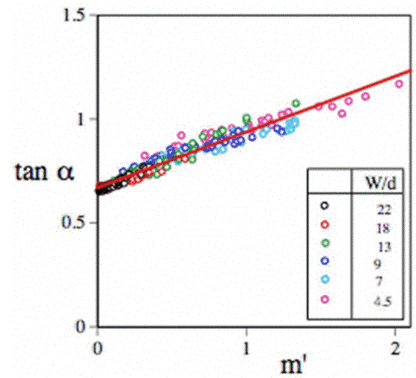
Global SSH behavior



$$\tan\alpha = \mu_w\left(\frac{h}{W}\right) + \tan\alpha_{\min}$$

$$\mu_w \approx 0.11$$

$$\alpha_{\min} \approx 30.7^\circ$$

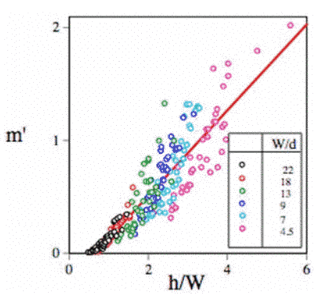


$$\tan\alpha = \tan\alpha_{stop} + \left(\frac{\dot{m}'}{\dot{m}'_0}\right)$$

$$\dot{m}'_0 \approx 3.7$$

$$\alpha_{stop} \approx 33.9^\circ > \alpha_{\min}$$

Minimum SSH thickness



$$\left(\frac{\dot{m}'}{\dot{m}'_0}\right) = \mu_w\left[\left(\frac{h}{W}\right) - \underbrace{\left(\frac{\tan\alpha_{stop} - \tan\alpha_{\min}}{\mu_w}\right)}_{h_{stop}/W \approx 0.73}\right]$$

But $\left(\frac{h_{jam}}{d}\right) \approx 12 \pm 3$ can be smaller (or larger) than W/d.

- Anisotropy.
- h_{stop} unlike Pouliquen's definition.

Friction depth

$$\frac{\partial \tau_{xs}}{\partial s} + \frac{\partial \tau_{xz}}{\partial z} = -\rho_s g \nu \sin\alpha \qquad \frac{\partial \tau_{ss}}{\partial s} = +\rho_s g \nu \cos\alpha$$

$$\tau_{xz} = \mp\mu_\tau(s)\tau_{zz} \approx \mp\mu_\tau(s)\tau_{ss} \text{ at } z = \pm W/2$$

$$\mu_{eff}(s) \equiv -\frac{\tau_{xs}}{\tau_{ss}} = \tan\alpha - \frac{\sigma_\mu}{W} \qquad \sigma_\mu(s) = 2\frac{\int_0^s \mu_\tau(s'')\left(\int_0^{s''} \nu ds'\right) ds''}{\int_0^s \nu ds'}$$

$$\tan\alpha = \mu_w\left(\frac{h}{W}\right) + \tan\alpha_{\min} \qquad \begin{cases} h\mu_w = \lim_{s\to\infty}\sigma_\mu \\ \tan\alpha_{\min} = \lim_{s\to\infty}\mu_{eff} \end{cases}$$

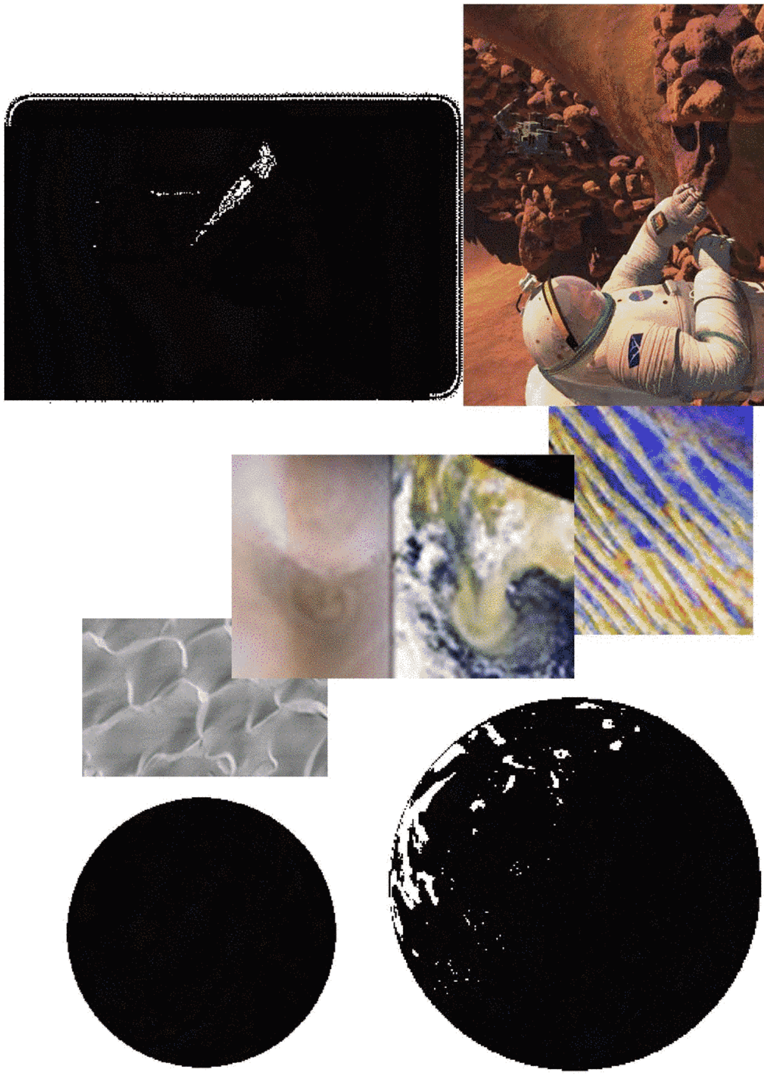
Louge, et al.
Powders & Grains (2005)

μ_τ cannot be constant with s

Required predictions

SSH flows

- have an exponential profile of ν ...
- … and a convex profile of u
- relate m’, tanα and h/W linearly
- are insensitive to d, except for h_{jam}



with special thanks to Norman Rockwell